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Experimentation, Information Sharing and Oligopoly Limit Pricing

by

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EXPERIMENTATION, INFORMATION SHARING, AND Oligopoly

Limit Pricing

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Abstract: The paper examines incumbents' incentives to share information in the presence of entry threat when incumbents face uncertainty about their cost functions. Similar to the experimentation and learning literature, firms in this model can learn more about their costs through output production which simultaneously produces information. I find that in the presence of entry threat, information sharing may not emerge as an equilibrium outcome although information sharing enables firms to actually deter entry through better coordination of their strategies. The learning effect of information production plays a crucial role in determining firms' incentives to share. Both the sharing and non-sharing equilibria are characterised by downward price distortions, whereas entry takes place with a positive probability only in a non-sharing equilibrium.

KEY WORDS: Information sharing, Bayesian learning, Entry threat, Cournot competition.

JEL Classification Numbers: D82, D83, L13

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1. INTRODUCTION

In this article, I examine firms' incentives to share information in the presence of an entry threat when information is produced endogenously through output production. Information sharing among firms in an industry is quite prevalent in many OECD countries. In an information sharing agreement, firms commit to share information in the future about relevant parameter values about which they face uncertainty. The process of sharing is usually carried out by 'Trade Associations' who collect and disseminate information among firms.¹

What determines firms' incentives to share information? Two trends of explanations are available to justify firms' incentives to share. One is that firms share information in order to strategically acquire rival's information (Vives (1990)). The other is that firms share information to indirectly collude and maintain their anti-competitive behaviour (Novshek and Sonnenschein (1982), Clarke (1983), Vives (1984), Gal-Or (1985)). Information sharing does not have to be bad: in fact information sharing can lead to a better decision making which is beneficial for both firms and the society. However, if information sharing facilitates collusion then it benefits firms but causes social loss. The more disaggregated the information is and/or the more exclusive² the sharing is, the higher is the likelihood of collusion. Kuhn and Vives in their European Commission report (1995) even suggest that there can hardly be any benign incentives for firms to share information other than to maintain their anti-competitive behaviour.³ Obviously anti-trust authorities are concerned with restricting collusion while promoting social welfare.

On the theoretical front, most analysis on firms' incentives for information sharing has been carried out assuming that the source of information is exogenously given. This

¹See e.g. Vives (1990), Creane (1998), Novshek and Thoman (1998) for examples of such industries.

²where information is available only to its members.

³see Novshek and Thoman (1998) for an exact quote.
assumption however may be restrictive since it ignores the learning effect of information production. The learning effect is crucial because depending upon how beneficial the learning is, firms’ incentives to share information may be altered. Replacing the assumption of exogenous information by endogenous information production, the aim of this paper is to examine whether maintaining market power provides firms with sufficient incentives to share their private information.\(^4\)

Using tools from the experimentation and learning literature (e.g. Blackwell (1951), Grossman et al. (1977), Mirman et al. (1994), Harrington (1995)), I model incumbents as Bayesian agents with unknown cost functions. The mechanism behind information production and learning is as follows. The observed total cost has a systematic component and a noise component with a known distribution. A higher output means that the systematic component is increased and enables firms to estimate their costs more accurately. I carry out the analysis in a Cournot duopoly setting where incumbents face a threat of entry from a potential rival. I then examine the effect of information production on incumbents’ incentives to share information and maintain their anti-competitive behaviour through entry deterrence. The paper thus contributes to the information sharing literature by providing an explicit analysis of the interactions between incumbents’ incentives to learn, to share information and to maintain their market power.

I find that the sharing of information enables incumbents to deter entry by coordinating their strategies more accurately. Despite this, information sharing may not emerge as an equilibrium outcome in the presence of an entry threat. An incumbent’s incentives to exchange information is largely motivated by the value it places on the information it produces as well as the one it receives from its rival. First I show that regardless

\(^4\)One exception to the assumption of exogenous information is the recent work by Creane (1995, 1998). The current paper however goes beyond Creane’s work by explicitly considering the interaction between effects of information production and incumbents’ incentives to indirectly collude.
of whether information is shared or not, a firm values self-produced information. Hence in order to learn more it produces more output than a myopic firm does under both sharing and non-sharing cases (proposition 3). Interestingly, a firm may not produce more information and output under the commitment to share relative to the no-sharing case even though it values self-produced information more under the commitment to share (proposition 4). This is an example where learning can sometimes be harmful. Given that a firm will receive information from its rival, producing more information may be costly if that implies it must also sacrifice more of its future profits to deter entry. Thus an incumbent may actually prefer to learn less by producing less under the commitment to share. This result contrasts with Creane (1995) where firms always produced more information under the commitment to share. A firm’s incentives to share information however depends not just on the value of self-produced information but also on how strongly it values the rival’s information. I show that although a firm values its rival’s information, it may still refuse to obtain information from its rival unless by doing so it can sufficiently enhance its future expected profits. Thus information sharing may not emerge as an equilibrium outcome (proposition 5). The analysis of this paper thus shows that incorporating information production and a threat of entry in an information sharing game can change some of the earlier results e.g. in a Cournot competition with private-value uncertainty sharing of information emerges as an equilibrium outcome (Gal-Or (1986), Shapiro (1986)).

By incorporating the learning effect of information production the paper also contributes to the literature on oligopoly limit pricing (Harrington (1987), Bagwell and Ramey (1991), Martin (1995), Schultz (1997)). The main essence of this literature is that limit pricing arises as an asymmetric information equilibrium when incumbents have better information about their production technology which the potential entrant is unable to observe. How incumbents acquire this superior knowledge about the technology however has not been modelled by the previous work. The current paper is most closely related to Harrington (1987) who also considers a homogenous product quantity-setting framework as in this paper. The
limit pricing result of my model however differs from that of Harrington (1987) where entry was always deterred by raising the price above above the equilibrium one. In his paper, the entrant immediately learns the production technology of the incumbents (which is the same for all incumbents) upon entry and starts producing at the same cost as the incumbents’. Hence to deter entry, incumbents had to raise their prices to signal high costs. In my paper, since information about costs must be learned through production and experience which the potential entrant is yet to acquire, to prevent entry the incumbents lower prices (by producing more) in an attempt to signal the cost advantage that they have. Thus, similar to the Milgrom-Roberts (1982) monopoly model, prices are distorted downwards. Further, even if the incumbents are engaged in limit pricing, unlike Harrington (1987), entry deterrence is not guaranteed in this model unless information is shared.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 analyses the entry deterrence game. Section 4 models experimentation, information production, and incumbents’ incentives to share information based on the value of self-produced and rival’s information. Section 5 concludes. All proofs are in the appendix.

2. BASICS

Consider a three-period homogenous product model where there are two incumbents with non-negative outputs $q^i$ and $q^j$.\(^5\) A potential entrant may enter the industry at the beginning of the third period. In each period $t$ ($t = 0, 1, 2$), the inverse demand function is given by $P_t = a - bQ_t$ where $P_t$ is the market price in period $t$ and $Q_t$ ($= q^i_t + q^j_t$) is the industry output in period $t$. The demand is nonstochastic and commonly known.

Firms have unknown linear costs. With production in each period, firm $i$ observes its

\(^5\)For the rest of the paper, I will take firm $i$ as the representative firm as firms are symmetric. Firm $j$ will denote the rival firm.
output \( q^i \) and the total cost \( C^i = \gamma q^i \epsilon_i \) where \( \gamma \) is an unknown positive-valued parameter (\( \gamma_i \) denotes the value of \( \gamma \) accruing to firm \( i \)) and \( \epsilon_i \) is firm \( i \)'s realization of a random variable \( \epsilon \) whose distribution is characterized by a continuously differentiable density function \( f(\epsilon) \) with 
\[
\int_{-\infty}^{+\infty} \epsilon f(\epsilon) d\epsilon = 0.
\]
I assume that 

A1. \( \gamma_i \in \{\bar{\gamma}, \bar{\gamma}\} \) and firm \( i \) does not know whether its cost of production is high \((\bar{\gamma})\) or low \((\bar{\gamma})\).

A2. The density function \( f(\epsilon) \) has monotone likelihood ratio property (MLRP), i.e., \( f'(\epsilon)/f(\epsilon) \) is continuous and strictly decreasing on \( \{\epsilon: f(\epsilon) \neq 0\} \).

A3. Error terms \( \epsilon_i \) are independent.

Assumptions A1 and A2 together imply that higher cost observations lead a firm to believe that it has high cost. Assumption A3 implies that by observing its total cost \( C^i \) firm \( i \) can make no inferences about its rival’s cost.

Let \( \mu_0 \in (0, 1) \) denote the prior belief of a firm that it has high cost, i.e., \( \mu_0 = \Pr[\gamma = \bar{\gamma}] \). These prior beliefs are common and common knowledge. In each period, firm \( i \) observes \( C^i \) and \( q^i \) but not the actual realization of \( \epsilon_i \). Firms act as Bayesian agents and observations \( C^i \) and \( q^i \) lead firms to revise their beliefs according to Bayes’ rule. Observations \( C^i \) and \( q^i \) imply two possible realizations of the error term: \( \epsilon^i = C^i - \bar{\gamma} q^i \) or \( \epsilon^i = C^i - \bar{\gamma} q^i \). Hence the posterior belief of firm \( i \) in period \( t \) that it is high cost is given by

\[
\mu^i_t = \Pr[\gamma = \bar{\gamma} | C^i, q^i] = \frac{\mu^i_{t-1} f(C^i - \bar{\gamma} q^i)}{\mu^i_{t-1} f(C^i - \bar{\gamma} q^i) + (1 - \mu^i_{t-1}) f(C^i - \bar{\gamma} q^i)}
\]  

(1)

Thus, incumbents may not have complete information about their cost structures even though they may have better information.

An ‘outside agency’ (e.g., a Trade Association) conducts the transmission of private information. After firms learn about their costs they report it to the agency who then makes the information available to the members of the Trade Association. In this article I consider the sharing of information to be complete i.e. through sharing of information each firm learns
what the other learns.\textsuperscript{6} Further, I assume that all firms, incumbents and the entrant, have rational beliefs i.e. each firm expects its next period belief to be the same as its belief in the current period: $E_{i}(\mu_{t+1}^{\prime}) = \mu_{t}^{\prime}$, and without any additional information each firm expects (at best) its opponent’s belief to be the same as the prior.

A standard information sharing game consists of two stages: in the first stage, before obtaining any private information, incumbents commit (or not) to share information. In the second stage, after they have received and exchanged (or not) their private information, incumbents compete in the market place. In this paper, competition in the second stage is complicated by the fact that a potential entrant may try to enter the market in the final period. I therefore envision the following sequence of events:

*Period 0.* At the beginning of period 0, incumbents decide whether to share information in the future. They then start production. Through output production, they learn about their costs and exchange information about their costs according to the agreement made.

*Period 1.* Incumbents produce in period 1 according to their revised beliefs and shared information. A market price is then realized. The potential entrant observes the market price of period 1, updates beliefs about the cost of the incumbents, and decides whether or not to enter the market in period 2.

*Period 2.* If entry occurs, then three-firm Cournot competition takes place. Otherwise incumbents remain as duopolists.

As usual in a sequential game, I first examine the entry deterrence subgame of period 1 and 2.

\textsuperscript{6}The assumption of truthful revelation of private information is commonly made in many papers on information sharing literature. See Ziv (1993) and Gal-Or (1986) for issues on incentives to cheat and partial revelation of private information.
3. THE ENTRY DETERRENCE SUBGAME

In the pre-entry period, incumbents simultaneously choose output. By producing in
period 1 incumbents will have more information about their cost structures. Because that
information will be available after production takes place, that new information will have no
value as far as the strategic entry deterrence game of period 1 and 2 is concerned. Further,
incumbents do not like entry in period 2 i.e. for both i and j, \( \pi_{2|NE} \geq \pi_{2|E} \) where \( \pi_{2|NE} \) and
\( \pi_{2|E} \) are respectively the profits earned in period 2 without and with entry. The period 1
strategy of incumbents can then be defined as a function that maps values of their private or
shared beliefs about costs into the set of possible output. Denote \( \mu_0 \tilde{\gamma} + (1 - \mu_0) \gamma \) by \( \tilde{\gamma} \).
Letting \( s_i(\mu_1^i, \mu_1^j, \tilde{\gamma}) \) represent the strategy of firm i in period 1, \( s_i(\mu_1^i, \mu_1^j, \tilde{\gamma}) : \{\mu_1^i, \mu_1^j, \tilde{\gamma} \} \rightarrow [0, \infty) \). At the time of entry, the entrant observes the realized market price which is the
only new information available to the entrant at the start of the game. It can then easily
calculate the total output of the industry via market demand in period 1. The strategy of the
potential entrant therefore is a mapping from the pre-entry information set to a decision on
entry, i.e. \( T(Q_1) : [0, \infty) \rightarrow \{\text{enter, do not enter} \} \) where \( Q_1 \) is the total industry output
produced in period one.

Entrant’s behaviour.

To enter the market, the entrant must incur a one-time irrecoverable cost \( K \) of
industry-specific capital. The value of \( K \) is common knowledge. Because the entrant is yet to
learn about its production technology, it expects that with probability \( \mu_0 \) it is going to be
high cost. The entrant knows that incumbents may have superior knowledge about their costs
because of their production experience. However, being yet an outsider it does not know
incumbents’ private information and is unable to observe their individual output rate.\(^7\)
Observing the market price \( P_1 \), the entrant therefore gets a signal of incumbents’ average cost
statistic. Let that signalled value of cost be denoted by \( c^8 \). With rational beliefs, the entrant
expects incumbents’ period 2 cost to equal \( c^8 \). Let \( V_c = [(P_2 - \tilde{\gamma})q_e - K] \) denote the post-entry
expected profit of the entrant under three-firm Cournot competition where the subscript \( e \) denotes the entrant. The entrant’s reaction function is given by:

\[
q_e = (a - \bar{\gamma} - bE_e Q_e) / 2b
\]

where \( Q_e = q_2^i + q_2^j \) and \( E_e Q_e = 2(a - c^8 - bq_e) / 3b \). In Cournot-Bayesian (henceforth C-B) equilibrium, the entrant expects to produce

\[
q_e = (a + 2c^8 - 3\bar{\gamma}) / 4b
\]

and expects the incumbents to produce

\[
Q_c = (a + \bar{\gamma} - 2c^8) / 2b
\]

Hence the entrant expects its post entry profit to equal

\[
V_e = \{(a - 3\bar{\gamma} + 2c^8)^2 / 16b\} - K
\]

(2)

To ensure that there exists positive probability of entry being unattractive, I assume that \( \bar{\gamma} \in (3\bar{\gamma} - a + 4(bK)^{1/2}) / 2 < \bar{\gamma} \). Denote \( (3\bar{\gamma} - a + 4(bK)^{1/2}) / 2 \) by \( c^d \) and assume that the parameter values are such that \( c^d > 0 \). Hence,

\[
\forall c^8 \in [\bar{\gamma}, c^d] \text{: entry is deterred, and}
\]

\[
\forall c^8 \in (c^d, \bar{\gamma}] \text{: entry takes place.}
\]

For the rest of the analysis, I also assume the interval of \( c^8 \) for which entry is deterred contains \( \bar{\gamma} \), i.e. \( \bar{\gamma} < c^d \); since the entrant has to incur the fixed cost \( K \) in addition to its expected cost \( \bar{\gamma} \), entry may not be profitable unless the signalled cost is sufficiently high.

**Incumbent’s Behaviour.**

The period 1 strategy of firm \( i \) is to choose \( q^i \) to maximize

\[
E \pi_1^i = [a - b(q_1^i + E_1 q_1^j)] q_1^i - \{\mu_1^i \bar{\gamma} + (1 - \mu_1^i) \bar{\gamma} \} q_1^i
\]

For notational convenience, let \( \{\mu_1^i \bar{\gamma} + (1 - \mu_1^i) \bar{\gamma} \} = \gamma^i \) and \( \{\mu_1^j \bar{\gamma} + (1 - \mu_1^j) \bar{\gamma} \} = \gamma^j \), i.e. \( \gamma^i \) and \( \gamma^j \) denote expected costs of production in period 1 after each firm has updated its belief.

The reaction function of firm \( i \) is given by:

\[
q_1^i = \{a - bE_1 q_1^j - \gamma^i \} / 2b
\]

Now, when firm \( i \) computes equilibrium values, it does so based on what it knows about its rival’s beliefs and

\footnote{The assumption that the entrant is unable to observe individual output has also been made by Harrington (1987) in the context of a homogenous product market (like in this paper). Harrington however assumes that incumbents are perfectly and symmetrically informed about their costs which is not the case this model.}
what it knows about what its rival knows. Without information sharing, i expects j’s expected cost to equal $\tilde{\gamma}$ and i knows that j also expects i’s expected cost to equal $\tilde{\gamma}$. Therefore, without information sharing, firm i expects the following values to prevail in C-B equilibrium: $q_1^i = (2a + \tilde{\gamma} - 3\gamma^j)/6b$, $E_iq_1^j = (a - \tilde{\gamma})/3b$ and expects the industry output to be $E_1Q_1 = 2(a - (\tilde{\gamma} + 3\gamma^j)/4)/3b$. Hence, without information sharing, firm i expects the signalled cost $c^g$ to equal $(\tilde{\gamma} + 3\gamma^j)/4$.

With information sharing, a firm learns what its rival learns, i.e. $E_i\gamma^j = \gamma^j$ and $E_j\gamma^i = \gamma^i$. Hence, with information sharing, firm i expects the following equilibrium values to prevail: $q_1^i = (a + \gamma^j - 2\gamma^i)/3b$, $q_1^j = (a + \gamma^j - 2\gamma^j)/3b$, $E_1Q_1 = 2(a - (\gamma^j + \gamma^j)/2)/3b$ and therefore expects the signalled cost $c^g$ to equal $(\gamma^j + \gamma^i)/2$.

_Bayesian Nash Equilibria._

A profile of strategies constitutes a solution to the game if it forms a Bayesian Nash equilibrium (BNE), i.e. each player’s strategy must maximise its expected payoff given other players’ strategy for all information sets along the equilibrium path. I shall further specify that any conditional beliefs derived during the play of the game must be consistent with the optimising behaviour of all other players as well as Bayes’ rule. Hence, in equilibrium each player will have no incentive to change his strategies or set of beliefs, given strategies of other players.

Let $\mu_d$ be the belief level such that $\{\mu_d\gamma + (1 - \mu_d)\gamma\} = c^d$. Thus if $\mu^i_1 \in [0, \mu_d]$, then i’s output alone will signal a cost which is $\leq c^d$. The next two propositions illustrate incumbents’ incentives to deviate from their C-B strategies depending on their belief levels.$^8$

**Proposition 1.** Without information sharing there exists a belief level $\mu_c$ satisfying $0 < \mu_d < \mu_c < 1$ such that the following strategies form a BNE in the entry deterrence subgame.

$^8$In the following propositions, I give equilibrium strategies for the incumbent i only, as that for the incumbent j follows immediately using symmetry.
Incumbent: \[ s_i(\mu^i_1, \mu^i_2, \tilde{\gamma}) = \begin{cases} 
\frac{(2a + \tilde{\gamma} - 3\gamma^i)}{6b} & \text{for } \mu^i_1 \in [0, \mu_c] \\
\frac{(a + \tilde{\gamma} - 2c^d)}{3b} & \text{for } \mu^i_1 \in (\mu_c, 1] 
\end{cases} \]

Entrant: \[ T(Q_1) = \begin{cases} 
\text{enter} & \text{if } Q_1 < \frac{(3a - 3\gamma - 4(bK)^{1/2})}{3b} \\
\text{do not enter} & \text{if } Q_1 \geq \frac{(3a - 3\gamma - 4(bK)^{1/2})}{3b} 
\end{cases} \]

where \( \mu_c \) is such that \( \{\mu_c\gamma + (1 - \mu_c)\tilde{\gamma}\} = (4c^d - \tilde{\gamma})/3. \)

Proposition 1 says, without information sharing, the interval \((\mu_c, 1]\) denotes the range of beliefs over which an incumbent deviates from its C-B output level \((2a + \tilde{\gamma} - 3\gamma^i)/6b\) and produces \((a + \tilde{\gamma} - 2c^d)\) as, by doing so, it expects the signalled cost to be \(c^d\) and the entry to be deterred. Since \(0 < \mu_d < \mu_c < 1\), the range of beliefs over which firm \(i\) does not deviate even when \(\gamma^i > c^d\) is \([\mu_d, \mu_c]\). This is because, given \(E_i(\mu^i_1) = \mu_0\), firm \(i\) expects \(c^8\) to equal \((\tilde{\gamma} + 3\gamma^i)/4 \leq c^d\) and expects entry to be deterred even without deviation.

Interestingly, even if the incumbent believes that \(E_i c^8\) will deter entry, the entrant may still enter. The reason for that is without information sharing, lack of information about each others beliefs prohibit incumbents from coordinating their strategies. Hence, even if they act in their best interests, entry may not be deterred. There are three possibilities to consider: (i) both firms have incentives to deviate i.e. \(\mu^i \in (\mu_c, 1]\) and \(\mu^j \in (\mu_c, 1]\); (ii) neither has any incentives to deviate i.e. \(\mu^i \in [0, \mu_c]\) and \(\mu^j \in [0, \mu_c]\); and (iii) firm \(i\) (say) has incentives to deviate while firm \(j\) does not i.e. \(\mu^i \in (\mu_c, 1]\) while \(\mu^j \in [0, \mu_c]\). In (i) each firm then produces \((a + \tilde{\gamma} - 2c^d)/3\) so that the actual signal which the entrant gets is \((2c^d - \tilde{\gamma}) > c^d\), and hence enters. In (ii) each produces their C-B level of output so that \(Q_1 = 2(a - (3(\gamma^i + \gamma^j)/4 - \tilde{\gamma}/2))/3b\) and the \(c^8\) which the entrant gets is \(3(\gamma^i + \gamma^j)/4 - \tilde{\gamma}/2\). Suppose \(\gamma^i = (4c^d - \tilde{\gamma})/3 - \nu_i\) and \(\gamma^j = (4c^d - \tilde{\gamma})/3 - \nu_j\), \(\nu_i \geq 0, \nu_j \geq 0\) so that \(c^8 = 2c^d - \tilde{\gamma} - 3(\nu_i + \nu_j)/4\). Then entry will actually be deterred if and only if \(c^d \leq \tilde{\gamma} + 3(\nu_i + \nu_j)/4\) which is not guaranteed to hold. Lastly in (iii) the signalled cost \(c^8\) which the entrant observes is \([c^d + 3(\gamma^j - \tilde{\gamma})]/4\). Entry will not take place in this case if \(\gamma^j \leq \tilde{\gamma}\) which is of course possible for a sufficiently low value of \(\mu^j\). Since
entry deterrence is not guaranteed, I have

**Remark 1.** In a non-information sharing equilibrium entry takes place with a positive probability.

The next proposition shows that with information sharing, each firm knows its rival’s beliefs which enables them to choose their strategies more accurately to deter entry.

**Proposition 2.** With information sharing, there exists a belief level \( \mu_{c'} > \mu_{c'} > \mu_d > 0 \), which may or may not be the same as \( \mu_c \), such that following strategies constitute a BNE in the entry deterrence subgame.

\[
\begin{align*}
\text{Incumbent: } s^i_{\mu^i_1, \mu^i_2, \gamma} = & \begin{cases} 
(a + \gamma^i_2 - 2\gamma^i_1)/3b & \text{if } \mu^i_1 \in [0, \mu_d] \\
(a + \gamma^i_1 - 2\gamma^i_2)/3b & \text{if } \mu^i_1 \in (\mu_d, \mu_c')
\end{cases} \\
& \quad \text{for any } \mu^i_2 \in [0, 1] \\
& \quad \{ \text{if } \mu^i_1 \in [0, \mu_d] \\
& \quad (a + 3\gamma^i_2 - 4\gamma^i_1)/3b & \text{if } \mu^i_1 \in (\mu_c', 1] \\
& \quad (a - \gamma^i_1)/3b & \text{if } \mu^i_1 \in (\mu_d, 1] \text{ and } \mu^i_2 \in (\mu_d, 1]
\end{cases}
\]

\[
\text{Entrant: } T(Q_1) = \begin{cases} 
\text{enter} & \text{if } Q_1 < (3a - 3\gamma - 4(bK)^{1/2})/3b \\
\text{do not enter} & \text{if } Q_1 \geq (3a - 3\gamma - 4(bK)^{1/2})/3b
\end{cases}
\]

where \( \mu_{c'} \) is such that \( \{\mu_{c'} \gamma + (1 - \mu_{c'})\gamma\} = 2c^i - \gamma^i. \)

Proposition 2 shows that the knowledge of a rival’s information enables a firm to choose its strategies in a manner such that the extent of firm i’s deviation now depends precisely upon firm j’s beliefs. If firm i believes it is going to be low-cost with sufficiently high probability (\( \mu^i_1 \in [0, \mu_d] \)), it continues to produce its C-B output level \( (a + \gamma^i_2 - 2\gamma^i_1)/3b \) irrespective of the value of \( \mu^i_2 \). If, however, \( \mu^i_1 > \mu_d \), then the interval of deviation for firm i

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\(^9\)For example, if \( \mu^i \) and \( \mu^j \) belong to the interval \([\mu_d, \mu_c]\) so that \( c^d - \gamma > 3\nu_i \geq 0, c^d - \gamma > 3\nu_j \geq 0 \) then entry will be deterred if and only if \( \nu_j \geq (c^d - \gamma)/3 + (c^d - \gamma) - \nu_i \) which immediately contradicts the above condition as it requires \( (c^d - \gamma) < \nu_i \). Thus entry takes place.
is given by \((\mu_c, 1)\) where the length of the interval varies depending upon how high (or low) \(\mu_1^j\) is (given \(\mu_1^i < \mu_d\)). If for both firms \(\mu_1^i > \mu_d\) and \(\mu_1^j > \mu_d\), then non-cooperative behaviour implies that both end up producing the same level of output (although the extents of deviation may differ) which just deters entry. It is important to note that such behaviour is individually rational as firms are acting non-cooperatively. Therefore, in contrast to the no-sharing case where the deterrence of entry was not guaranteed even with deviation, with information sharing incumbents are able to coordinate their strategies more accurately to actually prevent entry. Hence I have

\textit{Remark 2.} Entry is deterred in an information sharing equilibrium.

In addition, propositions 1 and 2 together imply the following:

(i) Unlike Harrington (1987) where entry was always deterred by raising the market price to signal high cost for the industry as a whole, in my paper, attempts to prevent entry cause the market price to be distorted downward. In Harrington (1987), the industry cost parameter determined the cost incurred by each firm including the entrant. Consequently, to deter entry incumbents had to manipulate the entrant’s belief about the cost it was going to incur. In my model, because information about production cost must be gathered through experience and learning and is firm-specific, incumbents try to manipulate the entrant’s beliefs \textit{not} about the cost it is about to incur, but the cost advantage that the incumbents have. Thus, similar to Milgrom and Roberts’ (1982) monopoly model, it is the low price which deters entry.\(^{10}\)

(ii) The range of beliefs over which deviation occurs, varies depending upon whether information is shared or not. In particular, the deviation range for one case can be larger or smaller than the other. For \(\mu_1^i > \mu_d\), the deviation range for the no-sharing case is \((\mu_c, 1]\), which can be either larger or smaller relative to the information sharing case. For \(\mu_1^j < \mu_d\),

\(^{10}\)Note that the inability of incumbents to coordinate deception and the nature of equilibrium in this paper differs fundamentally from Bagwell and Ramey (1991) where coordination was rendered impossible due to the entrant’s ability to observe individual firm’s behaviour causing the equilibrium to be non-distortionary in nature.
the deviation interval \((\mu_{c_i}, 1]\) for the sharing case is smaller only if \(\mu_{_1}^j\) is also smaller than \(\mu_0\) (< \(\mu_d\)). For both \(\mu_{_1}^j > \mu_d\), \(\mu_{_1}^i > \mu_d\) however, the deviation interval for the sharing case is actually larger relative to the no-sharing case. Thus, with information sharing an incumbent can potentially benefit if it knows that its rival's expected cost is sufficiently low. If not, then learning about rival's information may hurt the firm as then it may have to sacrifice more of its present profit.

For both the sharing and non-sharing cases however the posterior beliefs \(\mu_{_1}^i\) and \(\mu_{_1}^j\) themselves are influenced by production decisions of period 0. Therefore by varying output in period 0, a firm can vary the amount of information to be created ex post and hence \(\mu_{_1}^i\) and \(\mu_{_1}^j\) which then affects a firm's incentives to share information in the first place and the outcome of the strategic entry deterrence game. In the following section, I examine firms’ incentives to share information at the beginning of period 0 in the light of the strategic entry deterrence game of period 1, based on the learning effects of information production.

4. EXPERIMENTATION, VALUE OF INFORMATION, AND INFORMATION SHARING

A firm can produce information through output production. A learning effect comes from the fact that by varying output production a firm can vary the amount of information to be produced and can choose how much to learn. In order to learn more a firm may adjust its level of first period output away from its myopically optimal level thus sacrificing current profits. This is similar to the phenomenon of 'experimentation' (Grossman, Kihlstrom and Mirman (1977)) or 'information manipulation' (Mirman, Samuelson and Schlee (1994)).

I divide the analysis of this section into incentives to experiment and incentives to share information, created by the strategic entry deterrence subgame of period 1 and 2. I do this in order to explore separately the effect information sharing may have on the extent of experimentation and how the magnitude of experimentation itself, through its effects on firms’ current and future profits, influences incumbents’ incentives to commit to share information.
Let $V_i$ denote the value function of firm $i$ in period 1. Then $V_i = \pi_1 + \delta EP_2$ where $\pi_1$ and $\pi_2$ are the profit functions of periods 1 and 2 and $\delta \in (0, 1]$ is the discount factor. Firm $i$'s problem at the beginning of period 0 can be stated as: $\max \pi_0^i + \delta EV_i$, where $\pi_0^i$ and $q_0^i$ are the first period profit and output respectively and $E$ is the expectation operator. A myopic firm chooses $q_0^i$ to maximize current profit i.e. for a myopic firm: $\frac{d\pi_0^i}{dq_0^i} = 0$; whereas a far-sighted firm chooses $q_0^i$ to maximize $V_0^i = \pi_0^i + \delta EV_i$ i.e. for a far-sighted firm: $\frac{d\pi_0^i}{dq_0^i} + \frac{dEV}{dq_0^i} = 0$.

**Value of Self-Produced Information.**

In the analysis to follow, I use Blackwell's (1951) result presented (without proof) in the form of lemma 1 to examine how incumbents adjust their first period output depending on the value they place on the information they produce. The following well-known result is analogous to that in the risk-literature: the expected value of future profits which, if convex in beliefs, increases with a mean preserving spread of posteriors, i.e. information.

**Lemma 1.** If the value function is convex (concave) in beliefs then information has a net positive (negative) value.

Since the value function $V_i$ takes different forms depending upon whether the firm deviates or not, to distinguish, let $\overline{V}_i$ and $\overline{V}_i(c_0)$ denote respectively the value functions of firm $i$ when it does not, and does deviate (from its C-B output levels) and produces as if it has an expected cost equalling $c_0$. Because incumbents believe that entry will be deterred by undertaking the strategies as given by propositions 1 and 2, they compute their value functions as follows: $\overline{V}_i = \pi_1(\gamma^i) + \delta E\pi_2(\gamma^i)|_{\text{NE}}$ and $\overline{V}_i(c_0) = \pi_1(c_0, \gamma^i) + \delta E\pi_2(\gamma^i)|_{\text{NE}}$ where $\pi_1(\gamma^i)$

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11For the analysis which follows, I consider those values of $\delta$ satisfying $\delta \geq \max\{\delta_{ns}, \delta_s\}$ which sustain firms' incentives to deviate under both non-sharing and sharing cases where $\delta_{ns}$ and $\delta_s$ are respectively the cut-off values of $\delta$ for the no-sharing and the sharing cases (see the proofs of propositions 1 and 2 in the appendix). This of course accords with the fact that firms are 'far-sighted'.
and \( \pi_1(c_0, \gamma^1) \) denote profits earned without and with deviation when the true expected cost is \( \gamma^1 \). With rational beliefs the second period expected profit without entry \( E\pi_2(\gamma^1) = \pi_1(\gamma^1) \), the C-B equilibrium profit. The value functions are functions of posterior beliefs where the posteriors (information) \( \mu^1, \mu^1 \) (omitting subscripts from now on) are themselves random variables through their dependence on \( C \) via equation (1). Table 1 lists different values that the value functions take for the no-sharing and sharing cases.

Insert Table 1 here

The following result is used in later expressions to analyse the effect of the current output on future profits.

**Lemma 2.** Regardless of whether information is shared or not, a firm values self-produced information both when it does and does not deviate. Further, with information sharing, a firm values self-produced information more than without sharing both for the cases of no-deviation and deviation.

Self-produced information is valuable if the incumbents learn that they do not have to deviate from their C-B strategies. Self-produced information is also valuable if incumbents learn that they need to deviate, though it is less valuable than that without deviation. This is because although information is valueless as far as the current period profit is concerned \( (d^2\pi(., \gamma^1)/d(\mu^1)^2 = 0) \) as it makes them forego their current profit, knowing more however enables them to prevent entry in the future. Therefore information has a positive value overall. Also, information is valued more with information sharing as knowing rival’s information enables a firm to estimate its payoff more precisely. Lastly, with information sharing the value of information is the same regardless of whether a firm deviates alone or along with its rival (see the proof of lemma 2): since obtaining rival’s information does not alter a firm’s own production decision by the independence of cost functions, the value of own information is not affected by the fact whether firm i deviates alone or together.
Incentives for Experimentation.

With the value and mechanics of information established, I turn to examine firms’ incentives to experiment. Let \( h(\epsilon_i) \) and \( g(\epsilon_j) \) denote firm i’s and its rival’s density functions respectively. It is useful to define density functions in terms cost observations. Using \( C^i = \gamma_i q^i + c^i \), let \( h(C^i, q^i) = \mu_0 f(C^i - \gamma q^i) + (1 - \mu_0) f(C^i - \gamma q^i) \). Define the sets \( \mathcal{A} \) and \( \mathcal{B} \) as the sets of cost observations such that \( \mathcal{A} = \{C^i: \mu_i \in [0, \mu_c]\} \) and \( \mathcal{B} = \{C^i: \mu_i \in [0, \mu_{d}]\} \) and let \( \mathcal{A}^c \) and \( \mathcal{B}^c \) denote complements of sets \( \mathcal{A} \) and \( \mathcal{B} \) respectively. With information sharing, the values of \( C^i \) for which firm i deviates from its C-B level of output depends upon firm j’s posterior \( \mu_j^i \) which is a function of firm j’s observation of \( C^j \). Further denote \( \mathcal{C} = \{C^j: \mu_j \in [0, \mu_{c_j}]\} \) where (recall from proposition 2) \( \mu_{c_j} \) satisfies \( \mu_{c_j} \gamma + (1 - \mu_{c_j}) \gamma = 2c^j \gamma \), and let \( \mathcal{C}^c \) be the complement of \( \mathcal{C} \). Denoting the no-sharing and sharing cases by ‘ns’ and ‘s’, the expected value functions can now be written as

\[
EV_{i|\text{ns}} = \int_{\mathcal{A}} \left\{ \int_{\mathcal{C}} \bar{V}_i h(C^i, q^i) dC^i + \int_{\mathcal{C}^c} \bar{V}_i ((4c^d - \gamma)/3) h(C^i, q^i) dC^i \right\} g(\epsilon_j) d\epsilon_j
\]

(6)

\[
EV_{i|\text{s}} = \int_{\mathcal{B}} \left\{ \int_{\mathcal{C}} \bar{V}_i h(C^i, q^i) dC^i + \int_{\mathcal{C}^c} \bar{V}_i (2c^d - \gamma) h(C^i, q^i) dC^i \right\} g(\epsilon_j) d\epsilon_j +
\]

\[
\int_{\mathcal{B}^c} \left\{ \int_{\mathcal{C}} \bar{V}_i h(C^i, q^i) dC^i + \int_{\mathcal{C}^c} \bar{V}_i (c^d q^i) h(C^i, q^i) dC^i \right\} g(\epsilon_j) d\epsilon_j
\]

(7)

where the corresponding values of \( \bar{V}_i \) and \( \bar{V}_i(\cdot) \) are given in Table 1. Since the value functions depend upon cost observations obtained at the end of period 0 (conditional on beliefs), expectations are taken with respect to firm i’s possible cost observations as well as its rival’s.

Denote \( f(C^i - \gamma q^i) \) by \( \tilde{f} \), \( f(C^i - \gamma q^i) \) by \( f \). Differentiating \( EV_i \) with respect to \( q_0^i \), using MLRP, integrating by parts, and manipulating, I obtain the following marginal value functions for the no-sharing case (equation (8)) and the sharing case (equation (9)) where \( \bar{V}_i^{'} = d^2 \bar{V}_i/d(\mu^i)^2 \) and \( \bar{V}_i^{''}(\cdot) = d^2 \bar{V}_i(\cdot)/d(\mu^i)^2 \).
\[
\frac{dEV}{d\theta_i} \bigg|_{ns} = (\gamma - \gamma) \int_{A} \left\{ \int_{A} \{ \tilde{f} \mu_0 (1 - \mu_i) \overline{V}_i \mu_i d\mu_i \} dC^i + \int_{A^c} \{ \tilde{f} \mu_0 (1 - \mu_i) \overline{V}_i \mu_i d\mu_i \} dC^i \right\} g(\epsilon_i) d\epsilon_i
\]

(8)

While deciding how much to produce and to learn under the commitment to share, a firm takes its rival's revelation decision as given. Hence, by the independence of cost functions

\[
\frac{dEV}{d\theta_i} \bigg|_{s} = (\gamma - \gamma) \int_{B} \left\{ \int_{C} \{ \tilde{f} \mu_0 (1 - \mu_i) \overline{V}_i \mu_i d\mu_i \} dC^i + \int_{C^c} \{ \tilde{f} \mu_0 (1 - \mu_i) \overline{V}_i \mu_i d\mu_i \} dC^i \right\} g(\epsilon_i) d\epsilon_i
\]

\[
+ \int_{B^c} \left\{ \int_{B} \{ \tilde{f} \mu_0 (1 - \mu_i) \overline{V}_i \mu_i d\mu_i \} dC^i + \int_{B^c} \{ \tilde{f} \mu_0 (1 - \mu_i) \overline{V}_i \mu_i d\mu_i \} dC^i \right\} g(\epsilon_i) d\epsilon_i
\]

(9)

Using lemma 2 and equations (8) and (9), I establish in proposition 3 that there exists incentives for firms to experiment when they value the information they produce.

**Proposition 3.** An incumbent increases output beyond the level produced by a myopic firm both in cases where information is shared and not.

Irrespective of whether or not information is shared, a firm values self-produced information. Therefore, to learn more a firm increases the randomness of \(\mu_i\) (i.e. information) by producing more output than a myopic firm does. By doing so, a firm sacrifices its current profit but increases its future expected profit.

Will a firm produce more output and information under the commitment to share since it values self-produced information more (lemma 2) when it shares? Interestingly, the following proposition illustrates even if the value of information is higher when information is shared, it need not be true that the firm prefers to experiment and learn more under the commitment to share.

**Proposition 4.** Ex-ante commitment to share information does not imply that the incumbent produces more output and information compared to the no-information sharing case.
The proposition suggests that a firm may actually prefer to experiment and learn less under the commitment to share. In the proof I show whether \( q^i_{obs} \geq (\prec) q^i_{obs} \) depends upon whether the no-deviation expected marginal value of information with sharing exceeds (or is not too short of) that without sharing. With information sharing, the expected marginal value of (own) information itself depends upon what the incumbent learns about its rival's information. If this (non-deviation) expected marginal value of information, conditional on rival's information, is sufficiently small relative to the (non-deviation) expected marginal value of information under no sharing, the incumbent will actually end up producing and learning less under the commitment to share.

The basic intuition behind the proposition is similar to the notion that "no news is good news" (or indeed that "ignorance is bliss"). With information sharing, there are two sources of learning which interact with one another: learning through own experimentation and learning from the rival. Given that with information sharing the incumbent will learn from its rival, producing more information through own production may be costly if the firm learns it needs to deviate more and forego more of its current profits (in expected terms). Thus the knowledge that information is forthcoming from outside sources may actually reduce a firm's incentives to create more information since it simultaneously increases the riskiness associated with learning in terms of foregone profits. Thus a firm may actually prefer to produce and learn less under the commitment to share.

The above result stands in contrast to Creane (1995) where commitment to reveal information always increased a firm's output production and learning. In Creane's model, there was no threat of entry and commitment to share information gave firms more incentives to produce and learn since learning was never 'hurtful' as is possible in this model. Some of the previous work on information sharing (e.g. Li et. al (1987), Vives (1988)) have shown that if firms knew they would be forced to reveal information, they would reduce their information production in the first stage. In the context of a differentiated-products duopoly, Harrington
(1995) shows that firms may reduce experimentation if experiments are publicly observed. My result however differs fundamentally from those because impediment to information production in my model comes not from the strategic advantage the information gives to the rival when it is shared but because firms may themselves prefer to learn less.

**Value of Rival’s Information and Incentives to Share Information.**

Analysis of the previous section suggests that, for a given level of rival’s information, agreeing to reveal (and hence share) information increases a firm’s output production and future expected profit although the increase may not necessarily be larger than those of a non-sharing firm. A firm’s incentive to share information however is greatly influenced not just by its own production and revelation decision but also by its rival’s decisions. A firm may not want to agree to share information in the first place if it perceives obtaining rival’s information will affect its current and future expected profits adversely. First, I examine how receiving rival’s information affects a firm’s future expected profit.

Denote \( f(C^j, \gamma^j) \) by \( f_j \), \( f(C^j, \gamma^j) \) by \( f_j \), \( \nabla_{ij} = d^2 \bar{V}_i / d(\mu^j)^2 \), \( \nabla_{ij}^j(\cdot) = d^2 \bar{V}_i(\cdot) / d(\mu^j)^2 \), and let \( g(C^j, \gamma^j) = \mu_0 \bar{f}_j + (1 - \mu_0) f_j \). Differentiating \( EV_i \) with respect to rival’s first period output \( q_0^j \), using MLRP, integrating by parts, using Leibniz’ rule and simplifying, I obtain equation (10) which captures the effect of a rival’s production decision on firm i’s future expected profits where the independence of cost functions again substantially simplify calculations.

\[
\frac{dEV_i}{dq_j} = (\gamma - \gamma) \int_{B} \left\{ \int_{C} \left( \bar{f}_j \mu_0 (1 - \mu^j) \nabla_{ij} dC^j \right) + \int_{C} \left( \bar{f}_j \mu_0 (1 - \mu^j) \nabla_{ij} (2c^d - \gamma) \frac{dC^j}{dC^j} \right) \right\} h(c_i) \, dc_i
\]

\[
+ \int_{B} \left\{ \int_{B} \left( \bar{f}_j \mu_0 (1 - \mu^j) \nabla_{ij} dC^j \right) + \int_{B} \left( \bar{f}_j \mu_0 (1 - \mu^j) \nabla_{ij} dC^j \right) \right\} h(c_i) \, dc_i
\]

**Proposition 5.** A firm values rival’s information. Therefore agreeing to receive information from the rival increases a firm’s future expected profit.
If a firm does not share information then \( \text{dEV}_i/\text{d} q_0^j = 0 \) whereas a firm who shares information has \( \text{dEV}_i/\text{d} q_0^j > 0 \) as it values rival's information (see the proof). Therefore receiving information from the rival increases a firm's future expected profit over and above the increase attained through its own experimentation.

Will a firm enter into an information sharing agreement because it values rival's information? A firm will have incentives to commit to share information if and only if the expected value of total profit with commitment \( \pi^i_s|_s + \delta \text{EV}_i|_{\text{ins}} \) (weakly) exceeds the expected value of total profit without commitment \( \pi^i_{\text{ins}} + \delta \text{EV}_i|_{\text{ins}} \). However, a firm may not agree to commit ex-ante. The reasoning is as follows. Regardless of whether information is shared or not, incentives to experiment prompt incumbents to produce beyond the profit maximising output level of a myopic firm (proposition 3). On one hand, the more they experiment the more they sacrifice their current profit (relative to the myopic level) where the exact amount of profit foregone depends upon the extent of firm's own experimentation as well as its rival's since goods are strategic substitutes. On the other hand, more experimentation (own and rival's) causes future expected profit to increase more since \( \text{dEV}_i/\text{d} q_0^j > 0 \) and \( \text{dEV}_i/\text{d} q_0^j > 0 \). Depending upon how learning affects production under the commitment to share, there are three cases to consider by proposition 4:

*Case (i):* \( q_0^i|_s = q_0^i|_{\text{ins}} \). In this case \( \pi^i_s|_s = \pi^i_{\text{ins}} \) but \( \text{EV}_i|_s > \text{EV}_i|_{\text{ins}} \) since a firm values rival's information. Therefore firms will commit in this case.

*Case (ii):* \( q_0^i|_s > q_0^i|_{\text{ins}} \). In this case firms regard learning as more beneficial under the commitment to share. Therefore they produce more and sacrifice more of their current profit. Thus \( \text{EV}_i|_s > \text{EV}_i|_{\text{ins}} \) as \( \text{dEV}_i/\text{d} q_0^i|_s > \text{dEV}_i/\text{d} q_0^i|_{\text{ins}} \) and by proposition 5, but \( \pi^i_s|_s < \pi^i_{\text{ins}} \). A firm will then commit to share if and only if \( \delta \geq (\pi^i_{\text{ins}} - \pi^i_s|_s)/(\text{EV}_i|_s - \text{EV}_i|_{\text{ins}}) \) which may not hold unless the future gain from sharing is considerably large relative to the current loss.

*Case (iii):* \( q_0^i|_s < q_0^i|_{\text{ins}} \). In this case firms regard learning as relatively harmful under the commitment to share and therefore reduce production. Thus \( \text{dEV}_i/\text{d} q_0^i|_{\text{ins}} > \text{dEV}_i/\text{d} q_0^i|_s \) but
\[ \pi_{i|s}^1 > \pi_{0|s}^1 \] as they sacrifice less current profit with sharing. Therefore, depending upon how highly firm \( i \) values firm \( j \)'s information, the magnitude of the difference \((EV_{i|ns} - EV_{i|a})\) will determine a firm's incentive to share.

The above analysis immediately gives rise to the following result.

**Proposition 6.** Uncertainty in firm-specific costs does not imply that firms will commit to share information in Cournot competition with an entry threat when information is produced endogenously.

Previous authors (e.g. Li (1985), Gal-Or (1986), Shapiro (1986)) have established that in Cournot competition with private-value uncertainty, firms' tendency is to share information. In particular, Gal-Or (1986) establishes that the equilibrium is in dominant strategy when the uncertainty is about firm-specific costs. Those papers however did not model the learning effect of information production. Analysis of this paper therefore suggest that some of the earlier results may need to be revised when firms' incentive to share information is determined by the interactions between their willingness to retain market power and willingness to produce information.

5. CONCLUSION

In this paper I have examined how the threat of entry influences incumbents' incentives to share information in order to maintain their anti-competitive behaviour when information is produced endogenously through output production, and the effect of information sharing on the outcome of a strategic entry deterrence game. Firms in this model face uncertainty about their cost functions and acting as Bayesian agents they learn more about their costs through output production.

I find that, with endogenous information production, maintaining market power may not be the primary reason for incumbents to share information even though through
information sharing firms can retain their market power by successfully deterring entry. A firm’s incentive to share information is greatly influenced by the learning effect of its experimentation where the perceived benefits of learning is highly sensitive to whether the results of experimentation remain private or is shared. Commitment to share information can dampen a firm’s incentives to create information if knowing rival’s information makes learning through own production more costly. In this case a firm may prefer to learn less by producing less. If learning turns out to be too costly under the commitment to share, a firm may prefer to remain ignorant about its rival’s information and rely more on the self-produced information. Thus information sharing may not emerge as an equilibrium outcome. In a non-sharing equilibrium however entry takes place with a positive probability.

The present model can be extended in at least two directions. First, it would be very useful to explore the welfare implications of information sharing in the presence of entry threat and learning. With endogenous information production, Creane (1995) showed that consumers will be made worse off if anti-trust authorities block the sharing arrangement since firms produce more output under the commitment to share. Such implications however are not so obvious in my model. It is clear that consumers will benefit in the future under non-sharing equilibrium since entry deterrence is not guaranteed. Consumers benefit from price distortions in the intermediate period under both sharing and non-sharing equilibria. The effect on consumers’ welfare in the first period however depends upon the type of the prevailing equilibrium and how much output is produced under such an equilibrium. This indicates that a comprehensive welfare analysis is needed to determine appropriate policies of the anti-trust authorities. Second, in this paper I have conducted the analysis assuming goods are strategic substitutes and when the uncertainty is about private-value parameters. An obvious avenue for future research will be to examine firms’ incentives to share information under a similar model set-up when goods are strategic complements and/or the uncertainty is about common-value parameters (e.g. uncertainty about the market demand).
References


Schultz, C. "Limit Pricing when Incumbents have Conflicting Interests" Discussion Paper, June 1997, Centre for Industrial Economics, University of Copenhagen.


APPENDIX

Proof of Proposition 1: First, I verify that \( Q_1 \geq (3a - 3\tilde{\gamma} - 4(bK)^{1/2})/3b \) deters entry. The industry output observed by the entrant is \( Q_1 = 2(a - c^s)/3b \). Entry is deterred for \( c^s \leq c_d \Rightarrow \) entry is deterred for all \( Q_1 \geq (3a - 3\tilde{\gamma} - 4(bK)^{1/2})/3b \). Without information sharing, \( E_i^1 \gamma^j = \tilde{\gamma} \), \( F_i^s = (\tilde{\gamma} + 3\tilde{\gamma})/4 \) and note that \( \mu_d < \mu_c \) as \( \tilde{\gamma} < c_d \). For \( \mu_1^i \in [0, \mu_c] \), \( E_i^1 c^s \leq c_d \). Hence firm i expects entry to be deterred naturally. Therefore, producing C-B output level is the best response of player i given any strategy of other players; likewise for firm j for \( \mu_1^j \in [0, \mu_c] \).

For \( \mu_1^i \in (\mu_c, 1] \), \( E_i^1 c^s > c_d \). Therefore, i expects entry to occur under C-B strategy. Because firm i believes that j will have no incentive to deviate as \( E_i^1 \gamma^j = \tilde{\gamma} < c_d \), to prevent entry if firm i deviates, it will deviate just enough to make \( E_i^1 c^s = c_d \), i.e. it will deviate and produce as if it has an expected cost equaling \( (4c_d \cdot \tilde{\gamma})/3 \). Incumbent i will have incentive to deviate if and only if the gain from deviation \( G_1 = \delta \{E \pi_{2|\mathrm{NE}} - E \pi_{2|E} \} \geq \{E \pi_{C-B} - E \pi((4c_d \cdot \tilde{\gamma})/3)\} = C_i \), the cost of deviation where \( \delta \in [0, 1] \) is the discount factor. Given \( E_i^1 \mu_d^j = \mu_1^i, E \pi_{2|E} = E \pi_{C-B} = (2a + \tilde{\gamma} - 3\tilde{\gamma})/36b \). Also, \( E \pi_{2|E} = (2a + 3.5\tilde{\gamma} - 5.5\tilde{\gamma})/64b \), \( E \pi((4c_d \cdot \tilde{\gamma})/3) = (a + 2c_d - 3\tilde{\gamma})(a + \tilde{\gamma} - 2c_d)/9b \). Now, note that \( (G_1 - C_i) < 0 \) for \( \delta = 0 \), \( (G_1 - C_i) > 0 \) for \( \delta = 1 \) with ‘a’ large\(^{12} \) and that the difference \( (G_1 - C_i) \) is strictly increasing in \( \delta \). Hence there must exist a \( \delta_{ns}, \delta_{ns} > 0 \) such that \( \forall \delta \geq \delta_{ns}, (G_1 - C_i) > 0 \). To show that there exists incentives for deviation for any value of \( \gamma^j \) in the interval \((4c_d \cdot \tilde{\gamma})/3, \tilde{\gamma} \), note that

\[
\lim_{\gamma^j \to (4c_d \cdot \tilde{\gamma})/3} \partial(G_1 - C_i)/\partial \gamma^j \simeq 59a - 59\tilde{\gamma} = 57(4(bK)^{1/2}) \text{will be} > 0 \text{for } a > \tilde{\gamma} + (57/59)4(bK)^{1/2} \text{given that } c_d > 0 \text{ and that } (a - c_d)^3 > 0, \text{the preceding condition will hold for a large \textquoteleft a\textquoteright \textquoteleft} and that \( \partial^2(G_1 - C_i)/\partial(\gamma^j)^2 < 0 \Rightarrow \) a maximum will be obtained somewhere over the range of deviation. For a large, this maximum will be obtained sufficiently close to \( \tilde{\gamma} \).

Hence, for \( \mu_1^i \in (\mu_c, 1] \), i will have incentive to deviate, likewise for firm j. Hence the above strategies will constitute a BNE.  

\[Q.E.D.\]

\[\text{\textsuperscript{12}} \partial(G_1 - C_i)/\partial a = \{14(a - c_d) - 0.5(c_d - \tilde{\gamma}) + 49.5(\gamma^j - c_d)/144 > 0 \text{ (since the deviation output } (a + \tilde{\gamma} - 2c_d)/3b > 0 \text{ and } \gamma^j > c_d).} \]
Proof of Proposition 2: With information sharing, $E_1\gamma^j = \gamma_j$, $E_1c^d = (\gamma_j + \gamma^j)/2$ under C-B strategies. It is easy to verify that for $\mu_1^j \in [0, \mu_d^j]$, $\mu_1^j \in [0, \mu_d^j]$, the actual $c^d \leq c^d$ and hence the strategy profile $\tilde{q}_{C-B}, \tilde{q}_{C-B}$ do not enter constitutes a BNE. For $\mu_1^j \in [0, \mu_d^j]$, if $\mu_1^j > \mu_d^j$, then $c^d = (\gamma^j + \gamma^j)/2 < c^d$ will still be met if and only if $\gamma_j < \gamma^j$. Thus for $\gamma_j < \gamma^j < 2c^d - \gamma^j$, there exists $\mu_c^j > \mu_d^j$ such that for $\mu_1^j \in (\mu_d^j, \mu_c^j]$ (and $\mu_1^j \in [0, \mu_d^j]$), the strategy profile $\tilde{q}_{C-B}, \tilde{q}_{C-B}$ do not enter constitutes a BNE. Next, I show that for $\mu_1^j \in (\mu_d^j, 1]$, $\mu_1^j \in (\mu_d^j, 1]$, firms have incentives to deviate. Now, $E_2 \pi_2^* = (a + \gamma_j - 2\gamma^j)/9b$, $E_2 \pi_2^* = (a + \gamma_j - 2\gamma^j)(a + \gamma_j - 2\gamma^j)/16b$, and $E_2 \pi_2(c^d) = (a + 2c^d - 3\gamma^j)(a - c^d)/9b$. Again, note that $C_1 > G_1$ for $\delta = 0$, $G_1 > C_1$ for $\delta = 1$, and that the difference $(G_1 - C_1)$ increases monotonically in $\delta$. Hence there must exist a $1 > \delta_0 > 0$ such that $\forall \delta \geq \delta_0$, $(G_1 - C_1) \geq 0$.

Also, $\lim_{\gamma^j \rightarrow 4c^d} \partial(G_1 - C_1)/\partial \gamma^j > 0$ for a large, and $\partial^2(G_1 - C_1)/\partial \gamma^j < 0$ and hence $\gamma^j$ reaches a maximum. For a big, this maximum will prevail very close to $\gamma$. Thus for $\mu_1^j \in (\mu_d^j, 1]$, $\mu_1^j \in (\mu_d^j, 1]$, firms will deviate. Lastly, if a firm deviates when $\mu_1^j \in (\mu_d^j, 1]$, $\mu_1^j \in (\mu_d^j, 1]$, it will do so when $\mu_1^j \in (\mu_c^j, 1]$, $\mu_1^j \in [0, \mu_d^j]$ as $\mu_c^j > \mu_d^j$. Q.E.D.

Proof of Lemma 2: First, consider the case when information is not shared. Let $\bar{V}_i'' = d^2 \bar{V}_i/d(\mu_i)^2$. From Table 1, when a firm does not deviate $\bar{V}_i'' = (1 + \delta)(\bar{\gamma} - \gamma)^2/2b > 0$, and when it does deviate $\bar{V}_i'' = \delta(\bar{\gamma} - \gamma)^2/2b > 0$. Since value functions are convex in beliefs, by Lemma 1, a firm values self-produced information when information is not shared. Next, consider the case when information is shared. Again it follows directly from Table 1, that $\bar{V}_i'' = 8(1 + \delta)(\bar{\gamma} - \gamma)^2/9b > 0$, $\bar{V}_i''(a - c^d - \gamma^j) = 8\delta(\bar{\gamma} - \gamma)^2/9b > 0$, and $\bar{V}_i''(a^d) = 8\delta(\bar{\gamma} - \gamma)^2/9b > 0 \Rightarrow$ by Lemma 1 a firm values self-produced information with information sharing. Since $\bar{V}_i'\gamma_0' > \bar{V}_i'\gamma_0'$ and $\bar{V}_i''(c_0)' > \bar{V}_i''(c_0)'$ a firm values information more with sharing than without. Q.E.D.

Proof of Proposition 3: In expressions (8) and (9) $\frac{d\mu_i}{dC^d} = \mu_0(1 - \mu_0)(\tilde{f}_i'[-f^d]/D^2$ where $D =$

13Since $\partial(G_1 - C_1)/\partial a = 12a + 2(a - c^d) + 18(c^d - \gamma) + 6\gamma^j > 0.$
\( \mu_q \hat{f} + (1 - \mu_q)f \). By MLRP, \( \hat{f}(c)/f(c) \) must be nondecreasing in \( c \). Hence \( \frac{d\mu_i}{dC^i} > 0 \). By lemma 2, \( \overline{V}_i'' \) and \( \overline{V}_i'''(c) \) are positive. Therefore, \( d\text{EV}_i/dq_0|_k > 0 \), \( k = n_s, s \). Denote output levels of a myopic and a far-sighted firm by \( q_0|_m \) and \( q_0|_k \) respectively. Then, \( \frac{d\pi_0}{d\pi_0|_m} = \frac{d\pi_0}{d\pi_0|_k} \) since for a far-sighted firm, \( d\text{EV}_i/dq_0|_k > 0 \Rightarrow q_0|_m > q_0|_m \) as the profit function is concave. \( \text{Q.E.D.} \)

**Proof of Proposition 4:** Given the concavity of profit functions, \( q_0|_s \geq (<) q_0|_n \) according as \( d\text{EV}_i/dq_0|_s \geq (<) d\text{EV}_i/dq_0|_n \). Note that \( \mathcal{A} \cup \mathcal{C} = \mathcal{B} \cup \mathcal{C} \) and denote this set by \( \Omega \). Substituting for \( \overline{V}_i'' \) and \( \overline{V}_i'''(c) \) from lemma 2 and simplifying, the following difference is obtained:

\[
\frac{d\text{EV}_i}{dq_0|_s} - \frac{d\text{EV}_i}{dq_0|_n} = (\gamma - \zeta)^3 \left\{ \int_{\Omega} \int_{\mathcal{B} \cup \mathcal{C}} \frac{7}{18} \mu_0(1 - \mu_i) \frac{d\mu_i}{dC^i} dC^i dG(c) + \int_{\Omega} \int_{\mathcal{B} \cup \mathcal{C}} \frac{8}{9} \mu_0(1 - \mu_i) \frac{d\mu_i}{dC^i} dC^i dG(c) \right\} 
\]

The first term on the RHS is strictly positive reflecting the potential future benefits of a firm from information sharing as entry is prevented. The second two terms together represent the no-deviation expected marginal value of information in period 1 with sharing. The last term represents the no-deviation expected marginal value of information in period 1 without sharing. If, depending upon the magnitude of the no-deviation range \( C \) (which again depends upon the distribution of \( \epsilon_j \)) and other parameter values, the no-deviation expected marginal value of information without sharing is large enough to offset the first two effects then \( d\text{EV}_i/dq_0|_s \leq d\text{EV}_i/dq_0|_n \Rightarrow q_0|_n \geq q_0|_s \) will hold. \( \text{Q.E.D.} \)

**Proof of Proposition 5:** \( \overline{V}_{ij} = 2(1 + \delta)(\gamma - \zeta)^2/9b > 0 \), \( \overline{V}_{ij}(2\epsilon_i^d - \gamma) = 2\delta(\gamma - \zeta)^2/9b > 0 \), and \( \overline{V}_{ij}(c^d) = 2\delta(\gamma - \zeta)^2/9b > 0 \Rightarrow \) value functions are convex in rival's beliefs both in cases
of no-deviation and deviation. Using MLRP, \( \frac{d\mu^j}{dc^j} = \mu_0(1 - \mu_0)(\tilde{f}_j f_j - f_j^2) / D_j > 0 \) where \( D_j = \mu_0 \tilde{f}_j + (1-\mu_0)f_j \). It then follows straight from equation (10) that \( d\bar{V}_i/dq_i \) is > 0.

\[ Q.E.D. \]

**Table 1. Value functions**

<table>
<thead>
<tr>
<th>No-deviation ( \bar{V}<em>i = \pi_1(\gamma^i)</em>{NE} + \delta E \pi_2(\gamma^i)_{NE} )</th>
<th>Deviation ( \bar{V}<em>i(c_0) = \pi_1(c_0, \gamma^i)</em>{NE} + \delta E \pi_2(\gamma^i)_{NE} )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>No-sharing</strong></td>
<td><strong>sharing</strong></td>
</tr>
<tr>
<td>( (1 + \delta)(2a - 3\gamma^i + \tilde{\gamma})^2 ) ( \frac{36b}{36b} ) for ( \mu^i \in [0, \mu_c] )</td>
<td>( \left{ \frac{1}{9b}(a + 2c^d - 3\gamma^i)(a + \tilde{\gamma} - 2c^d) \right} ) for ( \mu^i \in (\mu_c, 1] )</td>
</tr>
<tr>
<td>( 1 + \delta ) ( \frac{(a + \gamma^i - 2\tilde{\gamma})^2}{9b} ) if ( \mu^i \in [0, \mu_d] ) regardless of ( \mu^j );</td>
<td>( \left{ \frac{1}{9b}(a + 2c^d - 3\gamma^i)(a + 3\gamma^j - 4c^d) \right} ) if ( \mu^j \in [0, \mu_d]; \mu^i \in (\mu_c, 1] )</td>
</tr>
<tr>
<td>Or, ( \mu^j \in [0, \mu_d] ) but ( \mu^i \in (\mu_d, \mu_c] )</td>
<td>( \left{ \frac{1}{9b}(a + 2c^d - 3\gamma^i)(a - c^d) \right} ) if ( \mu^i \in (\mu_d, 1]; \mu^j \in (\mu_d, 1] )</td>
</tr>
</tbody>
</table>