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Effects of Noncompliance and Tax Evasion

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Abstract: This paper uses the unique Taxpayer Compliance Measurement Program (TCMP) micro data to study the equity effects of noncompliance. We access four years of TCMP data, 1979, 1982, 1985, and 1988. The TCMP data allows use to observe income and taxes before and after a tax audit. In order to generate a range of scalar estimates of the redistributive impact of more complete compliance we employ the family of extended Gini and concentration coefficients. We find that the vertical equity effects of noncompliance are very small or negative; however, there is a considerable amount of horizontal inequity generated by noncompliance and in this sense more complete auditing of tax returns could improve the fairness of the tax system.

1. Introduction

The US tax income tax system is based on voluntary compliance. Of course, compliance with the tax code is imperfect. For example, the IRS recently reported that the estimated size of the tax gap in 1996 was 195 billion dollars. One way in which the IRS investigates the phenomenon of noncompliance is through the Taxpayer Compliance Measurement Program (TCMP). The TCMP is a special IRS examination procedure designed to measure voluntary compliance with Federal taxes in the entire taxpaying population. Recently, the IRS has allowed limited access to the TCMP microdata. This paper uses the TCMP data to investigate the redistributive impact of noncompliance. Using the unique data of the TCMP we are able to address the following set of hypothetical questions: What impact would more complete compliance have on income inequality? How would horizontal equity be effected if the tax authorities were able to more fully implement the existing tax code? What impact would increasing the compliance rate have on vertical equity?

While we do not claim that the TCMP audits represent complete compliance, do believe that they accurately represent the likely outcome of expanded compliance efforts. Stepping up compliance efforts will change the redistributive effect of the tax system ; our analysis provides estimates of the marginal benefit (cost) of improved (reduced) vertical and horizontal equity. In order to generate a range of scalar estimates of the redistributive impact of more complete compliance we employ the family of extended Gini and concentration coefficients of Yitzhaki (1983).¹

¹ Bishop, Chow, Formby, and Ho (1994) use the TCMP data to study noncompliance in a Lorenz dominance framework. The Lorenz curve, while the most general measure of inequality available, does not (directly) allow for scalar estimates of the redistributive impact of noncompliance as presented here.

2. Measuring the Redistributive Effect of Noncompliance

Let the reported income of an income unit be X and let the tax paid be T (which could be modelled as $\tau(X)$, where $\tau(\cdot)$ is a tax schedule). Further, let unreported income be U and let the correct tax be $T + S$ (in the case of schedule $\tau(\cdot)$, we would have $S = \tau(X+U) - \tau(X)$).

Let A and B denote any two attributes of an income unit (e.g. declared income, undeclared income, tax due, *etc.*) and denote the concentration curve for attribute A with respect to the ordering of income units by attribute B by $C_{A|B}$. That is, for each $p \in [0,1]$, $C_{A|B}(p)$ is the share in the total ΣA going to the 100p% of income units who are the poorest in the distribution of B . Denote the Lorenz curve for A by L_A (or $C_{A|A}$).²

The raw data yields **pre-audit** Lorenz and concentration curves:

$$L_X, L_{X-T} \text{ and } C_{X-T|X}$$

for, respectively, before tax income, after tax income and after tax income relative to the ordering of income units by levels of reported income before tax. With unreported income included, and unpaid taxes deducted, the corresponding **post-audit** curves are:

$$L_{X+U}, L_{X+U-T-S} \text{ and } C_{X+U-T-S|X+U}$$

The concentration curves:

$$C_{X+U|X} \text{ and } C_{X+U-T-S|X-T}$$

will also be needed to measure compliance effects relative to the pre-audit distributions of income before and after tax.

² There is, in fact, a slight problem here. When two or more income units enjoy a common value of attribute B , say b_0 , but *different* values of attribute A , then strictly, the concentration curve $C_{A|B}$ is not well-defined at percentiles p which include some but not all of those having b_0 . For the empirical implementation of measures developed here, it will be enough to define $C_{A|B}$ at deciles. In each of the TCMP data sets used, there is in excess of 47,000 income units. In the 1988 data set, for example, there are 42,756 distinct values of before tax pre-audit income and 43,184 distinct values of after tax post-audit income. Hence the errors introduced by numerical integration and smoothing procedures are unlikely to be fatal. See part 1 of the Appendix

The **redistributive effects** of the income tax, pre-and post-audit, are measured respectively by the distance between the Lorenz curve for post-tax income, and the one which would obtain under a proportional (distributionally neutral) tax with the same yield. This latter hypothetical curve is of course the before tax Lorenz curve, pre-or post-audit. Hence redistributive effect measures the inequality reducing characteristic of the income tax:

$$\text{Pre-audit} \quad RE_0 = L_{X-T} - L_X$$

$$\text{Post-audit} \quad RE_1 = L_{X+U-T-S} - L_{X+U}$$

The determinants of the redistributive effect of a tax system generally have been analysed by Kakwani (1984). A full empirical analysis of the determinants of pre- and post-audit redistributive effect, RE_0 and RE_1 in the present context, can be found in Bishop, Chow, Formby and Ho (1994).³

The audit has both vertical and horizontal impacts. The vertical impact of the audit arises through the modification of people's relative income differentials - both before tax, by the inclusion of unreported income, and after tax, by the inclusion of unreported income and additional tax due. As an addition to reported income X , the unreported income U will exert an equalizing impact if U is an increasing proportion of X in cross-section, and a disequalizing impact if it is a decreasing proportion. Similarly, the impact of the audit on after tax income differentials depends on whether the increment $U-S$ to after tax income is an increasing or decreasing proportion of pre-audit after tax income $X-T$. The two measures following capture the **vertical impact of the audit**:

$$\text{Before tax} \quad V_b = C_{X+U|X} - L_X$$

$$\text{After tax} \quad V_a = C_{X+U-T-S|X-T} - L_{X-T}$$

Each is ambiguously positive for all $p \in (0,1)$ if the relevant increment is an increasing proportion of income, and negative if it is a decreasing proportion.

The principle of horizontal equity in income taxation demands the "equal treatment of equals". We approach horizontal inequity caused by tax cheating by letting equals be defined by the tax law and focus our attention on the reranking caused by noncompliance. Regarding "equals" as "equals before the tax law", it is clear that evasion and non-compliance generates horizontal inequities. That is, tax evasion reranks taxpayers in such a way that those who are regarded as equals by the law no longer bear the same tax burden. Furthermore,

³ See also the Appendix, part 2, where the relevant theory is sketched out.

evasion and non-compliance affects horizontal inequity at any given *money* income level as well as to “equals before the tax law”.⁴

We therefore measure the **horizontal impact of the audit** by the way the ordering of income units is modified - before tax by the inclusion of unreported income, and after tax by the inclusion also of additional tax due:

$$\begin{aligned} \text{Before tax} \quad H_b &= C_{X+U|X} - L_{X+U} \\ \text{After tax} \quad H_a &= C_{X+U-T-S|X-T} - L_{X+U-T-S} \end{aligned}$$

Finally, the **inequality effect of the audit** is captured by the impact on the Lorenz curves of before and after tax income of including unreported income and additional tax due:

$$\begin{aligned} \text{Before tax} \quad IE_b &= L_{X+U} - L_X \\ \text{After tax} \quad IE_a &= L_{X+U-T-S} - L_{X+U-T-S} \end{aligned}$$

Each of these can be expressed simply in terms of the relevant (pre- or post-audit) vertical and horizontal components:

$$\begin{aligned} \text{Before tax} \quad IE_b &= V_b - H_b \\ \text{After tax} \quad IE_a &= V_a - H_a \end{aligned}$$

The impact of the audit on redistributive effect is, simply, its net inequality effect or, net vertical impact minus net horizontal impact:

$$RE_1 - RE_0 = IE_b - IE_a = [V_a - V_b] - [H_a - H_b]$$

As this equation confirms, the loss of redistributive effect due to evasion and non-compliance is the difference between the vertical impact which compliance would have and the horizontal inequity generated by non-compliance.

All curves and differences defined above are multi-valued measures, defined at percentiles $p \in (0,1)$ in the relevant distribution of income (before or after tax, pre- or post-audit). They are, of course, population statistics for the TCMP data set, and they provide sample statistics for the economy-wide effects of evasion and non-compliance. As global estimates, they of course raise problems of statistical inference, which we do not deal with here.⁵ In order to aggregate into scalar indices, we select a consistent weighting function $w(p)$ for all measures. The particular choice $w(p) = v(v-1) (1-p)^{v-2}$, $v \geq 1$, yields the parametric family of so-called *extended Gini and concentration coefficients* of Yitzhaki (1983).

For the Lorenz curve L_A and concentration curve $C_{A|B}$ generally, these coefficients may be written:

$$G_A(v) = 1 - v(v-1) \int_0^1 (1-p)^{v-2} L_A(p) dp$$

and

⁴ Atkinson (1980) and Plotnick (1981) seek, more contentiously, to capture horizontal inequity *in the tax system* (if any) by reranking effects upon the distribution of *equivalent income*. Aronson, Johnson and Lambert (1994) distinguish the pure horizontal inequity and reranking effects of tax systems (see the Appendix). We find it very persuasive that reranking in money incomes captures the horizontal aspect of tax cheating..

⁵ The statistical inference issue is examined theoretically in Bishop, Chow and Formby, 1994, where some inferences are drawn from the TCMP data set for 1985, which is also one of data sets used in this paper. In particular, the vertical impact of evasion and non-compliance in the population of U.S. taxpayers as a whole is estimated to be significantly negative in 1985 (see page 48).

$$G_{A|B}(v) = 1 - v(v-1) \int_0^1 (1-p)^{v-2} C_{A|B}(p) dp$$

respectively. Increasing the value assigned to the parameter v shifts the focus of the index progressively towards the lower end of the distribution. The value $v=2$ yields conventional Gini and concentration coefficients, representable as areas on the Lorenz diagram.

In index form, our measures are as follows. For **redistributive effect**, we have:

$$\begin{aligned} \text{Pre-audit} \quad \rho_0(v) &= G_X(v) - G_{X-T}(v) \\ \text{Post-audit} \quad \rho_1(v) &= G_{X+U}(v) - G_{X+U-T-S}(v) \end{aligned}$$

For the **vertical impact of compliance**, we have:

$$\begin{aligned} \text{Before tax} \quad v_b(v) &= G_X(v) - C_{X+U|X}(v) \\ \text{After tax} \quad v_a(v) &= G_{X-T}(v) - C_{X+U-T-S|X-T}(v) \end{aligned}$$

and for the **horizontal inequity generated by non-compliance**:

$$\begin{aligned} \text{Before tax} \quad \eta_b(v) &= G_{X+U}(v) - C_{X+U|X}(v) \\ \text{After tax} \quad \eta_a(v) &= G_{X+U-T-S}(v) - C_{X+U-T-S|X-T}(v) \end{aligned}$$

The **inequality effect of the audit** becomes:

$$\begin{aligned} \text{Before tax} \quad l_b(v) &= G_X(v) - G_{X+U}(v) \\ \text{After tax} \quad l_a(v) &= G_{X-T}(v) - G_{X+U-T-S}(v) \end{aligned}$$

and the decomposition, explaining the sources of change in the inequality reducing characteristic of the income tax due to evasion and non-compliance, is:

$$\rho_1(v) - \rho_0(v) = [v_a(v) - v_b(v)] - [\eta_a(v) - \eta_b(v)]$$

or, simply:

$$\Delta\rho(v) = \Delta l(v) = \Delta v(v) - \Delta\eta(v)$$

3. Empirical Results

The data for this study is the Taxpayer Compliance Measurement Program (TCMP). The TCMP survey of individual returns consists of a stratified probability sample of filed returns that are thoroughly audited to check all information on the returns and relevant records that the taxpayers are asked to provide. The examinations reveal understatements and overstatements of income, exemptions, statutory adjustments, deductions and credits by comparing what taxpayers report on their returns with what the IRS examiners determine should have been reported on those same returns.

In recent years the TCMP survey of individual returns has generally been conducted every three years with 1988 being the most recent sample available. The survey is a national random (weighted) sample of approximately 50,000 returns drawn from the population of filed tax returns. The samples include all individual income tax

returns, both business and non-business, with positive incomes for 1979, 1982, 1985, and 1988. A detailed description of the TCMP data is provided by the IRS's *Taxpayer Compliance Measurement Program Handbook* (1989).⁶

Table 1 provides the Gini ($v=2$), extended Ginis ($v=1.5, 3.0$, and 5.0), and concentration indices for alternative income concepts and taxpayer unit orderings.⁷ The first two entries, the pretax (G_X) and post-tax (G_{X-T}) Gini coefficients, are based on unaudited incomes and are commonly used to evaluate changes in income inequality and tax progressivity. Column entries 3 and 4 provide the Gini coefficients based on audited incomes, where G_{X+U} is the audited pretax Gini and $G_{X+U-T-S}$ is the audited post-tax Gini. Column 5 provides the pre-audit, post-tax concentration index, $C_{X-T|X}$, which differs from G_{X-T} by maintaining the pretax ordering. Column 6 presents $C_{X+U|X}$, which differs from G_{X+U} in that the concentration Gini maintains the unaudited income positions. Column 7 gives, $C_{X+U-T-S|X+U}$, which is post audit income ordered by actual income, $X+U$. Finally, column 8 provides post audit income ordered by preaudit income, $C_{X+U-T-S|X-T}$.

Table 1 provides all the data necessary to evaluate the inequality, vertical, and horizontal effects of the TCMP audit. We use the (extended) Ginis of Table 1 (columns 1-4) to examine the impact of taxes and audits on inequality and changes in inequality over time. We create a separate table, Table 2, in order to examine the redistributive effects. The first row for each year provides the ordinary Gini coefficients ($v = 2$). Following the ordinary Gini in each year are the extended Ginis based on alternative weighting parameters, v .

We begin by examining the impact of taxes on inequality. The Ginis based on the TCMP data are larger than those typically reported by the Census Bureau. This is because the TCMP data include some very large incomes. As expected, all of the Ginis reported in Table 1 increase over time. For example, the Gini for post-tax incomes rises from 0.4671 in 1979 to 0.5276 in 1988. Not surprisingly, federal income taxes reduce income inequality--the post-tax Gini of column 2 is smaller than the pretax Gini of column 1. In addition, we find that the inclusion of unreported income has no measurable effect on income inequality. The differences between the pretax unaudited Gini (column 1) and the pretax audited Gini (column 3) are negligible for all v 's and all four years

⁶ Several caveats concerning the TCMP data warrant emphasis. First, the sample is restricted to taxpayers who file returns. Second, tax evaders often camouflage their activities and the TCMP audits do not uncover all of the tax evasion that is taking place. Finally, the TCMP audit does not include income and tax liabilities stemming from illegal activities.

⁷ The Ginis, extended Ginis and concentration indices are estimated from the percentile data given in Bishop, Chow, Formby, and Ho (1994) and additional similar author calculations using a cubic spline smoothing procedure.

considered. Finally, the inclusion of both unreported income and additional taxes owed also has a negligible impact on inequality, as the differences in column 2 and 4 are very small for all ν 's and all years considered.

Table 2 uses the (extended) Gini and concentration indices from Table 1 to construct distribution sensitive measures of the vertical, horizontal, and net redistributive effects of evasion and noncompliance. Columns 1 and 2 provide estimates of the net redistributive effect (NRE) of federal income taxes, pre and post audit. Columns 3 and 4 provide estimates of the vertical inequity (VE) of noncompliance, both before and after taxes. Columns 5 and 6 provide estimates of the horizontal inequity (HE) of noncompliance. Finally, columns 7, 8, and 9 provide the changes in the redistributive, vertical, and horizontal effects associated with the TCMP audits.

As expected, the NRE (columns 1 and 2) of federal taxes is positive. Both pre and post audit, the NRE rises with ν (as more weight is placed on the bottom of the income distribution) up to 3.0, then falls between 3.0 and 5.0. While there are some (mostly small) differences in the magnitude of the NRE estimates over time, the effect of changing ν is the same for each year.

As noted above, the net redistributive effect can be decomposed into vertical and horizontal equity effects. Columns 3 and 4 show that noncompliance reduces the vertical equity of the tax system. Column 3 shows the vertical equity effects of incorporating unreported income into pretax income. All entries in column 3 are positive implying that adding in unreported income improves vertical equity.⁸ Column 4 shows the vertical equity effects of incorporating both unreported income and additional taxes owed into post-tax income. Adding additional taxes to unreported income also improves vertical equity. Increasing the inequality aversion parameter, ν , generally results in an increase in VE, both before and after taxes. The exception to this finding is for $\nu \geq 3.0$ in 1988.

Column 5 shows the impact on horizontal inequity of incorporating unreported income into pretax income. The negative entries in column 5 indicate that adding unreported incomes reduces the horizontal inequity in the tax system. The entries in column 6 are also all negative, implying that reducing noncompliance on an after tax basis would also reduce horizontal inequity. The impact on horizontal equity of changing the weighting parameter is also unambiguous; as ν increases (i.e., more weight is given to the lower incomes) we find in all cases a decrease in horizontal inequity.

A ν value of less than 2.0 puts less weight on transfers among the poor than the ordinary Gini, while a ν value greater than 2.0 puts more weight on transfers among the rich than the ordinary Gini.

⁸ This conclusion is based on accepting the extended Gini as the correct measure of inequality. Bishop, Chow, and Formby (1994) use the TCMP data to illustrate stochastic dominance tests and find that taxpayers in the first decile underreport a larger than average share of their income.

Columns 7, 8, and 9 provide the changes in the redistributive, vertical, and horizontal effects associated with the TCMP audits. Our primary focus is on column 7--the change in NRE due to the audit. Recall that the NRE is simply the vertical equity VE effect minus the change in HE. In Table 2 this implies that column 7 is the difference between column 8 and column 9.

Table 2, column 7 shows that the changes in the NRE are all positive for years 1979, 1982, and 1985 implying that increased audit efforts would improve the redistributive aspects of the US tax system. However, something appears to have changed between 1985 and 1988. In 1988 we find little evidence that the TCMP audits change the NRE of taxes, with the exception of the extreme value of v (5.0); this implies that the largest redistributive impact of the 1988 audit is among the lowest income taxpayers.

Columns 8 and 9 isolate the effects of tax audits on VE and HE. Most striking is the finding that in nine of the sixteen 16 cases the net effect of additional taxes on VE is zero or close to zero. There are also three cases in 1988 where the change in VE due to additional taxes is negative and not close to zero. There are two cases with relatively large positive VE effect, $v = 2.0$ in 1979, and $v = 5.0$ in 1988. Finally, column 9 reports the change in HE due to the additional taxes. It is clear that for 1982 and 1985 all or nearly all of the change in the NRE is due to a reduction in horizontal inequity due to reducing noncompliance. For 1979, extreme values of v (1.5 and 5.0) result in the change in VE being larger than the change in HE. For 1988, an increase in HE is completely offset by a decline in VE in three of the four cases, resulting in no change in NRE.

In summary, there appears to be a change in the portion of the income distribution at which the TCMP audits are most effective in improving the equity of the tax system. In 1979, the largest redistributive and vertical impacts are in the upper tail of the distribution (see results for $v = 1.5$). By 1988, the largest redistributive and vertical impacts of the audit are in the lower tail of the distribution (see results for $v = 5.0$). Evaluating changes in VE over time and ignoring the extreme values of $v = 1.5$ and $v = 5.0$, we find that VE changes from near zero in 1979 to 1985 to negative in 1988. This negative VE finding at mid range v 's combined with a positive finding at $v = 5.0$ suggest that most of the vertical equity gains due to the audits are confined to the bottom of the income distribution. Finally, it appears that the largest, most consistent finding is that the audits are successful in reducing horizontal inequities due to noncompliance.

4. Conclusions

Tax equity is an important aspect of the design of any tax system. One aspect of tax equity rarely studied is the (in)equity of the tax system generated by noncompliance and evasion. This paper uses the unique Taxpayer Compliance Measurement Program (TCMP) micro data to study the equity effects of noncompliance. We access four years of TCMP data, 1979, 1982, 1985, and 1988. The TCMP data allows use to observe income and taxes before and after a tax audit. To parameterize the impact of noncompliance on tax system equity we use the generalized Gini coefficient.

To evaluate equity effects we consider both an inequality effect that is generated by noncompliance, as well as a net redistributive effect. We further decompose the net redistributive effect into its vertical and horizontal equity components. We find while there is no inequality effect due to noncompliance there is a net redistributive effect. This net redistributive effect, which combines the vertical and horizontal components, is declining over time. Much of this decline can be attributed to a decline in the vertical equity impact of the TCMP audits. For the ordinary Gini the vertical equity effects are very small or negative, resulting in most of the increase in net redistributive effect coming from a reduction in horizontal inequity. In sum, we find that there is a considerable amount of horizontal inequity generated by noncompliance and in this sense more complete auditing of tax returns could improve the fairness of the tax system.

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Appendix

1. Construction of concentration curves

When two or more income units enjoy a common value, say b_o , of attribute B but *different* values of attribute A, there are difficulties in defining the concentration curve $C_{A|B}$ at percentiles p , which include some but not all of those having b_o . There are two alternative ways to remedy this problem.

First, we may define a lexicographic ordering, in which income units are ordered first by levels of B, from lowest to highest, and second, at any given b_o , by levels of A, again from lowest to highest; and then, for each $p \in [0,1]$, define $\hat{C}_{A|B}(p)$ to equal the share in the total ΣA going to the 100p% of income units who come first in this lexicographic ordering.

Second, if $b_1 < b_2 < b_3 < \dots < b_n$ are the distinct values of attribute B occurring in the population, and if a proportion p_i of income units have B-values less than or equal to b_i , we may define $\check{C}_{A|B}(p_i)$ as the share in the total ΣA going to those with $b \leq b_i$, for $1 \leq i \leq n$, and use linear interpolation to define $\check{C}_{A|B}(p)$ for all intermediate values of p .

It is clear from the construction that $\hat{C}_{A|B}(p) \geq \check{C}_{A|B}(p) \forall p$, with equality at $p = p_i$, $1 \leq i \leq n$. Indeed, $\hat{C}_{A|B}(p_i)$ is the concentration curve which would obtain if, hypothetically, all those with any given B-value, b_o , also had the same A-value; and $\hat{C}_{A|B}(p) - \check{C}_{A|B}(p)$ measures the *disparity in A given B*.

The various curves of the form $C_{A|B}(p)$ referred to in the text may be taken to be of either form, $\hat{C}_{A|B}(p)$ or $\check{C}_{A|B}(p)$. The two coincide at the mass points $p = p_i$ through which, for the numerical integration procedure used in the empirics, smooth curves $C_{A|B}(p)$ are fitted, and so it hardly matters: if the number of mass points $p = p_i$ in the unit interval $0 \leq p \leq 1$ representing the income scale is large, and the disparities observed at mass points are small, then the extended concentration coefficient $G_{A|B}(v)$ computed from a fitted smooth curve $C_{A|B}(p)$, lying between $\hat{C}_{A|B}(p)$ and $\check{C}_{A|B}(p)$, will approximate closely to either of the formally correct ones $\hat{G}_{A|B}(v)$ and $\check{G}_{A|B}(v)$.

In each of the TCMP data sets used for the empirics, there is in excess of 47,000 income units. In the 1988 data set, for example, there are 42,756 distinct values of before tax pre-audit income and 43,184 distinct values of after tax post-audit income. Hence the errors introduced by the smoothing and numerical integration procedures used to produce our estimates of vertical and horizontal (non-) compliance effects are for the whole population of U.S. taxpayers is unlikely to predominate over sampling errors.

2. The determinants of the redistributive effects of taxes RE_0 and RE_1

As argued by Bishop, Chow, Formby and Ho (1994), multi-valued measures of **residual** and **liability progression** for the income tax may be defined as follows:

$$\begin{aligned} \text{Pre-audit} \quad RP_0 &= C_{X-T|X} - L_X \\ LP_0 &= L_X - C_{T|X} \\ \text{Post-audit} \quad RP_1 &= C_{X+U-T-S|X+U} - L_{X+U} \\ LP_1 &= L_{X+U} - C_{T+S|X+U}, \end{aligned}$$

where $C_{T|X}$ and $C_{T+S|X+U}$ measure proportions of taxes falling on percentiles of the relevant distribution of income before tax. As with all others, these two tax concentration curves may be of the $\tilde{C}_{A|B}(p)$ or $\tilde{C}_{A|B}(p)$ type.

The income tax system induces reranking among income units, in the transition from before to after tax. This stems from differences in tax treatment (*e.g.* of single persons, married couples, itemizers, non-itemizers), and is distinct from the reranking effects of evasion and non-compliance. There is no inequity connotation since this arises from society's agreed upon method of determining taxable capacity, which is reflected in the provisions of the tax code and IRS regulations. The following measures capture **tax system reranking**:

$$\begin{aligned} \text{Pre-audit} \quad RR_0 &= C_{X-T|X} - L_{X-T} \\ \text{Post-audit} \quad RR_1 &= C_{X+U-T-S|X+U} - L_{X+U-S} \end{aligned}$$

Following Kakwani (1984), the relationship between progression, reranking and **redistributive effect** can now be specified, pre- and post-TCMP audit:

$$\begin{aligned} \text{Pre-audit} \quad RE_0 &= RP_0 - RR_0 = [t/(1-t)] LP_0 - RR_0 \\ \text{Post-audit} \quad RE_1 &= RP_1 - RR_1 = [(t+s)/(1+u-t-s)] LP_1 - RR_1, \end{aligned}$$

where the variables t , u and s denote the aggregate levels of tax paid, of unreported income and of additional tax liability revealed by the audit, all relative to total declared income: $t = \Sigma T / \Sigma X$, $u = \Sigma S / \Sigma X$. The redistributive effect of the tax system is thus governed by tax level, tax progressivity and reranking.⁹

The tax level determinant, $[t/(1-t)]$ or $[(t+s)/(1+u-t-s)]$, measures total taxes relative to total post-tax income, pre- or post-audit, whilst tax progressivity measures disproportionality in tax payments independently of tax level. If the measures in these multi-valued decompositions are aggregated as in the text, the following determinants of $\rho_0(v)$ and $\rho_1(v)$ are obtained:

⁹ In Aronson, Johnson and Lambert (1994), a careful distinction between concentration curves of the $\tilde{C}_{A|B}(p)$ -type used in the analysis of redistributive effect is drawn. If the former type is used to define the RP_i and LP_i measures, $i = 0,1$ (*i.e.* taking discrepancies in people's taxes into account in measuring the reranking effect), then a 3-term decomposition of RE_i is achieved, $i = 0,1$, in which an extra term, involving a concentration curve difference of the type $\tilde{C}_{A|B}(p)$ or $\tilde{C}_{A|B}(p)$ -type, appears. In the case of a socially homogeneous population, this extra term captures precisely the pure (classical) horizontal inequity characteristic of the tax system, *i.e.* the extent of the "unequal tax treatment of pre-tax." In the case of a heterogeneous population, and, as here, analysis of non-compliance and evasion conducted in money income terms, there is nothing to be gained by pursuing this form of decomposition.

Pre-audit $\rho_0 = [t/(1-t)] \Pi_0^k(v) - \delta_0(v)$, and

Post-audit $\rho_1 = [(t+s)/(1+u-t-s)] \Pi_1^k(v) - \delta_1(v)$.

The tax progressivity index $\Pi_1^k(v)$ extends that of Kakwani (1977).¹⁰ The index $\delta_i(v)$ of reranking is that of Duclos (1994).

¹⁰ See Lambert (1993), page 175.

Table 1
Generalized Gini Coefficients and Concentration Indices
Pre and Post-Audit, 1979,1982, 1985 and 1988

Year/ Parameter	GX	GX-T	GX+U	GX+U-T-S	GX-U X	GX+U X	GX+U-T-S X+U	GX+U-T-S X-T
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1979								
v=2.0	.4671	.4363	.4658	.4324	.4352	.4590	.4320	.4273
v=1.5	.1760	.1495	.1763	.1443	.1457	.1711	.1440	.1407
v=3.0	.6254	.5933	.6227	.5892	.5931	.6137	.5888	.5816
v=5.0	.7595	.7315	.7563	.7276	.7315	.7457	.7275	.7166
1982								
v=2.0	.4753	.4471	.4734	.4428	.4469	.4667	.4431	.4385
v=1.5	.1827	.1559	.1819	.1532	.1557	.1772	.1531	.1501
v=3.0	.6345	.6060	.6316	.6000	.6060	.6226	.6010	.5941
v=5.0	.7685	.7439	.7653	.7364	.7439	.7536	.7382	.7290
1985								
v=2.0	.4871	.4608	.4853	.4567	.4602	.4783	.4567	.4522
v=1.5	.1937	.1680	.1930	.1656	.1674	.1877	.1656	.1622
v=3.0	.6455	.6199	.6427	.6140	.6191	.6335	.6140	.6080
v=5.0	.7758	.7543	.7726	.7468	.7536	.7609	.7468	.7394
1988								
v=2.0	.5276	.5024	.5252	.4999	.5022	.5183	.4999	.4954
v=1.5	.2418	.2169	.2404	.2153	.2164	.2352	.2153	.2115
v=3.0	.6776	.6532	.6745	.6500	.6531	.6647	.6500	.6422
v=5.0	.7893	.7775	.7950	.7743	.7775	.7816	.7743	.7644

Notes: Estimates are made using Internal Revenue Service's Taxpayer Compliance Measurement Program micro data. The measures are expressed in terms of changes in Gini and extended Gini based indices multiplied by a scalar. The variables ρ , v , and η refer to redistributive, vertical and horizontal equity effects of evasion and non-compliance, while v is Yitzhaki's (1983) distribution parameter. When $v=2.0$ the measures show the effects of evasion and non-compliance on ordinary Gini indices of vertical, horizontal and redistributive effects.

Table 2

**Extended Gini Index Measures of the Vertical, Horizontal and Redistributive Effects
of Tax Compliance and TCMP Audits -- 1979, 1982, 1985 and 1988**

Year/ Parameter	Net Redistributive Effect		Vertical Impact of Non-compliance		Horizontal Inequity Effects of Compliance		Decompositions		
	Post Audit $\rho_1(v)$ (1)	Pre Audit $\rho_0(v)$ (2)	Pre Tax $v_b(v)$ (4)	Post Tax $v_a(v)$ (4)	Pre Tax $\eta_b(v)$ (5)	Post tax $\eta_a(v)$ (5)	$\Delta\rho(v)$ (7)	$\Delta v(v)$ (8)	$\Delta\eta(v)$ (9)
a. $v = 2.0$									
1979	334	308	81	90	68	51	26	9	-17
1982	306	282	86	86	67	43	24	0	-24
1985	286	263	88	86	70	45	23	-2	-25
1988	253	252	93	70	69	45	1	-23	-24
b. $v = 1.5$									
1979	320	265	49	88	52	36	55	39	-16
1982	287	268	55	8	47	31	19	3	-16
1985	274	257	60	58	53	34	17	-2	-19
1988	251	249	66	54	52	38	2	-12	-14
c. $v = 3.0$									
1979	335	321	117	117	90	76	14	0	-14
1982	316	285	119	119	90	59	31	0	-31
1985	287	256	120	119	92	60	31	-1	-32
1988	245	244	129	110	98	78	1	-19	-20
d. $v = 5.0$									
1979	287	280	138	149	106	110	7	11	4
1982	289	246	149	149	117	74	43	0	-43
1985	258	215	149	149	118	74	43	0	-44
1988	207	118	7	131	134	99	89	54	-35

Notes: Estimates are made using the Internal Revenue Service's Taxpayer Compliance Measurement Program micro data. The measures are expressed in terms of changes in Gini and extended Gini based indices multiplied by a scalar. The variables ρ , v , and η refer to redistributive, vertical and horizontal equity effects of evasion and non-compliance, while v is Yitzhaki's (1983) distribution parameter. When $v = 2.0$ (part a) the measures show the effects of evasion and non-compliance on ordinary Gini indices of vertical, horizontal and redistributive effects.