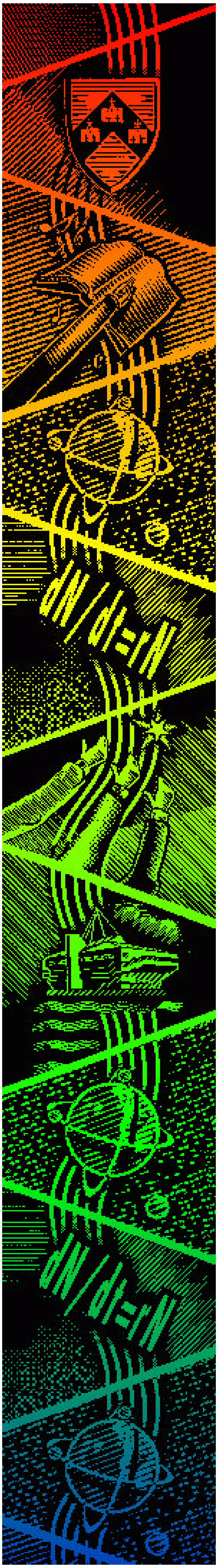

THE UNIVERSITY of York

Discussion Papers in Economics

No. 1998/11

Measuring and Implementing Equality of Opportunity for Income

by

Vitorocco Peragine

Department of Economics and Related Studies
University of York
Heslington
York, YO10 5DD

Measuring and Implementing Equality of Opportunity for Income

Vitorocco Peragine*
Department of Economics
University of York
Heslington, York Y01 5DD (UK)
Fax: 44-1904-433759
Phone: 44-1904-433788
E-mail: vp100@york.ac.uk

September 30, 1998

Abstract

Departing from the welfarist tradition, recent theories of justice focus on individual opportunities as the appropriate standard for distributive judgments. Justice is seen as requiring equality of opportunity, instead of outcomes, among the individuals. To explore how this philosophical conception can be translated into concrete public policy, we select the income as relevant outcome and the income tax as the relevant redistributive policy, and we address the following questions: (i) what is the degree of opportunity inequality in an income distribution? (ii) how to design an opportunity egalitarian income tax policy? Both positive and normative criteria for ranking income distributions on the basis of equality of opportunities are derived. Moreover, we characterize an opportunity egalitarian income tax policy and we formulate criteria for choosing among alternative tax systems.

*I am especially grateful to Peter Lambert for suggestions, encouragement and advice. Thanks are also due to participants at the 1998 Social Choice and Welfare Conference, Vancouver, for helpful questions and observations. Financial support from the C.N.R. (Rome) is gratefully acknowledged. Remaining errors are my responsibility.

1 Introduction

In defining the just allocations of a given amount of resources, in general theories of justice require equality of individuals with respect to some variable. However they differ from each other in the answer to the question: "equality of what?" Following Kolm [17], the use of the descriptive concept of equality is a requirement of internal rationality of a theory; whereas the choice of the relevant "space", in which equality among individuals has to be assessed, can be seen as the ethical or normative moment characterizing different theories of justice.

We can distinguish two extreme and opposite cases in this. First, we have the "Full Process Liberalism" (on this, see Kolm [17]), according to which the equality of formal opportunities is a necessary and sufficient condition for distributive justice. Equality of formal opportunities means that there is no legal bar to access to all positions and advantages. An example of this position is the Entitlements theory of Nozick [22]. The opposite case arises when equality is to be assessed in the space of welfare, utility or, in general, preference satisfaction; it is the case of the welfarist tradition. A third alternative position can be found in contemporary theories of justice which require equality to be assessed in a space that is situated in between the two polar cases presented above: Rawls [25], who first focused on the problem of choosing the correct space, proposes primary goods; Sen [30], [31]; [32] proposes capabilities to function; Dworkin [9]; [10] proposes resources; Cohen [8] proposes access to advantage; Arneson [2], finally, proposes opportunity for welfare.

What the proposals of Rawls, Dworkin, etc. share, is the common feature that the justice is seen as requiring equality of opportunities among the individuals, an opportunity being "a chance of getting a good if one seeks it" (Arnerson [2], p.85). The idea is that the equitability of a given distribution of outcomes[1] cannot be judged by observing only the degree of inequality present in that distribution. We need to extend the informational basis of our distributive judgements; we need information about the circumstances in which a given distribution arose. The same inequality in a distribution of outcomes can be sometimes judged equitable, sometimes not, depending on these circumstances.

What the principle of justice requires is not equality of individuals' final achievements; once the means or opportunities to reach a valuable outcome have been equally split, which particular opportunity, from those open to her, the individual chooses, is outside the scope of justice. In particular, the proposals of Dworkin, Cohen and Arneson, use the concept of individual responsibility and try to deduce from this the specific relevant equalizandum. Are to be considered opportunities all factors, influencing the individual final outcome, for which the

[1] In this introduction we use indifferently the expressions "individual final outcome" or "individual advantage", to express the idea of any person's final achievment which is valuable to her.

individual is not responsible.

Now, how to select in a precise way the factors which constitute external opportunities and those which are individual responsible choices? As it is evident, the central theoretical point, and consequently the main point of divergence among competing theories, is to decide where to draw the line. The main divergence concerns the status of individual preferences. Are they within or beyond the individual responsibility? Dworkin argues that justice requires equality of resources, and that preferences are irrelevant, in the sense that they are within the individual responsibility; his favourite examples involve the presence of expensive tastes. Whereas in Cohen and Arneson 's view the relevant cut is not between resources on the one hand and preferences on the other, but between factors within and outside the individual control. Hence, one has to consider the process whereby individual preferences are generated and, in particular, the presence of adaptive or endogenous preferences: individual tastes themselves can be partly determined by the external environment. An exhaustive discussion of the various positions within the opportunity egalitarian theories is surely beyond the object of this work.

The purpose of this paper is not to de¯ne the proper domain of individual responsibility; rather, we consider tools which could be applicable to such a discussion. Hence the focus will be on the development of techniques which can be of some utility for the implementation of a concrete public policy inspired by the opportunity egalitarian ethics; whatever the de¯nition of responsibility and opportunity a society decides to adopt.

Now, the choices that are likely to confront an opportunity egalitarian policy maker will involve the evaluation of di®erent public policies on the basis of the opportunity redistribution they introduce, which in turn requires comparing situations where individuals have di®erent opportunities. Hence, if we want to give operational content to the opportunity egalitarianism, we have to address the following issues: (i) how to measure the degree of inequality present in a distribution of opportunities? (ii) what redistribution mechanisms intended to increase the degree of 'opportunity equality'?

A natural answer, for the redistribution problem, would consist in equalizing the individual opportunity sets and, once this equalization is achieved, in letting the individuals choose from their opportunity set their preferred option[2]. This solution corresponds to performing a direct exercise of measurement of inequality in a distribution of opportunity sets[3] (in this direction are, for instance, the contributions by Alergi and Nieto [1], Herrero [16], Kranich [18], [19], Ok and Kranich [23]).

This approach is surely correct in principle; however, given the high levels of

[2] This is basically the program proposed by Dworkin [10].

[3] For a recent survey of the relevant literature, with attention also to the related issue of ranking opportunity sets, see Peragine [24]

measurability and comparability of opportunities required, it seems unlikely to be useful for operational purposes. In addition to the measurability limitations, consider that the elements entering a person's opportunity set will be, in general, social circumstances as well as individual native talents. Hence, it is likely that some of these opportunities are ¯xed, personal and cannot be redistributed.

These implementability constraints motivate an indirect approach to the opportunity egalitarian project, where the focus is not on the distribution (and redistribution) of opportunities "per se"; rather, it is on the consequences of a given distribution of opportunities on some form of individual advantage. A consistent reformulation of the opportunity egalitarian principle (OEP), could be the following: Inequalities in a given distribution of outcomes, which are due to di®erences in factors beyond the individual control (opportunities), are to be considered inequitable and are to be compensated; whereas inequalities, due to factors within the personal responsibility, are equitable and are not to be compensated.

According to this formulation, what we object to, on the basis of opportunity egalitarianism, is not the inequality in the distribution of factors beyond the individual control (the opportunity set) \per se"; rather, we object to the consequences of these inequalities on something which is valuable for us (some form of individual advantage). That is to say, we object to inequalities, in a given distribution of advantage, which are caused by inequalities in opportunities.

Consequently, considering in a uni¯ed perspective both the measurement and the redistribution issue, in this paper we adopt an indirect, pragmatic approach. We ¯rst address the question of how to measure inequalities, in a distribution of outcome, which are due to di®erences in factors beyond the individual responsibility. Or, more generally, how to rank di®erent "outcome distributions" on the basis of opportunity inequality. As for the prescriptive side of the analysis, we seek a public policy which compensates people for the outcome inequalities due to di®erences in opportunities, without interfering with the inequalities due to responsible choices.

The study of an opportunity egalitarian policy in formal economic models has been made, in particular, by Bossert [5], Bossert and Fleurbaey [6], Bossert et al. [7], Fleurbaey [11], [12], [13], Fleurbaey and Maniquet [14], Roemer [26], [27], [28].

One way of addressing the previous questions can be illustrated by the following (informal) argument. Consider a given population and a distribution of a particular form of advantage (income, utility, etc.). The advantage, for each individual, is causally determined by two classes of factors: factors beyond the individual's responsibility (or opportunities) and factors for which the individual is responsible. Now partition the population into types, a type being the group of people endowed with the same set of opportunities. Since the individual advantage is determined only by opportunities and responsibility, then, by de¯nition, any outcome inequality observed within a type can be unambiguously attributed to differences in the responsibility exercised. According to the OEP such inequalities are equitable and not to be compensated at the bar of justice; for, if they were, the individual responsibility would be denied. On the other hand, between-types outcome inequalities (a concept, this, to be precisely defined yet) can be interpreted as reflecting inequalities in opportunities.

Hence, roughly speaking, the focus of concern of an analysis on equality of opportunity becomes the measurement and the reduction of inequality, in terms of the selected outcomes, among different types.

To explain in details the strategy we propose, it is time to introduce a more formal model.

1.1 A formal model of equality of opportunities for income

We start our formal analysis by selecting the income as the relevant outcome. Income is a particularly useful example because much of actual redistributive policy in contemporary welfare states is designed in terms of income tax and income constitutes the standard basis for the measurement of economic inequality.

The population is represented by a finite set N of individuals. Each individual's income $x; x \in X;$ is causally determined by two kinds of factors: factors beyond the individual control, represented by a person's opportunity set $O; O \in \Omega;$ and factors for which the individual is fully responsible, represented by a scalar variable $w; w \in W$. Hence we have:

$$x = g(O; w)$$

where the function g is the same for all individuals.

Income x is supposed to be continuously distributed, with cumulative distribution function $F(x)$ and $X = [0; z]$.

A person's opportunity set O is observable; and we next assume that there is a general political agreement on the following complete ordering over all possible opportunity sets :

$$\Omega = \left\{ O \mid O^1 < O^2 < \dots < O^i < \dots < O^n \right\}$$

so that, in general, we have: $O^i < O^{i+1}$.

The responsibility variable w is unobservable. Individuals have the same degree of access to the same set of responsible choices W; and the individual income function g is supposed to be increasing in the responsibility variable.

Following Bossert [5] and Bossert and Fleurbaey [6], in our model the distribution of both O and w is not altered by the public policy enforced; that is, income is perfectly transferable. This is a quite natural assumption for the opportunities, especially if one thinks to the native talents as an important element of the individual opportunity set. As for the responsibility variable, the assumption could be justified by thinking that it would be hardly acceptable to held a person fully responsible for w; were this variable depending on the public policy.

The next step is to partition the population into types, where each type denotes the subset of the total population N having the same opportunity set: for $O \in \Omega$; a type is the set of individuals i such that $O^i = O$: Within each type there will be a distribution of income $F^i(x)$; with density function $f^i(x)$ and population n_i.

Consider any individual $k; k \in O^i$; who is endowed with opportunity O^i and, after exercising responsibility w_k; ends up with ex-post income $x_k^i = g(O^i; w_k)$: We can write the income level of individual k in type i as:

$$x_k^i = \mu^i + r_k$$

where μ^i is the mean income for type i; defined as:

$$\mu^i = \int x f^i(x)\, dx \tag{1}$$

and the residual terms r_k are such that, within each type i; $\int r f^i(x)\, dx = 0$:

Now consider that, once we have included in the specification of the type all relevant factors beyond a person control, the differences in the income level among people in the same type are, by definition, within their own responsibility. Every individual in a given type $i; i \in (1; ...; n)$ is endowed with the same opportunity set O^i. However, different individuals will make different choices, will exercise different degrees of responsibility and therefore will end up with different income levels; we interpret such income differences as completely determined by their own autonomous choices. Hence, we identify the within types income inequality with inequality in the responsibility exercised by the persons in that type.

On the other hand, it seems reasonable to interpret the average income reached within each type, as a characteristic of the type itself; that is to say that the average income is determined, within each type, by the common endowment of opportunity.

To express formally these ideas, we now introduce a crucial assumption saying that the individual income is additively separable in responsible and non-responsible factors[4].

Assumption 1:

$$x_k^i = \mu\left(O^i\right) + r(w_k) \tag{2}$$

In words, we assume the expected income μ^i, which is the same for all the individuals in each type, to be a function of the opportunity set; whereas the residual part r_k is determined by the responsibility exercised.

[4] A similar assumption of additively separability of the income (or outcome) function in responsible and non responsible characteristics, plays a fundamental role in Bossert [5]'s and Fleurbaey [11]'s models.

Consequently, all the income differentials observed in the same type, due to variation in the residual term r_k, are to be considered equitable and not to be compensated; whereas, differentials in the "mean terms" μ^i, due to differentials in opportunities, are inequitable and to be compensated.

Notice that we are assuming all individuals to have the same degree of access to the same set of possible responsible choices W, independent from the opportunities they are endowed with; given Assumption 1, this implies that equalizing the mean incomes across types ensures that the level of ex-post income a person can reach for any given choice of will is the same as for any other individual. To quote Arneson ([2], p. 84), "people are able to attain identical welfare levels with the same effort".

Considering that the opportunity set is intended as the set of factors influencing ab origine the individual chances to gain income, it seems reasonable to assume that those individuals with lower opportunity set have, in average, a lower income than those with higher opportunities; on the other hand, the function r is supposed to be a strictly increasing function of the responsibility exercised. Recalling that $\forall i \in (1;2;...;n)\,; O^i < O^{1+1}$ we have the following

Assumption 2:

$$\forall k \in O^i; \forall i \in (1;...;n)\,; \qquad (3)$$
$$r'(w_k) > 0 \quad \& \quad \mu\left(O^i\right) < \mu\left(O^{i+1}\right).$$

Let us denote by Ψ the set of distributions constructed as above.

It is evident that, in the framework we have introduced, the focus of concern of an analysis on equality of opportunity is the distribution of the type-means: we can compare any two income distributions $F, G \in \Psi$, on the basis of opportunity inequality, by comparing the inequality present in their relevant mean distributions; analogously, an opportunity egalitarian public policy will be intended to decrease the degree of income inequality present in the mean distribution.

Denoting by n_i^F the population in type i of distribution F, and by 1_i^F the unit vector of length n_i^F, we can formally define the types-mean distribution O_F, obtained from distribution $F(x)$, as follow:

$$O_F = (\mu^1 u^1; ...; \mu^i u^i; ...; \mu^n u^n). \qquad (4)$$

This is no other than the indirect expression of the opportunity distribution

$$\Omega = \left\{ O \mid O^1 < O^2 < ... < O^i < ... < O^n \right\}.$$

Moreover, notice that Assumption 2 implies that the type mean distribution O_F, obtained from any $F(x) \in \Psi$, is ordered:

$$\mu_1^F < ... < \mu_i^F < ... < \mu_n^F.$$

The exposition in the rest of the paper is organized as follows. In section 2 we address the problem of ranking income distributions in terms of equality of opportunity. The characterization of an opportunity egalitarian income tax policy, and the discussion of criteria for choosing among alternative tax systems, are contained in section 3; in the same section we address the problems of measuring the Horizontal Inequity (3.1) and the opportunity redistribution (3.2) introduced by an income tax. Section 4 provides concluding comments; the Appendix contains the proofs of the theorems.

2 Measuring income inequality due to opportunity inequality

We consider in this section the problem of ranking income distributions on the basis of "equality of opportunities".

In the standard, unidimensional problem of ranking income distributions on the basis of "income equality", the fundamental theorem of inequality economics states that (Hardy-Littlewood-Polya): one distribution of income Lorenz dominates another income distribution having the same mean if and only if the former can be obtained from the latter by a finite sequence of equalizing transfers, and if and only if the former is ranked higher than the latter by all inequality averse social welfare functionals.

We want to formulate an analogue of this theorem in the different context of opportunity inequality. To this end, we have first to formulate in this new framework the concepts of Lorenz partial ordering, of equalizing transfer and of inequality averse social evaluations functions.

Then we try to obtain a more complete ordering to rank a list of income distributions on the basis of opportunity equality.

We first develop our analysis by assuming that the relevant income distributions have the same types partition; however, later in the paper this assumption will be relaxed.

2.1 The Opportunity Lorenz Partial Ordering

Consider two income distributions $F; G \in \Omega$: We have denoted the mean income of type i in distribution F as μ_F^i, and the population size in i by n_i, so that $p_i = \frac{n_i}{N}$ is the population share in type i (remember we are assuming an equal type partition for F and G). Analogous notation for distribution G.

The ordinary Lorenz partial ordering ($\succcurlyeq_L$) applied to the mean distributions O_F and O_G obtained respectively from $F(x)$ and $G(x)$; is defined as:

$$O_F \succcurlyeq_L O_G \iff \sum_{i=1} \frac{p_i \int x f^i(x)\,dx}{\sum_{j=1} p_j \int x f^j(x)\,dx} \geq \sum_{i=1} \frac{p_i \int x g^i(x)\,dx}{\sum_{j=1} p_j \int x g^j(x)\,dx}; \forall k \in (1; ...; n): \tag{5}$$

Moreover, the ordinary Generalized Lorenz partial ordering ($\succcurlyeq_{GL}$) applied to the mean distributions O_F and O_G is defined as:

$$O_F \succcurlyeq_{GL} O_G \iff \sum_{i=1}^{k} p_i \int x f^i(x)\,dx \geq \sum_{i=1}^{k} p_i \int x g^i(x)\,dx; \forall k \in (1;...;n): \quad (6)$$

We can now define the Opportunity Lorenz partial ordering ($\succcurlyeq_{OL}$) and the Opportunity Generalized Lorenz partial ordering ($\succcurlyeq_{OGL}$):

Definition 1 For any $F(x); G(x) \in \Omega$; the ordering induced by $\succcurlyeq_{OL}$ is defined as follows:

$$F(x) \succcurlyeq_{OL} G(x) \iff O_F \succcurlyeq_{L} O_G; \quad (7)$$

and the ordering induced by $\succcurlyeq_{OGL}$ is defined as follows:

$$F(x) \succcurlyeq_{OGL} G(x) \iff O_F(x) \succcurlyeq_{GL} O_G(x): \quad (8)$$

That is, one income distribution Lorenz Dominates (Generalized Lorenz Dominates) another in opportunity terms if and only if the mean distribution of the former Lorenz dominates (Generalized Lorenz Dominates) the mean distribution of the latter.

2.2 Opportunity Equalizing Transfer Ordering

We aim to formulate an analog of the "principle of equalizing transfer" of the income inequality measurement theory. To this end, one needs first to define the concept of equalizing transfer in the current context. The relevant variable in terms of which to decide who are the richer agents and thus from which agents a transfer should be made is of course the opportunity set. Therefore, any agent k, with $k \in O^{i+1}$; is considered to be "richer" (in opportunity terms) than another agent h, with $h \in O^i$; because $O^i < O^{i+1}$; regardless of their relevant income levels. This has two implications: first, the equalizing transfer has to be thought as a transfer from one (rich) type to another (poor) type; second, all persons in the same type are equally rich (or poor) in opportunity terms, hence they have to be treated equally. Hence, an **Opportunity Equalizing Transfer** (OET) is a transfer of income which has to satisfy the principle of **progressivity** between types; as for the equal treatment of people in the same type, we can formulate two different solutions.

The first amounts to require anonymity within each type, and is the case of **lump sum** transfers for individual in the same type.

Formally, consider two types $i; j$ with $O^i \prec O^j$: An OET (in the "lump sum" version) is a quantity of income $\delta > 0$; with the relevant vector $d^i u^i$ (where

$d^i = \frac{\Delta}{n_i}$ and u^i is the unit vector of length n_i) such that the new types-mean incomes are:

$$\begin{aligned} \tilde{\mu}^i &= \mu^i + d^i = \mu^i + \frac{\Delta}{n^i} \\ \tilde{\mu}^j &= \mu^j - d^j = \mu^j - \frac{\Delta}{n^j} \end{aligned} \tag{9}$$

with $\tilde{\mu}^i \leq \tilde{\mu}^j$ (it is a rank-preserving equalizing transfer).

Given the anonymity requirement, this is equivalent to say that

$$\begin{aligned} \tilde{x}^i_k &= x^i_k + d^i; \forall k \in O^i \\ \tilde{x}^j_h &= x^j_h - d^j; \forall h \in O^j \end{aligned}$$

We can notice that the quantity Δ is the total (equal) amount transferred from the rich type to the poor one; however, the quantity received (given) by (to) each individual d^i differs according to the type population n_i. Considering any two types i, j we have that $d^i = d^j \frac{n_j}{n_i}$.

The second solution to the treatment of people in the same type amounts to require **proportionality** within types, and we require equal average transfer for individuals in the same type. Formally, denoting again by Δ the total transfer from a type to another, we now require the transfer to/from a person k in type i (call this individual transfer a^i_k) to satisfy the following conditions:

$$\forall k \in O^i; \forall i \in (1, ..., n); \quad \sum_{k=1}^{n^i} a^i_k = \Delta \tag{10}$$

$$\forall k, h \in O^i; \forall i \in (1, ..., n); \quad \frac{a^i_k}{x^i_k} = \frac{a^i_h}{x^i_h} \tag{11}$$

Thus, if the mean income of type i before the equalizing transfer is μ^i, the post-transfer mean income will be

$$\tilde{\mu}^i = \mu^i + \frac{1}{n^i} \sum_{k=1}^{n^i} a^i_k.$$

Hence, considering two types $i, j \in (1, 2, ..., n)$, with $O^i \prec O^j$, an OET (in the "proportionality" version) is a quantity of income $\Delta > 0$, with the relevant transfer to/from an individual k in type i being the already defined a^i_k, and such that the new types-mean incomes are:

$$\begin{aligned} \tilde{\mu}^i &= \mu^i + \frac{1}{n^i} \sum_{k=1}^{n^i} a^i_k = \mu^i + \frac{\Delta}{n^i} \\ \tilde{\mu}^j &= \mu^j - \frac{1}{n^j} \sum_{k=1}^{n^j} a^j_k = \mu^j - \frac{\Delta}{n^j} \end{aligned} \tag{12}$$

with $\tilde{\mu}^i \leq \tilde{\mu}^j$. Clearly, the lump sum and the proportional versions of OETs have exactly the same effect on the mean distribution; they differ each other only in the treatment of people within each type. Thus, considering a given mean distribution

$$O_F = (\mu^1 u^1; ...; \mu^i u^i; ...; \mu^n u^n)$$

after an OET involving types i and j; the new distribution will be

$$O_{\tilde{F}} = (\mu^1 u^1; ...; \mu^i u^i; ...; \mu^j u^j; ...; \mu^n u^n): \tag{13}$$

Distribution $O_{\tilde{F}}$ differs from distribution O_F by a single Opportunity Equalizing Transfer; moreover, we say that distribution $O_{\tilde{F}}$ is more "opportunity equal" than distribution O_F. We can extend this idea by saying that by going from one distribution O_F to another O_G, by a finite sequence of OETs so defined, we reduce the opportunity inequality. We can now formally define the binary relation "is obtained by a finite sequence of opportunity egalitarian transfers" ($\succcurlyeq_{OET}$).

Definition 2 For any equal means income distributions $F(x), G(x) \in \Gamma$ the ordering induced by $\succcurlyeq_{OET}$ is defined as follows:

$$F(x) \succcurlyeq_{OET} G(x)$$

if and only if the Opportunity Distribution O_F can be obtained from the Opportunity Distribution O_G by means of a finite sequence of OET's.

2.3 Opportunity Egalitarian SEF

In this section, we seek a Social Evaluation Function (SEF) expressing the Opportunity Egalitarian Principle, in order to rank a list of income distributions on the basis of opportunity equality. Consider the following additive, individualistic and symmetric form of SEF:

$$W = \sum p_i \int_0^z U^i(x) f^i(x) dx \tag{14}$$

where $U^i(x)$ is the social planner's imposed advantage function for type i, $f^i(x)$ is the income density function in type i, and $p_i = \frac{n_i}{N}$ is the population share in type i. Recall that we are assuming a discrete distribution for the opportunity sets O^i; on the contrary, income is continuously distributed, and we assume that the advantage function $U^i(x)$ is twice differentiable (almost everywhere) in the variable x.

We now try to capture the basic intuition beyond the opportunity egalitarian ethics, by restricting the class of social planner's imposed advantage functions. Hence we introduce three crucial conditions on the functions $U^i(x)$ in order to characterize the family of Opportunity Egalitarian SEFs. First, we assume that types advantage increases with income, whatever the opportunity level:

$$(C.1)\ \frac{dU^i(x)}{dx} \geq 0; \forall i \in (1, 2, ..., n); \forall x \in X.$$

Hence condition C:1, which is a common **monotonicity** assumption, guarantees that social welfare increases as a result of an income increment.

Next, we assume that our SEF is indifferent to income inequality due to individual responsibility; that is, for income inequality within the same type. We require **within-type income inequality neutrality**:

$$(C{:}2)\ \frac{d^2U^i(x)}{dx^2} = 0; \forall i \in (1; 2; \dots; n); \forall x \in X$$

This conditions says that, for fixed O^i; that is, when focusing on the group of people having the same opportunity set, the welfare gain resulting from a given total extra income, however distributed, is constant. Hence, a reduction in income inequality within a type, which leaves the mean income of the type unchanged, has no welfare effects. This welfare condition corresponds to the requirement of within types anonymity in the definition of an Opportunity Egalitarian Transfer.

Next, we want to formulate a condition expressing the aversion to inequality in the opportunity distribution. Inequality-aversion is usually expressed by assuming concavity of the utility function in the relevant variable. But the opportunity (actually, the expected income) distribution is a discrete distribution. Hence we seek an analog, in the discrete case, of the concavity assumption usually employed in the continuous case. The condition expressing **between-types income inequality aversion** is the following:

$$(C{:}3)\ \frac{dU^i(x)}{dx} \geq \frac{dU^{i+1}(x)}{dx}; \forall i \in (1; 2; \dots; n); \forall x \in X$$

which says that the marginal increase in welfare due to an increment of income, is a decreasing function of opportunity. Actually, in case of a continuum of opportunity types i and fully differentiable utility function $U(x; i)$, condition C.2 requires that the cross derivatives be non-positive. Condition (C:3) is a fundamental assumption which implies that a transfer of income from a richer to a poorer type (in opportunity terms), at a given income level, is welfare improving.

Defining the class of SEF which satisfy conditions (C.1) to (C.3) as SEF_{OE}; we have the following

Definition 3 For any $F(x); G(x) \in \Omega$; the ordering induced by $\succcurlyeq_{OEF}$ is defined as follows:

$$F(x) \succcurlyeq_{OEF} G(x) \iff W_F \geq W_G\ \forall W \in SEF_{OE}. \tag{15}$$

Some further remarks are in order with respect to the social evaluation function we are employing. It as an additive, individualistic and symmetric function. Now, considering the conditions (C:1 to C:3) we have introduced, we can ask: is the SEF we are employing a standard utilitarian one?

A fundamental component of utilitarianism, welfarism, requires that any social state be evaluated on the basis of (and only on the basis of) individual utilities. Any other variable can be considered relevant only in an instrumental sense: that is, only in its contribution to the individual utility. On the other hand, the opportunity egalitarian approach claims to be non-welfarist. Two reasons can justify the non-welfarist character of the opportunity approach. One is the fact that the individual's relevant advantages considered in the opportunity approach typically are not interpreted as individual utilities. However, this distinction is more semantic than substantial. The notion of utility used in economics is sufficiently flexible to be interpreted as any notion of individual valuable outcome.

The other factor, which distinguishes in a substantial sense the opportunity approach from welfarism, is the fact that, in the former, information about the individual's achievement does not suffice for comparing alternative social states for normative appraisal. Knowledge of the individual ex-ante opportunities is also important; better, opportunities are intrinsically important. Now, the individual advantage function (U) we are using, defined over individual income and opportunity set ($U = U(x; O^i)$); is not to be thought as an individual's utility function. In other words, we do not argue that the actual individual utility (or welfare) depends upon both the income and the opportunity level[5]; for this would imply to give an instrumental value to the opportunity, hence to make the opportunity approach consistent with welfarism. Rather, U is the evaluation function used by the social planner to evaluate a given social state from the distributive justice point of view: both the income an the opportunity levels are considered to be relevant for equity judgments purposes. It could well be the case that two persons with equal income, but with different ex-ante opportunity sets, get exactly the same level of welfare. Nevertheless, their situations are normatively different; for the relevant variable we are employing in distributive judgements is not welfare but opportunity. Hence, we are entering non-welfare information (namely, information about the ex-ante distribution of opportunities) in evaluating a social state; which is inconsistent with welfarism, hence with utilitarianism.

2.4 Equivalence results

We are now ready to obtain unambiguous social rankings of income distributions, based on the opportunity egalitarian principles. First we obtain that a given income distribution $F(x)$ is superior to another income distribution $G(x)$ on the basis of opportunity egalitarianism, if and only if the mean income distribution of

[5] From this specification it follows that, even if analytically similar to the "differences in needs" approach to the measurement of income inequality (see, in particular, Atkinson and Bourguignon [4], and the related literature), the current approach is very different in its economic meaning.

$F(x)$ (the indirect expression of the opportunity distribution) Generalized Lorenz Dominates (Lorenz Dominates) the Expected income distribution of $G(x)$.

Theorem 1 For any $F(x); G(x) \in \Psi$ the ordering induced by $\succeq_{OGL}$ is the same as the ordering induced by $\succeq_{OEF}$:

Proof. [In the appendix]

Therefore, if $F(x)$ and $G(x)$ have the same total income, then the ordering induced by $\succeq_{OL}$ is the same as the ordering induced by $\succeq_{OEF}$:

Theorem 1 can be seen as a modified form of Second Degree Stochastic Dominance in a two-dimensional setting, with the assumption that one dimension (opportunity) is only ordinally measurable. Analogously to the Atkinson (1970) theorem, it gives normative significance to the statistic-descriptive concept of Lorenz Dominance; the normative judgment being now based on the opportunity egalitarian ethics, instead of the utilitarian one (as in Atkinson).

Next we obtain that an income distribution Lorenz dominates another in opportunity terms if and only if, the former can be obtained from the latter by means of a finite sequence of opportunity equalizing transfers (OET's).

Theorem 2 For any equal means income distributions $F(x); G(x) \in \Psi$; the ordering induced by $\succeq_{OL}$ is the same as the ordering induced by $\succeq_{OET}$:

Proof. [In the appendix]

Theorem 1 and Theorem 2 yield the main result of this paper, which can be seen as the analogue of the fundamental theorem of inequality economics, in the current context of opportunity inequality: one distribution of income opportunity Lorenz dominates another income distribution having the same mean if and only if the former can be obtained from the latter by a finite sequence of opportunity equalizing transfers, and if and only if the former is ranked higher than the latter by all opportunity inequality averse SEFs.

Theorem 3 For any equal means income distributions $F(x); G(x) \in \Psi$; the following statements are equivalent:

(i) $F(x) \succeq_{OL} G(x)$;

(ii) $F(x) \succeq_{OET} G(x)$;

(iii) $F(x) \succeq_{OEF} G(x)$:

2.5 An extension to distributions with different type partitions

In this section we extend the dominance conditions derived in Theorem 1 to the case of distribution of income which have different type-partitions. This extension is of great importance in empirical applications. In fact, if the assumption of equal partition is appropriate when comparing distributions which refer to the same population but observed in di®erent moments in time, it is inadequate when observing di®erent population. For instance, when we want to compare the degree of opportunity inequality of di®erent countries, or we want to assess secular trends in the distribution for a country.

We keep the same de¯nition of Opportunity Lorenz partial ordering and the relevant Generalized version. As for the SEF, we need to introduce an additional assumption on the type advantage function. More precisely we consider the same form of SEF as before:

$$W_F = \sum p_F^i \int_0^z U^i(x) f^i(x) dx$$

where, of course, now the population share p_F^i varies with the distribution. To the conditions $C.1 - C.3$ already introduced and characterizing the family SEF_{OE}; we now add the following condition:

$$(C.4) \qquad U^i(z) = U^{i+1}(z); \forall i \in (1; 2; ...; n)$$

where z is the maximum income level in each type. By introducing condition $(C.4)$, the advantage functions $U^i(x)$ cease to be ordinal: any a±ne transformation such as, for example, $U^i \rightarrow a^i + bU^i$, now is supposed to be able to a®ect the results of social comparisons. This requirement is necessary in a context with di®erent types population. Condition $C.4$ implies the extension of our result. Let the set of SEF's satisfying conditions $C.1$ to $C.4$ be denoted by SEF_{OE}^*; and the relevant ordering (analogous of De¯nition 3) be denoted by $\succcurlyeq_{OEF^*}$. Then we obtain the following:

Theorem 4 For any $F(x)$ and $G(x) \in \Omega$; the ordering induced by $\succcurlyeq_{OGL}$ is the same as the ordering induced by $\succcurlyeq_{OEF^*}$:

Proof. [In the appendix]

Hence, unambiguous social rankings are achieved if, and only if, one distribution Generalized Lorenz dominates, in opportunity terms, another.

2.6 Complete Ordering

In this section we aim to obtain a criterion to rank a list of income distributions on the basis of "equality of opportunities", which is more complete than the ones developed in the previous sections.

According to the assumptions introduced in the previous sections, we can say that the within-types inequality is to be interpreted as inequality due to individual responsibility; whereas, the inequality in the distribution of types-mean income (or individual expected income), which is no other that the between types inequality, surely re°ects opportunity inequality.

Hence, we want to distinguish the Overall Inequality (OI) observed in a given distribution $F \in \Omega$ into: within-types inequality (W) and between-types inequality (B). We can use a Path Independent (Foster - Shneyerov [15]) measure of inequality (call it I); which is a relative index of inequality that can be decomposed as follow:

$$OI = B + W.$$

The terms B and W are calculated in the following way:

- $B = I\left(F^B\right)$ is given by the measure applied to the Smoothed Distribution F^B which replaces each income in a type with its representative income;
- $W = I\left(F^w\right)$ is given by the measure applied to the Standardized Distribution F^w which rescales types distributions until all types have the same representative income.

Expressing $I\left(F^B\right)$ as a residual:

$$I\left(F^B\right) = I(F) - I\left(F^W\right) \tag{16}$$

The interpretation in the current context is as follows. The Smoothed Distribution F^B, obtained by replacing each income with its type mean income μ_i, is no other than the distribution obtained by eliminating, at each income level x_k^i, the residual (responsibility) term r_k. Hence the inequality index applied to this distribution captures only and fully the income inequality due to opportunity inequality: $I\left(F^B\right)$ is our indirect index of opportunity inequality.

On the other hand, by rescaling all types distributions until all types have the same mean income (let us call it μ), we are left with a distribution in which the only inequality present is that due to differences in r_k, that is to individual responsibility. Thus, to obtain the Standardized Distribution:

$$x_k^i = \mu^i + r_k \rightarrow \left(x_k^i\right)^W = \mu + \frac{\mu}{\mu^i} r_k \tag{17}$$

Hence, $I\left(F^W\right)$ represents that part of income inequality due to individual responsibility, which has not to be taken into account when measuring the opportunity inequality.

Therefore, considering any two income distributions $F, G \in \Omega$, we'll say that the distribution F exhibits a lower degree of opportunity inequality than distribution G if and only if

$$I\left(F^B\right) < I\left(G^B\right).$$

If the index I we are employing is a relative index of inequality, which satisfies the scale invariance property (the measure is not affected by equiproportionate changes in all incomes), then $I\left(F^B\right)$ satisfies the Principle of Opportunity Equalizing Transfer, in its **proportionality** version. In fact, by definition, an OET applied to a given distribution X:

(i) reduces the inequality present in the relevant types mean distribution (due to the requirement of progressivity between types);

(ii) leaves the inequality within types unchanged (due to the requirement of equal average transfer across individuals in the same type).

Hence any OET applied to a distribution F leaves the value of $I\left(F^W\right)$ unchanged, while reduces the value of $I\left(F^B\right)$.

On the other hand, if we use an **absolute** index of inequality I_A, which is invariant to equal additions to all incomes, clearly $I_A\left(F^B\right)$ satisfies the Principle of Opportunity Equalizing Transfer, in its **lump sum** version.

3 Opportunity Redistribution by the Income Tax

In this section we consider the problem of defining a redistributive public policy inspired by the opportunity egalitarian ethics. Given the case of non-transferability of the individual opportunities and full transferability of income, we do not seek a mechanism to redistribute opportunities; rather, we try to characterize an income policy which compensates individuals for income inequalities due to differences in endowments of opportunities, without interfering with the inequalities due to autonomous choices.

Considering the previous analysis of opportunity inequality, it is natural to use the Lorenz criterion in comparing the income inequality in the pre the post-tax distributions; therefore, we can say that an opportunity redistributive public policy is a tax policy T such that, after its application to a given income distribution $F \in \Psi$, the post-tax distribution F_T:

1. Lorenz dominates, in opportunity terms, the pre-tax distribution F;

2. exhibits the same degree of inequality, according to the Lorenz criterion, as the pre tax distribution F, within any type i, $i \in (1, \ldots, n)$.

In more formal terms, denoting by $\succcurlyeq_L$ the Lorenz partial ordering applied to any two type i distributions F_i and G_i, $i \in (1, \ldots, n)$, belonging respectively to distribution $F, G \in \Psi$, and recalling the definition of Opportunity Lorenz partial ordering ($\succcurlyeq_{OL}$) given in section 2.1;we can introduce the following

Definition 4 A tax policy T is an Opportunity Redistributive Tax Policy if and only if,

$$\forall F \in \Psi, \forall i \in (1, \ldots, n), \; F^i \approx_L F_T^i \;\; \& \;\; F_T \succcurlyeq_{OL} F.$$

We now seek some normatively meaningful conditions in order to characterize an opportunity redistributive tax system.

In designing an equitable tax system, two basic principles are usually employed: the Vertical Equity (VE) and the Horizontal Equity (HE) principle. In this section we aim to reformulate these fundamental rules on the light of the Opportunity Egalitarian Ethics, and then to design a tax policy which is consistent with them.

The Horizontal Equity Principle (HE) requires the equal treatment of equals. Giving operative meaning to this command requires to define in a precise way the concepts of "equals" and of "equal treatment".

In the current context, it seems natural to evaluate the "equal position" in terms of opportunity: are "equals", in a normative sense, individuals with equal opportunities. Hence the HE rule in the present context refers to the treatment of people belonging the same type. If, consistent with the proportionality version of OET, we interpret the "equal treatment " as equal average tax rate $\left(\frac{T}{x}\right)$, and letting T_k^i be the tax paid by individual k in type i, we can formulate the following:

Axiom 1 (HE):

$$\forall i \in (1,2,...,n), \left(\forall k,h \in O^i\right), \frac{T_k^i}{x_k^i} = \frac{T_h^i}{x_h^i}.$$

Thus, an income tax $T(x,i)$ reflecting Axiom 1 (HE) should be proportional within each type:

$$(\forall x \in X), \forall i \in (1,2,...,n), \frac{d\left[\frac{t(x,i)}{x}\right]}{dx} = 0. \quad (18)$$

Hence, the HE principle can be interpreted in this context as a command requiring that the inequality in a given income distribution among people defined "equals"on the basis of a certain socially relevant characteristics, be unchanged after the public intervention.

An alternative formulation of the HE principle would amount to require equal payment (T) for all individuals in the same type. A formulation which would be consistent with the lump sum version of the OET.

On the other hand, the Vertical Equity Principle requires an "appropriately unequal treatment of unequals". In the current context it refers to the appropriate differentiation of treatment of people with different opportunities; that is, of people belonging to different types.

We interpret, as before, the "unequal treatment " as unequal average tax rate, and require also, as part of the Vertical Equity principle, that there be no reranking of means after the introduction of the taxes. Hence, letting a_i be the average tax rate for type i, and recalling that $O^i < O^{i+1}$, we can formulate the following:

Axiom 2 (VE):

$$(\forall i \in (1,2,\dots,n))\,; \left(\forall k \in O^i; \forall h \in O^{i+1}\right);$$

$$\frac{T_k^i}{x_k^i} < \frac{T_h^{i+1}}{x_h^{i+1}} \quad \& \quad (1-a_i)\mu^i < (1-a_{i+1})\mu^{i+1}.$$

Hence, an income tax $T(x;i)$ reflecting both the HE and the VE principles should be proportional within and progressive between types:

$$\left(\forall k,h \in O^i\right); \forall i \in (1,2,\dots,n)\,; \tag{19}$$

$$\begin{aligned} \frac{T_k^i}{x_k^i} &= \frac{T_h^i}{x_h^i} = a_i \\ a_i &< a_{i+1} \\ (1-a_i)\mu^i &< (1-a_{i+1})\mu^{i+1} \end{aligned}$$

The following result shows that Axioms HE and VE characterize the Opportunity Redistributive Tax Policy.

Theorem 5 The tax policy T satisfies Axioms HE and VE if and only if T is an Opportunity Redistributive Tax Policy.

Proof. [In the Appendix]

Following a similar reasoning, we can analyse the Opportunity Redistributive Tax Policy from a normative point of view; to this end, we employ the already defined $\succcurlyeq_{OEF}$ ordering. Letting T be a tax satisfying Axioms HE and VE, and denoting by F_P the post tax distribution obtained after the application of a fully proportional tax P raising the same revenue as T, we obtain the following

Theorem 6 For any distribution $F \in \Phi$; and the proportional tax P raising the same revenue as T, if T satisfies HE and VE then

$$F_T \succcurlyeq_{OEF} F_P. \tag{20}$$

Proof. [In the Appendix]

A consequence of these results is that we now have a criterion to discriminate among alternative tax systems on the basis of a clear normative judgement. In fact, given two tax systems T^1 and T^2, with the same total revenue, we say that T^1 is preferred to T^2 according to the Opportunity Egalitarian Ethics, if and only if the post-tax distribution (F_{T^1}) Opportunity-Lorenz dominates the post-tax distribution (F_{T^2}): $F_{T^1} \succcurlyeq_{OL} F_{T^2}$.

We now address two different important questions, both useful to fully appreciate the impact of a tax system on a given distribution: first, that of measuring violation of the Horizontal equity principle introduced by the tax; second, that of measuring the opportunity redistribution, which is no other that the between-types redistribution, introduced by the tax.

3.1 Measuring Horizontal Inequity

Before proposing a particular solution to the problem of measuring Horizontal Inequity (HI), let us de¯ne in a more precise way what we mean by Horizontal Inequity.

We have de¯ned the Horizontal Equity principle as the command requiring equal average tax rate within each type. This implies that, within each type, the degree of inequality present in the pre-tax distribution and that present in the post-tax distribution obtained by using a horizontally equitable tax, be the same according to every Lorenz-consistent inequality index (that is, every symmetric, relative, Principle of transfer-consistent, inequality measure). According to our de¯nition of Horizontal Equity, we cannot admit of progressive, proportional or regressive distinctions when measuring horizontal inequity; we have to focus on any disparity of treatment of those judged to be in the same position. More precisely, considering a given type i, according to the de¯nition introduced, there will be HI both in case of progressivity at some x:

$$\frac{d\left[\frac{t(x;i)}{x}\right]}{dx} > 0$$

and in case of regressivity at some x:

$$\frac{d\left[\frac{t(x;i)}{x}\right]}{dx} < 0$$

or, of course, when both happens (at di®erent x).

Moreover, we have to notice that the measurement of HI implies two distinct but interdependent levels of analysis. First, the HI as a local phenomenon: that is, the HI experienced within each type. Second, the problem of aggregating the local measures in an index of HI for the population as a whole. Both levels have to be considered in order to appreciate whether a given measure is appropriate or not.

A particular feature characterizing our scenario is that we de¯ne as \equals" individuals with equal opportunities; that is, individual belonging the same type. On the other and, as an e®ect of the di®erent degrees of individual responsibility exercised, within type there will be a distribution of income; it is therefore likely that people judged to be "equal" in a normative sense, are characterized by very di®erent income levels. Hence, our measure of HI cannot be thought as a simple measure of the dispersion in the post-tax incomes of those who were equals in the pre-tax income distribution. Instead, our measure of HI has to be based on a comparison between the degree of inequality in the post-tax distribution and that in the pre-tax distribution. There is a class of measures, proposed by the classical literature, which could be appropriate in the current context. Once the relevant group of equals has been identi¯ed, the HI experienced at local level is measured as difference between the post-tax and pre-tax dispersion among the group of equals[6]. Letting I^i_{pre} (I^i_{post}) be an inequality index applied to the pre-tax (post-tax) distribution of incomes belonging the same equals group i, the proposed measure of local horizontal inequity is:

$$LHI^i = I^i_{post} - I^i_{pre}.$$

Then it is proposed to aggregate these measures to obtain an overall measure of HI:

$$HI = \sum p^i \left(I^i_{post} - I^i_{pre} \right).$$

If this methodology is satisfactory when focusing on the local level, it is not necessarily so if we consider the aggregation issue. In fact, applying this procedure, there could be compensation across types: violations due to progressive tax in some types, could compensate violations in the opposite direction in other types.

Two ways of escaping this aggregation problem, and keeping the same measurement solution at local level, would be that of aggregating the local measures by using either:

- their absolute values:

$$HI = \sum p^i \left| I^i_{post} - I^i_{pre} \right|$$

- or their square:

$$HI = \sum p^i \left(I^i_{post} - I^i_{pre} \right)^2.$$

Selecting the "absolute value" solution, the application of this strategy to the current scenario is as follow. Let I^i_X (I^i_{X-T}) be any Lorenz-consistent inequality index applied to the pre-tax (post-tax) distribution of incomes of type i. A measure of the Horizontal Inequity experienced at type i is:

$$HI^i = \left| I^i_{X-T} - I^i_X \right| \tag{21}$$

It will be equal to zero when the degree of inequality in the pre-tax and post-tax distributions in type i is the same; for example, when a proportional tax has

[6] This class of indices has been proposed as response to the identification problem, which arises when, working with sample data, there can be very few perfect equals in the sample of pre-tax incomes, and therefore the application of classical HI measures can produce an under-estimation of the real HI present. In this case some authors (Lambert [20], Lambert & Ramos, [21]) have proposed to use the notion of "close equals groups", intended as groups of people whose incomes belong to a given range, and to measure the HI experienced at local level as difference between the post-tax and pre-tax dispersion among the group of close equals. Different measures of inequality are used: the "mean logarithmic deviation", the "absolute Gini index", and the normative "cost of inequality index" of Atkinson [3].

been applied within the type[7]. It will be positive when there has been some redistribution, either progressive or regressive, within the type; that is, when the income tax was not proportional everywhere in the type.

As for the aggregate measure of Horizontal Inequity, we simply weight the local measures by the relevant types population shares:

$$HI = \sum_{i=1} p^i HI^i \tag{22}$$

The index of Horizontal Inequity will be equal to zero when a proportional tax has been applied within each type. If positive, we can say that there has been some redistribution, either progressive or regressive, within some types.

3.2 Measuring Opportunity Redistribution

The standard way of measuring the redistributive effect introduced by an income tax is that of looking at the variation in the degree of inequality from the pre to the post-tax distribution. Hence, measuring the opportunity redistribution introduced by an income tax corresponds, in our framework, to measuring the reduction of inequality from the pre to the post-tax mean distribution. As the inequality in the mean income distribution is our indirect expression of the opportunity inequality, the between-types redistribution is our indirect expression of the opportunity redistribution. Having already defined the Lorenz criterion, we can use a Lorenz-based index of inequality, apply this index to the relevant mean distribution, and express the opportunity redistribution as the difference between the inequality in the post and pre-tax distribution as measured by that index. Letting G_{μ} be the Gini index applied to the pre-tax mean distribution of F, and $G_{\mu T}$ the Gini index of the post tax mean distribution of F_T, we can express the opportunity redistribution as:

$$OR = G_{\mu} - G_{\mu T} \tag{23}$$

which measures the reduction in the between-types inequality from the pre to the post-tax distribution (given that there is not reranking). It will be equal to zero when the degree of inequality in the pre-tax and post-tax mean distributions is the same; and possibly in some other scenarios where the pre-tax and post-tax Lorenz curves cross. If positive, there has for sure been some opportunity redistribution.

Alternatively, we could use any additively decomposable inequality measure I:

[7] A zero value of the index is not sufficient to say that a proportional tax has been applied; this is due to the possibility of crossing of Lorenz curves before and after tax, which would not be detected by using such index. Actually, the Lorenz dominance is equivalent to unanimity among all the Lorenz consistent indices.

$$OR = I_{(F)^B} - I_{(F-T)^B}$$

and then obtain the decomposition[8]:

$$\left[I_{(F)^B} - I_{(F-T)^B} \right] = \left[I_{(F)} - I_{(F-T)} \right] + \left[I_{(F-T)^W} - I_{(F)^W} \right] \tag{24}$$

where the first term in the right hand side indicates the decrease in the overall income inequality, and the second term measures the increase in the within-types dispersion; however, as already said, the last term cannot be interpreted as violation of the HE principle, because of potential compensations across types.

4 Concluding Remarks

In this paper we have tried to explore how an opportunity egalitarian theory of justice can be translated into a concrete public policy. After selecting the income as the relevant form of advantage and the income tax policy as the relevant redistributive policy, the following questions have been addressed: i) what is the degree of opportunity inequality in a given distribution of income? (ii) how to design an income tax policy consistent with the opportunity egalitarian ethics?

As for the first question, dominance criteria for unambiguous social ranking of income distributions have been proposed (section 2). In particular, an analogue (Theorem 3) of the fundamental theorem of inequality economics in the context of equality of opportunity has been derived.

Then (section 3), appealing properties have been formulated and used to characterize an opportunity egalitarian income tax. Finally, solutions to the problems of measuring the opportunity redistribution and the Horizontal Inequity introduced by an income tax have been discussed.

It is possible to indicate (at least) three possible extensions of this work. First, by relaxing the crucial assumption of additive separability of the income function, therefore studying measurement techniques and redistributive policy that could be applied to any income function;second, by considering the effect of taxation on the individuals' choices of responsibility, thereby analysing the incentive properties of alternative policies; third, by considering the case of unobservable level of responsibility.

5 Appendix

We first state and prove the following Lemma:

[8] This decomposition (which applies to any additively decomposable inequality measure) is proposed by Lambert [20] in a paper concerning the measurement of Horizontal Inequity, and is interpreted as : Overall Redistribution = Vertical Redistribution - Horizontal Inequity.

Lemma 1 $\sum_{k=1} v_k w_k \geq 0$ for all sets of numbers $\{v_k\}$ such that $v_k \geq v_{k+1} \geq 0$, $\forall k \in (1;2;...;n)$; if and only if $\sum_{i=1} w_i \geq 0$, $\forall k \in (1;2;...;n)$:

Proof. of Lemma 1.

Applying Abel's decomposition:

$$\sum_{k=1} v_k w_k = \sum_{k=1} (v_k - v_{k+1}) \sum_{i=1} w_i :$$

It is obvious that if $\sum_{i=1} w_i \geq 0;\ \forall k \in (1;2;...;n)$; then $\sum_{k=1} v_k w_k \geq 0$:

As for the necessity part, suppose that $\sum_{k=1} v_k w_k \geq 0$ for all sets of numbers $\{v_k\}$ such that $v_k \geq v_{k+1} \geq 0$, but $\exists j \in (1;2;...;n) : \sum_{i=1} w_i < 0$: Consider what happens when $(v_k - v_{k+1}) \searrow 0;\ \forall k \neq j$:

$$\sum_{k=1} v_k w_k \to (v_j - v_{j+1}) \sum_{i=1} w_i < 0$$

which is the desired contradiction. ■

Proof. of Theorem 1

We want to prove:

$$\sum_{i=1} p_i \mu^i_F \geq \sum_{i=1} p_i \mu^i_G ;\forall k \in (1;2;...;n) \iff (W_F - W_G) \geq 0;\ \forall W \in SEF_{OE}$$

where:

$$(W_F - W_G) \geq 0 \iff \left(\sum p_i \int_0^z U^i(x) f^i(x)dx - \sum p_i \int_0^z U^i(x) g^i(x)dx \right) \geq 0:$$

Using integration by parts, we obtain

$$(W_F - W_G) \geq 0 \iff \sum \frac{dU^i}{dx}\Big|_z S^i(z) - \int_0^z \sum \frac{d^2U^i}{dx^2} S^i(x)\,dx \geq 0$$

where $S^i(x) = \int_0^x [p^i G(y) - p^i F(y)]\,dy$:

Now, considering that, by condition (C.2), $\frac{d^2U^i(x)}{dx^2} = 0$; we obtain that:

$$(W_F - W_G) \geq 0 \iff \sum_{i=1} \frac{dU^i}{dx}\Big|_z S^i(z) \geq 0:$$

Considering that, by condition (C.1), $\frac{dU^i(x)}{dx} \geq 0$; and by condition (C.3) $\frac{dU^i(x)}{dx} - \frac{dU^{i+1}(x)}{dx} \geq 0$; we can apply Lemma 1. Hence we obtain that $(W_F - W_G) \geq 0$ if and only if:

$$\sum_{i=1}^{k} S^i(z) \geq 0\ ;\forall k \in (1;2;...;n):$$

Considering that $S^i(z) = p^i \mu^i_F - p^i \mu^i_G$; we finally obtain

$$\sum_{i=1}^{k} p_i \mu^i_F \geq \sum_{i=1}^{k} p_i \mu^i_G\ ;\forall k \in (1;2;...;n) \iff (W_F - W_G) \geq 0;\ \forall W \in SEF_{OE}:$$

∎

Proof. of Theorem 2[9]

That the $\succcurlyeq_{OET}$ dominance implies $\succcurlyeq_{OL}$ dominance, comes directly from the definition of OET and Lorenz dominance in the Opportunity distributions.

We only prove that: $F(x) \succcurlyeq_{OL} G(x) \Longrightarrow F(x) \succcurlyeq_{OET} G(x)$:

First notice that we are comparing two equal mean distributions with the same type partition. To simplify the notation, we write

$$x^i = n_i \mu^i_F$$

$$y^i = n_i \mu^i_G$$

and denote the mean distributions of F(x) and G(x) respectively by X and Y. Hence in this new notation

$$F(x) \succcurlyeq_{OL} G(x) \iff X \succcurlyeq_L Y \iff \sum_{i=1}^{j} x^i \geq \sum_{i=1}^{j} y^i\ ;\forall j \in (1;2;...;n):$$

Thus we know that $X \succcurlyeq_L Y$, we want to show how to get X from Y by a finite sequence of OET's. We consider in this proof the lump sum version of OET. However, the proof holds also in the proportionality case. Hence, let k be the first type where $x^k \neq y^k$: Since $X \succcurlyeq_L Y$ then $x^k > y^k$: (Recall that the distributions are ordered in an increasing way : $x^k < x^{k+1}$ and $y^k < y^{k+1}$):

Now obtain a new distribution $\tilde{Y}(k)$ by an OET (d^k) which makes $\tilde{y}^k(k) = x^k$, so that we have $\left(\tilde{y}^k(k) - y^k\right) = \left(x^k - y^k\right) = d^k$; and lowers the income y^{k+1} of an amount $d^{k+1} = d^k \frac{n^k}{n^{k+1}}$:

Notice that the total amount transferred from type $k+1$ to type k is $\Delta = d^k n^k = d^{k+1} n^{k+1}$: The new distribution $\tilde{Y}(k)$ has the properties:

$$\tilde{y}^i(k) = x^i \geq y^i;\forall i \leq k$$

[9] For this proof we follow the approach of Rothschild and Stiglitz [29].

$$\sum_{i=1}^{k+1} \tilde{y}^i(k) = \sum_{i=1}^{k+1} y^i(k)$$

$$\tilde{y}^j(k) = y^j;\ \forall j > k+1$$

Thus, $\tilde{Y}(k) <_L Y$ and $\tilde{Y}(k)$ agrees with X in k places. We can use the same procedure to find an OET which, applied to the distribution $\tilde{Y}(k)$; produces a new distribution $\tilde{Y}(k+j) <_L Y$ and which agrees with X in $k+j$ $(\geq k+1)$ places. Continuing with this procedure, we can produce X from Y by a sequence of (less than n) OET's. ■

Proof. of Theorem 4

In this case,

$$(W_F - W_G) \geq 0 \Leftrightarrow \left(\sum p_F^i \int_0^z U^i(x) f^i(x) dx - \sum p_G^i \int_0^z U^i(x) g^i(x) dx \right) \geq 0.$$

Using integration by parts, we obtain

$$(W_F - W_G) \geq 0 \Leftrightarrow$$

$$\sum p_F^i \left[U^i(x) F^i(x) \right]_0^z - \sum p_F^i \int_0^z \frac{dU^i}{dx} F^i(x)\, dx - \sum p_G^i \left[U^i(x) G^i(x) \right]_0^z +$$
$$+ \sum p_G^i \int_0^z \frac{dU^i}{dx} G^i(x)\, dx \geq 0$$

Now we know that $F^i(z) = G^i(z) = 1$; hence

$$\sum \left[p_F^i - p_G^i \right] U^i(z) + \sum \int_0^z \frac{dU^i}{dx} \left[p_G^i G^i(x) - p_F^i F^i(x) \right] dx \geq 0$$

Integrating by parts again:

$$\sum \left[p_F^i - p_G^i \right] U^i(z) + \sum \frac{dU^i}{dx} \int_0^z \left[p_G^i G^i(x) - p_F^i F^i(x) \right] dx -$$
$$- \sum \int_0^z \frac{d^2 U^i(x)}{dx^2} \left[\int_0^x \left[p_G^i G^i(y) - p_F^i F^i(y) dy \right] \right] dx \geq 0$$

Now considering that, by condition (C.2), $\frac{d^2 U^i(x)}{dx^2} = 0$; that by condition (C.4) $U^i(z) = U^j(z)$; and that $\sum_{i=1} p_F^i = \sum_{i=1} p_G^i = 1$; we obtain:

$$(W_F - W_G) \geq 0 \Leftrightarrow \sum_{i=1} \frac{dU^i}{dx} \Big|_z S^i(z) \geq 0$$

where $S^i(z) = \int_0^z [p_G^i G^i(x) - p_F^i F^i(x)]\, dx.$

Now considering that, by condition (C.1), $\frac{dU^i(x)}{dx} \geq 0$; and by condition (C.3), $\frac{dU^i(x)}{dx} - \frac{dU^{i+1}(x)}{dx} \geq 0$; we can apply Lemma 1 as in the proof of Theorem 1. Hence we obtain that $(W_F - W_G) \geq 0$ if and only if:

$$\sum_{i=1} S^i(z) \geq 0 \; ; \forall k \in (1; 2; :::; n):$$

Considering that in this case, $S^i(z) = p^i_F \mu^i_F - p^i_G \mu^i_G$; we finally obtain:

$$\sum_{i=1} p^i_F \mu^i_F \geq \sum_{i=1} p^i_G \mu^i_G \; ; \forall k \in (1; 2; :::; n) \iff W_F \geq W_G \text{ for all } W \in SEF_{OE}:$$

■

Proof. of Theorem 5

First let us focus on the effect of the tax T on the means distribution O_F. Let F_T be the post-tax distribution obtained after the application of the tax T. We have to prove that $O_{F_T} \succcurlyeq_L O_F$ for all $F \in \Omega$; if and only if T satisfies axioms HE and VE. Now, $O_F = (\mu^1 u^1; :::; \mu^i u^i; :::; \mu^n u^n)$ and $O_{F_T} = ((1 - a_1)\mu^1 u^1; :::; (1 - a_i)\mu^i u^i; :::; (1 - a_n)\mu^n u^n)$: Considering the between types progressivity imposed by axiom VE; and the absence of reranking, the result is ensured by the Jackobson-Fellman theorem.

As for the second claim of Theorem 5, again the Jakobsson-Fellman theorem ensures that the within type proportionality required by axiom HE is a necessary and sufficient condition for $F^i \sim_L F^i_T$; $\forall i \in (1; :::; n)$: ■

Proof. of Theorem 6

Considering that by Theorem 5: $F - T \succcurlyeq_{OL} F$; and that, formally, $F - P \sim_{OL} F$, we obtain that

$$F - T \succcurlyeq_{OL} F - P:$$

Now, considering that, by Theorem 1, for any equal mean distributions $F; G \in \Omega$; $F \succcurlyeq_{OL} G \iff F \succcurlyeq_{OEF} G$, we finally obtain that $F - T \succcurlyeq_{OEF} F - P$: ■

References

[1] Alergi R and Nieto J (1998) Equality of Opportunities: Cardinality-Based Criteria, mimeo.

[2] Arneson R. (1989) Equality of Opportunity for Welfare. Philosophical Studies, 56: 77-93.

[3] Atkinson A. B. (1970) On the measurement of Income Inequality, J Econ Theory, 2: 244-263.

[4] Atkinson A. B. and Bourguignon F. (1987) Income distribution and differences in needs. In Feiwel G.R. (ed.), Arrow and the Foundations of the Theory of Economic Policy, Macmillan, London.

[5] Bossert W. (1995) Redistribution mechanisms based on individual characteristics. Math Soc Sciences 29, 1-17.

[6] Bossert W. and Fleurbaey M. (1996) Redistribution and compensation. Soc Choice Welfare 13, 343- 355.

[7] Bossert W., Fleurbaey M., Van de gaer (1997) On second-best compensation, mimeo.

[8] Cohen G. A. (1989) On the currency of egalitarian justice. Ethics 99, 906-944.

[9] Dworkin R. (1981a) What is equality? Part1: Equality of welfare. Philos Public A®airs 10, 185- 246.

[10] Dworkin R. (1981b) What is equality? Part2: Equality of resources. Philos Public A®airs 10, 283-345.

[11] Fleurbaey M. (1994) On fair compensation. Theory Decision 36, 277-307.

[12] Fleurbaey M. (1995a) Three solutions for the compensation problem. J Econ Theory 65, 505- 521.

[13] Fleurbaey M. (1995b) The requisites of equal opportunity. In Social Choice, Welfare, Ethics. Barnett W.A., Moulin H., Salles M., Scho¯eld N. (eds.) Cambridge University Press, Cambridge.

[14] Fleurbaey M and Maniquet F (1996) Fair Allocations with unequal productions skills: The no-envy approach to compensation. Math Soc Sciences, 32: 71-93.

[15] Foster J.E. and Shneyerov A.A. (1996) Path Independent Inequality Measures. Vanderbilt University, Working Paper No. 96 - W04.

[16] Herrero C (1997) Equitable opportunities: an extension. Economic Letters 55, 91-95.

[17] Kolm S.-K. (1996) Modern Theories of Justice. The MIT Press. Cambridge, Mass.

[18] Kranich L. (1996) Equitable opportunities: an axiomatic approach. J Econ Theory 71, 131-147.

[19] Kranich L. (1997) Equitable opportunities in economic environments. Soc Choice Welfare 14, 57-64.

[20] Lambert P.J. (1995) On the Measurement of Horizontal Inequity. International Monetary Fund, Working Paper No. WP/95/135.

[21] Lambert P.J. and Ramos X. (1997) Horizontal Inequity and Vertical Redistribution. Forthcoming in International Review of Public Finance.

[22] Nozick R. (1974) Anarchy, State and Utopia. New York: Basic Books.

[23] Ok E.A. and Kranich L. (1995) The measurement of opportunity inequality: a cardinality-based approach, Soc Choice Welfare 15: 263-287.

[24] Peragine V (1998) The Distribution and Redistribution of Opportunity, J Economic Surveys (forthcoming).

[25] Rawls J. (1971) A Theory of Justice. Cambridge: Harvard University Press.

[26] Roemer J.E. (1993) A pragmatic theory of responsibility for the egalitarian planner. Philos Public A®airs 22, 146-166.

[27] Roemer J.E. (1996) Theories of Distributive Justice. Cambridge: Harvard University Press.

[28] Roemer JE (1998) Equality of Opportunity, Cambridge, MA: Harvard University Press.

[29] Rothschild and Stiglitz (1973) Some further Results on the Measurement of Inequality. J Econ Theory, 6: 188-204.

[30] Sen A.(1980) Equality of what? In McMURRIN (ed.) The Tanner Lectures on Human Values. Vol 1. Salt Lake City: University of Ut. Press.

[31] Sen A. (1985) Commodities and Capabilities. Amsterdam: North-Holland.

[32] Sen A. (1992) Inequality Reexamined. Cambridge, Mass.: Harvard University Press.