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Horizontal Inequity can be a Good Thing

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Horizontal Inequity Can Be A Good Thing

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Abstract A switch from any given income tax schedule to a differentiated tax structure in which two groups of taxpayers are treated differently, each still facing the same local degree of progression, can induce an increase in welfare despite causing horizontal inequity. We demonstrate this result in a number of special cases and make a general conjecture, the thrust of which is that society's acceptance of horizontal inequity will be second-best whenever the government must operate with a limited bundle of income tax instruments such as allowances, thresholds and marginal rates.

1. Introduction

Let t(x) be an income tax schedule, where x is pre-tax income. A switch from this schedule to a differentiated tax structure in which two groups of taxpayers are treated differently, each still facing the same local degree of progression, can induce an increase in (egalitarian) welfare despite causing horizontal inequity (henceforth **HI**).[1] We demonstrate this result at a number of levels, beginning with the simplest case in which the welfare improvement can be secured for virtually *any* two groups, but in terms of a restrictive welfare function, and terminating with the conditions under which an overall Lorenz improvement may be achieved. We conjecture that, given any income tax schedule, it will always be possible to secure a Lorenz improvement by taxing those at the same income level differently in each of two mutually exclusive and exhaustive subgroups

[1] It is trivial that within-group tax reforms which are separately progression-enhancing can give rise to **HI** overall. Just consider a population in which two persons, each with 100, are in different groups. In group A, the person with 100 receives a transfer from a richer person, whilst in group B, the person with 100 makes a transfer to someone poorer. Vertical equity is improved within each group, and welfare is increased, but equals have been treated unequally. Our interest here is strictly in reforms which preserve the tax system's vertical stance within groups.

whilst maintaining the tax's vertical stance within each group. Whilst society's acceptance of **HI** may not be first-best, it is second-best whenever the government must operate with a limited bundle of income tax instruments such as allowances, thresholds and marginal rates.

2. Preliminaries

If t(x) is the income tax schedule, the average and marginal tax rates experienced at income level x are a(x) = t(x)/x and m(x) = t'(x) respectively. The two measures of local progression we shall use are the liability progression LP(x) = m(x)/a(x) and the residual income progression $RP^*(x) = [1-a(x)]/[1-m(x)]$.[2] We shall examine both LP-neutral and RP-neutral tax changes in this paper. The former modifies all people's tax liabilities proportionately; the latter is such that post-tax incomes are modified proportionately.[3]

Let μ be mean post-tax income and let F(y) be the distribution function for post-tax income. The mean logarithmic deviation (henceforth MLD) is $J = \int_0^\infty \ell n\,(\mu/y)dF(y)$ and the Gini coefficient is $G = \int_0^\infty F(y)[1-F(y)]dy/\mu$. The extended Gini coefficient G(ν), ν>1, satisfies $\mu[1-G(\nu)] = \int_0^\infty [1-F(y)]^\nu dy$. If p = F(z), the Lorenz curve is $L_F(p) = \int_0^z ydF(y)/\mu$ and the generalized Lorenz curve is $GL_F(p) = \mu.L_F(p) = \int_0^z ydF(y)$.[4]

Let A be any subgroup of the population with mean post-tax income μ_A. If B is the complement of A, with mean income μ_B, then $\mu = \pi.\mu_A + (1-\pi).\mu_B$ where π is the proportion of the population belonging to A. If the distribution functions for post-tax income in A and B are $F_A(y)$ and $F_B(y)$, then $F(y) = \pi.F_A(y) + (1-\pi).F_B(y)$. The MLD decomposes into between- and within-group inequality measures: $J = \pi.J_A + (1-\pi).J_B + J^*$, where J^* is the MLD for the smoothed income distribution in which the members of A get μ_A each and the members of B get μ_B each.[5]

3. Two simple results

[2] See Lambert (1993), chapters 6-7, for more on the measurement of progression.

[3] The two types of reform have been compared from various angles by Pfähler (1984).

[4] See Yitzhaki (1983) for the extended Gini coefficient.

[5] See Bourguignon (1979). The Gini coefficient does not decompose in this way: see *e.g.* Lambert and Aronson (1993).

Let A be any subgroup of the population for which $\mu_A < \mu$, and B its complement (so that $\mu_B > \mu$). It will always be possible to select such subgroups unless the income distribution is perfectly equal. Now change the taxes revenue-neutrally, giving a small RP-neutral tax cut to group A and a small RP-neutral tax hike to group B. Post-tax incomes become $(1+\theta)[x-t(x)]$ in A and $(1-\lambda)[x-t(x)]$ in B, where θ and λ are constants such that $\pi\theta\mu_A = (1-\pi)\lambda\mu_B > 0$. If there is any overlap between the pre-tax income ranges for A and B, this introduces **HI**. It does not affect post-tax inequality within either group, and reduces inequality between groups.[6] Therefore the MLD of post-tax income falls. By Atkinson's (1970) theorem, at least one utilitarian SWF records an increase in welfare despite the introduction of **HI**.0. As shown in Pfähler (1984), the RP-neutral tax hike for B could even be improved upon, with a reduction in inequality J_B and therefore a further increase in overall welfare, if it were supplanted by an LP-neutral tax hike raising the same revenue.[7]

This simple result, approving of the introduction of **HI** where there was none before, is not peculiar to the MLD. It works for the Gini coefficient too, provided the people in A are 'generally poorer' than those in B. Namely, if $[\mu_A f_B(y) - \mu_B f_A(y)]$ is first negative and then positive, where $f_A(y)$ and $f_B(y)$ are the frequency density functions for post-tax income in A and B, then both the Gini and extended Gini coefficients are reduced by the combination of a (marginal) RP-neutral tax cut in A and tax hike in B - see the Appendix. Again, it means that at least one utilitarian SWF records an increase in welfare despite the introduction of **HI**.0. In addition, the non-utilitarian SWFs $\mu(1-G)$ and $\mu(1-G(\nu))$ developed by Sen (1973) and Muliere and Scarsini (1989) both favour this sort of tax reform.

4. A Lorenz improvement

In certain circumstances the combination of an RP-neutral tax cut for one group and an RP-neutral tax hike for the other leads to a Lorenz improvement overall. Suppose group A has poor and middle-income people only, and group B has middle-income and rich people only. More specifically, suppose that in A, a fraction q of the population have incomes v_P and (1-q) have v_M

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[6] This only requires that $(1+\theta)\mu_A$ and $(1-\lambda)\mu_B$ are closer together than μ_A and μ_B. If $\lambda = \pi[\mu_B-\mu_A]/\mu_B$ and $\theta = (1-\pi)[\mu_B-\mu_A]/\mu_A$, between-group inequality is eliminated entirely.

[7] This would involve tax liabilities for B of the form $(1+\psi)t(x)$ for an appropriate constant $\psi > 0$.

$> v_P$, and that in B a fraction r have incomes v_M and (1-r) have $v_R > v_M$. Again impose a small RP-neutral tax cut in A and hike in B, so that the incomes become $(1+\theta)v_P$ and $(1+\theta)v_M$ in A and $(1-\lambda)v_M$ and $(1-\lambda)v_R$ in B.[8] The necessary and sufficient condition for a Lorenz improvement is $rqv_Pv_R \geq (1-r)(1-q)v_M^2$ (see Appendix). Thus the introduction of **HI** for middle-income people whilst preserving residual income progression within both groups is unambiguously a good thing in this simple case.[9]

The condition on the income distributions in A and B which is in general necessary and sufficient for a marginal RP-neutral tax cut in A and hike in B to be Lorenz-improving is that $L_B(F_B(y)) \leq L_A(F_A(y))$ for every post-tax income level y : this result is derived in another context in Lambert (1992). The condition holds in some interesting cases in the real world. For example, it holds for the Sri Lankan money income distribution in 1978/9 and 1981/2, with A and B as the rural and urban subpopulations, and for the UK equivalent income distribution in 1984/5 with A as the married and B the single (see *ibid*). If it were found to hold between the richer north and poorer south in the United States, for example, then progression-neutral differentiation in the federal income tax could be recommended.

5. Conjecture and concluding discussion

Given any unequal distribution of post-tax income, it is of course possible to identify two mutually disjoint and exhaustive population subgroups A and B such that $L_B(F_B(y)) \leq L_A(F_A(y))$ $\forall y$. For example, one could simply partition the income distribution into poorer and richer subgroups. We conjecture, however, that more interesting partitions than this will always be possible. Specifically, our conjecture is this: whatever the tax code and distribution of post-tax income, assumed unequal, a choice of subgroups A and B can be made such that *(a)* the condition $L_B(F_B(y)) \leq L_A(F_A(y))$ $\forall y$ holds post-tax, and *(b)* the pre-tax income ranges in A and B overlap.

Consider what this conjecture implies, if true. It means that however well-designed a given income tax may be, it can be improved *yet further* by appropriate differentiation - carrying with

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[8] Assume in addition that $(1+\theta)v_P < (1-\lambda)v_M$ and $(1+\theta)v_M < (1-\lambda)v_R$ so that the tax reform induces no reranking between groups. The condition for revenue neutrality is that $(1-\pi)\lambda/\pi\theta = [qv_P + (1-q)v_M]/[rv_R + (1-r)v_M]$.

[9] The result can be generalized to allow non-degenerate distributions of low and high incomes in A and B respectively, retaining the single middle-income value v_M in both groups, which is the source of the **HI**.

it some **HI**. Let $t(x|\underline{\alpha})$ be the income tax schedule, in which the parameters $\underline{\alpha}$ stand for allowances, thresholds and marginal rates. For clarity of discussion, define $n(x|\underline{\alpha}) = x - t(x|\underline{\alpha})$ as the implied post-tax income 'schedule'. If our conjecture is true, then $n(x|\underline{\alpha})$ can be bettered by imposing an RP-neutral tax hike in one group, B, and using the revenue to finance an RP-neutral tax cut in the other group, A. The post-tax income schedules now become $(1+\theta)n(x|\underline{\alpha})$ for A and $(1-\lambda)n(x|\underline{\alpha})$ for B. This reform improves welfare, reduces inequality and necessarily introduces **HI**. Should governments seek out such reforms, legislating for increased vertical equity, whilst ensuring no individual anywhere faces increased progression, at the cost of introducing **HI**?

The thrust of much current research into measuring **HI** is that, given any income tax code with **HI**, a further welfare improvement can be secured by averaging taxes at each pre-tax income level to eliminate the **HI**.[10] We started with the **HI**-free schedule $n(x|\underline{\alpha})$ and argued for differentiation, leading to schedules $(1+\theta)n(x|\underline{\alpha})$ for A and $(1-\lambda)n(x|\underline{\alpha})$ for B and introducing **HI**. Now, we assert, a further welfare improvement should come from averaging these taxes across the two groups, removing the **HI** again! What is going on here? The conundrum can be resolved by considering informational requirements. Let p_x be the proportion of those having x who are in group A. Averaging would create a unified schedule $p_x(1+\theta)n(x|\underline{\alpha}) + (1-p_x)(1-\lambda)n(x|\underline{\alpha})$ for everybody, which is **HI**-free and superior to the differentiated structure $(1+\theta)n(x|\underline{\alpha})$ for A and $(1-\lambda)n(x|\underline{\alpha})$ for B, which has **HI** and is, in turn, superior to the original and **HI**-free $n(x|\underline{\alpha})$. The problem is that the unified schedule has heavy informational requirements; a tax code of the form $p_x(1+\theta)n(x|\underline{\alpha}) + (1-p_x)(1-\lambda)n(x|\underline{\alpha})$ could hardly be announced, published and understood by the public, due to the ever-changing p_x, whereas a bundle $\underline{\alpha}$ of conventional tax parameters can, and so could a differentiation parameter θ (or λ). However unfair the differentiation parameter might seem, especially to the losers, it brings an overall welfare gain relative to the original $n(x|\underline{\alpha})$. In a world where we can cope with only limited amounts of information, it can be better to have **HI** than no **HI**. We believe that this is a new insight.

Appendix

Let $F(y|\theta)$ be the distribution function and $G(\theta)$ the Gini coefficient for post-tax income

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[10] The papers of Aronson *et al.* (1994), Lambert and Ramos (1997) and Duclos and Lambert (1999) all measure **HI** by its welfare cost, compared with the **HI**-free alternative of smoothed tax liabilities.

after the tax reform described in Section 3. Then $F(y|\theta) = \pi.F_A(y/[1+\theta]) + (1-\pi).F_B(y/[1-\lambda])$ and $\mu G(\theta) = \int_0^\infty F(y|\theta)[1-F(y|\theta)]dy$ whence $G'(0) < 0 \Leftrightarrow \int_0^\infty g(y)F(y)dy \geq 0$ where $g(y) = \partial F(y|\theta)/\partial\theta|_{\theta=0}$ $= \pi y[-\mu_B f_A(y) + \mu_A f_B(y)]/\mu_B$. If there are poorer people in A than in B, then g(y) is first negative. If g(y) stays positive once it becomes positive, then because F(y) is increasing, G'(0) < 0. For the extended Gini, similar steps show that $\partial G(\nu)/\partial\theta|_{\theta=0} < 0 \Leftrightarrow \int_0^\infty g(y)[1-F(y)]^{\nu-1}dy < 0$: the extended Gini is also reduced by a marginal tax reform of the kind described, if A is the 'generally poorer' group in the sense given.[11] The generalized Lorenz curve for the scenario in Section 4 is given by $GL(öq) = \pi q v_P$, $GL(\pi q+(1-\pi)(1-r)) = \pi q v_P + (1-\pi)(1-r)v_M$, $GL(1-r(1-\pi)) = \pi q v_P + \{\pi(1-q)+(1-\pi)(1-r)\}v_M$ and of course $GL(1) = \mu$, with linear interpolation in between. The ordinates increase by $\Delta GL(\pi q) = \pi q\theta v_P > 0$, $\Delta GL(\pi q+(1-\pi)(1-r)) = \pi q\theta v_P - (1-\pi)(1-r)\lambda v_M$ and $\Delta GL(1-r(1-\pi)) = \lambda(1-\pi)rv_R > 0$. These are all non-negative if and only if $rqv_P v_R \geq (1-r)(1-q)v_M^2$.

[11] The same result can be derived using an approach outlined by Yitzhaki and Slemrod (1991) for the analysis of commodity tax reform, by asusming that the two regions are 'commodities' and neglecting labour supply and migration effects.

References

Aronson, R., P. Johnson and P.J. Lambert (1994). Redistributive effect and unequal income tax treatment. *Economic Journal*, vol. 104, pp. 262-270.

Atkinson, A.B. (1970). On the measurement of inequality. *Journal of Economic Theory*, vol. 2, pp. 244-263.

Bourguignon, F. (1979). Decomposable inequality measures. *Econometrica*, vol. 47, pp. 901-920.

Duclos, J.-Y. and P.J. Lambert (1999). A normative and statistical approach to measuring classical horizontal inequity. *Canadian Journal of Economics*, forthcoming.

Lambert, P.J. (1992). Rich-to-poor income transfers reduce inequality: a generalization. *Research on Economic Inequality*, vol. 2, pp. 181-90.

Lambert, Peter J. (1993). *The Distribution and Redistribution of Income: A Mathematical Analysis* (2nd edition). Manchester: University Press.

Lambert, P.J. and J.R. Aronson (1993). Inequality decomposition analysis and the Gini coefficient revisited. *Economic Journal*, vol. 103, pp. 1221-27.

Lambert, P.J. and X. Ramos (1997). Vertical redistribution and horizontal inequity.*International Tax and Public Finance*, vol. 4, pp. 25-37.

Muliere, P. and M. Scarsini (1989). A note on stochastic dominance and inequality measures. *Journal of Economic Theory*, vol. 49, pp. 314-323.

Pfähler, W. (1984). 'Linear' income tax cuts: distributional effects, social preferences and revenue elasticities. *Journal of Public Economics*, vol. 24, pp. 381-388.

Sen, A. (1973). *On Economic Inequality*. Oxford: Clarendon Press.

Yitzhaki, S. (1983). On an extension of the Gini index. *International Economic Review*, vol. 24, pp. 617-628.

Yitzhaki, S. and J. Slemrod (1991). Welfare dominance: an application to commodity taxation. *American Economic Review*, vol. 81, pp. 480-496.