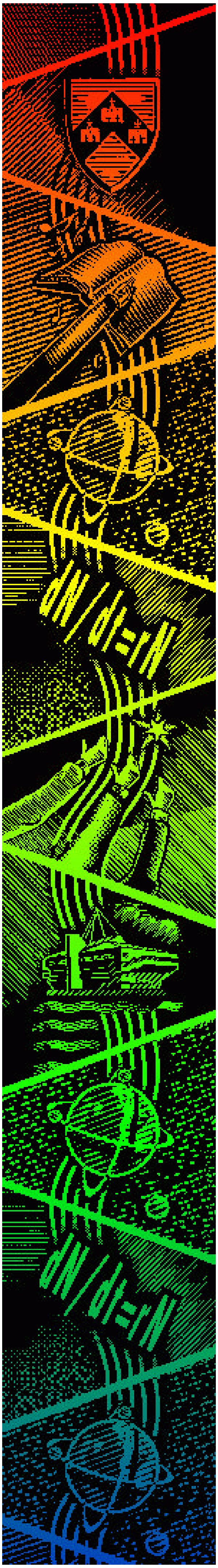

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Constitutional Design and Investment in Cooperatives and Investor-owned Enterprises

by

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Constitutional Design and Investment in Cooperatives and Investor-Owned Enterprises

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Abstract

This paper studies the role of the corporate governance systems in cooperatives and in investor-owned enterprises. The difference between the two forms is reduced to the type of majority needed to take decisions: one head one vote for cooperatives and proportional to capital invested in investor-owned firms. We show that the institutional form chosen matters for the initial investment decision of the agents: in particular we find that members of a cooperative invest less than they would in an investor-owned enterprise. This finding tallies with empirical evidence suggesting that cooperatives are undercapitalised.

1 Introduction

A non negligible portion of the economic activity in industrialised countries is organised in cooperative firms.

In view of this, it is surprising that the theoretical literature on the organisational forms of productive activity has devoted relatively little attention to their costs and benefits vis-a-vis other organisational types[1].

This paper performs an abstract comparison between a cooperative enterprise and a investor-owned enterprise[2]. We focus on the fundamental

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[1]For example, the excellent review by Holmström and Tirole (1989) does not analyse cooperatives. A notable exception is given by Hansmann' book (1996).

[2]We borrow the term 'investor-owned' from Hansmann (1996) to indicate an organisational form where votes are apportioned according to the capital contributed to the firm (one-share-one-vote rule).

constitutional difference between these two organisational forms, viz. the mechanism to arrive at decisions. This is majority voting in both cases, but in a cooperative each member has one vote, irrespective of her capital contribution, whereas in a investor-owned enterprise a member's voting power is proportional to her contribution to the capital of the organisation.

We therefore deliberately rule out any other difference between these two organisational forms; in particular, the technological possibilities and the individual utility functions are unaffected, as are the prices which the partnership faces[3].

We study the problem faced by a group of individuals who can enter a partnership to engage in a productive activity, and, beforehand, must choose the institutional design which will govern their future interaction.

We take an incomplete contract approach: if the set of contracts that individuals can write (and that the courts can costlessly enforce) were unconstrained then the choice of organisation would be irrelevant: the individuals involved could simply specify what each has to do in every possible circumstance and there would be no need to devise rules to take decisions after the initial contract has been agreed upon. Since we assume that complete contracts are not feasible, the members of the organisation can only decide on a way in which future decisions are to be taken: that is, they can design the constitution.

The main result of our paper is that the constitution matter: individuals are affected by the way in which the partners choose to take future decisions, even in the highly stylised set-up we consider. We can in fact be more specific, and use the model to illustrate the important under-investment drawback cooperatives suffer in practice (see for example, Bonin et al (1993, esp pp 1307-12). Because in our model an individual's financial contribution cannot be contracted upon, it must be individually rational for an individual to contribute financial capital to the partnership. We show that, in almost all circumstances, the equilibrium value of the combined individual contributions is unambiguously higher in a investor-owned enterprise than in a cooperative, and conversely that in no circumstances this value is unambiguously higher in a cooperative than in a investor-owned enterprise.

The intuition for this finding is relatively subtle. In our model, the part-

[3]Thus it is not the case that workers behave differently in a cooperative, for example by accepting lower wages or exerting more effort, nor that the agents choosing the organisation form derive utility directly from the type of organisation they work in ("advocates of workers ownership often suggest that participation in control of the firm through democratic processes is of value in itself, ... and a similar argument is sometimes made on behalf of consumer cooperatives" Hansmann 1996, p. 43), nor that inputs can be bought more cheaply in one particular organisation.

nership produces some local (to the partners) public good and a standard free rider problem emerges: in a cooperative, partners who (ex-post) derive low (private) benefit from the production of the public good are deterred from (ex-ante) contributing to the production of this good. Why does the investor-owned enterprise ameliorate this problem? Because it can happen that a minority of partners can take decisions which are opposed by the remaining majority. This of course can happen so long as they have a majority of the shares. This illustrates that there is a incentive for individuals to contribute their financial capital in order to, as it were, "buy the power to make decisions". This incentive counteract the free riding problem, which affects the investor-owned enterprise as well as the cooperative.

Given this argument, it should therefore come as no surprise that whenever the partners contribute collectively more to a investor-owned enterprise than they would to a cooperative, then their shareholding in the partnership capital is asymmetrically distributed: with symmetric capital shares, there is no circumstance in which voting can the differ in the two organisational forms.

Existing theoretical arguments for underinvestmet in cooperatives rely on the lesser 'tradeability' of their ownership rights[4]. Often the property rights in cooperatives are associated with membership status and cannot be freely traded in the market. Therefore, in assessing whether to contribute to the venture, existing members will not maximise the actual value of the future stream of profits. Instead they will consider whether their investment could result profitable over the period they remain members. As a consequence, the internal rate of return of investment in cooperatives must be higher than in investor-owned forms in order to be member-financed[5], resulting in a lower capital/labour ratio.

Kremer (1997) presents a paper in which partners contribute a fixed amount to the capital of a cooperative. Once they are members of the cooperative they receive an ex-ante noncontractible shock to their ability, and subsequently they decide on a wage schedule as a function of output. If the ability of the median voter is lower than the average ability, the wage schedule will tend to redistribute earnings from the high ability to the low ability workers. High ability workers cannot walk out from the cooperative, because they would lose their initial contribution. This model explains the relative wage compression that exists in cooperatives compared compared with investor-owned enterprises, and highlights a factor that may limit the

[4]For a survey of the literuature on workers' owned cooperatives, see Bonin, Jones and Putterman (1993).

[5]At the same time, extensive external financing of investment is hindered by the fact that existing members could divert the resulting resourses to their own benefit.

presence of cooperatives in the economy. It does not address the issue of investment levels in different organisational forms, because the initial contribution is taken as given. Furthermore, a necessary condition for inefficiency in cooperatives is the noncoincidence of the preferences of the median voter with the average preferences[6]. In our paper we find that inefficiency may arise even if mean and median preferences coincide[7].

Hart and Moore (1998) also study the relative efficiency of organising production in a cooperative and in a investor owned enterprise. There are two major differences with our paper: firstly, they assume that if the enterprise is investor owned then the investor is an outsider, not a member, and in this case the productive agents, instead of being members of a cooperative are employees of the outside investor. Secondly, as Kremer, Hart and Moore also take the extent of the initial contribution as given and do not, therefore, address the issue of underinvestement. Contract incompleteness causes inefficiency both in our paper (in the form of underinvestement) and in Hart and Moore's (in the form of inefficient quality).

The paper is organised as follows. Section 2 presents the main assumption of the model, while in section 3 we state the results in the first best (complete contracts), and for the cooperative and investor owned organisational types. Section 3.4 presents a comparison between the two organisational forms and section 4 concludes.

2 The model

2.1 Technology.

Our model studies the choices of three agents who can engage in an economic activity. These agents are (ex-ante) identical, own a limited amount of financial resources. They can engage in production according to a technology which has two crucial features: firstly, the labour input of each of three agents is necessary: their personal characteristics (skill, talent, etc.) are indispensable for any production to take place at all. Secondly, the technology can be used to produce both a private, marketable good, and a local public good. Local public good is taken here to mean that it is non-rival and non-excludable only among the three partners, and it is furthermore not marketable (i.e. it has no price for any other agent).

[6] Hart and Moore (1996) get to the same conclusion.

[7] This is because the members of the cooperative (and of the investor-owned enterprise) choose *ex-ante* an inefficient capitalisation level, irrespective of the actual realisation of their preferences.

There are a number of examples for this situation. One can think of medical or legal partnerships, where the marketable good is given by the services sold to client, while the local private good could be the social commitment of the partnership, or the quality of the working environment. In research partnership, the marketable good is given by the research projects carried out for third parties, and the public good by the contribution to academic research. In a farming cooperative, the marketable good is simply the production of the cooperative, and the public good the environmental protection of the cultivated area. In a sport club that owns its own premises, the marketable good is the rent of the facilities to the public, and the public good the quality of the service enjoyed by the members of the club [8].

The technology obeys a production function *F(input, public good, marketable good)* which we simplify as follows:

$$T - (E + X) = 0 \tag{1}$$

where T is the amount of resources available to the enterprise, stemming from the voluntary contributions of the agents, and E and X are respectively the amount of public good and marketable good produced. We assume a linear technology, and without further loss of generality, we normalise units so that the marginal rate of substitution between input and output and between the two inputs are both one.

2.2 Preferences

The preferences of the agents are described by a utility function whose arguments are the local public good produced by their partnership and a hicksian commodity. This utility function is affected by an idiosyncratic shock, ω. Thus we have:

$$U_i = U(c_i, E, \omega_i), \qquad i = 1, 2, 3$$

where c_i is agent i's (private) consumption of a hicksian commodity, E the public good and ω_i the realisation of the stochastic taste variable for agent i.

We take an extremely simple characterisation of the function U; given that our aim is to illustrate that investor-owned enterprise and cooperatives differ in the amount of investment they choose because of their different constitutional design, using the simplest possible utility function brings out our conclusions more starkly. Thus we postulate linear preferences:

[8] Hart and Moore (1996) discuss at length this case.

$$U = \begin{cases} c + \omega^H E & E < e_{\min} \\ c + \omega^H e_{\min} + \omega^j (E - e_{\min}) & E \geq e_{\min} > 0 \end{cases} \tag{2}$$

where the shock ω therefore measures the strength of an agent's desire for public good in excess of the minimum amount $e_{\min}$. We assume that the shock ω can take one of only two values, ω^H, high preference for the public good, and ω^L, low preference. ω is subject to the probability distribution (the same for the three agents):

$$\begin{cases} \Pr(\omega^j = \omega^H) = p \\ \Pr(\omega^j = \omega^L) = 1 - p \end{cases} \tag{3}$$

We assume furthermore that $\omega^H > \rho > \omega^L > 0$. Here ρ is the ratio between the output and the input price. In view of the linearity of both technology and preferences, there is no loss in generality in normalising this ratio to one.

The relative preference between marketable and public good is thus influenced by uncertainty over the realisation of ω: while all agents want in any case to have at least $e_{\min}$ of the public good, each wants to have more only with probability p. Each agent wants to consume only the public good if the realised value of his taste parameter is high, and just a quantity $e_{\min}$ if the realisation is low. We denote by ω_i^j, the realised value of ω for agent i, with $j = H, L$, and $i = 1, 2, 3$. We note that (3) implies that the realisations of the state of nature for the three agents are independent.

Finally, we assume that there is an exogenous ceiling M in the resources each agent can invest. This may represent the maximum that the agent is able to invest in the partnership, given the need to maintain a certain consumption level for himself and his family.

2.3 The constitutional choice

Setting up a form of productive collaboration of the type we envisage here, usually involves two types of decisions, which we can label the constitutional design and the day-to-day running. The former is the design of a constitution in the standard legal sense: the determination of the way in which day-to-day decisions are to be taken. In many cases, for private productive activities, individual agents are restricted in their choice of constitutional design to selecting from a given set of "pre-designed" constitutional designs[9].

[9]This is to protect third parties: they need to know the basic rules governing the organisation they are dealing with (decision making procedures, liability), and that these

In our model we restrict the agents' choice to selecting one of the following alternatives, which differ with regard to the nature of the majority required to reach decisions.

- Cooperative. In this case the voting principle is one member one vote, irrespective of the amount each has contributed.
- investor-owned enterprises. Here each member has a share of the votes equal to his share of the total private good contribution of the three agents.

Given the choice between these two structures, it should become evident that three is the minimum number of agents which is necessary to have in order to obtain a meaningful choice: with two agents, the voting process becomes trivially simple.

2.4 Timing

The timing of agents' interaction is depicted in figure 1.

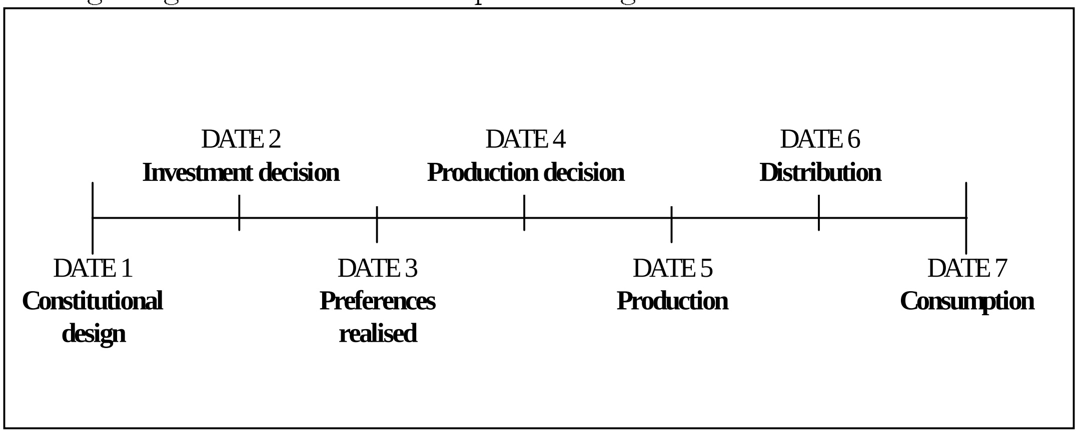

Figure 1: the timing of the model.

The constitutional design occurs at date 1. Once the constitution is in place, economic decision can be taken. At date 2, each agent, simultaneously and independently of the other two, chooses how much of his endowment of the private good to put into the partnership. We assume that any constitutional rule can require each agent to contribute at least $\frac{e_{\min}}{3}$ to the partnership[10], and we denote this minimum quantity with m, so that $m \equiv \frac{e_{\min}}{3}$.

rules will be stable over time. This of course is unlike the constitutional design for political decision making, which is free.

[10]This guarantees the resources necessary to produce the minimum amount of the public good that the agents desire, and allows us to skip the consideration of free rider problems that would arise in the case of no minimum contribution.

Formally, agent i chooses $t_i \in [m, M]$, $i = 1, 2, 3$. The total resources available to the enterprise are thus $T = \sum_{i=1}^{3} t_i$, and the amount of the endowment retained by agent i is $M - t_i \geqslant 0$.

Next, at date 3, the values of ω_i^j are realised, and observed by all three agents. That is, agents learn their own and their partners' preferences[11]. In a stylised example, each of the three partners is married, but neither has any children at the time of the constitutional design (date 1). If at the time the production decision is made, a partner has children (his realisation of ω^j is ω^L, which occurs with probability $1 - p$), he will want (need) to have as high a level of the hicksian composite commodity as possible: therefore he will want to produce only the marketable good. If instead he does not have children (his realisation of ω^j is ω^H, which occurs with probability p), he will want to devote the partnership resources to some "social" activity, such as involvement in the local community.

Next, at date 4, the amount of resources to be devoted to the production of the public good is selected, according to a vote which must of course be taken in accordance to the constitutional design.

At date 5, production of the marketable good and the public good take place,according to the decision taken in period 4 and the technological constraint (1), which can be written as:

$$E = \sum_{i=1}^{3} t_i - X$$

At date 6 the amount of money raised from the sale of the marketable good produced is distributed to the participants in proportion to their initial monetary contribution. Agent i thus receives:

$$x_i = \frac{t_i}{T} X = t_i \left(1 - \frac{E}{T}\right) \tag{4}$$

Finally, at date 7 each agent can consume the hicksian commodity (recall that we have set the ratio between the output and input price equal to one). The *ex-post* utility for agent i is then:

$$U_i = (M_i - t_i) + t_i \left(1 - \frac{E}{T}\right) + \left[3\omega_i^H m + \omega_j^i (E - 3m)\right] \tag{5}$$

[11]Because there is no subsequent opportunity for strategic signalling, the assumption of symmetry of information entails little loss in generality, and maitains the model at a tractable level of complexity.

The first two terms in (5) represent the utility deriving from the consumption of the hicksian composite commodity (where the first term is the initial wealth minus initial investment and the second is the share to agent i of the revenues for the production of the marketable good). The term in the square brackets represents agent i's utility deriving from the consumption of the public good.

2.5 Contracting

It is natural to assume that, at the time of the constitutional design, the partners cannot write a contract which conditions their (date 2) investment decision on the realised values of the ω's. A reason for this is that to write such a contract may prove too costly to the partners[12] because it is not possible for a court or other contract enforcing agency to observe the realisation of the taste parameters for the three agents[13].

The parties can of course agree to set a minimum investment required in order to take part in the partnership. Note, however, that, in view of the assumption that each partner's individual participation is necessary for any production to take place, it may not be incentive compatible to impose a minimum above what each agent would be willing to invest irrespective of his realisation of ω^j. This because each agent can simply deny having the money. In other words, the individual contribution must be incentive compatible, in the sense of constituting a Nash Equilibrium: since nothing happens between date 1 and date 2, no rational agent would, at date 1, sign an agreement committing him to contribute more than he would choose to contribute at date 2.

It follows that there is relatively little contracting to do. The only thing that a court can verify is whether any decision taken by the members has respected the (internal) constitutional design. In practice whether the necessary majority existed in favour of the decision taken.

[12]See for example Anderlini e Felli (1997).

[13]Indeed this asymmetry of information between the courts on the one side and the parties (who share the same information set) on the other side forms the basis of the recent literature on invomplete contracts.This approach has been critisised by Maskin and Tirole (1998), who argue that if the partners are unboundedly rational and can committ to no renegotiation of the contract, the inability of a court to enforce the contract becomes irrelevant.

3 Results

3.1 The first best

As a benchmark, we consider first the case in which the constraints on contracts imposed in section 2.5 do not hold: at date 1, the partners can write contracts:

- they commit to any level of date 1 individual contribution;
- they can make the date 4 production decision dependent on the values of their preference parameters realised at date 3.

We refer to this as the complete contracting situation, or first best[14].

With complete contracts constitutional design is irrelevant: there is no need to specify the rules according to which decisions will be made. All decisions can be made at date 0. The main results are stated in the following proposition:

Proposition 1 *Suppose it is possible to write a contract specifying each partner's initial contribution and the amount of the public good to be produced in any state of the world. Then:*

$$t_i = M \qquad i = 1, 2, 3$$

$$E = \begin{cases} 3m & if \quad \omega_i^j = \omega_i^L, \quad i = 1, 2, 3 \quad and \quad \omega^L < \frac{1}{3} \\ 3M & \qquad\qquad otherwise \end{cases}$$

That is, at the first best the investment of each agent is equal to the maximum possible (i.e. $t_i = M$, $i = 1, 2, 3$) giving total resources available for production equal to $T = 3k$. The production of the public good is always $E = 3k$, except for the case when all agents have low marginal utility of the public good and $\omega^L < \frac{1}{3}$, in which case $E = 3m$.

Proof. Since the partners can write a contract contingent on the realisation of the random variable ω, they will command production of the public good whenever the realisation of ω is such that its *ex-ante* expected utility is higher than for the production of the marketable good, and viceversa. As a consequence it is always optimal *ex-ante* to contribute the maximum capital of the organisation: $t_i = M$, $\forall i$. This proves the first part of proposition 1.

[14]This is the set of allocations which could be chosen by a benevolent dictator whose objective is the maximisation of the agents' total utility.

Next consider the case when $\omega^L > \frac{1}{3}$. Any partner's utility (given by equation 5) results is increasing in E both if ω^L and ω^H are the realised value of ω. Therefore the partners always want the maximum possible production of the public good, i.e. $E = 3M$.

Now consider the case when $\omega^L < \frac{1}{3}$. The contract will provide for the production of the public good if its *ex-ante* marginal utility is higher than the marginal utility of the marketable good. The redistribution of the marketable good among the agents can also be contracted *ex-ante*. Let us denote with $\{x_1, x_2, x_3\}$ the quantities of the marketable good to the partners, which must respect the technological constraint (1): $\sum_{i=1}^{3} x_i = 3M - E$.

If the realisation of ω is such that all three partners have the low marginal utility of the public good ω^L, they clearly prefer to produce the marketable good, since $1 > 3\omega^L$. Therefore production of the public good will be $E = 3m$ and production of the private good $\{x_1, x_2, x_3\} = \{M - m, M - m, M - m\}$.

If the realisation of ω is such that one partner has the high marginal utility of the public good ω^H and the other two the low ω^L, *ex-ante* utility for partner i is given by:

$$U_i = \frac{1}{3}\left(\omega^H E + (M - x_i)\right) + \frac{2}{3}\left(\omega^L E + (M - x_i)\right)$$

Summing over the partners and using the constraint, we obtain:

$$\sum\nolimits_{i=1}^{3} U_i = \omega^H E - \frac{1}{3}E + 2\omega^L E - \frac{2}{3}E = E(\omega^H + \omega^L - 1)$$

Therefore the expected utility of the three partners is increasing in E, and the contract will be $E = 3M$ and $\{x_1, x_2, x_3\} = \{0, 0, 0\}$.

The same results apply *a fortiori* if two or three people have a high marginal utility for the public good, completing the proof of proposition 1 ■

The fact that contracts can be made dependent on the realisation of the random variable ω makes its probability distribution not relevant in the agents' decisions. This is not the case when complete contracts are ruled out, as we shall see in the next section.

3.2 Investment and Voting in coops

In practice, the type of contracts which are necessary for the achievement of the first best, requiring as they do the observability by an enforcer of the realised value of the preference parameter, are not feasible. The main point of the paper is to show that when contracts are incomplete, the allocation of power within the enterprise and the rules for making decisions matter. In this section we consider the one-head one-vote rule (simple majority voting) that characterises cooperatives. The next section will deal with investor-owned organisations, where the partner's voting power is proportional to their monetary investment (one-share-one-vote rule).

Consider date 4. At this time all decisions regarding contributions are made, and the partners vote on the amount of public good the partnership should undertake. After the vote is taken, everything is straightforward; therefore, in order to decide how to vote, each agent simply calculates the utility he receives for any feasible level of production of the public good (and therefore of the marketable good). Given contributions $\{t_1, t_2, t_3\}$, the set of feasible levels of public good production is $[0, t_1+t_2+t_3]$. In practice we can restrict the choice to the interval $[3m, t_1+t_2+t_3]$, since all agents, regardless of the realisation of ω^j strictly prefer $3m$ to any $E < 3m$. Agent i's utility as a function of the level of public good production (for $E \geq 3m$) is given by equation (5).

Taken as a function of E only, the right hand side of (5) always reaches a maximum at a corner: either $\omega^{\ i} = \omega^{\ L}$, in which case the agent strictly prefers $E = 3m$, or $\omega^{\ i} = \omega^{\ H}$, in which case he strictly prefers $E = t_1 + t_2 + t_3$. It is here that the simple structure of our model is seen to be very useful, as there are only two possible patterns of voting: for any realisation of $(\omega_1^j, \omega_2^j, \omega_3^j)$ either a strict majority or a strict minority of partners prefers that all resources be invested in the public good.

In table 1 we summarise the production quantities and payoffs to the agents for all realisations of the random variable, together with the associated probability. The table shows the possible realisations of the random variable ω^j for the three partners (column 1) and the associated probabilities (column 2). Column 3 and 4 show, for each case, the amount produced of the public and the private good respectively and the last two columns report the payoff associated with such production choice for an agent who has low (column 5) or high (column 6) preference for the public good. As an example, consider the case when realised preferences are $\{\omega^L, \omega^L, \omega^H\}$ (row 4). In this case, which happens with probability $(1-p)^2 p$, the majority of the partners derives a low marginal utility from the public good, and thus the minimum quantity $3m$ is produced. A quantity $(t_1 + t_2 + t_3 - 3m)$ of the marketable good is produced and partner i is allocated a share $\frac{t_i}{t_1+t_2+t_3}$ of her initial contribution. The payoff for each of the three partners is thus $(M_i - t_i) + t_i \left(1 - \frac{3m}{t_1+t_2+t_3}\right) + 3\omega^H m$.

Insert Table 1 about here

A more compact presentation of the results in table 1 can be obtained by noting that from agent i's point of view there are four possible cases:

he may have a high preference for the public good and be in the majority, have a high preference for the public good and be in the minority, have a low preference for the public good and be in the majority, and finally, he may have a low preference for the public good and be in the minority. The associated probabilities are shown in table 2.

	Realised preference for agent i	
	ω_i^L	ω_i^H
ω^L is majority	$(1-p)^3+2p(1-p)^2$	$p(1-p)^2$
ω^H is majority	$p^2(1-p)$	$p^3+2p^2(1-p)$

Table 2: probabilities of the possible outcomes for agent i.

Bearing this in mind, we can now go back to date 2 and compute the utility for agent i as a function of the contribution level of all three partners, given the correct anticipation of future voting decisions. Let $\widehat{EU}_i^C$ be the expected utility for agent i after the cooperative form is chosen (date 1) and before the contribution decision is made (date 2):

$$\begin{aligned}\widehat{EU}_i^C &= \left[(1-p)^3+2(1-p)^2 p\right]\left[(M_i-t_i)+t_i\left(1-\frac{3m}{t_1+t_2+t_3}\right)+3\omega^H m\right] \\ &\quad +(1-p)p^2\left[(M_i-t_i)+3\omega^H m+(t_1+t_2+t_3-3m)\,\omega^L\right] \\ &\quad +(1-p)^2 p\left[(M_i-t_i)+t_i\left(1-\frac{3m}{t_1+t_2+t_3}\right)+3\omega^H m\right] \\ &\quad +\left(2(1-p)p^2+p^3\right)\left[(M_i-t_i)+(t_1+t_2+t_3)\,\omega^H\right]\end{aligned} \tag{6}$$

Simplifying we obtain

$$\begin{aligned}\widehat{EU}_i^C &= \left(1-3p^2+2p^3\right)\left(1-\frac{3m}{t_1+t_2+t_3}\right)t_i \\ &\quad +(t_1+t_2+t_3-3m)\,p^2\left[(1-p)\,\omega^L+(2-p)\,\omega^H\right]+(M-t_i)+3\omega^H m\end{aligned} \tag{7}$$

The assumption of linear utility and production functions implies that each individual invests either the minimum or the maximum amount possible, thus considerably simplifying the analysis. To see this, we simply take the second derivative of the utility function w.r.t. the decision variable t_i (for agent i):

$$\frac{\partial^2 \widehat{EU}_i^C}{\partial t_i^2}=6m\left((1-p)^3+3(1-p)^2 p\right)\frac{(t_1+t_2+t_3-t_i)}{(t_1+t_2+t_3)^3}>0$$

Therefore any stationary point of the individual maximisation problem is a minimum, and the solution can be only at the extreme points of the interval, with $t_i \in \{m, M\}$. This implies that the date 2 decision can be described (given the assumption that the players anticipated correctly future actions) by a 2 action, 3 player normal form symmetric game (see figure 2). This is shown in table 3, where player 1 chooses the row, player 2 the column and player 3 the matrix. Let us denote with $EU^C_{h(kl)}$ the expected utility of an agent who has invested h in the cooperative when the other two have invested k and l, with $h, k, l \in \{m, M\}$. The superscript C denotes the cooperative organisational form. For example, $EU^C_{M(mM)}$ denotes the expected utility of an agent who has invested the maximum amount M when the other two have invested m and M.

Player 3: m

Player 2 \ Player 1	m	M
m	$EU^C_{m(mm)}$, $EU^C_{m(mm)}$, $EU^C_{m(mm)}$	$EU^C_{m(mM)}$, $EU^C_{M(mm)}$, $EU^C_{m(mM)}$
M	$EU^C_{M(mm)}$, $EU^C_{m(mM)}$, $EU^C_{m(mM)}$	$EU^C_{M(mM)}$, $EU^C_{M(mM)}$, $EU^C_{m(MM)}$

Player 3: M

Player 2 \ Player 1	m	M
m	$EU^C_{m(mM)}$, $EU^C_{m(mM)}$, $EU^C_{M(mm)}$	$EU^C_{m(MM)}$, $EU^C_{M(mM)}$, $EU^C_{M(mM)}$
M	$EU^C_{M(mM)}$, $EU^C_{m(MM)}$, $EU^C_{M(mM)}$	$EU^C_{M(MM)}$, $EU^C_{M(MM)}$, $EU^C_{M(MM)}$

Figure 2: the normal form game in the cooperative case.

We begin by calculating the values of the different utilities. Substituting in 7 the appropriate values of $t_1 \in \{m, M\}$ we have:

$$EU^C_{m(mm)} = (M - m) + 3\omega^H m \tag{8}$$

$$\begin{aligned} EU^C_{m(mM)} = {} & m\left(1 - \frac{3m}{2m + M}\right)\left(1 - 3p^2 + 2p^3\right) \\ & + (M - m)\, p^2 \left[(1 - p)\,\omega^L + (2 - p)\,\omega^H\right] + (M - m) + 3\omega^H m \end{aligned} \tag{9}$$

$$EU^C_{M(mm)} = M\left(1-\frac{3m}{2m+M}\right)\left(1-3p^2+2p^3\right) + p^2\left[(1-p)\,\omega^L+(2-p)\,\omega^H\right](M-m)+3\omega^H m \tag{10}$$

$$EU^C_{m(MM)} = m\left(1-\frac{3m}{m+2M}\right)\left(1-3p^2+2p^3\right) + 2\,(M-m)\,p^2\left[(1-p)\,\omega^L+(2-p)\,\omega^H\right]+(M-m)+3\omega^H m \tag{11}$$

$$EU^C_{M(mM)} = M\left(1-\frac{3m}{m+2M}\right)\left(1-3p^2+2p^3\right) + 2\,(M-m)\,p^2\left[(1-p)\,\omega^L+(2-p)\,\omega^H\right]+3\omega^H m \tag{12}$$

$$EU^C_{M(MM)} = M\left(1-\frac{m}{M}\right)\left(1-3p^2+2p^3\right) + 3\,(M-m)\,p^2\left[(1-p)\,\omega^L+(2-p)\,\omega^H\right]+3\omega^H m \tag{13}$$

A key parameter in our model is the ratio between the minimum amount to be invested by each agent, m, and the resources available to each player, M. We denote it by $\lambda \equiv \frac{m}{M}$. It is clearly $0 \leqslant \lambda \leqslant 1$

The following proposition characterises the pure strategy equilibria existing for the cooperative:

Proposition 2 *If the cooperative institutional form is chosen, there exists p_1 and p_2, whose values depend on the parameters of the model, with $p_1 \leqslant p_2$, such that:*

if $0 < p < p_1$ the unique pure strategy NE is $\{m, m, m\}$;

if $p_1 < p < p_2$ there are two pure strategy NE: $\{m, m, m\}$ and $\{M, M, M\}$;

if $p_2 < p < 1$ the unique pure strategy NE is $\{M, M, M\}$.

Moreover, if $\lambda = 0$, it is $p_1 = p_2$ and there is only one pure strategy equilibrium in the game.

For the proof see appendix A.1.

An important consequence of proposition 2, which follows immediately from a comparison with proposition 1, is that contract incompleteness involves a lower investment in the firm when the cooperative type is chosen with respect to the first best. Recall that in the first best $t_i = M$, $i = 1, 2, 3$ $\forall\ \lambda, \omega^L, \omega^H$. This is not the case in the cooperative organisational form.

As an example, figure 3 shows the possible equilibria in the λ, p space for $\omega^1 = 0.1$ and $\omega^2 = 1.9$.

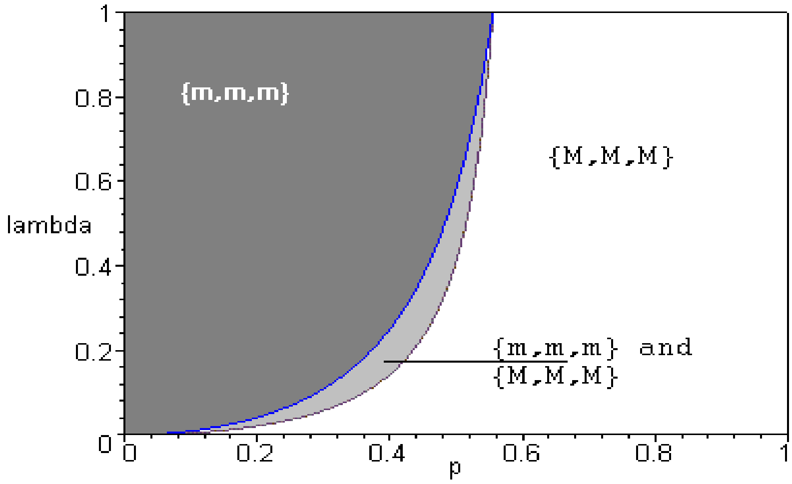

Figure 3: equilibria sets in the cooperative case for $\omega^L = 0.1$ and $\omega^H = 1.9$.

There are two reasons for this result. First of all, agents face the risk of having a low preference for the public good at time 3 and being in the minority (so that all the capital of the firm is used to produce the public good). In equilibrium this induces less investment than in the complete contract case if p is small enough. We call this uncertainty effect.

Furthermore, when the private good is produced (i.e. when the majority has a low preference for the public good), the rule specified in (4) implies redistribution from the players who invest most to those who invest less. To see this, consider the situation in which one player makes an initial investment of $t_i > m$ while the others invest just m. If the private good is not produced at time 4, the share of the private good to player i according to (4) amounts to $x_i = t_i(1 - \frac{3m}{T}) < t_i(1 - m)$ while the other two players receive $x_{-i} = m(1 - \frac{3m}{T}) > 0$. This redistribution effect is likely to discourage higher levels of investment of an agent compared to the others.

In fact, no redistribution effect takes place in equilibrium in the cooperative, since investment levels are always symmetric. However, the presence of this effect influences the nature of the equilibria: if no redistribution is possible, the equilibrium of the game is always unique: $\{m, m, m\}$ is $p < p_1 = p_2$ and $\{M, M, M\}$ if $p > p_1 = p_2$.

We can isolate the uncertainty effect by devising a distributional rule that does not imply redistribution among agents in case the private good is produced. This is obviously the case if there is no minimum amount of the public good to be produced for any realisation of the preferences (i.e. when $3m = 0$ and consequently $\lambda = 0$)[15].

The intuition behind this case is straightforward. Since in the cooperative the production decisions at time 4 are taken democratically, and thus do not depend on the amount of resources invested by the players. If $3m = 0$, the resources invested by each player are influenced only by the probability of having a high or low ex post valuation of the public good. This implies that the optimal investment decision of agent i is independent from the investment of the other agents (i.e. agent i has a dominant strategy that depends only on the exogenous probability values of ω^L and ω^H and on the probability p). The problem faced by the other two agents is identical to that of agent i, and thus their optimal strategy is the same (this ensures symmetry). The cut off point $p_1 = p_2$ gives the probability value (as a function of the other parameters of the problem) for which the expected utility of a high initial investment becomes higher than the expected utility of a lower initial investment. A low investment equilibrium is due in this case purely to the uncertainty regarding the realisation of the random variable ω.

3.3 Investment and Voting in the investor-owned firm

In the investor-owned company the voting power of the members are weighted according to their initial contribution to the capital of the organisation. The median voter theorem still applies, but in this case the "median voter" is the one who possesses the median share of the company's capital, rather than the median physical person. This implies that, unlike in the coop, the investment decision of, say, agent 1 will generally depend on the investment decision of agents 2 and 3 as well: as an individual's contribution increases or decreases, the identity of the median voter may change. If this happens, the balance of power within the organisation shifts, with implications on the nature of the equilibria of the game.

As for the cooperative, after the vote on production is taken, everything is straightforward in the investor-owned firm. Therefore, each agent calculates the utility he receives for any feasible production of the public and marketable good. The set of feasible levels of public good production we are interested

[15] It is possible to imagine redistribution-neutral rules for the distribution of the private good produced. For example, each agent could receive an amount proportional to her initial contribution minus the minimum contribution. While qualitative results would not change from the case where $e = 0$, it seems to us that such a rule does not describe satisfactorily the distributive aggrements in partnerships.

in is $[3m, t_1 + t_2 + t_3]$[16]. Agent i's utility as a function of the level of public good production (for $E \geq 3m$) is given by equation (5).

The sign of $\frac{\partial EU_i}{\partial E}$ in (5) is negative if $\omega^{\ i} = \omega^{\ L}$ (in this case agent i strictly prefers $E = 3m$), and positive if $\omega^{\ i} = \omega^{\ H}$ (in case agent i strictly prefers $E = t_1 + t_2 + t_3$). There are three possible patterns of voting: for any realisation of $(\omega_1^j, \omega_2^j, \omega_3^j)$ either a strict majority or a strict minority or exactly fifty per cent of the voting rights wants that all resources be invested in the public good. In order to get rid of the indeterminacy determined by an equal split of voting, we assume that whenever such a situation arises the decision is based on the *number* of agents favouring a certain allocation of the resources.

In the investor-owned organisation, however, the relevant majority is the majority of the shares in the enterprise, rather than the majority of the voters. Indeed, it is now necessary to consider the case where one person can form the majority, by virtue of its larger holding. Table 5, which is analogous to table 1 (cooperative case), shows production quantities and payoffs to the agents for the realised values of the random variable ω, taking into account the types of majority that can emerge in the investor-owned organisation. Take as an example the case where the realisation of the random variable ω^j is $\{\omega^L, \omega^L, \omega^H\}$, which occurs with probability $(1-p)^2 p$. In this case the agent with a high marginal utility of the public good (say, agent 3) may or may not hold the majority of the shares in the organisation[17]. If he does not, only the minimum amount of the public good is produced, and the payoffs are as specified in row 2a. If he does, the maximum quantity of the public good is produced and the payoff are as specified in row 2b.

Insert Table 5 about here

>From table 5 it is clear that the expected utility of each agent will depend on the relative size of her investment compared to those of the other partners. Let us consider the investment of, say, agent 1, taking as given t_2 and t_3 (and assuming, without loss of generality, $t_3 \geqslant t_2$). If $t_1 + t_2 < t_3$, agent 3's investment constitutes more than half of the total, and therefore agent 3 has alone the power to decide over production: he is the 'dictator' in the organisation. If $t_3 - t_2 \leqslant t_1 \leqslant t_3 + t_2$ no one member has alone a clear

[16]Recall that all agents strictly prefer $3m$ to any $E < 3m$, regardless of the realisation of ω^j.

[17]Of course, it may happen that the shares are distributed in such a way that no absolute majority is formed, wereas fifty per cent of the voting rights are held by agents with a high marginal utility of the public good and the other fifty by agents with a low utility of the public good.

majority of the votes, and the median vote coincides with the median voter. If $t_1 > t_2 + t_3$ agent one has absolute majority and is the dictator.

If the amount of public good the agents want to produce irrespectively of their realisation of preferences in period 3 is sufficiently high, i.e. if $2m \geqslant M$, the first and third case above trivially do not apply, since it is always $t_1 \in [t_3 - t_2, t_3 + t_2]$. In this case the investor-owned institutional forms cannot be distinguished from the cooperative form, since the median vote always coincides with the median voter, and the analysis replicates that of section (3.2). We therefore concentrate only on the more interesting case where $2m < M$[18].

If $2m < M$, the expected utility of the players has to be computed for all three cases, in order to pick the relevant actions for the game to be played. In appendix A.2 we show that it is still the case that there is no internal maximum for the maximisation problem, and thus we can restrict our attention to the binary choice between the minimum and the maximum initial investment, i.e. $t_i \in \{m, M\}$. As for the cooperative case, let us denote with $EU^P_{h(kl)}$ the expected utility of an agent who has invested h when the other two have invested k and l, with $h, k, l \in \{m, M\}$. The subscript P denotes the investor-owned organisational form.

The resulting game (figure 4) is very similar to the game for the cooperative case.

Player 3: m

Player 1 \ Player 2	m	M
m	$EU^P_{m(mm)}, EU^P_{m(mm)}, EU^P_{m(mm)}$	$EU^P_{m(mM)}, EU^P_{M(mm)}, EU^P_{m(mM)}$
M	$EU^P_{M(mm)}, EU^P_{m(mM)}, EU^P_{m(mM)}$	$EU^P_{M(mM)}, EU^P_{M(mM)}, EU^P_{m(MM)}$

Player 3: M

Player 1 \ Player 2	m	M
m	$EU^P_{m(mM)}, EU^P_{m(mM)}, EU^P_{M(mm)}$	$EU^P_{m(MM)}, EU^P_{M(mM)}, EU^P_{M(mM)}$
M	$EU^P_{M(mM)}, EU^P_{m(MM)}, EU^P_{M(mM)}$	$EU^P_{M(MM)}, EU^P_{M(MM)}, EU^P_{M(MM)}$

Figure 4: the normal form game in the investor-owned case.

[18] As a consequence, $0 \leqslant \lambda \leqslant \frac{1}{2}$.

In Appendix A.2 we show that $Z^P_{m(mm)} = Z^C_{m(mm)}$, $Z^P_{m(MM)} = Z^C_{m(MM)}$, $Z^P_{M(mM)} = Z^C_{M(mM)}$, $Z^P_{M(MM)} = Z^C_{M(MM)}$. This is natural, since expected payoff differ between the two organisational forms only if the majority of votes is held by a single agent, and this happens only if one agent contributes the maximum possible while the other two contribute the minimum.

However, the equilibria of the game will differ significantly. The next proposition states our main result regarding the investor-owned case.

Proposition 3 *If the investor-owned institutional form is chosen, there can exist a variety of equilibria. There exist three points p_1, p_4, p_5 such that the possible equilibria are:*

- *if $p_5 < p_4 < p_1$, then*

  if $0 < p < p_5$ the unique pure strategy NE is $\{m, m, m\}$;
  if $p_5 < p < p_4$ there are two pure strategy NE: $\{m, m, m\}$ and $\{m, M, M\}$;
  if $p_4 < p < p_1$ the unique pure strategy NE is $\{m, M, M\}$;
  if $p_1 < p < 1$ the unique pure strategy NE is $\{M, M, M\}$.

- *if $p_4 < p_5 < p_1$, then*

  if $0 < p < p_4$ the unique pure strategy NE is $\{m, m, m\}$;
  if $p_4 < p < p_5$ the unique pure strategy NE is $\{m, m, M\}$;
  if $p_5 < p < p_1$ the unique pure strategy NE is $\{m, M, M\}$;
  if $p_1 < p < 1$ the unique pure strategy NE is $\{M, M, M\}$.

- *if $p_4 < p_1 < p_5$, then*

  if $0 < p < p_4$ the unique pure strategy NE is $\{m, m, m\}$;
  if $p_4 < p < p_1$ the unique pure strategy NE is $\{m, m, M\}$;
  if $p_1 < p < p_5$ there are two pure strategy NE: $\{m, m, M\}$ and $\{M, M, M\}$;
  if $p_5 < p < 1$ the unique pure strategy NE is $\{M, M, M\}$.

- *if $p_5 < p_1 < p_4$*

  if $0 < p < p_5$ the unique pure strategy NE is $\{m, m, m\}$;
  if $p_5 < p < p_1$ there are two pure strategy NE: $\{m, m, m\}$ and $\{m, M, M\}$;
  if $p_1 < p < p_4$ there are two pure strategy NE: $\{m, m, m\}$ and $\{M, M, M\}$;
  if $p_4 < p < 1$ the unique pure strategy NE is $\{M, M, M\}$.

- *if $p_1 < p_5 < p_4$*

if $0 < p < p_1$ *the unique pure strategy NE is* $\{m, m, m\}$*;*
if $p_1 < p < p_4$ *there are two pure strategy NE:* $\{m, m, m\}$ *and* $\{M, M, M\}$*;*
if $p_5 < p < 1$ *the unique pure strategy NE is* $\{M, M, M\}$*.*

- *if* $p_1 < p_4 < p_5$

if $0 < p < p_1$ *the unique pure strategy NE is* $\{m, m, m\}$*;*
if $p_1 < p < p_4$ *there are two pure strategy NE:* $\{m, m, m\}$ *and* $\{M, M, M\}$*;*
if $p_4 < p < p_5$ *there are two pure strategy NE:* $\{m, m, M\}$ *and* $\{M, M, M\}$*;*
if $p_5 < p < 1$ *the unique pure strategy NE is* $\{M, M, M\}$*.*

For a proof, see appendix A.2.1. Note that p_1 is the same as in the cooperative case.

Proposition 3 is rather pedantic. For what concerns our interest, it says that if the investor-owned form is chosen, thare can be a greater variety of equilibria with respect to the cooperative case. The possible equilibria sets in the investor-owned case are:

- the unique pure strategy NE $\{m, m, m\}$;
- the unique pure strategy NE is $\{m, m, M\}$;
- the unique pure strategy NE is $\{m, M, M\}$;
- the unique pure strategy NE is $\{M, M, M\}$;
- the two pure strategy NE: $\{m, m, M\}$ and $\{M, M, M\}$
- the two pure strategy NE: $\{m, m, m\}$ and $\{M, M, M\}$
- the two pure strategy NE: $\{m, M, M\}$ and $\{m, m, m\}$

Also in the investor-owned case, contract incompleteness involves a lower investment in the firm with respect to the first best, where $t_i = M$, $i = 1, 2, 3$ $\forall\ \lambda, \omega^L, \omega^H$[19].

As an example, figure 5 shows the possible equilibria in the λ, p parameter space where $\omega^L = 0.1$ and $\omega^H = 1.8$.

[19]Cfr. proposition 1.

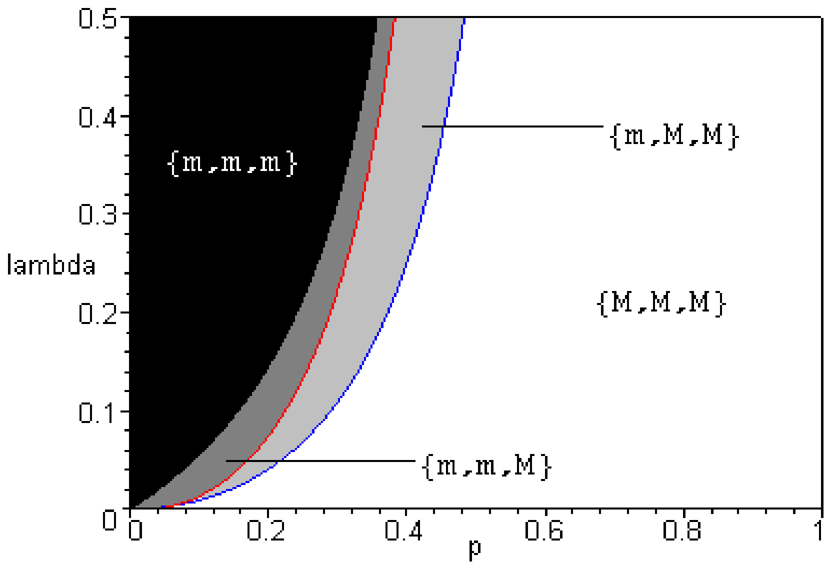

Figure 5: equilibria set in the investor-owned case for $\omega^L = 0.1$ and $\omega^H = 1.9$

The reasons for underinvestment are similar to those for the cooperative case. On the one hand there is the uncertainty effect (agents face the risk of having a low preference for the public good at time 3 and being in the minority, so that all the capital of the firm is used to produce the public good). On the other hand there is the redistribution effect: if the public good is not produced above the minimum level $3m$, an agent investing more than the others will lose part of her contribution.

However, the strategic role of investment in shaping the majority in the investor-owned firm, changes the relative importance of these two effect. Indeed, while in the cooperative case there are only symmetric equilibria, in the investor owned case it is possible to obtain asymmetric equilibria as well. This is because in the latter case initial investment assumes a strategic dimension, related to the distribution of votes among the members.

In the equilibrium $\{m, m, M\}$ just one agent invests the maximum M while the other two invest just m, there is no uncertainty effect for the agent investing M. This is because she has absolute majority of the votes in the organisation, and thus is free to choose production according to her revealed preference at date 4. However, the redistribution effect is still present: if the realised present of the agent investing M is ω^L, she will suffer a loss *ex-post*.

The higher is λ, the more costly this trade off, and the higher must be, *ceteris paribus*, the probability of having high marginal valuation of the public good to make the risk worthwhile (see fig. 5).

A slightly more subtle reasoning applies to the asymmetric equilibrium $\{m, M, M\}$. Even if the investment levels are not symmetric, a(ny) majority of two members out of three is needed to decide over production. In this case, therefore, the reason to invest the maximum M does not depend directly on the possibility of eliminating the uncertainty effect. In fact, it is exactly the opposite argument that applies: an agent wants to invest M when the other two invest m and M respectively, exactly to avoid that a clear majority of one only arises. To do this, an agent is ready to suffer a potential loss due to the redistribution effect in the case her revealed preference is ω^L*and* she is in the minority. Again, the higher is λ, the more costly the trade off, and the higher must, *ceteris paribus*, be the probability of having high marginal valuation of the public good to make the risk worthwhile (see fig. 5).

We can somehow simplify the situation by setting $3m = 0$, and thus $\lambda = 0$. In this case there is no redistribution effect. The following corollary summarizes the results.

Corollary 4 *Let $\lambda = 0$. If the investor-owned form is chosen, there are two point p_1 and p_5 such that:*

- *if $p_1 < p_5$*

Proposition 5 *if $0 < p < p_1$ the unique pure strategy NE is $\{m, m, M\}$;*
if $p_1 < p < p_5$ there are two pure strategy NE: $\{m, m, M\}$ and $\{M, M, M\}$;
if $p_5 < p < 1$ the unique pure strategy NE is $\{M, M, M\}$.

- *if $p_5 < p_1$*

if $0 < p < p_5$ the unique pure strategy NE is $\{m, m, M\}$;
if $p_5 < p < p_1$ the unique pure strategy NE is $\{m, M, M\}$;
if $p_1 < p < 1$ the unique pure strategy NE is $\{M, M, M\}$.

For the proof see appendixA.2.2.

As it is to be expected, when there is no redistribution effect, the equilibrium $\{m, m, m\}$ never occurs. In fact any agent is better off investing M when the other two invest m, in order to acquire the control of the organisation. This is not the case in the cooperative case, where there is no link between the amount invested and the decisional power.

3.4 Comparison between organisational forms

In this section we want to carry out a comparison between the two organisational forms studied in the previous sections. In particular, we want to see how different constitutional design influences the investment decision and the provision of the public good.

>From proposition 1 it is clear that the optimal level of initial investment is always equal to the maximum, i.e. $t_i = M$, $i = 1, 2, 3$. The comparison is thus very simple: we can affirm that the organisational form that induces the higher level of initial investment is, *ceteris paribus*, more efficient.

The same can be said regarding the production of the public good. In the first best, the only case in which production of the public good above the minimum level $E = 3m$ is not optimal is when the realised preferences are all for all 3 agents: $\{\omega^L, \omega^L, \omega^L\}$, while in all other cases optimal public good production is $E = 3k$. Since for any constitutional design production of the public good is $E = 3m$ when realised preferences are $\{\omega^L, \omega^L, \omega^L\}$, we can consider as more efficient the organisational type that induces a higher production of the public good.

In order to simplify the comparison, we adopt the following criteria to identify which equilibrium is chosen when there are multiple equilibria for a given set of parameters' value. We will assume that *when, in case of multiple equilibria, one Pareto-dominates the others, the Pareto dominant equilibrium is chosen.* This assumption seems natural given the assumptions of our model, and allows us to restrict our attention to a smaller set of possible equilibria. If no NE Pareto-dominates the other, we will refer to the multiple equilibria.

The following proposition states the results for the Pareto-dominant equilibria:

Proposition 6

- *If the cooperative is formed, whenever in the equilibria set there are the two pure strategy NE $\{m, m, m\}$ and $\{M, M, M\}$, the NE $\{M, M, M\}$ is Pareto-dominating.*

- *If the investor-owned firm is formed:*

whenever in the equilibria set there are the two pure strategy NE $\{m, m, m\}$ and $\{M, M, M\}$, the NE $\{M, M, M\}$ is Pareto-dominating;

whenever in the equilibria set there are the two pure strategy NE$\{m, M, M\}$ and $\{m, m, m\}$, the NE $\{m, M, M\}$ is Pareto-dominating;

When in the equilibria set there are the two pure strategy NE$\{M, M, M\}$ and $\{m, m, M\}$, the NE $\{m, m, M\}$ is never Pareto-dominating. However, there are parameter values for which $\{M, M, M\}$ is not dominating either.

For a proof., see appendix A.3.

From the comparison of propositions 2 and 6, it follows immediately that if the cooperative institutional form is chosen, there exist only one Pareto-dominant pure strategy NE. Furthermore, there exists a value $p_1$such that:

- if $0 < p < p_1$ the unique Pareto-dominant pure strategy NE is $\{m, m, m\}$;
- if $p_1 < p < 1$ the unique Pareto-dominant pure strategy NE is $\{M, M, M\}$.

Analogously, from the comparison of propositions 3 and 6, if follows that is the investor-owned if formed the possible types of Pareto-dominant pure strategy equilibria sets are:

- the unique pure strategy NE $\{m, m, m\}$;
- the unique pure strategy NE is $\{m, m, M\}$;
- the unique pure strategy NE is $\{m, M, M\}$;
- the unique pure strategy NE is $\{M, M, M\}$;
- the two pure strategy NE: $\{m, m, M\}$ and $\{M, M, M\}$.

The equilibrium set actually occurring is obviously a function of the parameters of the model: $\lambda, p, \omega^L, \omega^H$.

We can now proceed to a graphical comparison between the two organisational forms. To do so, we fix a given value for the utility parameters ω^L and ω^H, and plot the resulting equilibria sets in the space, p, λ.

Figure 6 shows the equilibria set for $\omega^L = 0.7$ and $\omega^H = 1.6$.

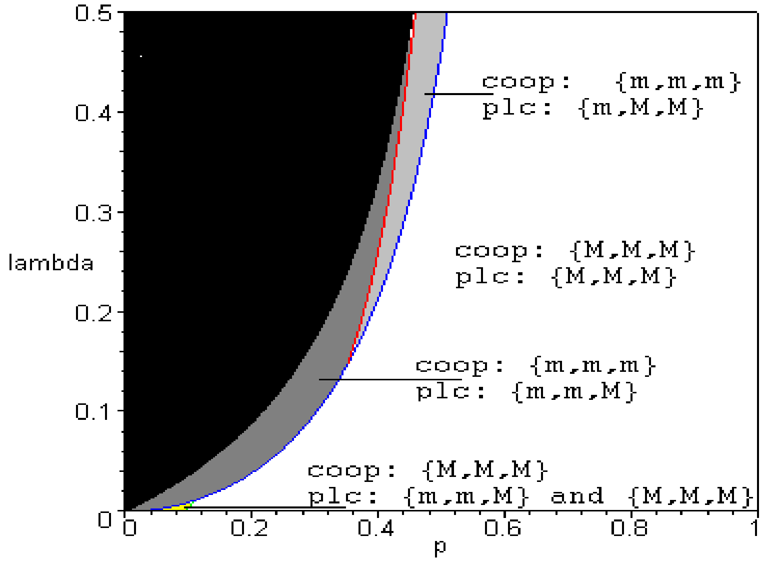

Figure 6: comparison of equilibria sets for $\omega^L = 0.7$ and $\omega^H = 1.6$.

It is possible to notice that whenever the unique dominant Nash equilibrium in the cooperative is $\{M, M, M\}$, the same equilibrium exists for the investor owned organisation as well (white and black areas). This is a general result of the model. However, the may be some parameter values for which $\{M, M, M\}$ is not unique in the investor-owned case: in the black area the maximum investment equilibrium coexists with the equilibrium $\{M, m, m\}$. The latter is not Pareto-dominated by the former since the agent who invests M alone is in expected terms better off when he decides alone over production allocation, then when investment levels are symmetric and decisions are taken democratically. Investment levels will generally be equal to the first best (although this may not be the case in the black area for the investor-owned firm). However, production of the public good may not be optimal: if the majority (or the only one investing m in the $\{m, m, M\}$ equilibrium) but not the totality of the members have a realised low preference for the public good, its production is *ex-post* lower than in the first best case.

The most interesting cases arise when the only Pareto-dominant investment equilibrium in the cooperative is the minimum (grey areas). While for the coopearive only the symmetric equilibrium $\{m, m, m\}$ is possible, the investor-owned organisation can present asymmetric equilibria that increase total initial investment, with a result closer to the first best situation. The possible unique Pareto-dominant Nash asymmetric equilibria are $\{m, m, M\}$ and $\{m, M, M\}$. Again, actual production levels of the public good depend on the realised preferences.

As the preference parameters ω^L and ω^H vary, the curves of figure 6 move, and some equilibria may disappear, but the qualitative results are substantially unchanged. If ω^L decreases, the curves tend to move towards the right, and the area with lower investment levels become *ceteris paribus* larger. A low ω^L deters investment, since the loss from a realised low preference of the public good is potentially higher. Conversely, an increase in ω^H makes *ceteris paribus* the investment more likely, since it increases the possible benefits of the production of the public good.

4 Conclusions

In this paper we carry out an abstract comparison between a cooperative and an investor-owned enterprise in a world with incomplete contracts. We identify organisational forms with their decision making mechanism: one-head-one-vote for cooperatives and one-share-one-vote for investor-owned.

Our main result is that constitutional design matters: different decision making rules imply different equilibrium investment levels for the cooperative and investor-owned institutional forms.

In both cases undeinvestment with respect to the first best occurs (this is a more general consequence of contract incompleteness). However, the investor-owned form seems to induce more investment than the cooperative form. In fact, we show that the total investment in an investor-owned firm is almost always unambiguously than in the cooperative, while it is never unambiguously higher in the cooperative.

This is because initial investment has in the investor-owned case a strategic role in shaping the majority of votes: an agent may wish to invest more in order to gain control of the enterprise, or in order to prevent another agent from gaining control.

This strategic role of investment gives rise to asymmetric pure strategy Nash equilibria that do not arise in a cooperative, since the size of investment does not influence control.

A Appendix

A.1 Proof of proposition 2

Dividing the equations 8 to 13 by M (recall that $\lambda = \frac{m}{M}$) and defining $Z^C_{h(kl)} \equiv \frac{EU^C_{h(kl)}}{M}$ we have:

$$Z^C_{m(mm)} = (1-\lambda) + 3\omega^H\lambda \tag{14}$$

$$Z^C_{m(mM)} = \lambda\left(1-\frac{3\lambda}{2\lambda+1}\right)\left(1-3p^2+2p^3\right) + (1-\lambda)\,p^2\left[(1-p)\,\omega^L+(2-p)\,\omega^H\right]+(1-\lambda)+3\omega^H\lambda \quad (15)$$

$$Z^C_{M(mm)} = \left(1-\frac{3\lambda}{2\lambda+1}\right)\left(1-3p^2+2p^3\right) + (1-\lambda)\,p^2\left[(1-p)\,\omega^L+(2-p)\,\omega^H\right]+3\omega^H\lambda \quad (16)$$

$$Z^C_{m(MM)} = \lambda\left(1-\frac{3\lambda}{\lambda+2}\right)\left(1-3p^2+2p^3\right) +2\,(1-\lambda)\,p^2\left[(1-p)\,\omega^L+(2-p)\,\omega^H\right]+(1-\lambda)+3\omega^H\lambda \quad (17)$$

$$Z^C_{M(mM)} = \left(1-\frac{3\lambda}{\lambda+2}\right)\left(1-3p^2+2p^3\right) +2\,(1-\lambda)\,p^2\left[(1-p)\,\omega^L+(2-p)\,\omega^H\right]+3\omega^H\lambda \quad (18)$$

$$Z^C_{M(MM)} = (1-\lambda)\left(1-3p^2+2p^3\right)+3\,(1-\lambda)\,p^2\left[(1-p)\,\omega^L+(2-p)\,\omega^H\right]+3\omega^H\lambda \quad (19)$$

The symmetric structure of the game depicted in table 3 implies that there are only four possible Nash Equilibria in pure strategies, which are defined by the number of agents who choose the maximum contribution: this can be 0,1,2,3. The conditions for each of these type of equilibrium equilibria to hold are shown in table A1.

NE	Representative payoff vector	Representative strategy vector	Conditions
0	$Z^C_{m(mm)}, Z^C_{m(mm)}, Z^C_{m(mm)}$	$\{m, m, m\}$	$Z^C_{m(mm)} > Z^C_{M(mm)}$
1	$Z^C_{M(mm)}, Z^C_{m(mM)}, Z^C_{m(mM)}$	$\{m, m, M\}$	$Z^C_{M(mm)} > Z^C_{m(mm)}$ $Z^C_{m(mM)} > Z^C_{M(mM)}$
2	$Z^C_{M(mM)}, Z^C_{m(MM)}, Z^C_{M(mM)}$	$\{m, M, M\}$	$Z^C_{M(mM)} > Z^C_{m(mM)}$ $Z^C_{m(MM)} > Z^C_{M(MM)}$
3	$Z^C_{M(MM)}, Z^C_{M(MM)}, Z^C_{M(MM)}$	$\{M, M, M\}$	$Z^C_{M(MM)}, > Z^C_{m(MM)},$

Table A1: the Nash Equilibria of the game

Thus, for example, the equilibrium $\{Z^C_{m(mm)}, Z^C_{m(mm)}, Z^C_{m(mm)}\}$ where all players contribute the minimum amount obtains if nobody is better off by contributing the maximum, given that the other two contribute the minimum. Therefore it must be in this case: $Z^C_{m(mm)} > Z^C_{M(mm)}$.

Define the following functions (using equations 14 to 19):

$$\begin{aligned} cen(\omega^L,\omega^H,\lambda,p) &= Z^C_{m(mm)} - Z^C_{M(mm)} = \\ & (1-\lambda) - \left(1-\frac{3\lambda}{2\lambda+1}\right)\left(1-3p^2+2p^3\right) \\ & -p^2\left[(1-p)\,\omega^L + (2-p)\,\omega^H\right](1-\lambda) \end{aligned} \tag{20}$$

$$\begin{aligned} cef(\omega^L,\omega^H,\lambda,p) &= Z^C_{M(MM)} - Z^C_{m(MM)} = (1-3\lambda)\left(1-3p^2+2p^3\right) \\ & +3p^2\left[(1-p)\,\omega^L + (2-p)\,\omega^H\right] - 3\lambda\left(1-\frac{3\lambda}{\lambda+2}\right)\left(1-3p^2+2p^3\right) \\ & -2\,(1-\lambda)\,p^2\left[(1-p)\,\omega^L + (2-p)\,\omega^H\right] - (1-\lambda) \end{aligned} \tag{21}$$

$$\begin{aligned} cem(\omega^L,\omega^H,\lambda,p) &= Z^C_{m(mM)} - Z^C_{M(mM)} = \\ & \frac{1}{3}\lambda\left(1-\frac{3\lambda}{2\lambda+1}\right)\left(1-3p^2+2p^3\right) + p^2\left[(1-p)\,\omega^L + (2-p)\,\omega^H\right](1-\lambda) \\ & +(1-\lambda) - \left(1-\frac{3\lambda}{\lambda+2}\right)\left(1-3p^2+2p^3\right) \\ & -2\,(1-\lambda)\,p^2\left[(1-p)\,\omega^L + (2-p)\,\omega^H\right] \end{aligned} \tag{22}$$

From table A.1 it is then clear that:

- $\{m,m,m\}$ is NE if $cen(\omega^L,\omega^H,\lambda,p) < 0$;
- $\{M,M,M\}$ is NE if $cef(\omega^L,\omega^H,\lambda,p) > 0$;
- $\{m,m,M\}$ is NE if $cen(\omega^L,\omega^H,\lambda,p) < 0$ and $cem(\omega^L,\omega^H,\lambda,p) > 0$;
- $\{m,M,M\}$ is NE if $cef(\omega^L,\omega^H,\lambda,p) < 0$ and $cem(\omega^L,\omega^H,\lambda,p) < 0$.

It is possible to find analytical solutions for p such that 20, 21 and 22 are equal to zero. It turns out that the functions have generally only one real solution in p [20] (only for $\lambda = 0$[21] all three roots of the three equations above are real, with two roots coinciding at $p = 0$ and the third at $p > 0$). Call p_1 the value for which $cef(\omega^L,\omega^H,\lambda,p) = 0$, p_2 the value for which $cen(\omega^L,\omega^H,\lambda,p) = 0$ and p_3 the value for which $cem(\omega^L,\omega^H,\lambda,p) = 0$.

Note that $cef(\omega^L,\omega^H,\lambda,p) > 0$ for $p > p_1$, $cen(\omega^L,\omega^H,\lambda,p) > 0$ for $p < p_2$ and $cem(\omega^L,\omega^H,\lambda,p) > 0$ for $p < p_3$. The conditions for the equilibria thus result:

[20] We spare to the reader the analitical solution, which is quite cumbersome and does not add much insight to the problem.

[21] For the case when $\lambda = 0$ see below.

- $\{m, m, m\}$ is NE if $p < p_2$;
- $\{M, M, M\}$ is NE if $p > p_1$;
- $\{m, m, M\}$ is NE if $p > p_2$ and $p < p_3$;
- $\{m, M, M\}$ is NE if $p < p_1$ and $p > p_3$.

In order to prove proposition A.1 we need to show that $p_1 \leqslant p_3 \leqslant p_2$. We will first show analytically that $p_3 \leqslant p_2$, and then resort to a complete scan of the parameter range to show that it is always $p_1 \leqslant p_3$.

A complete numerical scan of the parameter space shows that the curve $cen()$ is downward sloping in the neighbourhood of p_2 and that the curve $cef()$ is upward sloping in the neighbourhood of p_1. To prove that $p_3 \leqslant p_2$ we will show that $cen(\omega^L, \omega^H, \lambda, p) \geqslant cem(\omega^L, \omega^H, \lambda, p)$, $\forall p$. If this is true, $cem()$ crosses the horizontal axis to the left of $cen()$ and therefore it is $p_3 \leqslant p_2$. We know from 20 and 22 that $cen() \geqslant cem()$ if $Z^C_{m(mm)} - Z^C_{M(mm)} \geqslant Z^C_{m(mM)} - Z^C_{M(mM)}$, i.e. $Z^C_{m(mm)} + Z^C_{M(mM)} \geqslant Z^C_{m(mM)} + Z^C_{M(mm)}$.

Call $p^2\left((1-p)\,\omega_L + (2-p)\,\omega_H\right) = P$ and $\left(1 - 3p^2 + 2p^3\right) = Q$ (note that it is always $P > 0$ and $Q > 0$) to obtain from 14 and 18 :

$$\begin{aligned}
A &= Z^C_{m(mm)} + Z^C_{M(mM)} = (1-\lambda) + \omega^H\lambda + \left(1 - \frac{3\lambda}{\lambda+2}\right) Q + 2\,(1-\lambda)\,P + 3\omega^H\lambda = \\
&= \left(1 - \frac{3\lambda}{\lambda+2}\right) Q + (2P+1)\,(1-\lambda) + 6\omega^H\lambda
\end{aligned}$$

and from 15 and 16:

$$\begin{aligned}
B &= Z^C_{m(mM)} + Z^C_{M(mm)} = \lambda\left(1 - \frac{3\lambda}{2\lambda+1}\right) Q + P\,(1-\lambda) + (1-\lambda) \\
&\quad +3\omega^H\lambda + \left(1 - \frac{3\lambda}{.\lambda+1}\right) Q + P\,(1-\lambda) + 3\omega^H\lambda \\
&= \left[\lambda\left(1 - \frac{3\lambda}{2\lambda+1}\right) + \left(1 - \frac{3\lambda}{2\lambda+1}\right)\right] Q \\
&\quad +2P\,(1-\lambda) + (1-\lambda) + 6\omega^H\lambda \\
&= \left[\lambda - \frac{9\lambda^2}{6\lambda+3} + 1 - \frac{9\lambda}{6\lambda+3}\right] Q + (2P+1)\,(1-\lambda) + 6\omega^H\lambda
\end{aligned}$$

Now subtracting the latter from the former and simplifying we obtain:

$$\begin{aligned}
A-B &= \left[1-\frac{9\lambda}{3(\lambda+2)}-\lambda+\frac{9\lambda^2}{3(2\lambda+1)}-1+\frac{9\lambda}{3(2\lambda+1)}\right]Q= \\
&= \left[-\frac{3}{(\lambda+2)}-1+\frac{3\lambda}{(2\lambda+1)}+\frac{3}{(2\lambda+1)}\right]\lambda Q= \\
&= \frac{-3\,(2\lambda+1)-(\lambda+2)\,(2\lambda+1)+3\lambda(\lambda+2)+3(\lambda+2)}{(\lambda+2)\,(2\lambda+1)}\lambda Q= \\
&= \frac{-6\lambda-3-2\lambda^2-\lambda-4\lambda-2+3\lambda^2+6\lambda+3\lambda+6}{(\lambda+2)\,(2\lambda+1)}\lambda Q= \\
&= \frac{\lambda^2-2\lambda+1}{(\lambda+2)\,(2\lambda+1)}\lambda Q=\frac{(\lambda-1)^2}{(\lambda+2)\,(2\lambda+1)}\lambda Q\geqslant 0
\end{aligned}$$

The difference between the two functions is always positive, unless $\lambda = 0$, in which case they coincide. Therefore we have $p_3 \leqslant p_2$.

A complete numerical scan of the parameter space shows that $p_1 \leqslant p_2$, $\forall \lambda, \omega^H, \omega^L$.

We show now that when $3m = 0$, and thus $\lambda = 0$, $p_1 = p_2$, $\forall \lambda, \omega^H, \omega^L$.

For the equilibrium $\{M, M, M\}$ to exist the condition $cef(\omega^L, \omega^H, \lambda, p) = Z^C_{M(MM)} - Z^C_{m(MM)} > 0$ must be respected. Substituting for $\lambda = 0$ in 19 and 17 we obtain the following condition:

$$-3p^2+2p^3+3p^2\left((1-p)\,\omega_L+(2-p)\,\omega_H\right)-2p^2\left((1-p)\,\omega_L+(2-p)\,\omega_H\right)>0$$

which implies $p > p_1 = \frac{2\left(1-\omega^H\right)+\left(1-\omega^L\right)}{\left(1-\omega^H\right)+\left(1-\omega^L\right)}$.

Similarly, for the equilibrium $\{0, 0, 0\}$ to exist the condition $cen(\omega^L, \omega^H, \lambda, p) = Z^C_{m(mm)} - Z^C_{M(mm)} > 0$ must be respected. Substituting for $\lambda = 0$ in 14 and 16 we obtain the following condition:

$$\left(1-3p^2+2p^3\right)+p^2\left((1-p)\,\omega^L+(2-p)\,\omega^H\right)-1<0$$

which implies $p < p_2 = \frac{2\left(1-\omega^H\right)+\left(1-\omega^L\right)}{\left(1-\omega^H\right)+\left(1-\omega^L\right)}$. Clearly, $p_1 = p_2$.

A.2 The investor-owned case

The game in the investor-owned case is slightly more complicated than in the cooperative case, since we have to take into account the effect of investment in shaping the majority of the votes.

We use the following notation: EU^{P_1} is the expected utility of, say, agent 1 when $t_1 \in [m, t_3 - t_2)$, EU^{P_2} is the expected utility of agent 1 when $t_1 \in [t_3 - t_2, t_3 + t_2]$, and EU^{P_3} is the expected utility of agent 1 when $t_1 \in (t_3 + t_2, M]$.

When $t_1 \in [m, t_3 - t_2)$ the relevant lines of table 5 are: 1, 2b , 3a, 4a, 5a, 6a, 7b, 8. Expected utility results:

$$
\begin{aligned}
EU^{P_1} &= \left[(1-p)^3 + 2(1-p)^2 p + (1-p)p^2\right] \left[(M_1 - t_1) + t_1 \left(1 - \frac{3m}{t_1+t_2+t_3}\right) + 3\omega^H m\right] \\
&+ \left[(1-p)^2 p + (1-p)p^2\right] \left[(M_1 - t_1) + 3\omega^H m + (t_1+t_2+t_3-3m)\,\omega^L\right] \\
&+ \left((1-p)p^2 + p^3\right) \left[(M_1 - t_1) + (t_1+t_2+t_3)\,\omega^H\right]
\end{aligned}
$$

Simplifying we obtain:

$$
\begin{aligned}
EU^{P_1} &= (1-p)\left(M_1 - \frac{3t_1 m}{t_1+t_2+t_3} + 3\omega^H m\right) \\
&+ (1-p)p\left[(M_1 - t_1) + 3\omega^H m + (t_1+t_2+t_3-3m)\,\omega^L\right] \\
&+ p^2\left[(M_1 - t_1) + (t_1+t_2+t_3)\,\omega^H\right]
\end{aligned}
$$

When $t_1 \in [t_3 - t_2, t_3 + t_2]$ the relevant lines of table 5 are: 1, 2, 3, 4, 5, 6, 7, 8. Note that in this case the resulting expected utility is the same as in the cooperative case (cfr. equation 6).

$$
\begin{aligned}
EU^{P_2} &= \left[(1-p)^3 + 3(1-p)^2 p\right] \left[(M_1 - t_1) + t_1 \left(1 - \frac{3m}{t_1+t_2+t_3}\right) + 3\omega^H m\right] \\
&+ (1-p)p^2\left[(M_1 - t_1) + 3\omega^H m + (t_1+t_2+t_3-3m)\,\omega^L\right] \\
&+ \left(2(1-p)p^2 + p^3\right) \left[(M_1 - t_1) + (t_1+t_2+t_3)\,\omega^H\right]
\end{aligned}
$$

Simplifying results:

$$
\begin{aligned}
EU^{P_2} &= \left(1 - 3p^2 + 2p^3\right)\left(M_1 - \frac{3t_1 m}{t_1+t_2+t_3} + 3\omega^H m\right) \\
&+ (1-p)p^2\left[(M_1 - t_1) + 3\omega^H m + (t_1+t_2+t_3-3m)\,\omega^L\right] \\
&+ (2-p)p^2\left[(M_1 - t_1) + (t_1+t_2+t_3)\,\omega^H\right]
\end{aligned}
$$

When $t_1 \in (t_3 + t_2, M]$ the relevant lines of table 5 are: 1, 2a ,3a, 4b, 5b, 6a, 7a, 8; and the resulting expected utility is:

$$
\begin{aligned}
EU^{P_3} &= \left[(1-p)^3 + 2(1-p)^2 p + (1-p)p^2\right]\left[(M_1 - t_1) + t_1\left(1 - \frac{3m}{t_1+t_2+t_3}\right) + 3\omega^H m\right] \\
&+ \left[(1-p)^2 p + 2(1-p)p^2 + p^3\right]\left[(M_1 - t_1) + (t_1+t_2+t_3)\,\omega^H\right]
\end{aligned}
$$

Simplifying:

$$
\begin{aligned}
EU^{P_3} &= (1-p)\left(M_1 - \frac{3t_1 m}{t_1+t_2+t_3} + 3\omega^H m\right) \\
&+ p\left[(M_1 - t_1) + (t_1+t_2+t_3)\,\omega^H\right]
\end{aligned}
$$

Note that in all three cases the derivative of the expected utility of agent 1 with respect to his own investment depends only on the first term of the right hand side. The sign of the second derivative is thus the same as the sign of

$$
\frac{d^2}{dt_1^2}\left(-\frac{3t_1 m}{t_1+t_2+t_3}\right) = 6m\frac{t_2+t_3}{(t_1+t_2+t_3)^3} > 0
$$

Therefore any maximum of the expected utility has to be either at one of the extremes or at a discontinuity point. The set of these point is given by $\{m, t_3 - t_2, t_2 + t_3, M\}$. However, it can be shown that the actual investment will be in the subset $t_i \in \{m, M\}$.

We calculate now the values of the different utilities. Substituting in 7 the appropriate values of $t_1 \in \{m, M\}$ we have:

$$
EU^P_{m(mm)} = EU^{P_2}(m,m,m) = M - m + 3\omega^H m \tag{23}
$$

$$
\begin{aligned}
EU^P_{m(mM)} &= EU^{P_1}(m,m,M) = m\left(1 - \frac{3m}{2m+M}\right)(1-p) \\
&+ (M-m)\left[(1-p)\,p\omega^L + p^2\omega^H\right] + (M-m) + 3\omega^H m
\end{aligned} \tag{24}
$$

$$EU^P_{M(mm)} = EU^{P_3}(M, m, m) = M\left(1 - \frac{3m}{2m+M}\right)(1-p) + p\,(M-m)\,\omega^H + 3\omega^H m \tag{25}$$

$$\begin{aligned} EU^P_{m(MM)} &= EU^{P_2}(m, M, M) = m\left(1 - \frac{3m}{m+2M}\right)\left(1 - 3p^2 + 2p^3\right) \qquad (26) \\ &\quad +2\,(M-m)\,p^2\left[(1-p)\,\omega^L + (2-p)\,\omega^H\right] + (M-m) + 3\omega^H m \end{aligned}$$

$$\begin{aligned} EU^P_{M(mM)} &= EU^{P_2}(M, m, M) = M\left(1 - 3p^2 + 2p^3\right)\left(1 - \frac{3m}{m+2k}\right) \qquad (27) \\ &\quad +2\,(M-m)\,p^2\left[(1-p)\,\omega^L + (2-p)\,\omega^H\right] + 3\omega^H m \end{aligned}$$

$$\begin{aligned} EU^P_{M(MM)} &= EU^{P_2}(M, M, M) = \left(1 - 3p^2 + 2p^3\right)(M-m) \qquad (28) \\ &\quad +3\,(M-m)\,p^2\left[(1-p)\,\omega^L + (2-p)\,\omega^H\right] + 3\omega^H m \end{aligned}$$

A.2.1 Proof of proposition 3

Dividing the above equations by M and redefining $Z^P_{h(kl)} \equiv \frac{EU^P_{h(kl)}}{M}$ we have (recall that $\lambda = \frac{m}{M}$):

$$Z^P_{m(mm)} = (1-\lambda) + 3\omega^H\lambda \tag{29}$$

$$\begin{aligned} Z^P_{m(mM)} &= \lambda\left(1 - \frac{3\lambda}{2\lambda+1}\right)(1-p) \qquad (30) \\ &\quad +(1-\lambda)\left[(1-p)\,p\omega^L + p^2\omega^H\right] + (1-\lambda) + 3\omega^H\lambda \end{aligned}$$

$$Z^P_{M(mm)} = \left(1 - \frac{3\lambda}{2\lambda+1}\right)(1-p) + p\,(1-\lambda)\,\omega^H + 3\omega^H\lambda \tag{31}$$

$$\begin{aligned} Z^P_{m(MM)} &= 3\lambda\left(1 - \frac{3\lambda}{\lambda+2}\right)\left(1 - 3p^2 + 2p^3\right) \qquad (32) \\ &\quad +2\,(1-\lambda)\,p^2\left[(1-p)\,\omega^L + (2-p)\,\omega^H\right] + (1-\lambda) + 3\omega^H\lambda \end{aligned}$$

$$Z^P_{M(mM)} = \left(1-3p^2+2p^3\right)\left(1-\frac{3\lambda}{\lambda+2}\right) + 2\left(1-\lambda\right)p^2\left[\left(1-p\right)\omega^L+\left(2-p\right)\omega^H\right]+3\omega^H\lambda \tag{33}$$

$$Z^P_{M(MM)} = \left(1-3p^2+2p^3\right)\left(1-\lambda\right) + 3\left(1-\lambda\right)p^2\left[\left(1-p\right)\omega^L+\left(2-p\right)\omega^H\right]+3\omega^H\lambda \tag{34}$$

Note that $Z^P_{m(mm)} = Z^C_{m(mm)}$, $Z^P_{m(MM)} = Z^C_{m(MM)}$, $Z^P_{M(mM)} = Z^C_{M(mM)}$, $Z^P_{M(MM)} = Z^C_{M(MM)}$.

As for the cooperative, there are only four possible Nash Equilibria in pure strategies, which are defined by the number of agents who choose the maximum contribution This can be 0,1,2,3. The conditions for each of these type of equilibrium equilibria to hold are shown in table A2.

NE	Representative payoff vector	Representative strategy vector	Conditions
0	$Z^P_{m(mm)}, Z^P_{m(mm)}, Z^P_{m(mm)}$	$\{m, m, m\}$	$Z^P_{m(mm)} > Z^P_{M(mm)}$
1	$Z^P_{M(mm)}, Z^P_{m(mM)}, Z^P_{m(mM)}$	$\{m, m, M\}$	$Z^P_{M(mm)} > Z^P_{m(mm)}$ $Z^P_{m(mM)} > Z^P_{M(mM)}$
2	$Z^P_{M(mM)}, Z^P_{m(MM)}, Z^P_{M(mM)}$	$\{m, M, M\}$	$Z^P_{M(mM)} > Z^P_{m(mM)}$ $Z^P_{m(MM)} > Z^P_{M(MM)}$
3	$Z^P_{M(MM)}, Z^P_{M(MM)}, Z^P_{M(MM)}$	$\{M, M, M\}$	$Z^P_{M(MM)} > Z^P_{m(MM)}$

Table A2: the Nash Equilibria of the game in the investor-owned case.

Since $Z^P_{m(mm)} = Z^C_{m(mm)}$ and $Z^P_{m(MM)} = Z^C_{m(MM)}$, the condition for the full investment equilibrium $\left\{Z_{M(MM)}, Z_{M(MM)}, Z_{M(MM)}\right\}$ is the same between the two organisational forms. However, the conditions for the other three equilibria are different. Let us define the following function:

$$pen(\omega^L, \omega^H, \lambda, p) = Z^P_{m(mm)} - Z^P_{M(mm)} = \left(1-\lambda\right)+3\omega^H\lambda - \left[\left(1-\frac{3\lambda}{2\lambda+1}\right)\left(1-p\right)+p\left(1-\lambda\right)\omega^H+3\omega^H\lambda\right] \tag{35}$$

$$pem(\omega^L, \omega^H, \lambda, p) = Z^P_{m(mM)} - Z^P_{M(mM)} = \lambda\left(1-\frac{3\lambda}{2\lambda+1}\right)\left(1-p\right)+\left(1-\lambda\right)\left[\left(1-p\right)p\omega^L+p^2\omega^H\right]+\left(1-\lambda\right) - \left(1-3p^2+2p^3\right)\left(1-\frac{3\lambda}{\lambda+2}\right)-2\left(1-\lambda\right)p^2\left[\left(1-p\right)\omega^L+\left(2-p\right)\omega^H\right] \tag{36}$$

From table A2 it is then clear that:

- $\{m, m, m\}$ is NE if $pen(\omega^L, \omega^H, \lambda, p) > 0$;
- $\{M, M, M\}$ is NE if $cef(\omega^L, \omega^H, \lambda, p) > 0$;
- $\{m, m, M\}$ is NE if $pen(\omega^L, \omega^H, \lambda, p) < 0$ and $pem(\omega^L, \omega^H, \lambda, p) > 0$;
- $\{m, M, M\}$ is NE if $cef(\omega^L, \omega^H, \lambda, p) < 0$ and $pem(\omega^L, \omega^H, \lambda, p) < 0$.

It is possible to find analytical solutions for p such that 35 and 36 are equal to zero. It turns out that the functions have generally only one real solution in for $0 \leqslant p \leqslant 1$. Let us denote with p_4 and p_5 the values of p for which $pen(\omega^L, \omega^H, \lambda, p) = 0$ and $pem(\omega^L, \omega^H, \lambda, p) = 0$ respectively, in the interval $p \in [0, 1]$. Recall from appendix 2 that p_1 is the value of p for which $cef(\omega^L, \omega^H, \lambda, p) = 0$ in the interval $p \in [0, 1]$. A complete numerical scan of the parameter space shows that the curve $pen()$ and $pem()$ are downward sloping in the neighbourhood of p_4 and p_5 respectively. The following in then true:

- $\{m, m, m\}$ is NE if $p < p_4$;
- $\{M, M, M\}$ is NE if $p > p_1$;
- $\{m, m, M\}$ is NE if $p_4 < p < p_5$;
- $\{m, M, M\}$ is NE if $p_5 < p < p_1$.

Proposition 3 results considering the possible orderings between p_1, p_4, and p_5.

A.2.2 Proof of corollary 4

Substituting $\lambda = 0$ in 35, 36 and 21, we obtain:

$$pen(\omega^L, \omega^H, p) = p\left(1 + \omega^H\right)$$

$$pem(\omega^L, \omega^H, p) = (1-p)\,p\omega^L + p^2\omega^H + 3p^2 - 2p^3 - 2p^2\left[(1-p)\,\omega^L + (2-p)\,\omega^H\right]$$

$$cef(\omega^L, \omega^H, p) = Z^C_{M(MM)} - Z^C_{m(MM)} = p^2\left[-3 + 2p + (1-p)\,\omega^L + (2-p)\,\omega^H\right]$$

It is easily seen that $pen(\omega^L, \omega^H, p) = 0$ only for $p = 0$. Thus we have $p_4 = 0$. Corollary 4 follows.

A.3 Proof of proposition 6

We prove first that the equilibrium $\{M, M, M\}$ always dominates the equilibrium $\{m, m, m\}$.We can show this as a general result, and it will hold *a fortiori* for the parameters for which multiple equilibria arise. To do so, we just need to demonstrate that the payoff of any agent of contributing M when the other two do the same is higher than contributing m when the other two do the same.

This amounts to showing that $Z^C_{M(MM)} > Z^C_{m(mm)}$ for the cooperative and $Z^P_{M(MM)} > Z^P_{m(mm)}$ for the investor-owned firm. Note however that since $Z^C_{M(MM)} = Z^P_{M(MM)}$ and $Z^C_{m(mm)} = Z^P_{m(mm)}$, we need to demonstrate that the result holds in one case only. Consider $Z^C_{M(MM)} - Z^C_{m(mm)}$:

$$\begin{aligned} Z^C_{M(MM)} - Z^C_{m(mm)} &= (1-\lambda)\left(1-3p^2+2p^3\right) + 3(1-\lambda)p^2\left[(1-p)\omega^L + (2-p)\omega^H\right] \\ &\quad +3\omega^H\lambda - \left[(1-\lambda) + 3\omega^H\lambda\right] \end{aligned}$$

Simplifying we have:

$$Z^C_{M(MM)} - Z^C_{m(mm)} = 3(1-\lambda)\left[(1-p)\omega^L + (2-p)\omega^H - 1\right] + 2p(1-\lambda)$$

which is always positive, given that $(1-p)\omega^L + (2-p)\omega^H > 1$ and $1 > \lambda$.

To show that the equilibrium$\{m, M, M\}$ always dominates $\{m, m, m\}$ we need to show first of all that an agent contributing m when the other two contribute M is better off than contributing m when the others contribute the same (i.e. $Z^P_{m(MM)} > Z^P_{m(mm)}$) and that an agent contributing M when the other two contribute M and m respectively is better off than contributing m when the others contribute the same (i.e. $Z^P_{M(mM)} > Z^P_{m(mm)}$). As for $Z^P_{m(MM)} - Z^P_{m(mm)}$, we have

$$\begin{aligned} Z^P_{m(MM)} - Z^P_{m(mm)} &= \lambda\left(1-\frac{3\lambda}{\lambda+2}\right)\left(1-3p^2+2p^3\right) + 2(1-\lambda)p^2\left[(1-p)\omega^L + (2-p)\omega^H\right] \\ &\quad + (1-\lambda) + 3\omega^H\lambda - \left[(1-\lambda) + 3\omega^H\lambda\right] \end{aligned}$$

which is clearly always positive. Let us consider now $pz^H_L - pz_{LLL}$

$$\begin{aligned} Z^P_{M(mM)} - Z^P_{m(mm)} &= \left(1-\frac{3\lambda}{\lambda+2}\right)\left(1-3p^2+2p^3\right) + 2(1-\lambda)p^2\left[(1-p)\omega^L + (2-p)\omega^H\right] \\ &\quad +3\omega^H\lambda - \left[(1-\lambda) + 3\omega^H\lambda\right] \end{aligned}$$

simplifying we obtain

$$Z^P_{M(mM)} - Z^P_{m(mm)} = \left(1 - \frac{3\lambda}{\lambda+2}\right)\left(1 - 3p^2 + 2p^3\right) + (1-\lambda)\left\{2p^2\left[(1-p)\,\omega^L + (2-p)\,\omega^H\right] - 1\right\}$$

which is also always positive.

As for the multiple NE $\{m, m, M\}$ and $\{M, M, M\}$, firstly we want to show that an agent investing M when the other two invest the same may is always better off than contributing m when the other two contribute M and m respectively (this proves that $\{m, m, M\}$ never Pareto-dominates $\{M, M, M\}$). A complete numerical scan of the parameter space shows that in the parameter range for which the multiple equilibria $\{m, m, M\}$ and $\{M, M, M\}$ exist, it is always $Z^P_{M(MM)} - Z^P_{m(mM)}$. Secondly, we want to show first that an agent investing M when the other two invest M may or may not be better off than an agent investing M when the other two invest m (i.e. $Z^P_{M(MM)} \lesseqgtr Z^P_{M(mm)}$). This proves that $\{m, m, M\}$ never Pareto-dominates $\{M, M, M\}$. Again, it can be numerically shown that in the parameter range where the multiple equilibria exist it is in some cases $Z^P_{M(MM)} < Z^P_{M(mm)}$.

This result is not entirely surprising. The agent investing M when the other two invest m retains the control of the organisation, and can decide alone on the production allocation. This may for some parameters' value offset the higher risk of being affected by redistribution towards the other agents if his preference for the public good is low, and thus make the lower investment equilibrium preferable.

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Table 1 -Cooperative

	Realised ω	Probability	Public good	Private good	Payoff to agent L	Payoff to agent H
1	$\omega^L,\omega^L,\omega^L$	$(1-p)^3$	$3m$	$t_i\left(1-\frac{3m}{t_1+t_2+t_3}\right)$	$(k_i-t_i)+t_i\left(1-\frac{3m}{t_1+t_2+t_3}\right)+3\omega^H m$	–
2	$\omega^L,\omega^L,\omega^H$	$(1-p)^2p$	$3m$	$t_i\left(1-\frac{3m}{t_1+t_2+t_3}\right)$	$(k_i-t_i)+t_i\left(1-\frac{3m}{t_1+t_2+t_3}\right)+3\omega^H m$	$(k_i-t_i)+t_i\left(1-\frac{3m}{t_1+t_2+t_3}\right)+3\omega^H m$
3	$\omega^L,\omega^H,\omega^L$	$(1-p)^2p$	$3m$	$t_i\left(1-\frac{3m}{t_1+t_2+t_3}\right)$	$(k_i-t_i)+t_i\left(1-\frac{3m}{t_1+t_2+t_3}\right)+3\omega^H m$	$(k_i-t_i)+t_i\left(1-\frac{3m}{t_1+t_2+t_3}\right)+3\omega^H m$
4	$\omega^L,\omega^H,\omega^H$	$(1-p)p^2$	$t_1+t_2+t_3$	0	$(k_i-t_i)+3\omega^H m+(t_1+t_2+t_3-3m)\omega^L$	$(k_i-t_i)+(t_1+t_2+t_3)\omega^H$
5	$\omega^H,\omega^L,\omega^L$	$(1-p)^2p$	$3m$	$t_i\left(1-\frac{3m}{t_1+t_2+t_3}\right)$	$(k_i-t_i)+t_i\left(1-\frac{3m}{t_1+t_2+t_3}\right)+3\omega^H m$	$(k_i-t_i)+t_i\left(1-\frac{3m}{t_1+t_2+t_3}\right)+3\omega^H m$
6	$\omega^H,\omega^L,\omega^H$	$(1-p)p^2$	$t_1+t_2+t_3$	0	$(k_i-t_i)+3\omega^H m+(t_1+t_2+t_3-3m)\omega^L$	$(k_i-t_i)+(t_1+t_2+t_3)\omega^H$
7	$\omega^H,\omega^H,\omega^L$	$(1-p)p^2$	$t_1+t_2+t_3$	0	$(k_i-t_i)+3\omega^H m+(t_1+t_2+t_3-3m)\omega^L$	$(k_i-t_i)+(t_1+t_2+t_3)\omega^H$
8	$\omega^H,\omega^H,\omega^H$	p^3	$t_1+t_2+t_3$	0	–	$(k_i-t_i)+(t_1+t_2+t_3)\omega^H$

Production quantities and payoffs to the agents for the realised values of the random variable ω.

Table 5 -Plc

	Realised ω	Probability	Majority	Public good	Private good	Payoff to agent L	Payoff to agent H
1	$\omega^L, \omega^L, \omega^L$	p^3		$3m$	$t_i\left(1-\frac{3m}{t_1+t_2+t_3}\right)$	$(k_i-t_i)+t_i\left(1-\frac{3m}{t_1+t_2+t_3}\right)+3\omega^H m$	–
2a	$\omega^L, \omega^L, \omega^H$	$(1-p)^2 p$	$t_1+t_2 \geq t_3$	$3m$	$t_i\left(1-\frac{3m}{t_1+t_2+t_3}\right)$	$(k_i-t_i)+t_i\left(1-\frac{3m}{t_1+t_2+t_3}\right)+3\omega^H m$	$(k_i-t_i)+t_i\left(1-\frac{3m}{t_1+t_2+t_3}\right)+3\omega^H m$
2b			$t_1+t_2 < t_3$	$t_1+t_2+t_3$	0	$(k_i-t_i)+3\omega^H m+(t_1+t_2+t_3-3m)\omega^L$	$(k_i-t_i)+(t_1+t_2+t_3)\omega^H$
3a	$\omega^L, \omega^H, \omega^L$	$(1-p)^2 p$	$t_1+t_3 \geq t_2$	$3m$	$t_i\left(1-\frac{3m}{t_1+t_2+t_3}\right)$	$(k_i-t_i)+t_i\left(1-\frac{3m}{t_1+t_2+t_3}\right)+3\omega^H m$	$(k_i-t_i)+t_i\left(1-\frac{3m}{t_1+t_2+t_3}\right)+3\omega^H m$
3b			$t_1+t_3 < t_2$	$t_1+t_2+t_3$	0	$(k_i-t_i)+3\omega^H m+(t_1+t_2+t_3-3m)\omega^L$	$(k_i-t_i)+(t_1+t_2+t_3)\omega^H$
4a	$\omega^L, \omega^H, \omega^H$	$(1-p)p^2$	$t_1 \leq t_3+t_2$	$t_1+t_2+t_3$	0	$(k_i-t_i)+3\omega^H m+(t_1+t_2+t_3-3m)\omega^L$	$(k_i-t_i)+(t_1+t_2+t_3)\omega^H$
4b			$t_1 > t_3+t_2$	$3m$	$t_i\left(1-\frac{3m}{t_1+t_2+t_3}\right)$	$(k_i-t_i)+t_i\left(1-\frac{3m}{t_1+t_2+t_3}\right)+3\omega^H m$	$(k_i-t_i)+t_i\left(1-\frac{3m}{t_1+t_2+t_3}\right)+3\omega^H m$
5a	$\omega^H, \omega^L, \omega^L$	$(1-p)^2 p$	$t_1 \leq t_3+t_2$	$3m$	$t_i\left(1-\frac{3m}{t_1+t_2+t_3}\right)$	$(k_i-t_i)+t_i\left(1-\frac{3m}{t_1+t_2+t_3}\right)+3\omega^H m$	$(k_i-t_i)+t_i\left(1-\frac{3m}{t_1+t_2+t_3}\right)+3\omega^H m$
5b			$t_1 > t_3+t_2$	$t_1+t_2+t_3$	0	$(k_i-t_i)+3\omega^H m+(t_1+t_2+t_3-3m)\omega^L$	$(k_i-t_i)+(t_1+t_2+t_3)\omega^H$
6a	$\omega^H, \omega^L, \omega^H$	$(1-p)p^2$	$t_1+t_3 \geq t_2$	$t_1+t_2+t_3$	0	$(k_i-t_i)+3\omega^H m+(t_1+t_2+t_3-3m)\omega^L$	$(k_i-t_i)+(t_1+t_2+t_3)\omega^H$
6b			$t_1+t_3 < t_2$	$3m$	$t_i\left(1-\frac{3m}{t_1+t_2+t_3}\right)$	$(k_i-t_i)+t_i\left(1-\frac{3m}{t_1+t_2+t_3}\right)+3\omega^H m$	$(k_i-t_i)+t_i\left(1-\frac{3m}{t_1+t_2+t_3}\right)+3\omega^H m$
7a	$\omega^H, \omega^H, \omega^L$	$(1-p)p^2$	$t_1+t_2 \geq t_3$	$t_1+t_2+t_3$	0	$(k_i-t_i)+3\omega^H m+(t_1+t_2+t_3-3m)\omega^L$	$(k_i-t_i)+(t_1+t_2+t_3)\omega^H$
7b			$t_1+t_2 < t_3$	$3m$	$t_i\left(1-\frac{3m}{t_1+t_2+t_3}\right)$	$(k_i-t_i)+t_i\left(1-\frac{3m}{t_1+t_2+t_3}\right)+3\omega^H m$	$(k_i-t_i)+t_i\left(1-\frac{3m}{t_1+t_2+t_3}\right)+\omega^H$
8	$\omega^H, \omega^H, \omega^H$	p^3		$t_1+t_2+t_3$	0	–	$(k_i-t_i)+(t_1+t_2+t_3)\omega^H$

Production quantities and payoffs to the agents for the realised values of the random variable ω.