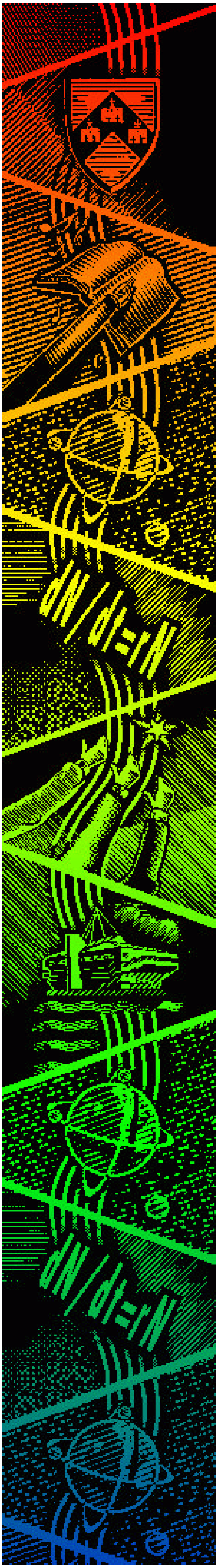

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Depreciation Rates and Capital Stocks

by

Karim Abadir and Gabriel Talmain

Department of Economics and Related Studies
University of York
Heslington
York, YO10 5DD

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University of York

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ABSTRACT: Suppose we have observations ranging over $t = 0, 1, \ldots T$ on real net investment, $\{I_{n,t}\}_0^T$, and on real gross investment, $\{I_{g,t}\}_0^T$. We derive a method of calculating the depreciation rate for each of the periods $\{\delta_t\}_1^T$, and estimating 'the' implied net capital stock $\{K_t\}_0^T$. We then provide empirical examples of the procedure, and analyse the results.

KEYWORDS: Depreciation, Capital Stock.

Corresponding author:

Professor Karim Abadir

Department of Economics

University of York

Heslington

YORK YO1 5DD

UK

(E-mail: kma4@york.ac.uk)

1. Introduction

Suppose one has observations ranging over $t = 0, 1, \ldots T$ on real net investment, $\{I_{n,t}\}_0^T$, and on real gross investment, $\{I_{g,t}\}_0^T$. Data on the depreciation rate for each of the periods $\{\delta_t\}_1^T$, and on 'the' net capital stock $\{K_t\}_0^T$ are typically not available to the public, if at all. When it exists, it is usually calculated by the Perpetual Inventory Method, PIM [e.g. see Statistics Directorate (1993) or Central Statistical Office (1985)], which is quite different from the way δ_t and K_t are defined in macroeconomic models; e.g. see Nickell (1978). Using the latter definitions, we derive a method of calculating the depreciation rate for each of the periods $\{\delta_t\}_1^T$, and estimating the implied capital stock $\{K_t\}_0^T$. Estimates of a variate will be denoted by a circumflex (hat), such as $\hat{K}_t$. The plan is to derive the method in Section 2, then illustrate it with three empirical examples in Section 3. Some comments conclude in Section 4.

2. The estimation procedure

There are two linear identities, containing two unknown series. The first is the one defining the depreciation

$$I_{g,t} - I_{n,t} \equiv \delta_t K_{t-1}, \tag{2.1}$$

and the second defining capital accumulation

$$K_t \equiv K_{t-1} + I_{n,t}. \tag{2.2}$$

Leading the first identity by one period, then substituting the second into it,

$$\begin{aligned} I_{g,t+1} - I_{n,t+1} &\equiv \delta_{t+1} K_t \\ &\equiv \delta_{t+1} \left(K_{t-1} + I_{n,t} \right). \end{aligned} \tag{2.3}$$

Subtracting (2.1) from (2.3),

$$\triangle I_{g,t} - \triangle I_{n,t} \equiv \left(\delta_{t+1} - \delta_t \right) K_{t-1} + \delta_{t+1} I_{n,t}, \tag{2.4}$$

where $\triangle$ is the forward difference operator such that $\triangle x_t \equiv x_{t+1} - x_t$. (Note that the backward difference operator $\bigtriangledown$ is such that $\bigtriangledown x_t \equiv x_t - x_{t-1}$.)

Two contiguous δ_t, δ_{t+1} will usually not differ by much, unless a major event (like war or a massive earthquake) has taken place. Take two uneventful points t_0 and $t_0 + 1$, to be characterized later in two lemmata. Then, (2.4) gives

$$\triangle I_{g,t_0} - \triangle I_{n,t_0} \simeq \delta_{t_0+1} I_{n,t_0}. \tag{2.5}$$

The only unobservable (unknown) component of (2.5) is δ_{t_0+1}, which may thus be calculated from this formula and the data. Once the estimate $\hat{\delta}_{t_0+1}$ is determined, the following sequence of calculations takes place.

1. Identity (2.1) gives $\hat{K}_{t_0}$.
2. Then, the whole sequence $\left\{\hat{K}_t\right\}_0^T$ is determined by (2.2).
3. As a consequence, $\left\{\hat{\delta}_t\right\}_1^T$ is obtainable from (2.1).

It is of interest to bound the error that results from (2.5). The following two lemmata do so, and give explicit guidance (in terms of observables) on choosing t_0.

Lemma 2.1. *The approximation error for the seed of the estimation procedure is*

$$\hat{\delta}_{t_0+1} - \delta_{t_0+1} \equiv \left(\frac{\delta_{t_0+1}}{\delta_{t_0}} - 1\right)\left(\frac{I_{g,t_0}}{I_{n,t_0}} - 1\right).$$

Proof. From identity (2.4), the error of estimating $\hat{\delta}_{t_0+1}$ by approximation (2.5) is

$$\begin{aligned} \hat{\delta}_{t_0+1} - \delta_{t_0+1} &\equiv \frac{\left(\delta_{t_0+1} - \delta_{t_0}\right) K_{t_0-1}}{I_{n,t_0}} \\ &\equiv \frac{\left(\delta_{t_0+1} - \delta_{t_0}\right)\left(I_{g,t_0} - I_{n,t_0}\right)}{I_{n,t_0}\delta_{t_0}}, \qquad \text{by (2.1)} \\ &\equiv \left(\frac{\delta_{t_0+1}}{\delta_{t_0}} - 1\right)\left(\frac{I_{g,t_0}}{I_{n,t_0}} - 1\right), \end{aligned} \tag{2.6}$$

where $I_{n,t_0} \neq 0$ and $\delta_{t_0} \neq 0$ almost surely (i.e. with probability 1). ■

The implication for our choice of t_0 is that one should select it such that $I_{g,t_0}/I_{n,t_0}$ is approximately minimized *ceteris paribus* (i.e. assuming that near t_0 there is little or no variation of the ratio δ_{t+1}/δ_t, which is weaker than requiring a fixed δ_t). In other words, we should choose a time where the depreciation charge is small relative to net investment. The ratio $I_{g,t_0}/I_{n,t_0}$ reflects the business cycle, and will therefore have many local minima, some of them not significantly different from each other. As a result, the previous characterization of t_0 is useful but incomplete. To choose the point t_0 from amongst these minima, we need another lemma. Define the symbol $\xrightarrow{p}$ for convergence in probability, and $O_p(.)$ for probabilistic orders of magnitude.

Lemma 2.2. *If* $K_T \xrightarrow{p} \infty$ *as* $T \to \infty$, *then*

$$\frac{\hat{K}_T}{K_T} \xrightarrow{p} 1 \text{ and } \frac{\hat{\delta}_T}{\delta_T} \xrightarrow{p} 1.$$

Furthermore, $K_T = O_p(f(T))$ *for some function* $f(.)$ *if and only if*

$$\frac{\hat{K}_T}{K_T} - 1 = O_p\left(\frac{1}{f(T)}\right) \text{ and } \frac{\hat{\delta}_T}{\delta_T} - 1 = O_p\left(\frac{1}{f(T)}\right) .$$

Proof. Without loss of generality, let $t_0 = 0$. This does not affect the proof, since taking any other fixed point t_0, then letting $T \to \infty$, leads to the same orders of magnitude as taking $t_0 = 0$. The estimation error is then obtained by tracing the original approximation error through the procedure.

From (2.2),

$$K_T \equiv K_0 + \sum_{t=1}^{T} I_{n,t}.$$

If $K_T \xrightarrow{p} \infty$, while K_0 is given, then it must be due to $\sum_{t=1}^{T} I_{n,t} \xrightarrow{p} \infty$. As a result,

$$\frac{\hat{K}_T}{K_T} \equiv \frac{\hat{K}_0 + \sum_{t=1}^{T} I_{n,t}}{K_0 + \sum_{t=1}^{T} I_{n,t}} \xrightarrow{p} 1,$$

and, by (2.1),

$$\frac{\hat{\delta}_T}{\delta_T} \equiv \frac{K_{T-1}}{\hat{K}_{T-1}} \xrightarrow{p} 1.$$

To establish the rate of convergence,

$$\frac{\hat{K}_T}{K_T} - 1 \equiv \frac{\hat{K}_0 - K_0}{K_0 + \sum_{t=1}^{T} I_{n,t}} \equiv \frac{\hat{K}_0 - K_0}{K_T} = O_p\left(\frac{1}{f(T)}\right),$$

and, correspondingly,

$$\frac{\hat{\delta}_T}{\delta_T} - 1 \equiv \frac{K_{T-1}}{\hat{K}_{T-1}} - 1 \equiv \frac{K_{T-1} - \hat{K}_{T-1}}{\hat{K}_{T-1}} \equiv \frac{K_0 - \hat{K}_0}{\hat{K}_{T-1}} = O_p\left(\frac{1}{f(T)}\right);$$

which completes the proof. ∎

The convergence rate is of negative-exponential order if and only if $K_T \xrightarrow{p} \infty$ exponentially. This seems to be the case for many macro series, whether the trend is stochastic or deterministic, as is evidenced by the ubiquitous logarithmic transformation in applied macro. The implication for our estimation procedure is that its accuracy improves exponentially as one moves away as much as possible from t_0 to $t > t_0$. This leads to choosing t_0 preferably near the beginning of the sample (which also leads to K_{t_0-1} being small relative to I_{n,t_0}; see (2.6)). This completes the characterization of t_0.

A final word about modelling is in order. The calculations of the series $\{\delta_t\}$ and $\{K_t\}$ are one-off exercises, whereas modelling the macroeconomy (e.g. consumption equations etc.) may entail re-estimating econometric models. It then pays off to take t_0 as far back in time as reasonably (in the light of earlier characterizations) possible. The following example illustrates. Suppose that data are available from 1950, but that the period to be covered in an empirical macromodel of interest starts in 1970. One should still permit t_0 to be chosen from 1950-70 in order to generate $\{\delta_t\}$ and $\{K_t\}$. The obtained series will be exponentially more reliable than the one generated by restricting the choice of t_0 to after 1970.

3. Three illustrative examples

All the data we analyse are in constant prices, obtained from the UK National Accounts and Datastream. We start with an example of aggregate UK data over the period 1948-1996. The choice of UK data for the first example is due to the availability of good data on investment, as well as readily-available PIM-based estimates of $\{\delta_t\}$ and $\{K_t\}$ which will be useful to compare to ours. In the second example, we compare results for different countries, all derived by our method. Finally, we estimate and examine sectoral depreciation rates in the UK.

3.1. Example 1

The first requirement for our procedure is to decide on which t_0 to start the numerical calculations from. Figure 1 shows the ratio $I_{g,t}/I_{n,t}$ which is to be minimized. Two points stand out. The first is the global minimum at 1968. The second is the local minimum of 1961, which is not very different from the global minimum value of $I_{g,t}/I_{n,t}$, but has the advantage of being located further back in time (see Lemma 2.2). We calculate and plot the resulting series for $\left\{\hat{\delta}_t\right\}$ and $\left\{\hat{K}_t\right\}$. These are in Figures 2 and 3, respectively, where we have also added the corresponding series from the national accounts. The two series that we have estimated are very close to each other, and are hardly distinguishable from one another. One has to caution that the reliability of the national accounts series for $\left\{\hat{K}_t\right\}$ is generally lower than for other series: there is up to 20% possible error for the aggregate series, and even more for the disaggregated one (not analysed here), as quoted by Central Statistical Office (1985, pp.198-203). Our series is inferred from an identity (not PIM), and is therefore likely to be more accurate. However, the profile of their estimated series and ours is similar. This is because both routines, apart from their different estimates of $\hat{K}_{t_0}$, build their subsequent series by means of $I_{n,t}$ as $\bigtriangledown K_t \equiv I_{n,t}$. A similar remark goes for the estimates of $\left\{\hat{\delta}_t\right\}$.

It is worth noting that the depreciation rate has been increasing steadily over time up to 1988. It suggests a capital stock that is ageing faster than before, and is being retired at an accelerated rate.

3.2. Example 2

The second example is a comparative study of $\left\{\hat{\delta}_t\right\}$ for Canada, Germany, Japan, UK, US. We start again by determining t_0 for each of these countries from Figure 4. We then derive the estimates of $\left\{\hat{\delta}_t\right\}$ and plot them in Figure 5. The most striking feature of the series is Japan's depreciation. It pictures a country whose industrial structure and profile is rapidly changing to that of a leading modern industrial power. The depreciation rate up to the early seventies has been very high, and has come down sharply since. There are other, less striking, features in the graph. Canada's depreciation rate has been increasing of late, to above 6%. The UK's rate, which we noted in the previous subsection to be increasing, is still below that of the other countries included in this example.

3.3. Example 3

The third example examines sectoral depreciation rates in the UK. As before, t_0 is determined from the series $\{I_{g,t}/I_{n,t}\}$ for each sector. We then estimate the $\{\delta_t\}$ series for each sector, and plot them in Figure 6. The jump in 1982 is due to a change of definition. Depreciation charges on leased assets are now attributed to the user of the capital, rather than its legal owner. This is a transfer, and therefore does not cause a corresponding jump in the aggregate (all sectors) $\{\delta_t\}$ series.

Depreciation rates for housing correspond to an asset life of approximately 100 years, as expected from typical leasehold contracts in the UK. Construction, agriculture, mining and transport are areas with higher-than-average depreciation, since they tend to rely more heavily on equipment that needs to be modernized more often than in other sectors. The private electricity sector is a relatively young one in the UK (due to recent privatisations), and its depreciation is still about average.

4. Concluding remarks

The rates of depreciation $\{\delta_t\}$ that we can calculate by our methods are not necessarily fixed over time. Furthermore, they are estimated from the data, rather than being calibrated. Both aspects are potentially useful for macromodels, especially growth models that are concerned with issues relating to modelling the (rate of) diffusion of new technology.

The other available methods, which are mainly variants of PIM, are ones where direct estimation of the stock of capital takes place. These methods require addressing questions regarding the aggregation of different vintages of capital. Our method avoids having to deal with this problem. It also implies that data-providing agencies should shift their focus in this area to the easier task of gathering depreciation charges (e.g. from the accounts of firms), rather than the direct valuation of the capital stock.

Department of Mathematics and Department of Economics, University of York, Heslington, York YO1 5DD, UK,

and

Department of Economics, University of York, Heslington, York YO1 5DD, UK.

REFERENCES

Central Statistical Office (1985): *UK national accounts, sources and methods*, 3rd edition. London: HMSO.

Nickell, S.J. (1978): *The investment decisions of firms.* Cambridge: CUP.

Statistics Directorate (1993): *Methods used by OECD countries to measure stocks of fixed capital.* Paris: OECD.