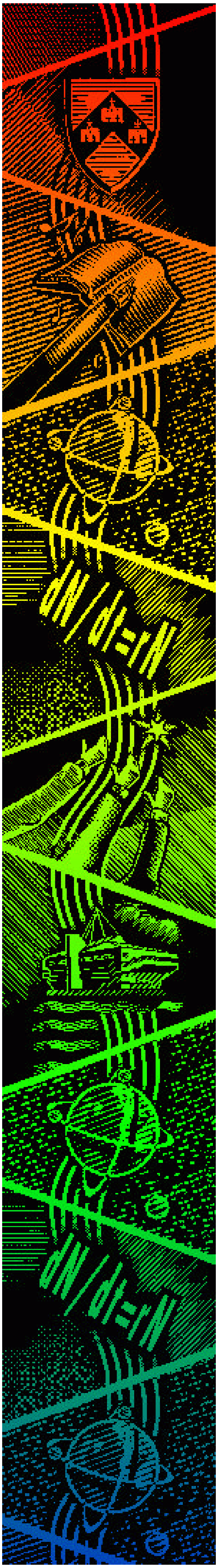

THE UNIVERSITY *of York*

Discussion Papers in Economics

No. 1999/01

Dual Labour Markets and Menu Costs: Explaining the Cyclicality of Productivity and Wage Differentials

by

Huw D Dixon, Claus Thustrup Hansen and Henrik Jacobson Kleven

Department of Economics and Related Studies
University of York
Heslington
York, YO10 5DD

Dual Labour Markets and Menu Costs: Explaining the Cyclicality of Productivity and Wage Differentials*

Huw David Dixon
University of York and CEPR

Claus Thustrup Hansen
University of Copenhagen and EPRU†

Henrik Jacobsen Kleven
University of Copenhagen and EPRU†

June 1999

Abstract

The conventional menu cost framework performs poorly with realistic labour supply elasticities; the menu costs required for price rigidity are very high and the welfare consequences of monetary disturbances are negligible. We show that the presence of dual labour markets greatly improves the performance of the framework both by reducing menu cost requirements and by boosting the welfare consequences. In addition, the introduction of dual labour markets provides an explanation of procyclical productivity and the shrinking of wage differentials during booms, in line with stylized facts on business cycles.

Keywords: Nominal Rigidity, Dual Labour Markets.
JEL Classifications: E30, J22, J42.

*We would like to thank Peter Birch Sørensen for helpful suggestions. Comments by seminar participants at the University of Copenhagen are also gratefully acknowledged.

†The activities of EPRU (Economic Policy Research Unit) are financed through a grant from the Danish National Research Foundation.

1 Introduction

One of the main points of the New Keynesian literature is that nominal inertia of prices and wages can be explained in an imperfectly competitive setting by the presence of lump-sum costs associated with price/wage changes (Mankiw, 1985; Blanchard and Kiyotaki, 1987), or due to bounded rationality (Akerlof and Yellen, 1985). With price or wage setters, nominal prices are expected to be at or near to the optimal levels, with no first order effects of price changes on payoff. Furthermore, with imperfect competition – implying a socially suboptimal equilibrium – nominal inertia gives rise to first order welfare effects as output and employment vary. Thus, nominal disturbances may generate large welfare effects due to the presence of small menu costs. Whilst the argument is simple and intuitive, the models run into a fundamental problem: the menu costs required are too large for realistic values of the labour supply elasticity. The weight of empirical evidence is that the labour supply elasticity is very small (Pencavel, 1986). If this is so, then an increase in employment will result in a significant upward pressure on wages which will bear directly on prices. Thus, in Blanchard and Kiyotaki (1987), with a zero elasticity of labour supply no finite level of menu costs can lead to nominal rigidity.

In addition, even if menu costs do result in a nominal shock increasing output, these models are still unable to explain the observed cyclicality of productivity and wage differentials. First, productivity is either acyclical or countercyclical depending on whether production exhibits constant or diminishing returns to scale, thus conflicting with the stylized fact that demand disturbances yield procyclical productivity (e.g., Rotemberg and Summers, 1990). Second, since labour markets are assumed to be completely symmetric, all wages are identical. Therefore, the models cannot capture the compression of wage differentials during booms as indicated by several empirical studies (see Melow, 1981; Freeman, 1984; Wunnava and Honney, 1991).

We believe that these shortcomings reflect the specific nature of the existing models rather than being a general feature of New Keynesian Economics.

In particular, the use of the representative sector framework, implying that all sectors are identical, is inimical to the explanatory power of menu costs. In this paper we focus on the presence of dual labour markets, which has become an established part of labour market theory since Harris and Todaro (1970) and is confirmed by a substantial amount of empirical evidence; see Dickens and Lang (1985) and the survey in Saint-Paul (1996, pp. 62-68). In our framework, there is a *primary* labour market with monopolistic wage setting and a *secondary* labour market characterized by perfect competition. Thus, in equilibrium there is a wage differential in favour of primary labour. The output market has a representative monopolistic sector, so that the special case of no secondary labour market corresponds to the Blanchard and Kiyotaki model.

The dual labour market equilibrium is characterised by two distortions: the aggregate *level* of employment is too low and the *allocation* of employment is distorted away from the primary labour market. The level effect is also present in representative sector frameworks, whereas the allocation effect is the result of labour market dualism. Labour market duality improves the performance of the menu cost model for realistic values of the labour supply elasticity, and this is exactly due to the allocation effect. We look at what happens in response to a monetary expansion in the presence of menu costs in both output markets and labour markets (as in Blanchard and Kiyotaki, 1987). We find that overall the level of menu costs required as a percentage of GDP is smaller with dual labour markets, and that the welfare consequences of nominal disturbances are larger. Furthermore, in accordance with empirical evidence wage differentials are countercyclical and productivity may be procyclical. Whilst there are other papers which explain procyclical productivity within the menu cost framework (Basu, 1995; Dixon and Hansen, 1999), these papers are unable to address the additional issue of wage compression during booms.

All the results stem from the same underlying interaction between output and employment in the two labour markets. It is well-known that small menu costs cannot prevent price or wage changes in a competitive market, implying that secondary wages are pushed up following a positive demand shock. By contrast if menu costs are sufficiently large, workers in the primary labour market choose to satisfy the extra demand instead of raising wages. Hence, the wage differential in favour of primary labour shrinks, and employment in the primary market increases relative to employment in the secondary market. Since productivity is higher in the primary market, such a change in the allocation of employment increases overall productivity, thereby boosting both GDP and welfare.

Finally, the menu costs needed for wage rigidity in the labour market decrease. To grasp the intuition for this result, note that in the standard framework an increase in production can come about only by an increase in aggregate employment, whereas in our framework there is the additional possibility of labour reallocation. Since it is less costly for the households to reallocate labour than it is to expand the aggregate number of work hours, the costs of keeping wages fixed are lower in the dual labour market case, thereby yielding lower menu cost requirements of workers. By contrast, the menu cost requirements of firms increase because of higher wages in the secondary market following a monetary expansion. However, the aggregate menu cost requirement of firms and workers is reduced by the introduction of a dual labour market.

The remainder of the paper is organized in the following way. In Section 2 we outline the basic structure of the model and the properties of the equilibrium with fully flexible prices. In Section 3 we look at the cyclicality of welfare, productivity, and wage differentials in the presence of menu costs for both wage and price setters. In both sections the model either has a closed form solution or standard approximations are used so that a quantitative evaluation of the model can be undertaken. Finally, Section 4 concludes.

2 The Model

A simple dual labour market, along the lines of Calvo (1978), is incorporated in a standard menu cost setting. To maintain the representative agent framework, we assume that each household works in two separate labour markets; (i) a primary market characterized by monopolistic competition and (ii) a secondary market characterized by perfect competition. In the first market, the wage is set by the household who is the only supplier of a particular skill. In doing so, the household takes into account that it is possible to work in the secondary market at the competitive wage.

The firms produce differentiated goods using labour input both from the primary and from the secondary labour market. Thus, the model is one of *intra*sectoral dualism, where high-wage jobs coexist with low-wage jobs within each firm. However, the model can easily be reinterpreted as one of *inter*sectoral dualism, where wages differ across sectors.

The relative importance of the two labour markets is determined by a parameter, η. Apart from a few trivial simplifications,[1] our framework has the Blanchard and Kiyotaki (1987) model as the special case of no secondary labour market, corresponding to $\eta = 1$. Furthermore, to facilitate comparison with the Blanchard and Kiyotaki framework we use the same notation.

2.1 Households

There is a continuum of households $j \in [0, 1]$. Each household j obtains the following utility level

$$U_j = \left(\int_{i=0}^{1} C_{ij}^{(\theta-1)/\theta} di \right)^{\theta/(\theta-1)} - N_j^{\beta}, \quad \beta > 1, \quad \theta > 1. \tag{1}$$

[1] We exclude money from the utility function and assume instead like Ball and Romer (1989, 1990, 1991) that a simple transactions technology determines the relation between aggregate spending and real money balances. We also assume that goods and workers are distributed on the unit interval. This removes a lot of constants from the Blanchard and Kiyotaki framework and avoids the critique of the Dixit-Stiglitz framework put forward by d'Aspremont et al. (1996).

The first term is a composite good consisting of a continuum of differentiated goods, C_{ij}, with an elasticity of substitution between any two goods equal to θ. The second term represents disutility of work, N_j, where the elasticity of marginal disutility of work, $\beta - 1$, is positive. The budget constraint of household j is given by

$$\int_{i=0}^{1} P_i C_{ij} di \leq W_j^P N_j^P + W^S N_j^S + \int_{i=0}^{1} V_{ij} di \equiv I_j, \tag{2}$$

where P_i is the price of good i, W_j^P and W^S are the wages in the primary and secondary labour market, respectively, N_j^P and N_j^S are the employment of the household in the two markets, and V_{ij} is the household's (lump sum) share of profits from firm i. Total employment of the household is given by $N_j = N_j^P + N_j^S$. Maximizing (1) with respect to C_{ij} subject to (2) gives the demand for good i of household j

$$C_{ij} = \left(\frac{P_i}{P}\right)^{-\theta} \frac{I_j}{P}, \tag{3}$$

where P is the consumer price index given by

$$P \equiv \left(\int_{i=0}^{1} P_i^{1-\theta} di\right)^{1/(1-\theta)}. \tag{4}$$

By using (1) through (4) we obtain the indirect utility function

$$S_j = \frac{W_j^P N_j^P + W^S N_j^S + \int_{i=0}^{1} V_{ij} di}{P} - \left(N_j^P + N_j^S\right)^{\beta}. \tag{5}$$

The household maximizes this expression with respect to N_j^S and W_j^P taking into account a negative relationship between wage and employment in the primary market. The first order conditions can be written in the following way

$$\frac{W^S}{P} = \beta \left(N_j^P + N_j^S\right)^{\beta - 1}, \tag{6}$$

$$\frac{W_j^P}{P}(1-\nu) = \beta \left(N_j^P + N_j^S\right)^{\beta - 1}, \tag{7}$$

where $\nu \equiv -\frac{\partial W_j^P}{\partial N_j^P}\frac{N_j^P}{W_j^P}$ is the degree of monopoly power measured by the Lerner index in the primary market. Thus, the household sets the primary wage as a mark-up over the marginal disutility of work, whereas the supply of labour in the secondary market is set such that the marginal disutility of work equals the real consumption wage in that market.

Finally, we assume that some transactions technology (e.g. a cash-in advance constraint) determines the relation between aggregate spending of the households and money balances

$$\int_{j=0}^{1} I_j dj = M. \tag{8}$$

2.2 Firms

The technology of firm i is described by

$$\begin{aligned} Y_i &= \left(\left(N_i^P\right)^{\eta}\left(N_i^S\right)^{1-\eta}\right)^{1/\alpha}, \qquad \alpha \geq 1, \quad 0 \leq \eta \leq 1, \\ N_i^P &= \left(\int_{j=0}^{1}\left(N_{ij}^P\right)^{(\sigma-1)/\sigma} dj\right)^{\sigma/(\sigma-1)}, \quad \sigma > 1, \end{aligned} \tag{9}$$

where N_{ij}^P is employment of worker j hired in the primary labour market, N_i^S is input hired in the secondary labour market, σ measures the degree of substitution between any two types of skill in the primary labour market, and α is a returns to scale parameter. The parameter η determines the relative importance of primary and secondary employment, where the limits correspond to having only one labour market characterized either by perfect competition ($\eta = 0$) or monopolistic competition ($\eta = 1$) as in Blanchard and Kiyotaki (1987).

Cost minimization implies that the firm demands labour according to

$$N_{ij}^P = \left(\frac{W_j^P}{W^P}\right)^{-\sigma} \eta \frac{WY_i^{\alpha}}{W^P}, \quad N_i^S = (1-\eta)\frac{WY_i^{\alpha}}{W^S}, \tag{10}$$

where

$$W \equiv \left(\frac{W^P}{\eta}\right)^{\eta}\left(\frac{W^S}{1-\eta}\right)^{1-\eta}, \quad W^P \equiv \left(\int_{j=0}^{1}\left(W_j^P\right)^{1-\sigma} dj\right)^{\frac{1}{1-\sigma}}. \tag{11}$$

By using equations (9) and (10) we can write profits as

$$V_i = P_i Y_i - W^P N_i^P - W^S N_i^S, \tag{12}$$

where the price and output of firm i are constrained by the aggregate demand of the households, derived from (3) and (8), i.e.

$$Y_i = \left(\frac{P_i}{P}\right)^{-\theta} \frac{M}{P}. \tag{13}$$

Maximizing profits subject to (9) and (13) yields the following price-setting rule of the firm

$$\frac{P_i}{P} = \left[\frac{\theta\alpha}{\theta - 1}\frac{W}{P}\left(\frac{M}{P}\right)^{\alpha-1}\right]^{\frac{1}{1+(\alpha-1)\theta}}. \tag{14}$$

2.3 General Equilibrium

It follows from (10) that the Lerner index, ν, in (7) is equal to $1/\sigma$. By using this relationship as well as the symmetry property $W_j^P = W^P$, we can combine the equations (6) and (7) so as to get

$$\frac{W^P}{W^S} = \frac{\sigma}{\sigma - 1}, \tag{15}$$

which determines the relative wages (RW) in the two labour markets as a function of the degree of substitution among workers in the primary labour market. The symmetry of the model and equation (10) determine a corresponding equation for the relative demand for labour (RD):

$$\frac{N^P}{N^S} = \frac{\eta}{1-\eta}\left(\frac{W^P}{W^S}\right)^{-1}. \tag{16}$$

The equations (15) and (16) are displayed in Figure 1, which illustrates the implications of having the dual labour market. The equilibrium is determined by the intersection of RW and RD given by point A. This solution is inefficient; the household has the same disutility of work in the two markets and so the efficient relative wage is equal to one, yielding the efficient relative employment at point C. Thus, the presence of a dual labour market implies that too much input is allocated to the secondary market relative to the primary market. This misallocation of labour input yields the welfare loss indicated by the shaded area in Figure 1.

< Figure 1 >

In addition to this misallocation effect, the model also captures the standard adverse effect of monopolistic competition on the aggregate level of employment. This is apparent if we derive the aggregate employment of the households. Equations (6), (8), (9), (10), (14), (15), (16), and $P_i = P$ imply (see appendix)

$$N = \left(\left(\frac{\theta - 1}{\theta} \right)^{\alpha} \left(\frac{\sigma - 1}{\sigma} \right)^{\eta} \left(\frac{\sigma - \eta}{\sigma} \right)^{\alpha - 1} \right)^{\frac{1}{\alpha\beta - 1}} \hat{N} \leq \hat{N}, \tag{17}$$

where $\hat{N} \equiv \left((\alpha\beta)^{-\alpha} (1 - \eta)^{1-\eta} \eta^{\eta} \right)^{\frac{1}{\alpha\beta - 1}}$ is the efficient (Walrasian) employment level. It is evident from this equation that equilibrium employment is below its efficient level and that the difference is larger when goods are less substitutable (smaller θ), when the skills of the workers are less substitutable (smaller σ), and when the labour supply is more elastic (smaller β). A higher η also reduces employment, since this expands the relative size of the labour market characterized by monopolistic wage setting. Finally, note that aggregate employment is at the efficient level when the labour supply is completely inelastic, corresponding to $\beta \to \infty$.

Thus, we have two distortions in the labour market: (i) The *level* of employment is too low due to imperfect competition in both output and labour markets, and (ii) the *allocation* of employment is distorted in favour of the secondary market due to the excessive primary wage. Both distortions are reflected in aggregate consumption/GDP. This is seen by combining equations (6), (8), (9), (10), (14), (15), (16), and $P_i = P$, thereby yielding (see appendix)

$$C \equiv \int_{i=0}^{1} \frac{P_i Y_i}{P} di = \left(\frac{\theta - 1}{\theta} \left(\frac{\sigma - 1}{\sigma} \right)^{\eta\beta} \left(\frac{\sigma - \eta}{\sigma} \right)^{1-\beta} \right)^{\frac{1}{\alpha\beta - 1}} \hat{C} \leq \hat{C}, \tag{18}$$

where $\hat{C} \equiv \left(\alpha\beta\eta^{-\eta\beta}(1-\eta)^{\beta(\eta-1)}\right)^{\frac{1}{1-\alpha\beta}}$ is the efficient level of consumption. Equation (18) shows that aggregate consumption is below the efficient level and – like to employment – consumption depends positively on the parameters θ, σ and β. To see the role of labour market dualism for aggregate consumption, let us look at the special case where labour supply is completely inelastic:

$$\lim_{\beta\to\infty} C = \left(\frac{\sigma-1}{\sigma}\right)^{\eta/\alpha}\left(\frac{\sigma}{\sigma-\eta}\right)^{1/\alpha}\hat{C} \leq \hat{C}. \qquad (19)$$

This equation highlights the difference between a standard menu cost framework, obtained by setting η equal to 0 or 1, and the generalized model with $0 < \eta < 1$. In the former case, the equilibrium level of consumption is at the efficient level when labour supply is inelastic, whereas in the latter case this is no longer sufficient for the equilibrium to be a social optimum. The presence of a dual labour market implies that productivity in the primary labour market is higher than productivity in the secondary labour market. Thus, by reallocating workers from secondary employment to primary employment the economy would reap a productivity gain, thereby increasing GDP and consumption without inflicting more disutility of work on the households. Clearly, such a reallocation increases welfare, and therefore the market equilibrium is not a social optimum even with inelastic labour supply.

Table 1 provides numerical examples of general equilibria. In the first three rows β is 1.6, corresponding to a wage elasticity of labour supply equal to $1\frac{2}{3}$, the most inelastic case considered by Blanchard and Kiyotaki (1987). In the first row we consider the conventional one sector framework, i.e. $\eta = 1$, and use a constellation of α, σ, and θ also examined by Blanchard and Kiyotaki (1987). Table 1 also displays the average (macro) degree of imperfect competition in the primary and secondary labour markets, measured by the average (macro) Lerner index

$$\xi \equiv \eta \cdot \frac{W^P/P - \beta\left(N^P + N^S\right)^{\beta-1}}{W^P/P} + (1-\eta)\cdot\frac{W^S/P - \beta\left(N^P+N^S\right)^{\beta-1}}{W^S/P} = \frac{\eta}{\sigma},$$

where $\beta \left(N^P + N^S\right)^{\beta-1}$ is the marginal disutility of work, and where the last equality follows from equations (6) and (7). The parameter values in the first row imply that the macro Lerner index is 0.20, and that employment and consumption are almost 50 per cent lower than their socially optimal levels. In the next row we change η from 1 to 0.5, keeping constant all other parameters. It is important to see that such a change involves two effects: (i) By decreasing the macro degree of imperfect competition, ξ, the distortion in the level of employment is reduced (affecting GDP and utility positively) and (ii) by introducing dualism in the labour market the allocation of employment deteriorates (affecting GDP and utility negatively). As a result of the first effect employment goes up by 18 per cent relative to the symmetric case, but since the composition of employment gets less efficient GDP goes up by only 15 per cent. To isolate the effect of a dual labour market, let us reduce η and at the same time adjust σ so as to keep the macro Lerner index constant. This is done in the third row, and it is evident that the introduction of asymmetry in the labour market reduces the levels of both employment, GDP, and welfare.

Table 1. Examples of general equilibria

β	σ	η	$\xi = \eta/\sigma$	$N/\hat{N}$	$C/\hat{C}$	$U/\hat{U}$
1.6	5.0	1.0	0.20	0.52	0.56	0.82
	5.0	0.5	0.10	0.62	0.64	0.88
	2.5	0.5	0.20	0.50	0.52	0.77
6.0	5.0	1.0	0.20	**0.92**	**0.92**	**0.98**
	5.0	0.5	0.10	0.94	0.94	0.98
	2.5	0.5	0.20	**0.91**	**0.89**	**0.95**
∞	5.0	1.0	0.20	1	1	1
	2.5	0.5	0.20	1	0.97	0.97

Note: In all examples we use $\alpha = 1.1$ and $\theta = 5$.

The value of β can be determined in two different ways; either one can set β in accordance with empirical estimates of the labour supply elasticity, or alternatively one can resort to a calibration procedure, i.e. to set β such that the model generates realistic equilibrium values of employment and GDP. As already mentioned, the case with a labour supply elasticity of $1\frac{2}{3}$ (i.e.

$\beta = 1.6$) is the most inelastic case examined in Blanchard and Kiyotaki (1987). However, this elasticity is much too high both from an estimation and a calibration point of view. A large body of empirical literature, see e.g. Pencavel (1986), suggests that the labour supply elasticity is well below 0.5, and in addition $\beta = 1.6$ generates a very unrealistic equilibrium with underutilization of labour of almost 50 per cent. Therefore, let us turn to the case with $\beta = 6.0$, implying a labour supply elasticity of 0.2. This case is in line with empirical estimates and, in addition, generates a much more realistic equilibrium with employment being 8 per cent lower than the socially optimal level in the symmetric setting with $\eta = 1$. The parameter values underlying the results in bold constitutes our benchmark comparison between the standard model and the dual model throughout the paper.

The qualitative effects of introducing dualism in the labour market are unaffected by the value of β. However, when the labour supply elasticity is lower, the allocation effect is stronger relative to the effect of decreasing the macro degree of imperfect competition. In fact, when η is reduced from 1 to 0.5, implying a reduction in the macro Lerner index from 0.20 to 0.10, the deterioration in the allocation of employment is strong enough to prevent an increase in welfare. In the next row we keep constant the macro Lerner index, implying that welfare falls 3 per cent below the level of the symmetric case. Finally, in the last two rows we set the elasticity of labour supply to zero. In the symmetric case employment, GDP, and welfare are at their socially optimal levels, whereas in the asymmetric case only employment is at its efficient level.

3 Nominal Rigidity and Fluctuations

The previous section analysed how dual labour markets influence the equilibrium of an economy without any nominal frictions. As with a simple, single labour market, money is neutral since all objective functions are defined on real variables. However, if price/wage setters face small adjustment costs (menu costs) of changing prices/wages, money may be non-neutral and changes in aggregate demand may give rise to fluctuations in real variables. One of the main insights from the New Keynesian literature is that imperfect competition mitigates the menu costs required for monetary non-neutrality and may cause nominal disturbances to yield larger welfare fluctuations than the adjustment costs causing them, see Mankiw (1985), Akerlof and Yellen (1985), Blanchard and Kiyotaki (1987), Rotemberg (1987), Ball and Romer (1989, 1990, 1991). However, these papers rely on models where labour markets are completely symmetric, i.e. either perfectly competitive or monopolistically competitive.[2] In this section, we ask how the presence of a dual labour market influences the menu cost requirements and the consequences of nominal disturbances. The first subsection deals with the size of menu costs needed to generate price stickiness, whereas the second subsection deals with fluctuations in welfare, productivity, and wage differentials due to nominal disturbances.

It is well-known that imperfect competition is a precondition for menu cost requirements to be small. Thus, the difference in competitiveness between the primary and secondary labour market implies that the two markets react differently to nominal disturbances and this is crucial for understanding the results. Therefore, a few notes on the two distinct labour markets are warranted before turning to the subsections. First, note that it is impossible for menu costs of any size to prevent wage changes in the competitive labour market.[3] Thus, the wage in the secondary market rises after an increase in labour demand due to a monetary expansion. Second, note from equation (16) that if menu costs are sufficiently high to prevent wage changes in the primary market then a monetary expansion always increases primary

[2]In fact, Akerlof and Yellen (1985) do emphasize dual labour markets as an important stylized view of the economy, but without going into the implications for the non-neutrality of money.

[3]This may be seen by deriving the menu cost requirement of a monopolistic worker and letting the elasticity of substitution between workers go to infinity. This corresponds to letting σ go to infinity in equation (22). Note also, by continuity, that menu costs will be very large even with a small amount of imperfect competition. Thus, the qualitative results that follow are not restricted to the knife-edge case of perfect competition in the secondary market.

employment relative to secondary employment. This reallocation effect, not present in the standard framework ($\eta = 1$), increases GDP and welfare, cf. the movement from A to B in Figure 1. However, the wage increase in the secondary market may also increase the menu cost requirement of the firms by increasing their marginal costs of satisfying the extra demand. Thus, the magnitude of the wage rise in the secondary market following a monetary expansion is crucial both for the menu costs requirements and for the welfare effects. Therefore, we start by finding the response of the secondary wage following a monetary expansion given that firms and wage setters in the primary labour market keep their respective prices and wages fixed. This is derived from (6), (8), (10), and (15), and is given in elasticity form by (see appendix)

$$\zeta \equiv \frac{dW^S/W^S}{dM/M} = \frac{\alpha\,(1-\xi)\,(\beta-1)}{\xi\,(1-\eta)\,(\beta-1)+1-\xi} > 0. \tag{20}$$

This elasticity indicates why standard New Keynesian results break down when labour supply becomes perfectly inelastic ($\beta \to \infty$). In this case ζ equals $\frac{\alpha(1-\xi)}{\xi(1-\eta)}$ which goes to infinity when the dual labour market is excluded from the analysis ($\eta \to 1$). In the absence of a dual labour market, a marginal increase in production can come about only by an increase in the number of work hours, and with $\beta = \infty$ more work hours imply an infinite increase in the marginal disutility of work (or, equivalently, the competitive wage). Furthermore, with infinite marginal disutility of work menu costs of any size will not make the households increase employment instead of changing the wages. These shortcomings of the standard theory do not arise in the presence of a dual labour market as the above elasticity is finite even for a perfectly inelastic labour supply. This is because an increase in production can be achieved – at a fixed number of total work hours – by substituting primary work hours (with high productivity) for secondary work hours (with low productivity). The following subsections analyse this issue in greater detail.

3.1 Menu Cost Requirements

This subsection derives the levels of menu costs of firms and workers that are sufficient to make price and wage rigidity a Nash-equilibrium.[4] For the firms to keep prices fixed, menu costs must be greater than or equal to the increase in profits resulting from adjustment of prices. The profit gain resulting from adjustment or, equivalently, the menu cost requirement of firms is derived by making a second order Taylor approximation on (12) around the initial equilibrium, assuming that other firms do not change their prices and that workers keep primary wages fixed. The aggregate menu cost requirement of firms in proportion of GDP equals (see appendix)

$$\frac{dV}{C} \approx F(\theta, \alpha, \eta, \zeta, m) \equiv \frac{1}{2}\left(\frac{\theta - 1}{1 - \theta + \theta\alpha}\right)\left[(1 - \eta)\zeta + \alpha - 1\right]^2 m^2, \qquad (21)$$

where $V \equiv \int_{i=0}^{1} V_i di$ and $m \equiv dM/M$. Equations (20) and (21) reveal that the menu cost requirement of firms is greater in the presence of a dual labour market compared to the case with a single monopolistic labour market. Since monopolistic wages are rigid, whereas competitive wages increase following a monetary expansion, marginal costs of firms increase more when a positive share of the labour market is characterized by competitive wage setting. Higher marginal costs imply that it is more costly for the firm to satisfy the extra demand, thereby driving up menu cost requirements. However, this presumes that workers do not adjust the wages in the primary labour market, and it is exactly the menu cost requirements in the labour market that create the difficulties for the standard theory.

Turning to the labour market, we derive the private gain of a worker from adjusting the wage in the primary labour market or, equivalently, the menu cost requirement of workers. This calculation is done by making a second order Taylor approximation on (5) around the initial equilibrium, assuming that other workers do not change wages in the primary labour market and

[4] We restrict the analysis to this approach adopted from Blanchard and Kiyotaki (1987). However, flexibility may also be an equilibrium unless menu costs are sufficiently large, as noted in Ball and Romer (1991).

that output prices are fixed. The aggregate menu cost requirement of workers in proportion of GDP is given by (see appendix)

$$\frac{dS}{C} \approx G(\sigma, \eta, \theta, \alpha, \zeta, m) \equiv \frac{1}{2}(\sigma - 1)\frac{\eta(\theta - 1)}{\alpha\theta}\zeta^2 m^2, \tag{22}$$

where $S \equiv \int_{j=0}^{1} S_j dj$. The menu costs needed for wage rigidity in the primary market are always increasing in the wage response of the secondary market, ζ. According to equation (20) and the definition of the macro Lerner index, ξ, the wage response, ζ, is always lower with a dual labour market, and therefore it follows from the above equation that the menu cost requirement of workers is also lower. Note that this holds independently of whether the comparison between the standard model and the dual model is made on basis of a constant macro Lerner index or not. Thus, we can state the following proposition on the menu cost requirements of firms and workers:

Proposition 1 *Comparing the dual labour market setting* $(0 < \eta < 1)$ *with the single, monopolistic labour market setting* $(\eta = 1)$ *we have:* (i) *The menu cost requirement of firms,* $F(\cdot)$*, is greater with a dual labour market, whereas* (ii) *the menu cost requirement of workers,* $G(\cdot)$*, is smaller with a dual labour market.*

Proof. (i) follows from (20), (21), and the definition of ξ. (ii) follows from (20), (22), and the definition of ξ. $\square$

To elaborate further on the implications of having a dual labour market we present some numerical examples in Table 2, illustrating the menu cost requirements following a 5 per cent monetary expansion. In the first three rows we consider a labour supply elasticity equal to $1\frac{2}{3}$ (i.e. $\beta = 1.6$). The first row considers the single, monopolistic labour market $(\eta = 1)$ analysed by Blanchard and Kiyotaki which yields very low menu cost requirements.[5] In the second row, we introduce a dual labour market by reducing η to 0.5.

[5] In fact, the menu cost requirements of workers are much lower than in Blanchard and Kiyotaki (1987) due to an error in the original article, see Hansen (1998).

As explained above, this increases the menu cost requirement of firms, but reduces the menu cost requirement of workers. However, this change of η involves a reduction of the macro degree of imperfect competition in the labour market, ξ. In the third row, we isolate the effect of a dual labour market by adjusting σ so as to keep constant the macro Lerner index. In this case there is a much larger reduction of the menu cost requirement of workers compared to the single market case.

Table 2. Menu cost requirements as percentage of GDP following a monetary expansion of 5%.

β	σ	η	ξ	Firms $F(\cdot)$	Workers $G(\cdot)$
1.6	5.0	1.0	0.20	0.00	0.16
	5.0	0.5	0.10	0.06	0.07
	2.5	0.5	0.20	0.06	0.03
6.0	5.0	1.0	0.20	**0.00**	**11.00**
	5.0	0.5	0.10	1.69	3.37
	2.5	0.5	0.20	**1.07**	**0.78**
	1.4	0.3	0.20	1.55	0.12
	3.3	0.7	0.20	0.56	2.55
	2	0.5	0.25	0.85	0.41
∞	5.0	1.0	0.20	0.00	∞
	2.5	0.5	0.20	6.65	5.20

Note: In all examples we use $\alpha = 1.1$ and $\theta = 5$.

A labour supply elasticity of $1\frac{2}{3}$ is clearly unrealistic as discussed in Section 2.3. Hence, the next six rows consider a much more realistic elasticity of 0.2 (i.e. $\beta = 6$). In the conventional symmetric model, the menu cost requirement of firms is still very low. In fact, with a single monopolistic labour market the labour supply elasticity is irrelevant for the menu cost requirement of firms; what matters is the returns to scale parameter, α, and when returns to scale are close to constant, menu cost requirements are always low. However, the menu cost requirement of firms is derived on the assumption that wages are rigid in the primary market, and this assumption is not a reasonable one, when labour supply is inelastic. In fact, the menu cost requirement of workers is equal to 11% of GDP, which is clearly unrealistic.

In the next row we introduce labour market dualism by reducing η from 1 to 0.5. In this case the menu cost requirement of firms is higher, since marginal costs are pushed up by higher secondary wages. However, the menu cost requirement of workers is significantly reduced. With a symmetric labour market workers can only accommodate the higher demand by increasing the number of work hours, and this is costly when labour supply is inelastic. By contrast, in the asymmetric setting the same effect can be achieved in a much less costly way by substituting high-productive, primary work for low-productive, secondary work. Of course, this example does not isolate the effect of a dual labour market, since reducing η also implies less imperfect competition on a macro level. The pure effect of a dual labour market is apparent by comparing the fourth and sixth rows (in bold), where η is 1 and 0.5 respectively, and where the macro degree of imperfect competition, ξ, is held constant at 0.2. It is evident that the menu cost requirement of workers is 14 times lower relative to the symmetric setting. In the dual labour market case, prices and wages are sticky when menu costs of firms (workers) are around 1 per cent (0.8 per cent) of GDP. The menu cost requirement of firms is in line with the empirical evidence in Levy et al. (1997) but to our knowledge no evidence exists on wage adjustment costs. However, there seems no reason to believe that the costs of wage adjustments are lower than those of price adjustments.

In the next rows, we analyse the sensitivity of menu costs with respect to η and ξ. According to the table, the lower η and the higher ξ, the lower is the menu cost requirement of workers. Finally, in the last two rows we look at the interesting special case, where the elasticity of labour supply is equal to zero. In the standard framework, Keynesian results break down; menu costs of any size cannot generate monetary non-neutrality (the menu cost requirement of firms is still low, but it is derived on the assumption that primary wages are rigid, which they won't be). By contrast, in the dual labour market model menu cost requirements are finite; if menu costs are sufficiently high, workers are willing to keep primary wages fixed and accommodate the higher demand for primary labour, and at the same time reduce the number of secondary hours such that the total number of working hours is constant. However, it should be noted that with a perfectly inelastic labour supply menu cost requirements are not within realistic limits.

3.2 Fluctuations in Welfare, Productivity, and Wage Differentials

This section is concerned with the consequences of monetary expansions, assuming that menu costs are indeed able to generate nominal rigidity. We show that labour market dualism boosts the welfare consequences of monetary expansions, and in addition offers a new explanation of the cyclicality of productivity and wage differentials.

Starting with the implications for wage differentials, note that a monetary expansion pushes up wages in the secondary labour market, whereas menu costs prevent wage adjustment in the primary labour market. This implies that the wage differential, $W^P - W^S$, narrows during booms and widens during recessions. This is in line with empirical investigations on the cyclicality of union-nonunion wage differentials by Melow (1981), Freeman (1984), Wunnava and Honney (1991) and others. In addition, this feature of the model is crucial for the consequences for welfare and productivity. The welfare effect of an increase in the money stock is derived by making a second order Taylor approximation on the utility function (1) around the initial equilibrium. This yields (see appendix)

$$\frac{dU}{C} \simeq W\left(\theta, \alpha, \xi, \beta, \zeta, m\right) = m - \frac{\theta - 1}{\theta\alpha}\frac{1-\xi}{\beta - 1}\left[\zeta m + \frac{1}{2}\zeta^2 m^2\right]. \qquad (23)$$

The welfare effect depends on η both through the macro Lerner index, $\xi = \eta/\sigma$, and through the wage response of the secondary labour market, ζ. The pure effect of a dual labour market is measured by changing η and at the same time adjusting σ so as to hold ξ constant. Since ζ is always lower in the case of a dual labour market than in the case of a single monopolistic labour market, equation (23) shows that the welfare effect of a monetary expansion is larger in the presence of labour market dualism. The intuition for this result is not difficult to grasp. Following a monetary expansion, the secondary wages relative to primary wages go up, and according to equation (16), this implies an expansion of primary employment relative to secondary employment. Since the allocation of labour is distorted away from primary employment in the initial equilibrium, such a reallocation creates a positive welfare effect in addition to the positive welfare effect resulting from a higher employment level also present in the standard framework.

Turning to the change in productivity following monetary disturbances, we define aggregate productivity $Q \equiv C/N$. As a measure of the cyclicality of productivity, we calculate the percentage change in productivity, Q, relative to the percentage change in GDP (see appendix)

$$X(\beta, \zeta) \equiv \frac{dQ/Q}{dC/C} = 1 - \frac{\zeta}{\beta - 1}. \tag{24}$$

In other words, X is the share of the increase in GDP accounted for by higher productivity. It is a well established empirical regularity that productivity is procyclical (see e.g. Rotemberg and Summers, 1987), and any satisfactory theory of business cycles should be able to capture this stylized fact. However, this is not the case for the Blanchard and Kiyotaki (1987) setup. By setting η equal to 1, it follows from equations (20) and (24) that productivity is countercyclical with decreasing returns to scale ($\alpha > 1$) and acyclical in the special case with constant returns to scale ($\alpha = 1$). By contrast, with a dual labour market ($0 < \eta < 1$) productivity may be procyclical with decreasing returns and is always procyclical with constant returns. Furthermore, comparing the standard framework with the dual labour market model, the productivity effect, X, is always greater in the latter case. Intuitively, during a boom employment of primary labour goes up relative to that of secondary labour, and since productivity is higher for primary labour, this reallocation contributes positively to productivity. We are now able to state the following proposition on the consequences of monetary shocks for wage differentials, welfare, and productivity:

Proposition 2 *Holding constant the macro Lerner index* ξ *we have:* (i) *The wage differential,* $W^P - W^S$*, is countercyclical with a dual labour market* $(0 < \eta < 1)$. (ii) *The welfare effect of a monetary expansion,* $W(\cdot)$*, is always greater with a dual labour market.* (iii) *With a single, monopolistic labour market* $(\eta = 1)$ *productivity is countercyclical, i.e.* $X(\cdot) \leq 0$. *Productivity is less countercyclical or procyclical with a dual labour market.*

Proof. (i) follows from the fact that primary wages are fixed due to the menu costs, whereas secondary wages increase according to (20). (ii) follows from (20) and (23). (iii) follows from (20) and (24). □

Table 3 displays the total menu cost requirement of firms and workers as well as the change in welfare and productivity following a monetary expansion of 5 per cent. In the first three rows we consider β equal to 1.6. In this situation, the Blanchard and Kiyotaki setup performs well; small menu costs (0.16 per cent of GDP) generate large welfare cycles (1.75 per cent of GDP). Yet, the model conflicts with empirical evidence by generating countercyclical productivity. Introducing labour market dualism, holding constant the macro Lerner index, ξ, the model performs better. Menu costs are smaller, the welfare effect is larger, and productivity is less countercyclical.

In the much more realistic case with a labour supply elasticity equal to 0.2 ($\beta = 6.0$), the Blanchard and Kiyotaki model performs very badly. It takes very large menu costs (11 per cent of GDP) to generate cycles of a much smaller magnitude (1.36 per cent of GDP), and in addition productivity is countercyclical. Reducing η from 1 to 0.5 significantly improves the results. The menu cost requirement is more than halved, the welfare effect is boosted, and productivity is now procyclical. Adjusting σ so as to capture the pure effect of a dual labour market, the picture looks even better. The total menu cost requirement is only 1.85 per cent of GDP, and the welfare effect is now larger than the menu costs causing it (2.86 per cent of GDP). Furthermore, 32 per cent of the expansion in GDP is accounted for by increased productivity. Thus, the introduction of dual labour markets significantly improves the ability of menu cost models to explain economic fluctuations.

Table 3. The consequences for welfare and productivity following a monetary expansion of 5%

β	σ	η	ξ	Menu Costs $F(\cdot)+G(\cdot)$	Welfare $W(\cdot)$	Productivity $X(\cdot)$
1.6	5.0	1.0	0.20	0.16	1.75	-10
	5.0	0.5	0.10	0.13	1.46	-6
	2.5	0.5	0.20	0.08	1.98	-2
6.0	5.0	1.0	0.20	**11.00**	**1.36**	**-10**
	5.0	0.5	0.10	5.06	1.88	14
	2.5	0.5	0.20	**1.85**	**2.86**	**32**
	1.4	0.3	0.20	1.66	3.17	41
	3.3	0.7	0.20	3.11	2.44	20
	2	0.5	0.25	1.26	3.24	40
∞	5.0	1.0	0.20	∞	–	–
	2.5	0.5	0.20	12.03	5.00	100

Note: In all examples we use $\alpha = 1.1$ and $\theta = 5$. Figures for Menu Costs, Welfare, and Productivity are expressed as percentages.

In the next rows, we present a sensitivity analysis with respect to the wage share for primary labour, η, and the macro Lerner index, ξ. The lower η and the higher ξ, the better the performance of the model. Finally, in the last two rows we look at the case of a perfectly inelastic labour supply. In the standard framework, menu costs of any size will not be able to generate price and wage rigidity, and accordingly there is monetary neutrality. By contrast, in the dual labour market model menu costs can, in principle, explain business cycles. Note that since the boom is now solely generated by a reallocation from low-productive, secondary employment towards high-productive, primary employment, all of the GDP increase is accounted for by procyclical productivity.

4 Concluding Remarks

In this paper, we have shown that the introduction of dual labour markets greatly improves the performance of the conventional menu cost framework: overall menu cost requirements are lower, welfare consequences of nominal disturbances are larger, and in addition labour market dualism provides an explanation of procyclical productivity and countercyclical wage differentials. The numerical examples suggest that the presence of dual labour markets does lead to menu cost requirements that are within realistic limits even when labour supply is very inelastic. However, it must be said that the welfare effect is still small relative to the menu costs required for price rigidity, implying that the efficiency loss of fluctuations is modest.

While we presented our model as one of *intra*sectoral dualism, where high-paying jobs coexist with low-paying jobs within each firm, several other writers have focused on the issue of *inter*sectoral dualism, where wages differ across sectors (e.g., Harris and Todaro, 1970; Calvo, 1978; Bulow and Summers, 1986). It can be shown that our model can be reinterpreted as a model of *inter*sectoral dualism. In this reinterpretation we consider an economy with two sectors producing different goods; one sector hires only monopolistic labour, whereas the other hires only competitive labour. This two sector model is formally equivalent to the one sector model of this paper when aggregate consumption is modelled as a Cobb-Douglas aggregate of the two types of goods in the same way as the two types of labour are aggregated in the one sector model. Thus, our results are not restricted to a specific type of labour market dualism.

The mechanism for boosting productivity in this paper is the reallocation of labour from the secondary to the primary labour market in response to a positive demand shock. This is analogous to the industrial policy advocated by Bulow and Summers (1986), namely the subsidising of the primary sector to increase its share of aggregate employment. This would also have the effect of compressing wage differentials. However, there is also an important difference. An on-going industrial subsidy programme will have a permanent effect, implying a permanent fiscal cost with associated distortionary taxation. By contrast, the productivity effect here is transitory and has no fiscal implications.

Clearly, the importance of menu costs is a matter of some debate. We can take a narrow literal view of menu costs, in which case they are perhaps not so important. However, if we take a broader view (including costs of decision and elements of bounded rationality), then they are more significant. Either way, they remain the main theory of nominal rigidities that we have. The question remains as to whether the basic simple intuition underlying the menu cost idea is compatible with plausible magnitudes of parameters. What this paper shows along with Basu (1995) and Dixon and Hansen (1999) is that the pessimism of early papers was due to the representative sector framework and is not generic to menu costs outside that framework.

References

[1] Akerlof, G. A. and Yellen, J. L. (1985): "A Near-Rational Model of the Business Cycle, with Wage and Price Inertia." *Quarterly Journal of Economics* 100, Supplement, pp. 823-838.

[2] Ball, L. and Romer, D. (1989): "Are Prices Too Sticky?" *Quarterly Journal of Economics* 104, pp. 507-524.

[3] Ball, L. and Romer, D. (1990): "Real Rigidities and the Nonneutrality of Money." *Review of Economic Studies* 57, pp. 183-203.

[4] Ball, L. and Romer, D. (1991): "Sticky Prices as Coordination Failure." *American Economic Review* 81(3), pp. 539-552.

[5] Basu, S. (1995): "Intermediate Goods and Business Cycles: Implications for Productivity and Welfare." *American Economic Review* 85(3), pp. 512-531.

[6] Blanchard, O. J., and Kiyotaki, N. (1987): "Monopolistic Competition and the Effects of Aggregate Demand." *American Economic Review* 77, pp. 647-666.

[7] Bulow, J. I. and Summers, L. H. (1986): "A Theory of Dual Labor Markets with Application to Industrial Policy, Discrimination, and Keynesian Unemployment." *Journal of Labor Economics* 4, no. 1, pp. 376-414.

[8] Calvo, G. A. (1978): "Urban Unemployment and Wage Determination in LCD's: Trade Unions in the Harris-Todaro Model." *International Economic Review* 19, no. 1, pp. 65-81.

[9] d'Aspremont, C. et al. (1996). "On the Dixit-Stiglitz Model of Monopolistic Competition." *American Economic Review*, vol. 86, no. 3, pp. 623-29.

[10] Dickens, W. T. and Lang, K. (1985): "A Test of Dual Labor Market Theory." *American Economic Review* 75(4), pp. 792-805.

[11] Dixon, H. D. and Hansen, C. T. (1999): "A Mixed Industrial Structure Magnifies the Importance of Menu Costs." Forthcoming in *European Economic Review.*

[12] Freeman, R. (1984): "Longitudinal Analyses of the effects of Trade Unions." *Journal of Labor Economics* 2(1), pp. 1-26.

[13] Hansen, C. T. (1998): "A Note on Blanchard and Kiyotaki (1987)." EPRU Working Paper 1998-06.

[14] Harris, J. and Todaro, M. (1970): "Migration, Unemployment and Development: A Two Sector Analysis." *American Economic Review* 60, no. 1, pp. 126-43.

[15] Levy, D., Bergen, M., Dutta, S. and Venable, R. (1997): "On the Magnitude of Menu Costs: Direct Evidence from Large U.S. Supermarket Chains." *Quarterly Journal of Economics* 112, no. 3, pp. 791-825.

[16] Mankiw, N. G. (1985): "Small Menu Costs and Large Business Cycles: A Macroeconomic Model of Monopoly." *Quarterly Journal of Economics* 100, pp. 529-539.

[17] Mellow, W. (1981): "Unionism and Wages: A Longitudinal Analysis." *Review of Economics and Statistics* 63, pp. 45-52.

[18] Pencavel, J. (1986): "Labor Supply of Men: A Survey." Chapter 1 of *Handbook of Labor Economics.* Eds.: O. Ashenfelter and R. Layard, Amsterdam: North-Holland, pp. 3-102.

[19] Rotemberg, J. J. (1987): "The New Keynesian Microfoundations." *NBER Macroeconomics Annual*, Cambridge: MIT Press, Vol. 2.

[20] Rotemberg, J. J. and Summers, L. H. (1990): "Inflexible Prices and Procyclical Productivity." *Quarterly Journal of Economics* 105(4), pp. 851-874.

[21] Saint-Paul, G. (1996): *Dual labour Markets: A Macroeconomic Perspective.* MIT Press, Cambridge, Massachusetts.

[22] Wunnava, P. V. and Honney, J. K. (1991): "The Union-Nonunion Wage Differential over the Business Cycle." *Economics Letters* 37, pp. 97-103.

A Derivation of equations (17) to (24)

A.1 Derivation of (17)

Eq. (14) and the symmetry of the firms in the model, i.e. $P_i = P\ \forall i$, imply

$$\frac{W}{P} = \frac{\theta - 1}{\theta\alpha}\left(\frac{M}{P}\right)^{1-\alpha},$$

and after using (9) and (13), we get

$$\frac{W}{P} = \frac{\theta - 1}{\theta\alpha}\left(\left(N^P\right)^{\eta}\left(N^S\right)^{1-\eta}\right)^{\frac{1-\alpha}{\alpha}}. \tag{25}$$

The relative employment in the two markets can be derived by combining (15) and (16):

$$\frac{N^P}{N^S} = \frac{\eta}{1-\eta}\frac{\sigma - 1}{\sigma}, \tag{26}$$

and after using the identity $N^P + N^S = N$, we have

$$N^S = N\frac{(1-\eta)\,\sigma}{\sigma-\eta}. \tag{27}$$

Inserting these two equations in (25) yields the following relationship between aggregate employment and the real wage

$$\frac{W}{P} = \frac{\theta-1}{\theta\alpha}\left(\left(\frac{\eta}{1-\eta}\frac{\sigma-1}{\sigma}\right)^{\eta}\frac{(1-\eta)\,\sigma}{\sigma-\eta}N\right)^{\frac{1-\alpha}{\alpha}}. \tag{28}$$

A corresponding relationship can be found by combing (6), (11), and (15). Inserting W^P from (15) into (11) yields

$$\frac{W}{P} = \left(\frac{\sigma}{\sigma-1}\frac{1}{\eta}\right)^{\eta}\left(\frac{1}{1-\eta}\right)^{1-\eta}\frac{W^S}{P},$$

which combined with (6) gives

$$\frac{W}{P} = \beta\left(\frac{\sigma}{\sigma-1}\frac{1}{\eta}\right)^{\eta}\left(\frac{1}{1-\eta}\right)^{1-\eta}N^{\beta-1}. \tag{29}$$

By setting eq. (28) equal to eq. (29) and simplifying one yields

$$N = \left(\left(\frac{\theta-1}{\theta}\right)^{\alpha}\left(\frac{\sigma-1}{\sigma}\right)^{\eta}\left(\frac{\sigma-\eta}{\sigma}\right)^{\alpha-1}\right)^{\frac{1}{\alpha\beta-1}}\hat{N},$$

where

$$\hat{N} \equiv \lim_{\theta\to\infty,\sigma\to\infty} N = \left((\alpha\beta)^{-\alpha}(1-\eta)^{1-\eta}\eta^{\eta}\right)^{\frac{1}{\alpha\beta-1}},$$

is the is the efficient (Walrasian) employment level. QED.

A.2 Derivation of (18)

The symmetry of the firms in the model, i.e. $P_i = P\ \forall i$, implies

$$C \equiv \int_{i=0}^{1}\frac{P_iY_i}{P}di = \int_{i=0}^{1}Y_i di = Y.$$

After inserting (26) and (27) in (9), we get

$$Y = \left(\left(\frac{\eta}{1-\eta}\frac{\sigma-1}{\sigma}\right)^{\eta}\frac{(1-\eta)\,\sigma}{\sigma-\eta}N\right)^{1/\alpha}.$$

Inserting (17) in this equation and simplifying implies

$$C \equiv \int_{i=0}^{1} \frac{P_i Y_i}{P} di = \left(\frac{\theta - 1}{\theta} \left(\frac{\sigma - 1}{\sigma} \right)^{\eta\beta} \left(\frac{\sigma - \eta}{\sigma} \right)^{1-\beta} \right)^{\frac{1}{\alpha\beta - 1}} \hat{C} \leq \hat{C},$$

where

$$\hat{C} \equiv \lim_{\theta \to \infty, \sigma \to \infty} C = \left(\alpha\beta\eta^{-\eta\beta} (1-\eta)^{\beta(\eta-1)} \right)^{\frac{1}{1-\alpha\beta}},$$

is the efficient level of consumption. QED.

A.3 Derivation of (20)

Note, that (7) no longer holds after a monetary shock where menu costs are sufficiently high to keep W_j^P fixed. We start by finding the aggregate labour demand in the secondary labour market from equations (10) and (13). This yields

$$N^S = \int_{i=0}^{1} N_i^S di = (1-\eta) \frac{W}{W^S} \left(\frac{M}{P} \right)^{\alpha}. \tag{30}$$

The aggregate labour supply in the secondary labour market follows from (6)

$$N^S = \int_{j=0}^{1} N_j^S dj = \left(\frac{1}{\beta} \frac{W^S}{P} \right)^{1/(\beta-1)} - N^P.$$

Rewriting this equation by inserting N^P derived from equation (10), (13), $P_i = P \; \forall i$, and $W_j^P = W^P \; \forall j$, we have

$$N^S = \left(\frac{1}{\beta} \frac{W^S}{P} \right)^{1/(\beta-1)} - \eta \frac{W}{W^P} \left(\frac{M}{P} \right)^{\alpha}. \tag{31}$$

By equilibrating supply, eq. (31), and demand, eq. (30), in the secondary labour market, we get the following equation determining the secondary wage level

$$\left(\frac{1}{\beta} \frac{W^S}{P} \right)^{1/(\beta-1)} - \eta \frac{W}{W^P} \left(\frac{M}{P} \right)^{\alpha} = (1-\eta) \frac{W}{W^S} \left(\frac{M}{P} \right)^{\alpha},$$

and after inserting (11) this yields

$$\left(\frac{1}{\beta} \frac{W^S}{P} \right)^{1/(\beta-1)} = \left(\frac{\eta}{1-\eta} \frac{W^S}{W^P} \right)^{-\eta} \left(\frac{M}{P} \right)^{\alpha} + \left(\frac{\eta}{1-\eta} \frac{W^S}{W^P} \right)^{1-\eta} \left(\frac{M}{P} \right)^{\alpha}. \tag{32}$$

Now, take the total derivative w.r.t. M and W^S

$$
\begin{aligned}
&\left(\frac{1}{\beta}\frac{W^S}{P}\right)^{1/(\beta-1)}\frac{1}{\beta-1}\frac{dW^S}{W^S} \\
= &\left(\frac{\eta}{1-\eta}\frac{W^S}{W^P}\right)^{-\eta}\left(\frac{M}{P}\right)^{\alpha}\left(-\eta\frac{dW^S}{W^S}+\alpha\frac{dM}{M}\right) \\
&+\left(\frac{\eta}{1-\eta}\frac{W^S}{W^P}\right)^{1-\eta}\left(\frac{M}{P}\right)^{\alpha}\left((1-\eta)\frac{dW^S}{W^S}+\alpha\frac{dM}{M}\right).
\end{aligned}
$$

Dividing on both sides with (32) gives which after inserting (15) becomes equal to

$$
\begin{aligned}
\frac{1}{\beta-1}\frac{dW^S}{W^S} = &\frac{1}{1+\frac{\eta}{1-\eta}\frac{\sigma-1}{\sigma}}\left(-\eta\frac{dW^S}{W^S}+\alpha\frac{dM}{M}\right) \\
&+\frac{1}{\left(\frac{\eta}{1-\eta}\frac{\sigma-1}{\sigma}\right)^{-1}+1}\left((1-\eta)\frac{dW^S}{W^S}+\alpha\frac{dM}{M}\right).
\end{aligned}
$$

Isolating dW^S/W^S in this equation yields

$$
\frac{dW^S}{W^S}=\frac{\alpha\left(1-\eta/\sigma\right)\left(\beta-1\right)}{\eta/\sigma\left(1-\eta\right)\left(\beta-1\right)+1-\eta/\sigma}\frac{dM}{M},
$$

or

$$
\zeta\equiv\frac{dW^S/W^S}{dM/M}=\frac{\alpha\left(1-\xi\right)\left(\beta-1\right)}{\xi\left(1-\eta\right)\left(\beta-1\right)+1-\xi},
$$

where $\xi\equiv\eta/\sigma$. QED.

A.4 Derivation of (21)

The profit gain resulting from adjustment of prices is derived by making a second order Taylor approximation on (12) around the initial equilibrium, assuming that other output prices as well as primary wages are rigid. Letting V^1 denote the approximation of profits when firm i chooses not to adjust the price, and letting V^2 denote the approximation of profits when firm i chooses to adjust the price, we have

$$
dV_i\equiv V^2-V^1\approx\frac{1}{2}\frac{\partial^2V\left(P_i,M\right)}{\left(\partial P_i\right)^2}\left(dP_i\right)^2+\frac{\partial^2V\left(P_i,M\right)}{\partial P_i\partial M}dP_idM. \tag{33}
$$

In (33) we have used the envelope theorem, i.e. $\partial V_i/\partial P_i = 0$. The profit function equals

$$V(P_i, M) = P_i \left(\frac{P_i}{P}\right)^{-\theta} \frac{M}{P} - W \left(\frac{P_i}{P}\right)^{-\theta\alpha} \left(\frac{M}{P}\right)^{\alpha}.$$

The first order derivative is given by

$$\frac{\partial V_i}{\partial P_i} = \left(\frac{P_i}{P}\right)^{-\theta} \frac{M}{P}(1-\theta) + \alpha\theta \frac{W}{P} \left(\frac{P_i}{P}\right)^{-\alpha\theta-1} \left(\frac{M}{P}\right)^{\alpha}.$$

The second order derivatives evaluated in the initial equilibrium are

$$\frac{\partial^2 V_i}{(\partial P_i)^2} = \frac{M}{P^2}(1-\theta)(1-\theta+\theta\alpha), \tag{34}$$

$$\frac{\partial^2 V_i}{\partial P_i \partial M} = \frac{1}{P}(1-\theta)(1-\alpha-(1-\eta)\zeta), \tag{35}$$

where

$$\zeta \equiv \frac{dW^S/W^S}{dM/M}.$$

From (14) we have

$$dP_i = P\frac{1}{1-\theta+\theta\alpha}\left[(1-\eta)\zeta+\alpha-1\right]m, \qquad m \equiv \frac{dM}{M}. \tag{36}$$

By inserting (34), (35), and (36) in (33), we yield

$$\begin{aligned} dV_i \quad \approx \quad & \frac{1}{2}(1-\theta)(1-\theta+\theta\alpha)\left(\frac{1}{1-\theta+\theta\alpha}\right)^2 \left[(1-\eta)\zeta+\alpha-1\right]^2 m^2 M \\ & +(1-\theta)(1-\alpha-(1-\eta)\zeta)\left(\frac{1}{1-\theta+\theta\alpha}\right)\left[(1-\eta)\zeta+\alpha-1\right]m^2 M, \end{aligned}$$

or equivalently

$$dV_i \approx \frac{1}{2}\left(\frac{\theta-1}{1-\theta+\theta\alpha}\right)\left[(1-\eta)\zeta+\alpha-1\right]^2 m^2 M.$$

Now, the aggregate menu cost requirement relative to GDP can be written in the following way

$$\frac{dV}{C} \approx F(\theta, \alpha, \eta, \zeta, m) \equiv \frac{1}{2}\left(\frac{\theta-1}{1-\theta+\theta\alpha}\right)\left[(1-\eta)\zeta+\alpha-1\right]^2 m^2,$$

where $V \equiv \int_{i=0}^{1} V_i di$. This equation corresponds to (21). QED.

A.5 Derivation of (22)

The utility gain of a worker resulting from adjusting the wage in the primary labour market is derived by making a second order Taylor approximation on (5) around the initial equilibrium, assuming that other primary wages as well as output prices are rigid, i.e.

$$dS_j \equiv S^2 - S^1 \approx \frac{1}{2}\frac{\partial^2 S\left(W_j^P, M\right)}{\left(\partial W_j^P\right)^2}\left(dW_j^P\right)^2 + \frac{\partial^2 S\left(W_j^P, M\right)}{\partial W_j^P \partial M} dW_j^P dM, \quad (37)$$

where S^1 denotes the approximation of utility when worker i chooses not to adjust the wage and S^2 denotes the approximation of utility when worker i chooses to adjust the wage. Note, that we have used the envelope theorem, i.e. $\partial S_j / \partial W_j^P = 0$. The indirect utility is given by

$$S_j = \frac{W_j^P}{P} N_j^P\left(W_j^P\right) + \frac{W^S}{P} N_j^S + \frac{\int_{i=0}^1 V_{ij} di}{P} - \left(N_j^P\left(W_j^P\right) + N_j^S\right)^\beta,$$

where

$$N_j^P\left(W_j^P\right) = \left(\frac{W_j^P}{W^P}\right)^{-\sigma} \eta \frac{W}{W^P}\left(\frac{M}{P}\right)^\alpha,$$

and

$$N_j^S = \left(\frac{1}{\beta}\frac{W^S}{P}\right)^{\frac{1}{\beta-1}} - N_j^P = \left(\frac{1}{\beta}\frac{W^S}{P}\right)^{\frac{1}{\beta-1}} - \left(\frac{W_j^P}{W^P}\right)^{-\sigma} \eta \frac{W}{W^P}\left(\frac{M}{P}\right)^\alpha.$$

By inserting these exressions into S_j we have

$$\begin{aligned} S\left(W_j^P, M\right) &= \left[\frac{W_j^P - W^S}{P}\right]\left(\frac{W_j^P}{W^P}\right)^{-\sigma} \eta \frac{W}{W^P}\left(\frac{M}{P}\right)^\alpha \\ &\quad + \frac{\int_{i=0}^1 V_{ij} di}{P} + \left(\beta^{\frac{-1}{\beta-1}} - \beta^{\frac{-\beta}{\beta-1}}\right)\left(\frac{W^S}{P}\right)^{\frac{\beta}{\beta-1}}. \end{aligned}$$

The first order derivative equals

$$\begin{aligned} \frac{\partial S_j}{\partial W_j^P} &= \frac{1}{P}\left(\frac{W_j^P}{W^P}\right)^{-\sigma}\left(\frac{W}{W^P}\right)\eta\left(\frac{M}{P}\right)^\alpha (1-\sigma) \\ &\quad + \sigma\left(\frac{W^S}{P}\right)\left(\frac{W_j^P}{W^P}\right)^{-\sigma-1} \frac{W}{\left(W^P\right)^2}\eta\left(\frac{M}{P}\right)^\alpha. \end{aligned}$$

The second order derivatives can be shown to be

$$\frac{\partial^2 S_j}{\left(\partial W_j^P\right)^2} = (1-\sigma)\frac{N_j^P}{PW^P}, \tag{38}$$

and

$$\frac{\partial^2 S_j}{\partial W_j^P \partial M} = (\sigma-1)\,\zeta\frac{N_j^P}{P}\frac{1}{M}. \tag{39}$$

Furthermore, from equations (6) and (7) we get

$$\frac{W_j^P}{W^S} = \frac{1}{1-\nu} \Rightarrow \frac{dW_j^P}{W_j^P} = \frac{dW^S}{W^S} \Leftrightarrow dW_j^P = W_j^P \zeta m. \tag{40}$$

Inserting the equations (38), (39), and (40) into (37) implies

$$dS_j \approx \frac{1}{2}(1-\sigma)\frac{N_j^P}{PW^P}\left(W_j^P\zeta m\right)^2 + (\sigma-1)\,\zeta\frac{N_j^P}{P}W_j^P\zeta m^2,$$

which can be rewritten as

$$dS_j \approx \frac{1}{2}(\sigma-1)\,\zeta^2 m^2 \frac{W_j^P N_j^P}{P}.$$

The aggregate menu cost requirement in proportion of GDP is given by

$$\frac{dS}{C} \approx \frac{1}{2}(\sigma-1)\,\zeta^2 m^2 \frac{W_j^P N_j^P/P}{C},$$

where $S = \int_{j=0}^{1} S_j dj$. From equations (7), (15), (16), (17), and (18), we get

$$\frac{W_j^P N_j^P/P}{C} = \frac{(\theta-1)\,\eta}{\theta\alpha}.$$

Thus, we have

$$\frac{dS}{C} \approx G(\sigma,\eta,\theta,\alpha,\zeta,m) \equiv \frac{1}{2}(\sigma-1)\,\zeta^2 m^2 \frac{(\theta-1)\,\eta}{\theta\alpha},$$

which corresponds to (22). QED.

A.6 Derivation of (23)

The aggregate utility of the households is given by

$$U(C,N) = C - N^{\beta}.$$

Letting U^1 denote the utility level before the monetary expansion and U^2 denote the utility level after the expansion, a second order Taylor expansion gives

$$dU = U^2 - U^1 \simeq \frac{\partial U}{\partial C} dC + \frac{\partial U}{\partial N} dN + \frac{1}{2}\left[\frac{\partial^2 U}{\partial C^2}(dC)^2 + \frac{\partial^2 U}{\partial N^2}(dN)^2 + 2\frac{\partial^2 U}{\partial C \partial N} dC dN\right].$$

Deriving the different derivatives of U with respect to C yields

$$dU \simeq dC + \frac{\partial U}{\partial N} dN + \frac{1}{2}\frac{\partial^2 U}{\partial N^2}(dN)^2.$$

Measuring relative to initial consumption gives

$$\frac{dU}{C} \simeq \frac{dC}{C} + \frac{1}{C}\frac{\partial U}{\partial N} dN + \frac{1}{C}\frac{1}{2}\frac{\partial^2 U}{\partial N^2}(dN)^2$$

$\Leftrightarrow$

$$\frac{dU}{C} \simeq \frac{dC}{C} + \frac{N}{C}\frac{\partial U}{\partial N}\frac{dN}{N} + \frac{1}{2}\frac{N^2}{C}\frac{\partial^2 U}{\partial N^2}\left(\frac{dN}{N}\right)^2.$$

Deriving the different derivatives of U with respect to N yields

$$\frac{dU}{C} \simeq \frac{dC}{C} - \frac{\beta N^{\beta}}{C}\frac{dN}{N} - \frac{1}{2}\beta(\beta-1)\frac{N^{\beta}}{C}\left(\frac{dN}{N}\right)^2,$$

where

$$C = \frac{M}{P} \qquad \Rightarrow \qquad \frac{dC}{C} = \frac{dM}{M} = m.$$

$$N = \left(\frac{1}{\beta}\frac{W^S}{P}\right)^{\frac{1}{\beta-1}} \qquad \Rightarrow \qquad \frac{dN}{N} = \frac{1}{\beta-1}\frac{dW^S}{W^S} = \frac{1}{\beta-1}\zeta m.$$

By inserting (18) and (17) we find

$$\frac{N^{\beta}}{C} = \frac{1}{\alpha\beta}\frac{\theta-1}{\theta}\frac{\sigma-\eta}{\sigma}.$$

Thus, we have

$$\frac{dU}{C} \simeq W(\theta,\alpha,\xi,\beta,\zeta,m) = m - \frac{\theta-1}{\theta\alpha}\frac{1-\xi}{\beta-1}\left(\zeta m + \frac{1}{2}\zeta^2 m^2\right),$$

which corresponds to (11). QED.

A.7 Derivation of (24)

Aggregate productivity is defined as $Q \equiv C/N$. This implies that the percentage change in productivity relative to the percentage change in GDP is given by

$$X(\beta,\zeta) \equiv \frac{dQ/Q}{dC/C} = 1 - \frac{dN/N}{dC/C}.$$

The relative change in aggregate consumption/production, $\frac{dC/C}{dM/M}$, is equal to one according to (13). The relative change in aggregate employment, $N \equiv N^P + N^S$, can be determined from (6) and (20)

$$\frac{dN/N}{dM/M} = \frac{1}{\beta - 1}\frac{dW^S/W^S}{dM/M} = \frac{\zeta}{\beta - 1}.$$

It then follows that

$$X(\beta,\zeta) = 1 - \frac{\zeta}{\beta - 1}.$$

QED.

Figure 1. Relative wages and employment.