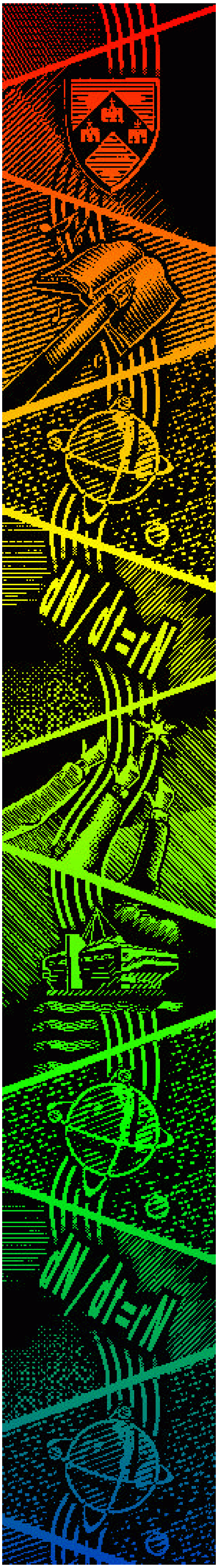

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Intersectoral Labour Reallocation and Employment Volatility: A Bayesian Analysis Using aVAR-GARCH-M Model

by

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Abstract

Based on the macroeconomic VAR model for total employment and sectoral employment shares developed by Campbell and Kuttner (1996) we extend the model to a multivariate ARCH in mean (ARCH-M) model. We investigate the question of whether volatile growth in sectoral employment shares has an impact on total employment. The estimation method we use is the Gibbs-Metropolis algorithm for a Bayesian vector ARCH (B-VAR) model. This model is a standard tool in financial econometrics and was developed by Engle (1986). The Bayesian estimation gives an exact small sample solution. It is found that a model incorporating a GARCH-M structure performs better than a simple VAR. Moreover sectoral shocks can account for more than 60% of the variance of total employment growth within a VAR-GARCH-M framework.

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1 Introduction

Since the appearance of Lilien's seminal paper (1982a) the study of the macroeconomic effects of reallocation shocks has flourished at an increasing pace. While theoretical contributions had a difficult take-off, empirical analysis has generated a massive amount of work. Much energy has been devoted to overcome the "observational equivalence" problem embedded in the use of Lilien's dispersion measure. Recently, Campbell and Kuttner (1996) have modelled the relationship between aggregate and sectoral employment explicitly using dynamic time series models. Our approach follows their lead but differs both in outlook and main purpose.

It is our aim to test for the presence of aggregate and sectoral shocks volatilities and to evaluate their relevance for sectoral shifts analysis. While carrying out this experiment we wish to start to develop a methodology which would bring to bear the potential nonlinearities inherent in sectoral shocks when they are modelled directly within a VAR structure. Methods for analysing these nonlinearities have been widely developed for financial markets econometrics (c.f. Bollerslev, Chou and Kroner, 1992; Campbell, Lo and MacKinlay, 1997) and it is then from this literature that we are borrowing the analytical tools necessary for implementing our experiment.

We extend previous research in two main directions: first, we structure the analysis in terms of a multivariate GARCH in mean (GARCH-M) model and our primary goal is to test for the presence of the implied volatilities and to evaluate the macroeconomic relevance of sectoral shocks; second, we frame our analysis in a Bayesian perspective using the Gibbs-Metropolis algorithm for a Bayesian vector ARCH (B-VAR) model ([1]) as estimation method and using Bayes factors for hypothesis testing; third, we use the same data set for aggregate and sectoral employment as Mills, Pelloni and Zervoyianni, MPZ henceforth, (1995) to see if our results can corroborate their findings.

We wish to point out that to the best of our knowledge, this is the first time that sectoral shifts analysis is developed with a multivariate ARCH framework. Moreover, this is also the first time, within or outside the boundaries of VAR-ARCH modelling, that empirical sectoral shifts analysis is approached from a Bayesian point of view. The combination of these two innovative steps has led to the extension of existing analytical techniques, such as the impulse response function and innovation analysis, to accomodate a B-VAR-GARCH-M structure.

In summary the value added of this paper to the existing literature is twofold [(2)]. On one side the non-linear impact of sectoral shifts on aggregate employment is explicitly introduced for the first time within a VAR framework by modelling shocks as ARCH processes. In doing so, we assimilate and extend Lilien's potentially heteroscedastic framework and model it so as to evaluate its macroeconomic effects. Though we do not provide a formal theoretical model to support our empirical investigation, we think there is a compelling intuition behind it (i.e. the size of sectoral shocks and its persistence can affect economic behaviour) and we hope that this paper may operate as a catalyst for future research. From a methodological point of view the use of a comprehensive and consistent Bayesian approach for estimation, model selection and innovation analysis, though convergent towards recent developments of macroeconometrics (Gordon and Boccanfuso, 1998), is new to this field and provides a frame for analytical advances.

The paper is organised as follows. In the next section we describe the framework that is used to model sectoral shocks non linearities, introduce notation and describe the data set. In section 3 we discuss the Bayesian methodology employed both for estimation and hypothesis testing. In section 4 we present the results obtained from modelling sectoral shocks volatility and its persistence using the VAR specification described in section 2. It also contains a discussion of these results and in particular of the explanatory power of allocative shocks for aggregate employment variations. Section 5 offers a few concluding remarks.

2 Sectoral shocks and volatility

2.1 The basic model

Lilien's dispersion hypothesis (1982a) claims that, in an economy with limited mobility of resources across markets (labour in particular), changes in the composition of employment demand will trigger a process of job reallocation which will affect aggregate employment. Lilien's (1982a) original turnover framework explicitly appeals to labour search as the underlying economic mechanism and Lucas and Prescott (1974) is invoked as the theoretical reference model. However, Lilien makes no attempt to frame his hypothesis within a fully developed theoretical model, but proceeds directly to work out the essential skeleton of a turnover structure where intersectoral shifts in demand composition operate as the basic driving force of unemployment fluctuations. Under reasonably mild assumptions he derives a reduced form unemployment equation characterized by a positive relationship between the unemployment rate and a measure of dispersion of employment demand conditions. For the empirical implementation Lilien proxies intersectoral dispersion with a weighted standard deviation of cross-sectoral employment growth rates.(3) He finds a strong positive correlation between U.S. unemployment rate and his dipersion measure, and interprets this result as evidence in favour of the sectoral shifts hypothesis. However this approach based on Lilien measure is open to severe criticism. In particular it has been pointed out that the sectoral shifts hypothesis embodies a problem of "observational equivalence" (Lilien, 1982b; Abraham and Katz, 1986). This problem arises because the positive correlation between unemployment and Lilien's dispersion proxy, instead of reflecting sectoral shocks, may be generated by aggregate shocks if cyclical responsiveness varies across sectors.

Different approaches have been proposed to overcome this difficulty in the measurement of sectoral shifts. These lines of research fall essentially into two classes: that using micro/panel data (Murphy and Topel, 1987; Loungani and Rogerson, 1989; Starr-McCluerr, 1993) and that based on aggregate data. This latter approach can itself be divided into four further subclasses: that exploring the correlation between observed dispersion and the vacancy rate (Abraham and Katz, 1986; Davis, 1987); the line of research based on constructing a dispersion index defined in terms of sectoral stock prices instead of sectoral employment (Loungani, Rush and Tave, 1990; Brainard and Cutler, 1993); the "purging" methodology (Lilien 1982b; Abraham and Katz, 1984; Neumann and Topel, 1991; MPZ, 1995, 1996), which is aimed at decomposing growth rates into a component measuring the sectoral response to aggregate shocks and a component measuring sector-specific factors; finally, the approach based on VAR systems free of Lilien's dispersion index which are aimed at modelling sectoral shocks directly ([4]). Prominent within this fourth subclass is the work by Campbell and Kuttner (CK henceforth).

CK construct a VAR system including the growth rates of aggregate employment and of manufacturing employment shares which they subsequently expand to include five more sectors. The underlying structure of their model is developed by imposing short-run and long-run identifying restrictions which explore their analysis of driving forces. The results of their analysis vary quite widely depending on the underlying restrictions. Sectoral shocks account for 6% of total employment variance under the short-run triangular bivariate system. While they can explain 82% of the latter under the long-run restriction when the bivariate system is extended to a seven dimensional system.

Since CK's model is linear and is characterized by a symmetric response of aggregate employment to sectoral shocks: a positive shock to the manufacturing sector will increase aggregate employment growth and vice versa for a negative shock. Thus CK assume that sectoral shocks operating like aggregate shocks in the sense that it is the shock's direction which counts in determining the aggregate response. This treatment of allocative disturbances somehow distorts the nature of sectoral shifts analysis ([5]). In fact, as stressed by Davis (1986), sectoral shocks cannot be seen as having a "positive-negative structure" but instead they should be understood as either "favourable" or "unfavourable" to the existing allocation of labour resources. In order to analyse the aggregate impact of labour market turbulence it is more pertinent to look at the magnitude than at the direction of allocative shocks. This stylized fact that the size of shocks matters independently of direction seems to suggest that sectoral shifts analysis should accommodate a non-linear framework. Thus it would be interesting to explore this specific feature of allocative shocks when modelling them directly.

In what follows, though we do not analyse the size of the shocks directly, we try to expand CK's approach to a non-linear model. Non-linearities may be introduced either through the mean or through the variance structure. A potential source of non-linearities can be identified in shocks volatility. In particular the analysis can be extended to allow for changes in the variance both of aggregate and sectoral shocks and explore how much their presence affects results.

The possible significance of aggregate shocks variability has long been recognized. For instance the so-called Lucas proposition (Lucas, 1973) claims that the effects of unanticipated nominal disturbances vary inversely with these shocks' volatility. Engle (1982, 1983), in developing ARCH models, also explored as well the inverse relationship between inflation variability and real output. Thus the inclusion of aggregate shocks volatility into this sort of analytical framework should, at least in principle, be uncontroversial.

In regard to the insight of including the volatility of a sector-specific shock, it seems to be an inherently cogent assumption to make when analyzing the aggregate effects of labour market turbulence. It is the essence of the sectoral shifts hypothesis that an uneven and idiosyncratic arrival process of information about the desired employment allocation across sectors would explain a large fraction of employment and unemployment variation. Because of this arrival pattern of news it is reasonable to expect that during a period of turbulence, large shocks may tend to be followed by large shocks. Basically we can expect that idiosyncratic information, reflecting changes in sector-specific fundamentals, reaches a sector in clusters, so that allocative shocks may present a profile of changing volatility which would persist also because of market dynamics in response to incoming news.

Our maintained hypothesis can be couched as follows. The sectoral shifts hypothesis claims that sector-specific shocks are reflected in changes of sectoral returns to human and physical capital and that because labour and capital are, at least in part, sector specific, movements between sectors will be time consuming. However in our view the pace of labour reallocation will be affected not only by large reallocation shocks but also by their persistence. Since we assume that large shocks of either sign will be followed by a large shock of either sign, the fact that the direction of labour reallocation may or may not change frequently is not a matter of concern for our model. What really matters is the presumption that the macroeconomic effects would emerge as a response to sizeable reallocation shocks and to a pattern of persistent volatility. Although we do not provide formal theoretical underpinnings to our insight, one can reasonably imagine that shocks' heteroscedasticity may have an influence on the decision rules of economic agents with a definite attitude towards risk and/or facing specific cost structures.

We reformulate Lilien's sectoral shifts hypothesis in terms of sectoral time series models containing volatility effects as follows:

a) The sectoral growth model with ARCH structure

Lilien's model (1982) is developed around a firm net hiring function characterised by a firm-specific component modelled as a random process with time-varying variance:

$$y_t^j = y_t + \varepsilon_{j,t}, \qquad t = 1, \ldots, T; \quad j = 1, \ldots, M \tag{1}$$

where $\varepsilon_{j,t} \sim (0, \sigma_t^2)$. Ignoring quits and letting the behaviour of a specific sector be reflected by the behaviour of its typical firm, equation (1) can be interpreted as the employment rate of change at sectoral level, which can be decomposed into an aggregate component common to all sectors, y_t, and a component, $\varepsilon_{j,t}$ specific to sector j. We can call (1) the sectoral growth model with heteroscedasticity. Given the state of the art at the time Lilien wrote its paper, he did not try to model his assumption of heteroscedasticity explicitly but used it to derive his intersectoral measure of labour mobility. Given the current state of econometric estimation, it is possible to add a proper dynamic dimension to Lilien's hypothesis of heteroscedasticity by explicitly modelling the variance driving process by using an ARCH structure. The hypothesis contained in equation (1) can then be enriched by introducing the following assumptions about sectoral heteroscedasticity:

$$\varepsilon_{j,t} = u_{j,t}\sqrt{h_t^j}, \tag{2}$$

$$h_t^j = \alpha + \sum_{i=1}^{q} \theta_i^j \varepsilon_{j,t-i}^2, \quad i = 1, \ldots, q, \tag{3}$$

$$\text{for } \alpha > 0, \theta_i \geq 0,$$

In this new specification of the model the sector-specific component is generated by equation (2), where $u_{j,t} \sim NiiD(0,1)$, $t = 1, \ldots, T$, and $h_t^j \equiv Var\left(\varepsilon_{j,t}|\mathrm{I}_{t-1}\right)$ is given by the process specified in (3). Thus the model suggests that the conditional variance, h_t, is a positive function of the past squared sectoral innovations, regardless of their signs, so that large errors tend to be followed by a large error and small errors by a small error. Modelling sectoral shocks as in (2) adds a new dimension to the model because it makes it possible to capture not only a time-varying variance but also a potential phenomenon of volatility clustering.

b) The sectoral growth model with ARCH-M effects

A further extension can be introduced by taking into account the possible impact of the conditional variance on the conditional mean. This can be accomplished by respecifying equation (1) as

$$y_t^j = y_t + \gamma_0^j + \sum_{i=1}^{r} \gamma_i h_{t-i}. \tag{4}$$

This assumption would entail the insight that the behaviour of economic agents, in the presence of reallocation shocks, would be affected not only by the size of these shocks but also by their volatile structure. For instance, risk averse agents could be inclined to move from high volatility sectors to low volatility ones. Equations (2), (3) and (4) provide then an ARCH in mean model for the employment growth rate of sector j.

c) A VAR-GARCH-M sectoral growth model

We base our analysis on the previous intuition. Assuming normal distributions, we have the model:

$$y_t \sim \mathrm{N}\left[\mu_t, \mathbf{H}_t\right], \qquad t = 1, \ldots, \mathrm{T}, \tag{5}$$

where y_t is a vector whose elements are the growth rates of aggregate employment and of sectoral employment shares. Thus the modelling strategy for our experiment is to implement a VAR model supplementing it with an ARCH structure:

$$y_t = \mu_t + \varepsilon_t = \beta_0 + \sum_{i=1}^{k} \mathbf{B}_i \mathbf{y}_{t-i} + \sum_{i=0}^{r} \mathbf{\Psi}_i h_{t-i} + \varepsilon_t \tag{6}$$

$$\text{vech } \mathbf{H}_t = \alpha_0 + \sum_{i=1}^{p} \mathbf{A}_i \text{ vech } \mathbf{H}_{t-i} + \sum_{i=1}^{q} \Theta_i \text{ vech } \left(\varepsilon_{t-i}\ \varepsilon_{t-i}^{'}\right) \tag{7}$$

In equations (6) and (7) we have an M-dimensional VAR(k)-GARCH (p,q)-M(r) process where y_t is a $(M \times 1)$ vector of observations of variables; $\mathbf{H}_t$ is a $(M \times M)$ diagonal conditional variance-covariance matrix; vech $\mathbf{H}_t$ is a $[(M(M+1)/2) \times 1]$ vector; h_t is a M-dimensional vector of conditional variances; ε_t is M-dimensional process of mutually and serially uncorrelated random errors and so vech $\left(\varepsilon_t\, \varepsilon_t^{'}\right)$ is a $(M(M+1)/2)$ dimensional vector; α_0 and β_0 are respectively $[(M(M+1)/2) \times 1]$ and $(M \times 1)$ vectors of time invariant intercept coefficients; $\mathbf{B}$,$\mathbf{\Psi}$,$\mathbf{A}$ and Θ are coefficient matrices, the first two are of dimension $(M \times M)$ while the other two have dimension $[(M(M+1)/2) \times (M(M+1)/2)]$; vech (.) denotes the column stacking operator for the elements of a symmetric matrix lying on and below the main diagonal.

We are using a variation of the multivariate GARCH-M model originally used in financial econometrics (Bollerslev, Engle and Wooldridge, 1988). According to our specification the conditional means are functions of the contemporaneous and lagged values of the conditional variances so as to verify whether the information content of the conditional variances is relevant in determining the estimates of the conditional mean values. In turn each conditional variance depends upon the past values of the squared shocks, its own lagged values and the lagged values of the conditional variances relative to the other equations.

We are suggesting that aggregate and sectoral shocks could display a changing (conditional) variance over time so that the larger the shocks experienced in the past, the larger the current volatility and its impact on behaviour. We could observe that a large change in variance this period will increase next period variance, thereby increasing the chance of a large shock in the next period.

Furthermore we wish to explore if volatility spillovers exist or if the conditional variance changes are sector-specific. Changes in conditional volatility could be brought about by shocks to sector-specific fundamentals without affecting other sectors volatility and mean values. Alternatively volatility changes in one sector could affect volatility changes in other sectors and influence their employment growth rates. Thus we wish to model sectoral shocks characterized by a changing, clustering volatility whose effects may spillover to other sectors.

We wish to stress that while Lilien's proxy was introduced to measure intersectoral employment dispersion and its macroeconomic effects, our attention is focused on adding an extra dynamic dimension when modelling reallocative shocks directly and on verifying if and how much this new class of models matters. No experiment has been implemented yet in order to quantify the volatility of sector-specific shocks and its relevance in explaining changes in sectoral employment shares and aggregate employment (unemployment) fluctuations. Thus in this respect our analysis somehow extends sectoral shifts analysis and goes beyond it.

2.2 The Prior

Our specification of the prior is aimed at introducing standard restrictions on the parameters of the mean and ARCH equations which should be fairly uncontroversial among researchers. Though we do not expect that everybody would agree with our initial beliefs, we think that a reasonable standard of consensus may exist about our assumptions.

First we introduce a ridge-type framework of the Litterman type so as to shrink the coefficients estimates toward zero and thus avoid problems of overfit when dealing with an unstructural VAR system. Second we impose standard stationarity conditions on the ARCH processes. More precisely, letting β denote the coefficients of the mean equations, we assume a shrinkage normal prior of the form $\beta^m \sim N\left(0, H_*^m\right)$, where $\beta^m$is the coefficient matrix and $H_*^m = diag\left(1, \frac{1}{2,}\ldots, \frac{1}{k}\right)$ is the covariance matrix. The assumption is that the further we move back in time the tighter is the distribution around zero so that our confidence of the expected value of the coefficients being zero increases as the lags become longer.

For the ARCH part of the model we use as a prior a truncated normal distribution so that $\alpha^m \sim N_0^{\infty}(\alpha_*^m, I_N/2)$, where $\alpha_*^m = 0.011_N$. This specification of the prior for the ARCH coefficients reflect the pure time series restriction of covariance-stationarity. We will not impose any further structure to the prior reflecting our prior belief of the relative importance of aggregate and sectoral components.

2.3 The data base

The crucial variables for our empirical analysis are total employment and the employment shares of the durable, nondurable, transport and manufacturing sectors so that we have a five-dimensional VAR. We shall let n_t^j denote the natural logarithm of aggregate employment when $j = 1$ and instead indicate the logarithms of employment shares in the durable, nondurable, transport and service sectors respectively when $j = 2, 3, 4, 5$. We take transformation $y_t^j = \triangle n_t^j = n_t^j - n_{t-4}^j = \log\left(N_t^j\right) - \log\left(N_{t-4}^j\right)$ so as to consider employment growth rates.

The results of our experiment are based on the United States quarterly employment data (1975Q1 - 1990Q4) from the OECD Data Base. This is the same data set used by MPZ (1995) for their experiment using dispersion measures. We resort to the same sample for aggregate and sectoral employment so as to check to see if our empirical analysis will confirm the support for the sectoral shift hypothesis that emerged from that work. By applying the same data set to a different model to investigate the same phenomenon, we subscribe to the methodological argument in Hendry, Leamer and Poirier (1990), according to which econometric modelling should be viewed as an incremental progressive accumulation of knowledge. The series Employment Civilian is the total employment in our model (n_t), and we also take the Durable Goods Manufacturing employment (the sum of S24, S25, S32, S33, S34, S35, S36, S37, S38 and S39), the Nondurable Goods Manufacturing employment (the sum of S20, S21, S22, S23, S26, S27, S28, S29, S30 and S31), the Transportation employment (the sum of S40, S41, S42, S44, S45, S46, S47), and the Services employment (the sum of SIC 70, 72, 73, 75, 76, 78-84, 86, 87 and 89) as the sector shares of employment.

3 The Econometric Methodology: Bayesian Analysis of a VAR-(G)ARCH-M Model

In this section we outline the econometric methodology we have followed in order to implement our experiment. A more extensive and detailed presentation of this methodology can be found in Polasek and Ren (1998).

3.1 Bayes-tests for VARCH-M models

In order to select the appropriate order of the VAR process and to choose between the linear and non-linear versions of our model we will use Bayes factors ([6]).

When comparing any two models the Bayes factor (BF) can be calculated using the marginal likelihood concept. In general terms letting y denote the relevant data set and θ_j be the appropriate set of parameters under model M_j, the marginal likelihood can be written:

$$f\left(y|M_j\right) = \int f\left(y|\theta_j, M_j\right) f\left(\theta_j|M_j\right) d\theta_j \tag{8}$$

If we denote by $f\left(y|\mathrm{M}_1\right)$, the marginal likelihood for model 1, and by $f\left(y|\mathrm{M}_2\right)$, the marginal likelihood for model 2, and if the two models are equally likely a priori, i.e. $\mathrm{P}\left(\mathrm{M}_1\right) = \mathrm{P}\left(\mathrm{M}_2\right) = 0.5$ we obtain the BF for M_2 versus M_1 by the ratio

$$\mathrm{BF}_{21} = \frac{f\left(y|\mathrm{M}_2\right)}{f\left(y|\mathrm{M}_1\right)}. \tag{9}$$

Using the usual rules for computing odds, we find the posterior probabilities for models M_1 and M_2 by suitable normalisations. In particular, one may want to use the so-called 9:19:99 rule for evaluating Bayes factors: $\mathrm{BF} > 9$ are remarkable, $\mathrm{BF} > 19$ are significant, and $\mathrm{BF} > 99$ are highly significant hypotheses.
Using logs we can transform this scheme to log-Bayes factors

$$\ln \mathrm{BF}_{21} = \ln f\left(y|\mathrm{M}_2\right) - \ln f\left(y|\mathrm{M}_1\right). \tag{10}$$

If M_1 is the best model and M_2 is the model under consideration one can use the differences of the log marginal likelihoods to judge the importance of models. Now $\ln f(y|M_2)$ is the log-marginal likelihood of model 2 and $\ln f(y|M_1)$ is the log-marginal likelihood of model 1. The above cut-off points for the log-BF are now: ln9=2.2, ln19=2.9, and ln99=4.6. Roughly speaking this means, we can use the numbers 2,3, and 4 to judge the improvements in model comparison by looking at the log marginal likelihoods.

Thus, in this paper we compare any two VARCH-M time series models of different orders by computing Bayes factors as the ratio of marginal likelihoods evaluated along the lines of Chib's decomposition (Chib, 1995).

It should be noted that equations (9) and (10) are meaningful if the marginal likelihoods can be interpreted as probabilities and the BF as the ratio of probabilities. In general that would be the case if the prior distribution is proper. However if the prior is improper then the marginal likelihood will also be improper and the BF will be meaningless. A wide literature aimed at handling this problem has recently emerged developing different types of BF [(7)]. When necessary in this paper we make use of posterior Bayes factors (Aitkin, 1992), computed as the ratio of the posterior means of the likelihood functions [(8)], or of fractional Bayes factors (O'Hagan, 1995), obtained by using the fractional marginal likelihood concept [(9)].

3.2 Unit root test with marginal likelihoods

Though not all Bayesian econometricians agree about pretesting for unit roots ([10]), we proceed to test the univariate properties of the relevant time series using marginal likelihoods. In this section we outline how the classical augmented Dickey-Fuller (DF) regression for unit roots can be used to calculate the marginal likelihoods for a Bayes test.

Let y_t be the time series we want to test for unit roots and let z_t be the first differences so that $z_t = y_t - y_{t-1}$. Following Dickey and Fuller's approach we consider four different autoregressive processes

Δ-AR (AR(p) for first differences):

$$z_t = \alpha_1 z_{t-1} + \ldots + \alpha_p z_{t-p} + u_t, \tag{11}$$

DF-AR (Dickey-Fuller regression):

$$z_t = \alpha_0 y_{t-1} + \alpha_1 z_{t-1} + \ldots + \alpha_p z_{t-p} + u_t. \tag{12}$$

DF-AR with mean:

$$z_t = \mu + \alpha_0 y_{t-1} + \alpha_1 z_{t-1} + \ldots + \alpha_p z_{t-p} + u_t \qquad (12a),$$

DF-AR with trend:

$$z_t = \mu + \beta t + \alpha_0 y_{t-1} + \alpha_1 z_{t-1} + \ldots + \alpha_p z_{t-p} + u_t \qquad (12b),$$

$$t = 1, \ldots, T; \qquad p = 1, \ldots, p_{max;}$$

Equation (11) defines the AR(p) non-stationary model in first differences in the sense that if the time series is I(1) the first differences should be a stationary AR(p) process. We denote this model as the $\triangle - \text{AR}$ model. Equations (12), (12a) and (12b) provide three alternative stationary models: the so-called Dickey-Fuller regression model (DF-AR model), Dickey-Fuller regression model with mean (DF-AR 2 model) and Dickey-Fuller regression model with mean and trend (DF-AR 3 model) respectively. We select the specification with highest probability according to the marginal likelihood criterion discussed in section 3.1. If the estimated marginal likelihood for the $\triangle$-AR model is higher than the marginal likelihoods of the alternative stationary models then the unit root hypothesis is supported. Of course if any of the alternative models is selected by the marginal likelihood, then the stationarity hypothesis is instead supported.

Thus in general using a fractional or an informative prior distribution, the marginal likelihoods for models (11) to (12b) can be calculated and the model (and the lag length) with the highest marginal likelihood will be chosen.

3.3 Estimation of the VARCH-M model

In this section we restrict our discussion to a diagonal system for the covariance matrix $\mathbf{D}_h = diag(h_1, \ldots, h_T)$ since previous estimates with a full covariance matrix have shown almost no significant contributions of the off-diagonal elements. In a compact way, the VARCH-M system can be written as

$$\mathbf{y}^m \quad \sim \quad N[\mathbf{X}^m \beta^m, \mathbf{D}_h^m = diag(h_{1m}, \ldots, h_{Tm})], \quad m = 1, \ldots, M \qquad (13)$$

with $h_{tm} = \mathbf{z}_{tm}^{'} \alpha_m$, where the $N \times 1$ vector $\mathbf{z}_{tm}$ contains the lagged variances and squared observation of the whole system and α_m is the ARCH parameter of the m-th equation.

The full conditional distributions (f.c.d.) are derived from the joint distribution of $\mathbf{Y}$ and the parameters for each equation $m = 1, \ldots, M$: a) The f.c.d. for β^m is obtained by a normal distribution

$$f(\beta^m | \alpha_m, \mathbf{Y}) = N[\beta^m_{**}, \mathbf{H}^m_{**}], \qquad (14)$$

with

$$(\mathbf{H}^m_{**})^{-1} = (\mathbf{H}^m_{*})^{-1} + \mathbf{X}^{m'} (\mathbf{D}_h^m)^{-1} \mathbf{X}^m, \qquad (15)$$

$$\beta^m_{**} = \mathbf{H}^m_{**} \left((\mathbf{H}^m_{*})^{--1} \beta^m_{*} + \mathbf{X}^{m'} (\mathbf{D}_h^m)^{-1} \mathbf{y}_m \right), \qquad (16)$$

where $\beta^m \sim N[\beta^m_*, \mathbf{H}^m_*]$ is the prior distribution for β^m, with $\beta_* = 0$, and the whole data set is given by $\mathbf{Y} = (\mathbf{y}^1, \ldots, \mathbf{y}^M)$.

b) The f.c.d. for the ARCH parameter α_m is obtained by a Metropolis step. The candidate distribution is a multivariate normal distribution where the mean is the old draw $\hat{\alpha}_m^{old}$ and the covariance matrix $\hat{\mathbf{D}}_h^m$ is obtained iteratively from smaller Metropolis runs (e.g. 3 times thousand runs) starting from the prior distribution

$$\alpha_m \sim N\left[\alpha_m^* = 0.01\mathbf{1}_N, \mathbf{I}_N/2\right]. \qquad (17)$$

The first runs from arbitrary prior distribution will have a bad accept/reject ratio, but after a few runs we calculate mean and variance matrix again and repeat the iterative proposal until a satisfactory accept/reject ratio is obtained.
If we define the current residual by $\varepsilon^m = \mathbf{y}^m - \mathbf{X}^m \beta^m$, then the f.c.d. is proportional to

$$f(\alpha_m | \beta^m, \mathbf{Y}) = |\prod_{t=1}^{T} h_t^m|^{--\frac{1}{2}} exp\left\{ -\frac{1}{2} \sum_{t=1}^{T} \frac{\varepsilon_{tm}^2}{h_{tm}} \right\}. \qquad (18)$$

Denoting the proposal distribution by $f(\alpha_m) = N[\hat{\alpha}_m, \hat{\sum}_m]$, where $\hat{\alpha}_m$ and $\hat{\sum}_m$ are the usual OLS estimators, then we accept a new value with probability $min(1, f(\alpha^{new})/f(\alpha^{old}))$.
This procedure is iterated over all equations until convergence.

3.4 Forecasting the VARCH-M model for impulse responses

1) Forecasting the future covariance matrices H_t

Using conditional expectations E_t for the error terms ε_t and the conditional matrices $\mathbf{H}_t$ we can calculate the future conditional covariance matrices (at time t for s step ahead and each sample point $m = 1, \ldots, M$)

$$\begin{aligned} vech\mathbf{H}_{t+s}^{(m)} &= \mathbf{a}_0^{(m)} + \sum_{i=1}^{q} \mathbf{A}_i^{(m)} vech E_t(\varepsilon_{t+s-i}\varepsilon_{t+s-i}^{'}) \\ &+ \sum_{j=1}^{p} \Theta_j^{(m)} vech\mathbf{H}_{t+s-j}^{(m)} \end{aligned} \tag{19}$$

with

$$E_t\varepsilon_{t+s} = \begin{cases} \hat{\varepsilon}_{t+s} & \text{for } s \leq 0, \\ 0 & \text{for } s > 0. \end{cases} \tag{20}$$

2) Forecasting future means

Using the previous results we can calculate the s-step ahead forecasts from the MCMC simulation of size M as:

$$\hat{\mathbf{y}}_{t+s} = \frac{1}{M}\sum_{m=1}^{M}\left[\beta_0^{(m)} + \sum_{i=1}^{k}\mathbf{B}_i^{(m)}E_t\hat{\mathbf{y}}_{t+s-i} + \sum_{i=1}^{r}\Psi_i^{(m)}vech\mathbf{H}_{t+s-i}^{(m)}\right], \tag{21}$$

$$E_t\mathbf{y}_{t+s} = \begin{cases} \mathbf{y}_{t+s} & \text{for } s \leq 0 \\ \hat{\mathbf{y}}_{t+s} & \text{for } s > 0. \end{cases} \tag{22}$$

3.5 The impulse response of the VARCH-M model

We consider the VAR(k) model with an ARCH in mean component

$$\mathbf{y}_t = \beta_0 + \sum_{i=1}^{k} \mathbf{B}_i \mathbf{y}_{t-i} + \sum_{j=1}^{r} \Psi_j vech\mathbf{H}_{t-j} + \varepsilon_t, \qquad t = 1, \ldots, T. \tag{23}$$

The impulse response function at time t is obtained by simulation in the following way (see e.g. Hamilton (1994, p.318)).

1) Set $\mathbf{y}_{t-1} = \mathbf{y}_{t-2} = \ldots = \mathbf{y}_{t-k} = \mathbf{0}$.
2) For the impulse in the j-th component set the j-th component to 1, i.e. $\varepsilon_{tj} = 1$ (or $\varepsilon_t' = (0, \ldots, 1, \ldots, 0)$), $\beta_0 = 0$, and $\varepsilon_{t+1} = \varepsilon_{t+2} = \ldots = 0$.
This procedure is a numerical approximation of the impulse response function of the non-linear vector time series model (24) which can be defined as the derivative of y_{t+s} with respect to the error term ε_t:

$$\frac{\partial y_{t+s}}{\partial \varepsilon_t'} = M_s, \quad s = 1, 2, \ldots \tag{24}$$

Having estimated the VAR-VARCH-M parameters via MCMC we can use the output of a MCMC run of length M to make a s-step ahead prediction at time t. We take the mean of the predictive distribution over the simulated sample:

$$\hat{\mathbf{y}}_{t+s} = \frac{1}{M} \sum_{m=1}^{M} \left[\beta_0^{(m)} + \sum_{i=1}^{k} \mathbf{B}_i^{(m)} \hat{\mathbf{y}}_{t+s-i} + \sum_{j=1}^{r} \Psi_j^{(m)} vech\mathbf{H}_{t+s-j}^{(m)} \right] \tag{25}$$

with

$$vech\mathbf{H}_t^{(m)} = \mathbf{a}_0^{(m)} + \sum_{i=1}^{q} \mathbf{A}_i^{(m)} vech(\varepsilon_{t-i}\varepsilon_{t-i}') + \sum_{j=1}^{p} \Theta_j^{(m)} vech\mathbf{H}_t^{(m)},$$

and $\beta_i^{(m)}$ and $\mathbf{H}_t^{(m)}$ is the m-th sample of the MCMC output.

The value of the vector $\hat{\mathbf{y}}_{t+s}$ at time $t+s$ of this simulation corresponds to the j^{th} column of the matrix M_s. By doing a separate simulation for impulses to each of the innovations ($j = 1, \ldots, n$), all of the columns of M_s can be calculated. In this case we use (19) and (21) but we replace (20) with

$$E_t \varepsilon_{t+s} = \begin{cases} \hat{\varepsilon}_{t+s} & \text{for } s < 0 \\ 1 & \text{for } s = 0 \\ 0 & \text{for } s > 0. \end{cases} \tag{26}$$

3.6 Approximate variance decomposition for the VAR-GARCH-M model

Given the Gibbs output of size $S, \left\{\theta^{(s)}\right\}, s = 1, \ldots, S$, the mean, variances and covariances of the forecasts for the VAR-ARCH-M model are given by

$$Ave(y_{t+1}^j) = \frac{1}{S}\sum_{r=1}^{S} y_{t+1}^{j(s)} = \bar{y}_{t+1}^j, \tag{27}$$

$$Var(y_{t+1}^j) = \frac{1}{S}\sum_{r=1}^{S} (y_{t+1}^{j(s)} - \bar{y}_{t+1}^j)^2, \tag{28}$$

and

$$Cov(y_{t+1}^j, y_{t+1}^m) = \frac{1}{S}\sum_{r=1}^{S} (y_{t+1}^{j(s)} - \bar{y}_{t+1}^j)(y_{t+1}^{m(s)} - \bar{y}_{t+1}^m)' \tag{29}$$

for all $j, m = 1, \ldots, M$.

Using $\left(\text{Var } \left(y_{t+1}^j\right)\right)$ we can calculate the contribution of the innovations to the j^{th}-variance of the one-step ahead forecast. This procedure can be generalised so as to compute the proportion of the j^{th} – variance due to each innovation for a one-step ahead forecast.

For a VAR model the impulse response function (IRF) is obtained from the MA-representation

$$y_t = \mu + \varepsilon_t + M_1\varepsilon_{t-1} + M_2\varepsilon_{t-2} + \ldots$$

and the M_i matrices can be obtained as in (24).

These impulse responses $\{M_{ij}, i, j = 1, \ldots, M\}$ are interpreted in the following way. If the residual ε_t is increased by one unit in the j-th component, then the value of the i-th variable at time $t + s\,(y_{i,t+s})$ is increased by M_{ij} (holding all other influences constant). Hamilton (1994, p.319) shows how these matrices can be obtained numerically. Set $y_{t-1} = y_{t-2} = \ldots = y_{t-p} = 0$ and choose $\varepsilon_t = e_j$ (the j-th unity vector), i.e. $\varepsilon_t = \varepsilon_{t+1} = \ldots = 0$ is set to zero. The value of the vector y_{t+s} at time $t + s$ of this simulation corresponds to the j-th column of the matrix M_s. The whole matrix M_s is obtained by running ε_t from e_1 to e_M over all unit vectors.

Since the impulse response function for the VAR-GARCH-M model cannot be given in closed form we obtain the IRF numerically in analogy to the VAR model. The variance decomposition is obtained in the following way:
1) We calculate the M_s matrices by choosing $\varepsilon_t = e_j$ in the VAR-GARCH-M model and calculate the forecasts y_{t+s}.
2) We calculate the covariance matrix $\sum = Var\left(y_{t+s}\right)$ from the s-step ahead predictions in (25) and we obtain from the Choleski decomposition the lower triangular matrix P, i.e. $\sum = PP'$.
3) For $h = 1, \ldots, h_{max}$ we calculate the orthogonalized impulse responses $A_{(h)} = M_h P$.
4) For the first equation we calculate, for $h = 1$, the relative variance decompositions. Denote the first row of $A_{(h)}$ by $\underset{\sim}{a}_1' = (a_{11}, \ldots, a_{1M})$ and the variance is given by $s_{11}^2 = \underset{\sim}{a}_1' \underset{\sim}{a}_1$. The relative variance decomposition is now $\left(a_{11}^2, \ldots, a_{1M}^2\right) / s_{11}^2$.
For $h = 2$ we need the matrices $A_{(1)}$ and $A_{(2)}$ and for the first equation we need the first rows $a_1^{(1)}$ and $a_1^{(2)}$. The variance is given by $s_{12}^2 = a_1^{(1)\prime} a_1^{(1)} + a_1^{(2)\prime} a_1^{(2)}$.

The relative variance decomposition now is given by

$$\left(a_{11(1)}^2 + a_{11(2)}^2, \ldots, a_{1M(1)}^2 + a_{1M(2)}^2\right) / s_{12}^2.$$

In this way we can proceed to the h-th equation obtaining

$$\sum_{i=1}^{h} \left(a_{11(i)}^2, \ldots, a_{1M(i)}^2\right) / s_{1h}^2$$

with

$$s_{1h}^2 = \sum_{i=1}^{h} \sum_{m=1}^{M} a_{1m(i)}^2, \quad h = 1, \ldots, h_{max}.$$

For the other equations we proceed in the same way as for the first equation.

4 Implementation and Results

4.1 Bayesian stationarity tests

Given the data set of section 2.2 we test for the univariate properties of the series. The output of the "Bayesian unit root test" in Table 1 should be interpreted along the lines of the methodology outlined in sections 3.1 and 3.2.

The tested time series, whose results are repeated in Table 1, are the growth rate of aggregate employment ("Total%") and the growth rates of the employment shares of the durable, nondurable, transportation and services sectors ("share%"). Previously, always applying the methodology of sections 3.1 and 3.2, we had tested the levels (i.e. $n_t^j = \log N_t^j$) of these variables and found all of them to be I(1).

Using Jeffrey's prior density we calculate the fractional maginal likelihoods for models (11) - (12b). Since our prior is improper the values reported in Table 1 are based on the fractional Bayes factors as defined by O'Hagan (1995) [11]. We see from the table that the marginal likelihoods are the largest for the stationary models with and without constants. The trend model and the first differences are never chosen. The shortest lag length is attained for the growth of total employment (lag 2), while the longest lag length is given for lag $p = 5$ and the shares of durable goods. The other 3 series exhibit the lag length 4 (note that these are quarterly data). Therefore we can conclude that the transformation to 4^{th} differences produces stationary time series.

Since unit root tests can be affected by structural breaks and outliers we run a Bayes test for a possible break point in the time series. Table 2 tests the unit root (differencing) model with unknown break point against the stationary DF regression with an unknown break point. Let us denote the break point with τ, then the break point model is

$$y_t = \begin{cases} \alpha_1^1 y_{t-1} + \ldots + \alpha_p^1 y_{t-p} + u_t^1 & \text{for } t \leq \tau, \\ \alpha_1^2 y_{t-1} + \ldots + \alpha_p^2 y_{t-p} + u_t^2 & \text{for } t > \tau, \end{cases} \tag{30}$$

and in a similar way for the augmented DF regression. We marginalise over all possible break points in the range of $p < \tau < \mathrm{T} - p$, i.e.

$$f(y|p) = \frac{1}{T - 2p} \sum_{\tau=p+1}^{T-p} f(p|y, \tau) \tag{31}$$

Comparing these marginal likelihoods for unit root tests with break points, we see that the stationary models for the DF-AR2 models (with trend) always turns out best. Also, by comparing the best DF-regression models in Tables 2 and 3, we see that the break point models with outliers are marginally favoured in a Bayesian test. This result is not "significant", but still surprising since the data exhibit a step change around 1983.

Additional to the break point model we can test for an outlier at an unknown location $t = 1, \ldots, \mathrm{T}$. This means we introduce a dummy variable $p_t = 1$ if t is the location of an outlier and $\mathrm{D}_t = 0$ for all other time points. The model for the $\triangle - \mathrm{AR}$ process is

$$y_t = \alpha_1 y_{t-1} + \ldots + \alpha_p y_{t-p} + \gamma \mathrm{D}_t + u_t \tag{32}$$

and for the augmented $\mathrm{DF} - \mathrm{AR1}$ regression

$$y_t = \alpha_0 n_{t-1} + \alpha_1 y_{t-1} + \ldots + \alpha_p y_{t-p} + \gamma \mathrm{D}_t + u_t \tag{33}$$

Similarly, we augment the $\mathrm{DF} - \mathrm{AR2}$ and the $\mathrm{DF} - \mathrm{AR3}$ regression model.

The marginal likelihood is the average over all unknown outlier points, i.e.

$$f(y|p) = \frac{1}{\mathrm{T}} \sum_{\tau=1}^{\mathrm{T}} f(y|p, \tau) \tag{34}$$

We see from Table 3 that the stationary (DF) outlier models always perform better than the non-stationary models. Surprisingly, all outlier models are slightly better than the original models in Table 1 or the break point model in Table 2. This leads us to the conclusion that the step changes observed in the beginning of the 80's are rather outlier effects than structural breaks.

4.2 Empirical results

In this section we present and discuss the main results emerging from estimating our five-dimensional VAR-GARCH-M model:

$$y_t^m = \beta_0^m + \sum_{j=1}^{5} \left(\sum_{i=1}^{k} \beta_i^{mj} y_{t-i}^j + \sum_{i=1}^{r} \psi_i^{mj} h_{t-i}^j \right) + \varepsilon_{mt} \tag{35}$$

$$h_t^m = \alpha_0^m + \sum_{j=1}^{5} \left(\sum_{i=1}^{p} \alpha_i^{mj} h_{t-i}^m + \sum_{i=1}^{q} \theta_i^{mj} \varepsilon_{m,t-i}^{2j} \right) \tag{36}$$

where m = 1, ..., 5.

The model has been estimated using the Gibbs-Metropolis algorithm for a Bayesian vector ARCH model. Because M = 5 is quite large a dimension, the seasonality is not picked up by a VAR(4), other coefficients of the VAR process have to make up the "case of the lost seasonality".

4.2.1 Testing the VAR-GARCH model

Model selection has been carried out using the marginal likelihood criterion outlined in section 3.1. Holding the prior distribution fixed we choose among alternative specifications of the likelihood by using Bayes factors. We use Chib's marginal likelihood decomposition for the calculation of posterior ordinates (Chib, 1995). Since the prior mean of the likelihood cannot be calculated in closed-form for the ARCH equations we always employ for them posterior marginal liklihoods. We wish to point out that only for the ARCH parameters we have to approximate the marginal likelihood ordinate by the ordinate of the posterior marginal likelihood; for the parameters of the mean equations we make use of the prior distribution in order to calculate this ordinate. Given that we use a data based prior as much as possible, the approximation will be quite good since our data based prior will be close to the posterior distribution.

From Table 4 we can see the marginal likelihood, dependent on k (the order of the VAR(k) model), p and q the order of the GARCH model and r the number of lags in the ARCH-M component. The best model (which we shall call model 2, M_2) is a VAR(2)-ARCH(2,2)-M(2) model with a marginal likelihood value of -244.50. We can compare the values in Table 4 with the marginal likelihoods of other models and perform a Bayes test. Let us first test if the VAR(1) model (top row of Table 4) is worse than the best reported VAR-GARCH model. The first row of Table 13 shows a marginal likelihood of -253.77 and the best remaining model is still the VAR(2)-ARCH(2,2)-M(2) model. Taking the difference yields $\log \mathrm{B}_{21} = 9.27$ ($\mathrm{B}_{21} \approx 10,615$). This means the VAR(1) model is 10,615 times less likely than the best VAR(2)-GARCH(2,2)-M(2) model.

If we compare the best VAR(2)-GARCH(2,2)-M(0) model, i.e. a model where there is no feedback from the variances to the mean of the time series, then the difference in log marginal likelihoods is $\log \mathrm{B}_{21} = 3.95$, which yields a Bayes factor of $\mathrm{B}_{12} = 0.0193$. This result implies that model M1 is about 51.94 times less likely than the best VAR-GARCH-M model. Thus the measure of relative support provided by the data for model 1 (M1) against model 2 (M2) is very strong against M1 in both cases.

Then the model has been run by imposing the following restrictions. We first consider the model where the first equation (aggregate employment) does not display an ARCH component while the other equations are characterized by an ARCH structure (restricted model 1). Then we reverse the experiment by imposing an ARCH component only for the first equation (restricted model 2). The results are summarised in Table (5) and Table (6).

Subject to the first restriction, the best model is again a VAR(2)-ARCH(2,2)-M(2) (Table 5). Calling this model M2, and comparing with the VAR(1) model, M1, we have a $\log \mathrm{B}_{21} = 10.64$, i.e. $\mathrm{B}_{21} \approx 41,773$. If we compare M2 with the VAR(2)-ARCH(2,2)-M(0), M1, model we obtain a logarithm of the Bayes factor for M2 versus M1 of 8.24 (i.e. $\mathrm{B}_{21} \approx 3,789$). The evidence provided by the data set would strongly favour the model incorporating the ARCH effects even when these are confined only to the sectoral components.

When we consider the model where ARCH effects are only present in the aggregate employment equation the best model is shown to be VAR(2)-ARCH(2,1)-M(2) (Table 6). Comparing as before with the VAR(1) we have a $\log \mathrm{B}_{21} = 8.18$ ($\mathrm{B}_{21} \approx 3,569$). When we compare against the best model without the "M-component" we observe a log-Bayes factor of 5.23 ($B_{21} \approx 187$). Thus the GARCH-M model is again supported when only the aggregate shocks display a volatile structure.

It should be noted that there is a drop in the value of the marginal likelihood of the best model when we move from the unrestricted model to the restricted ones and that the drop is larger for the model where the ARCH component is limited only to the first equation. This result seems to suggest that there would be a bigger loss of information if we were to ignore the volatility effects of sectoral shocks than those of aggregate disturbances.

4.2.2 Variance decomposition analysis

In this section we present the results of the forecast error variance decomposition obtained as described in section 3.6. The innovations have been triangularized according to a Choleski decomposition where aggregate employment has been ordered ahead of sectoral shares. This triangularization somehow betrays the essence of sectoral shifts analysis but it can be interpreted as a benchmark lower bound on the contribution of sectoral shocks to the explanation of total employment variance ([12]).

Sectoral shares are then introduced in the following order: durables, non-durables, services and transport.

Since we are interested in having aggregate employment ordered ahead of the sectoral shares, we have chosen one combination without attaching any particular economic interpretation to it.

Table 7 reports the results of the innovation accounting analysis carried out using the VAR(2)-ARCH(2,2)-M(2) model. The proportions of forecast error variance of aggregate employment growth accounted for by its own innovations and by innovations in the growth of sectoral shares in the first column are reported. If we consider a one-quarter horizon, the portion of the total variance in total employment due to reallocation shocks is approximately 30%. As s becomes larger the fraction of variance of total employment growth accounted for by sectoral reallocation shocks becomes larger. With a forecast horizon of four quarters, sectoral innovations contribute 65% to the aggregate employment variance. If we look at the forecast error variance of the sectoral components of the model, it should be noted that aggregate innovations cannot account for more than 28% of the sectoral variances whatever the forecast horizon. The one-step forecast error variance of sectoral components is largely (58-60%) accounted for by innovations within the own sector. However, as time is evolving the contributions of other sectors become larger. Thus, with a four-quarters horizon, other sectors innovations account for 53%, 34%, 41% and 47% of the variances of the durable, nondurable, transportation and services sectors respectively.

5 Summary and Conclusions

In this paper we have implemented a VAR-GARCH-M model to exlpore the macroeconomic effects of intersectoral labour reallocation. Our estimation method used the Gibbs-Metropolis algorithm for a Bayesian vector ARCH model which leads to an exact small sample distribution of the coefficients. The model selection has been carried out using the marginal likelihood criterion using Bayes factors for model comparisons. The ARCH structure has been introduced in order to capture the potential non-linearities due to shocks' volatilities which have been overlooked by previous studies also aimed at modelling sectoral shocks directly. Two major results have emerged. First, when comparing hypotheses, the computed Bayes factors suggest that the data support the VAR(2)-GARCH(2,2)-M(2) relative to the other models. This outcome indicates not only the presence of volatility clustering of the shocks, but also that the volatilities, feedback onto growth rates has to be taken into account.

Second, the innovation accounting analysis, carried out using a Cholesky decomposition where aggegate shocks are ordered ahead of sectoral innovations, shows that sectoral shocks account for approximately 65% of the total employment growth rate when we consider a one year forecast horizon. Thus, reallocative shocks, though we have embedded them in an unfavourable scenario, have a large and significant role in explaining aggregate employment behaviour.

The evidence in favour of sectoral shifts emerging from MPZ (1995), carried over the same sample period but with a different approach, is corroborated by our analysis. If we compare our findings with those of CK, taking into account the fact that they used a different sample, different sectoral decompositions and a different VAR modelling strategy, we find that our analysis provides stronger support to the role played by reallocative shocks in explaining aggregate employment behaviour. It seems that the ARCH structure can capture important characteristics of the system which seem to strengthen the role of sectoral disturbances. Thus from our analysis of the macroeconomic effects of reallocation shocks a new dimension emerges as a new characterizing feature of research in this field: Once the heteroskedasticity of sector-specific shocks is explicitly modelled, the potential relevance of sectoral shocks volatility surfaces as a source of non-linearity characterizing labour market turbulence.

Notes

(1) All the models can be estimated as part of the BASEL software package.
(2) A third potential contribution of this paper would emerge if it were interpreted as a test of "certainty equivalence" as well. In models displaying certainty equivalence, heteroscedasticity in the driving processes should not affect agents' decision rules. However, it is reasonable to conceive that certainty equivalence breaks down for firms operating under fixed employment adjustment costs or linear hiring - firing costs (Caballero, Engel and Haltiwanger, 1997; Campbell and Fisher, 1996). If our paper is viewed in this perspective then current and lagged variance terms should not "significantly" enter the employment growth equations if certainty equivalence holds.
(3) Lilien's famous dispersion proxy can be written in notation consistent with that used in this paper as follows: $\hat{\sigma}_t = \left[\sum_{i=1}^{M} (N_{jt}/N_t)(\triangle \log N_{jt} - \triangle \log N_t)^2\right]^{1/2}$, where N_{jt} is employment in sector j and N_t is aggregate employment. Lilien then estimates a reduced

form equation of the general form, $u_t = a + \sum_{i=1}^{T} b_i \sigma_{t-i} + \sum_{i=1}^{S} c_i x_{t-i} + \varepsilon_t$, where x is a vector of aggregate shocks and u is the unemployment rate.
(4) Long and Plosser (1987) can be seen as a forerunner of this approach.
(5) CK, on p.95 recognise how their symmetric treatment of sectoral shocks somehow departs from the standard view of sectoral shifts.
(6) For a discussion of Bayes factors c.f. Kass and Raftery (1995) and Poirier (1995). For Bayes factors and non-linear models see Koop and Potter (1997).
(7) Partial BF, local BF, pseudo BF, intrinsic BF, posterior BF, fractional BF, are the most recurrent concepts in the literature, c.f. Gelfand and Dey (1994), Kass and Raftery (1995), O'Hagan (1995) and the references therein.
(8) Using the same notation as in the main text, if we integrate with respect to the posterior we obtain the posterior mean of the likelihood function $f(y|M_j) = \int f(y|\theta_j, M_j) f(\theta_j|y, M_j) d\theta_j$, where $f(\theta_j|y, M_j)$ is the posterior distribution.
(9) Let us have a sample y of size n and a training sample of size n_1, where both n and n_1 are large, then for a a given fixed proportion $b = n_1/n$, it follows that $f(x|\theta_j, M_j) \approx [f(y|\theta_j, M_j)]^b$ and we can calculate the fractional marginal likelihood

$$f(b, y|M_j) = \frac{\int f(y|\theta_j, M_j) f(\theta_j|M_j) d\theta_j}{\int [f(y|\theta_j, M_j)]^b f(\theta_j|M_j) d\theta_j}$$

(10) See Phillips (1991) with discussion and Uhlig (1994).
(11) The results are qualitatively the same, if we would had used as an informative prior like the Minnesota prior (Litterman, 1993).
(12) CK adopt a similar device in order to implement their analysis.

Order	Total %			
p	$\Delta - AR$	$DF - AR\ 1$	$DF - AR\ 2$	$DF - AR\ 3$
1	190.1606	193.2524	193.2577	192.7290
2	191.4274*	195.4222*	197.7968**	194.3029*
3	191.2504	194.6288	197.1488	193.6504
Order	Durable goods(share %)			
p	$\Delta - AR$	$DF - AR\ 1$	$DF - AR\ 2$	$DF - AR\ 3$
1	148.8612	153.4889	154.1387	151.0513
2	149.2258	153.7205	155.9291	153.2917
3	149.5605	154.4872	160.2191	158.5116
4	161.9327	164.5190	165.4652	162.5935
5	163.2937*	165.3017*	165.8435**	162.7422*
6	162.6695	164.7458	165.7810	162.7358
Order	Nondurable goods(share %)			
p	$\Delta - AR$	$DF - AR\ 1$	$DF - AR\ 2$	$DF - AR\ 3$
1	178.3648	180.7102	185.5339	181.9984
2	178.5722	180.6508	184.6048	181.1417
3	182.8687	185.4732	190.9893	187.6614
4	185.2309*	187.2895*	191.2793**	187.8355*
5	185.2048	187.1330	191.0991	187.6716
Order	Transportation(share %)			
p	$\Delta - AR$	$DF - AR\ 1$	$DF - AR\ 2$	$DF - AR\ 3$
1	187.0468	191.5356	189.8160	186.3851
2	187.7452	193.3442	191.9324	188.7775
3	189.2631	196.9206	196.3297	195.6773
4	201.4560*	206.1169**	204.4229*	201.6194*
5	200.7137	205.2555	203.5438	200.7393
Order	Services(share %)			
p	$\Delta - AR$	$DF - AR\ 1$	$DF - AR\ 2$	$DF - AR\ 3$
1	194.2419	197.6030	199.4266	195.8750
2	195.0135	197.7052	201.7799	198.2570
3	195.4949	198.5406	204.1732	200.6853
4	208.4259*	210.4122*	212.5920**	208.9598*
5	208.1274	210.0818	211.0447	208.2900

Table 1. Bayesian stationarity test: The fractional log marginal likelihood of US aggregate employment and employment shares for AR(p) models from 1975 Q1 to 1990 Q4.

Order	Total %			
p	$\Delta - AR$	$DF - AR\ 1$	$DF - AR\ 2$	$DF - AR\ 3$
1	182.382	189.049	191.526	190.312
2	183.417	190.109	192.782	191.423
3	184.243*	190.906*	193.627**	192.002*
4	175.202	182.374	192.671	191.554
Order	Durable goods (share %)			
p	$\Delta - AR$	$DF - AR\ 1$	$DF - AR\ 2$	$DF - AR\ 3$
1	149.406	156.242	158.098	157.712
2	151.179	158.048	159.452	158.993
3	151.357*	158.071*	160.441**	159.762*
4	141.956	149.450	159.131	158.651
Order	Nondurable goods (share %)			
p	$\Delta - AR$	$DF - AR\ 1$	$DF - AR\ 2$	$DF - AR\ 3$
1	164.186	171.359	177.762	173.812
2	164.798*	171.976*	180.541**	174.011*
3	163.916	171.169	176.562	172.882
4	161.056	168.422	172.402	170.699
Order	Transportation (share %)			
p	$\Delta - AR$	$DF - AR\ 1$	$DF - AR\ 2$	$DF - AR\ 3$
1	188.646	196.097	194.412	191.891
2	190.237	196.141	195.102	192.512
3	190.970*	196.632**	195.982*	193.023*
4	181.075	189.082	193.674	191.086
Order	Services (share %)			
p	$\Delta - AR$	$DF - AR\ 1$	$DF - AR\ 2$	$DF - AR\ 3$
1	186.263	193.917	196.812	190.562
2	187.406*	194.632*	197.561**	191.677*
3	186.119	193.600	195.672	192.078
4	172.090	181.0447	193.007	191.229

Table 2. Bayesian stationarity test with break point: The fractional log marginal likelihood of US aggregate employment and employment shares for the heteroskedastic break point AR(p) models from 1975 Q1 to 1990 Q4.

(* maximum marginal likelihood)

Order	Total %			
p	$\Delta - AR$	$DF - AR\ 1$	$DF - AR\ 2$	$DF - AR\ 3$
1	192.5412	194.4611	194.7620	193.8298
2	193.6752*	196.7521*	198.5711**	195.7281*
3	192.6751	195.7819	197.6519	194.7820
Order	Durable goods (share %)			
p	$\Delta - AR$	$DF - AR\ 1$	$DF - AR\ 2$	$DF - AR\ 3$
1	157.8778	161.7372	162.6374	160.5621
2	158.7281	162.7762	164.7821	163.8289
3	156.2921	162.6721	169.6723	171.0732
4	173.7720	175.8820	178.6271	177.7627
5	174.8965*	177.8044*	179.5620**	178.6237*
6	173.9071	176.4152	179.6367	177.6121
Order	Nondurable goods (share %)			
p	$\Delta - AR$	$DF - AR\ 1$	$DF - AR\ 2$	$DF - AR\ 3$
1	186.0921	188.6218	195.6728	195.2031
2	186.7823	189.6327	197.8881	196.7821
3	189.6729	193.8812	198.7873	197.8825
4	193.6725*	195.7865*	199.7275**	198.7790*
5	192.7848	194.7761	196.1721	196.8817
Order	Transportation (share %)			
p	$\Delta - AR$	$DF - AR\ 1$	$DF - AR\ 2$	$DF - AR\ 3$
1	192.8092	195.7288	193.8856	190.7372
2	193.1928	197.7812	195.9921	192.6734
3	195.7063	201.7932	199.8328	200.8832
4	207.4625*	212.6831**	209.7828*	206.7221*
5	206.0833	210.7742	208.4542	206.8827
Order	Services (share %)			
p	$\Delta - AR$	$DF - AR\ 1$	$DF - AR\ 2$	$DF - AR\ 3$
1	197.8281	201.6732	204.7892	201.8837
2	198.9983	203.9588	208.9822	205.7882
3	199.8389	205.8943	211.5625	208.7872
4	217.0893*	218.8222*	224.6754**	220.5673*
5	216.7822	218.02112	223.8972	219.5642

Table 3. Bayesian stationarity test with break point and outliers: The fractional log marginal likelihood of US aggregate employment and employment shares for AR(p) models with outliers from 1975 Q1 to 1990 Q4.

k	p	q	r	full model	restricted model 1	restricted model 2
1	0	0	0	-253.77	-261.43	-263.54
1	1	1	1	-246.22	-253.91	-257.33
1	2	3	1	-244.72	-252.17	-258.76
2	1	1	2	-245.33	-253.09	-257.13
2	1	2	1	-245.53	-255.08	-257.58
2	1	3	2	-246.75	-258.44	-260.08
2	2	1	2	-247.22	-256.91	-255.36^{*}
2	2	2	0	-248.45	-259.03	-260.59
2	2	2	1	-245.76	-256.90	-256.56
2	2	2	2	-244.50^{*}	-250.79^{*}	-256.48
2	3	3	2	-247.89	-254.09	-257.68
3	3	3	3	-246.11	-255.12	-258.90
3	3	3	2	-246.10	-257.87	-260.08
3	1	1	2	-245.45	-258.43	-261.2
3	1	1	3	-245.09	-257.33	-260.31
3	1	2	3	-248.81	-258.08	-260.91
3	1	2	2	-249.87	-259.09	-260.88
3	1	3	2	-246.54	-260.32	-261.04
3	1	3	3	-249.78	-260.55	-261.68

Table 4. The log marginal likelihood for VAR(5)-ARCH(p,q)-M(r) model

Time	Variance decomposition in				
Quarter	Total	Durable	Nondur-able	Transpor-tation	Services
1	0.6959	0.2206	0.0361	0.1849	0.1685
	0.0459	0.5876	0.2634	0.0297	0.0556
	0.1609	0.1237	0.6015	0.1683	0.1391
	0.0852	0.0532	0.0881	0.6085	0.0572
	0.0121	0.0149	0.0109	0.0085	0.5795
2	0.5165	0.1678	0.0011	0.1824	0.1865
	0.1090	0.4620	0.1047	0.1066	0.0347
	0.2482	0.2431	0.4534	0.2599	0.0653
	0.1246	0.1257	0.1320	0.4480	0.0911
	0.0018	0.0014	0.3088	0.0031	0.6223
3	0.3835	0.1884	0.2573	0.2781	0.1780
	0.1183	0.2894	0.1297	0.1314	0.1195
	0.2027	0.2074	0.3099	0.2046	0.1785
	0.2151	0.2335	0.2273	0.2960	0.2349
	0.0804	0.0812	0.0759	0.0899	0.2891
4	0.3485	0.2166	0.2442	0.2621	0.2401
	0.0276	0.2578	0.0276	0.0316	0.0197
	0.2916	0.2389	0.4160	0.2848	0.2102
	0.2417	0.2285	0.2199	0.3246	0.2371
	0.0905	0.0583	0.0924	0.0969	0.2929

Table 5. Variance decomposition of US employment for VAR-GARCH-M model in percentage

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