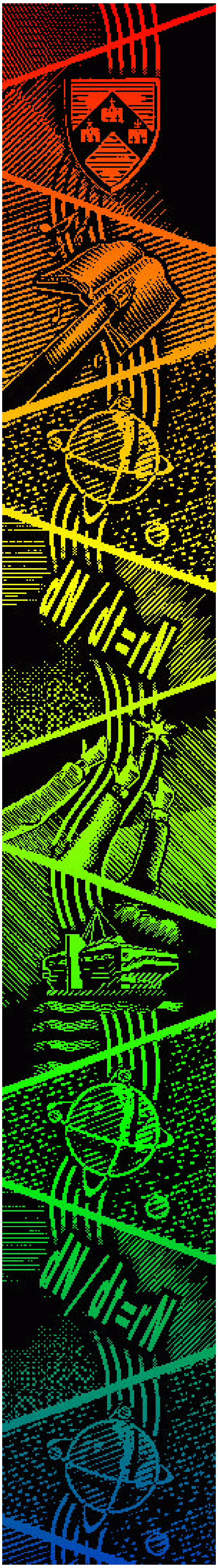

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A NORMATIVE AND STATISTICAL APPROACH TO MEASURING CLASSICAL HORIZONTAL INEQUITY

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Abstract This paper makes a new attack on the old problem of measuring horizontal inequity (HI) in an income tax system. A local measure of HI is proposed, and aggregated into a global index. Whilst other approaches have captured the welfare gain which would come from eliminating HI revenue-neutrally, our global index provides a measure of the revenue gain which would come from eliminating HI welfare-neutrally. When expressed as a fraction of mean post-tax income, the measure can be viewed as a negative component in the Blackorby and Donaldson (1984) index of progressivity, quantifying the loss of vertical performance arising from differences in the tax treatment of equals. We propose non-parametric estimation procedures to tackle the 'identification of equals' problem, providing, to our knowledge, the first consistent statistical solution to measuring classical HI. The method is applied to the Canadian distributions of market and net incomes between 1981 and 1994, to reveal both the changing profile of local HI along the income parade and also its aggregate significance as loss of progressivity. We also decompose total HI into within and between group components, and test its robustness to sampling variability and to the choice of equivalence scales and kernel bandwidths.

JEL: C14, D31, D63, H23

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1. Introduction

Horizontal equity (HE) demands that like individuals be treated alike by the direct tax system, and this principle can be extended to households and families. Violations of HE can arise, for instance, from the tax breaks granted for charitable giving, mortgage interest, capital gains, dividends, political contributions and retirement savings, and due to tax evasion, incomplete benefit take-up, arbitrariness in the allocation of state benefits, and differentiation in indirect tax rates[1]. Indeed it can be argued that any form of government intervention which impacts on relative prices in a world of heterogeneous individual consumption and investment preferences will lead to violations of HE.

In this paper, we outline a new procedure for measuring the extent of horizontal inequity (HI) in the income tax.[2] The starting point is a local HI measure capturing the dispersion of post-tax living standards among pre-tax equals using the 'cost-of-inequality' approach described by Kay and King (1984).[3] When this local measure is aggregated into a global index, using a weighting scheme which ensures that the importance attributed to a local inequity does not depend upon the living standard at which it is experienced, a global index results which, we show, measures the revenue gain *per capita* that would come from eliminating HI with no loss of social welfare in any equals group.

Two other recent approaches to measuring HI have been based on the local-to-global aggregation procedure, those of Aronson *et al.* (1994) and Lambert and Ramos (1997a). These have an opposite but symmetrical property: they capture the overall welfare gain that would come from eliminating HI revenue-neutrally within every equals group. We discuss the links with our own construction.

The new HI index enjoys a close connection with the tax progressivity index of Blackorby and Donaldson (1984). This, we show, decomposes into two components. One measures the distributional characteristic of the hypothetical (or reference) tax system in which the local inequities have been eliminated as described above; the other (negative) contribution is our HI index measured as a fraction of mean post-tax income. Using this decomposition, the analyst can describe the progressivity of the tax system in terms of vertical and horizontal contributions, and

[1] For such instances, see, e.g., Gravelle (1992), Bishop et al. (1994) and Duclos (1995b).

[2] Our methodology allows for negative income taxes, that is, for the presence of transfers. Our empirical application features a number of state benefits as well as a positive income tax.

[3] "How much commission would we pay Robin Hood to transfer £1 from the rich to the poor? The answer will depend on our view of inequality .. we can imagine a continuing series of such transfers which eventually bring us to a wholly egalitarian outcome and measure the amount of income which we would be willing to give up in order to bring about this result. This total amount is the "cost of inequality": the reduction in aggregate income which we would accept in order to achieve complete equality in its distribution .. the size of these costs depends on how much we are offended by inequality" *(op. cit., p. 221)*.

for different assumed values of inequality aversion.

In the 1980s, attention focused on the reranking approach to capturing HI, following the influential work of Feldstein (1976), Atkinson (1980) and Plotnick (1981).[4] Plotnick himself dismissed the classical approach to measuring HI, arguing that the banding of income data in sparse samples in order to create close equals groups for estimation purposes represents "an artificial way to salvage empirical applicability" *(1985, p. 241)*. We tackle the 'identification of equals' problem here, proposing a statistical solution in which a non-parametric estimation procedure is used to assess the distribution of classical horizontal inequity. The procedure is applied to the Canadian joint distributions of market and net equivalent incomes for 1981, 1985, 1990 and 1994.

The structure of the paper is as follows. In Section 2, the measurement system is specified and the theoretical results already indicated are made explicit. In Section 3, we discuss implementation difficulties and also statistical and modelling issues. Section 4 contains the application to Canada, and Section 5 concludes. A brief mathematical Appendix contains proofs of the theorems specified in Section 2.

2. The measurement system

Let $\mathbf{x}$ be an income distribution vector and $W(\mathbf{x})$ a homothetic social evaluation function, for which the equally distributed equivalent (henceforth EDE) income is ξ: $W(\xi\mathbf{1}) = W(\mathbf{x})$. For the empirical application to follow, $W(\mathbf{x})$ will be average utility, where $U_e(x)$ is a social decision-maker's (henceforth SDM's) utility-of-income function displaying constant relative inequality aversion with parameter e:

$$U_e(x) = x^{1-e}/(1-e) \quad \text{if} \quad 0 < e \neq 1 , \quad U_1(x) = \ln x \tag{1}$$

The cost of inequality in $\mathbf{x}$ is the amount the SDM would give up to have inequality eliminated with no loss of social welfare. In *per capita* terms, this is:

$$C = \mu - \xi \tag{2}$$

where μ is the mean of $\mathbf{x}$. In Figure 1, $U_e(x)$, μ, ξ and C are shown in the case of a simple income distribution comprising two income levels, $\mathbf{x} = (x_1, x_2)$. Point A shows the level of average utility $E[U_e]$, and point B shows that $U_e(\xi)=E[U_e]$.

The equals or like individuals in the HE command are, according to Feldstein (1976), those with the same utility. The injunction to treat like individuals alike has also been extended

[4] According to this view, HI shows in the rank reversals, if any, which occur in the transition from the pre- to the post-tax distribution of living standards. See also Plotnick (1982,1985) and Duclos (1993). For a criticism of the reranking approach, see Kaplow (1989) and Musgrave (1990). For a comparison of the reranking and classical approaches using simulation, see Lambert and Ramos (1997b).

to households and families.[5] The first step for our analysis of HI is to turn the business of identifying the equals into a unidimensional problem. We shall require income units' pre-tax incomes, or living standards, x^b, to be measured on a scale which identifies the equals: *equals will be those having the same pre-tax income* x^b. For all that is to follow, we assume that an appropriate scale has been devised[6]. The income unit may be the individual, the family, the household or the equivalent adult. For convenience we refer to an income unit as a person henceforth.

Let $\mathbf{x^b}$ and $\mathbf{x^a}$ be the vectors of pre- and post-tax income characterizing the tax system. If a scatterplot is drawn, these would typically evidence a strong though not perfect positive association. Figure 2 depicts a sample of Canadian individuals for the year 1990.[7] HI occurs whenever points on this scatterplot are vertically aligned: in this case, pre-tax equals have different post-tax incomes. For any fixed x in the pretax distribution $\mathbf{x^b}$, let Ω_x denote the group of persons having exactly x before tax: this is the 'equals group' located at point x. We conceptualize HI at x as *inequality introduced by the tax system at x*. Let the mean and EDE post-tax incomes for the equals group Ω_x be μ^a_x and ξ^a_x. The SDM would give up an amount:

$$H_x = \mu^a_x - \xi^a_x \tag{3}$$

of post-tax income *per capita* within Ω_x to have that group's HE violations removed with no loss of welfare. This is our measure of local HI at x.[8]

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[5] See Manser (1979, p. 224), Habib (1979, p. 286) and Steuerle (1983, p. 81).

[6] Money income would serve for x^b if the population under investigation consisted of people with identical tastes, needs and abilities. Imputable income and non-market sources of utility and disutility could also be incorporated. Manser (1979) discusses the modelling of household objectives including different leisure times of their members, and Rosen (1978) demonstrates an empirical procedure which, given rich enough microdata, will "generate two vectors, one of family utilities before tax and one of family utilities after tax", and he goes on to say that "the real problem in measuring horizontal equity is to summarize the differences between these vectors in a meaningful way" *(p. 314)*. Steuerle (1983) advocates equivalization as the means to provide "a working definition of equity" across family sizes. Jenkins (1988) argues against equivalizing, seeing the business of identifying the equals as an essentially multidimensional issue, and adopts instead a partial approach in which he refrains from making identifications across distinct socioeconomic subpopulations, capturing HI within each in terms of rank changes induced by the tax.

[7] The description of the data used to plot Figure 2 can be found in Section 4 ahead.

[8] Think of the horizontal axis in Figure 1 as measuring the post-tax incomes of pre-tax equals, and follow the veil of ignorance thought experiment of Harsanyi (1953). In choosing between societies, one faces a 2-stage gamble; first, of the pre-tax income level one may be assigned, and second of the tax treatment one may receive. If x is the pre-tax income level assigned at the first stage, resulting in post-tax income of either x_1 or x_2 depending on tax treatment, then ξ_x is the certainty equivalent of the second stage gamble, and H_x the risk premium.

The next step is to aggregate the H_x , $x\in\mathbf{R}$, into a global index, call it H, using a weighting scheme. There are plenty of possibilities; we choose to use population shares as weights. Thus:

$$H = \Sigma_x\, p_x.\, H_x \tag{4}$$

where p_x is the proportion of the overall population who are located at point x on the pre-tax income scale: $p_x = N_x / N$, where $N_x = |\Omega_x|$ and $N = \Sigma_x\ N_x$ .This construction ensures that the importance attributed to a local HE violation does not depend upon the income level at which it is experienced. That is, H is not polluted with vertical considerations, heeding Musgrave's (1990) warning to avoid "inappropriate comparisons between unequals" *(pp. 117-8)* in constructing a global HI index.[9] Now denote by $T_{wn}(x)$ the tax members of Ω_x would have to pay if the SDM were to eliminate their HI with no change of social welfare:

$$T_{wn}(x) = x - \xi^a_x \quad \forall x \tag{5}$$

We may call $T_{wn}(\bullet)$ the **welfare-neutral HE replacement tax**. Our first result shows that overall HI, as defined in (4), may be interpreted as the additional tax revenue *per capita* that would flow from replacing the actual tax system by this welfare-neutral replacement schedule:

Theorem 1

H measures the *per capita* gain in revenue that would come from substituting the tax system by $T_{wn}(\bullet)$.

For the proof of this and the subsequent theorems, see the Appendix. Of course there would be winners and losers from this hypothetical process of HI elimination. It is not a policy recommendation of this paper that we should identify and substitute $T_{wn}(\bullet)$ for the actual tax system; $T_{wn}(\bullet)$ serves as the yardstick against which the social cost of the HE violations in actual taxes can be assessed. An attractive feature of our index H is that it sets a *dollar value* upon HI, conditioned by the assumed inequality aversion e of the SDM: as e is increased, the SDM becomes willing to pay more to eliminate unequal tax treatment of equals, and measured HI therefore increases.

It is both natural and convenient to measure the cost of HI as a fraction of the total income in the post-tax distribution, say as:

$$H_1 = H/\mu^a \tag{6}$$

where μ^a is the overall mean post-tax income. H_1 is unit-free and lies between 0 and 1. In the absence of HI, $H_1 = 0$. If a very large number of people all had the same pre-tax income, and one of them got all of the post-tax income, then $H_1 \to 1$. A further attraction of this formulation is that measured HI can be interpreted as a subtraction from the vertical performance of the tax system, as quantified by the progressivity index of Blackorby and Donaldson (1984):

Theorem 2

[9] Global indices of HI proposed by Habib (1979), Berliant and Strauss (1985) and Aronson *et al.* (1994) use explicitly income-dependent weights for local inequities. For more on the last of these, see on.

Let Π and Π_{wn} be the Blackorby-Donaldson progressivity indices for the actual tax system and for $T_{wn}(\bullet)$ respectively[10]. Then:

$$\Pi = \theta \bullet \Pi_{wn} - H_1 \tag{7}$$

where θ is the ratio of mean income after $T_{wn}(\bullet)$ to mean post-tax income μ^a ($\theta \leq 1$).

In their derivation of their progressivity index, Blackorby and Donaldson (1984, p.696) state that their formulation "does not assume horizontal equity". Most other authors, when designing progressivity indices, do indeed assume away HI. Whilst allowing for HI, Blackorby and Donaldson (1984) gave no guidance as to how it would affect their index. Theorem 2 shows precisely that.

There are striking similarities here with two other recent approaches to measuring HI using a local-to-global methodology. In Aronson *et al.* (1994) and Lambert and Ramos (1997a), HI is captured as a weighted sum of local inequality indices (respectively, the Gini and mean logarithmic deviation), and in each case the global index can be viewed as a loss of redistributive effect. Suppose HI were eliminated with no effect upon *revenue* (rather than *welfare*) in each equals group Ω_x. This would mean averaging the tax liabilities of the group members; the tax each member of Ω_x would have to pay would be:

$$T_{rn}(x) = x - \mu^a_x \quad \forall x \tag{8}$$

We may call $T_{rn}(\bullet)$ the **revenue-neutral HE replacement tax**. It is clear from Figure 1 that such averaging out produces a welfare gain for the membership of Ω_x. Hence, comparing the two HE replacement tax schedules $T_{rn}(\bullet)$ and $T_{wn}(\bullet)$, the one delivers a better welfare performance than the actual system and the same revenue, whilst the other delivers the same welfare and more revenue. The global HI indices of Aronson *et al.* (1994) and Lambert and Ramos (1997a), call them H_{AJL} and H_{LR}, capture this welfare gain from hypothetically replacing the actual tax system by $T_{rn}(\bullet)$. Their relationship to redistributive effect is the same as ours to the Blackorby-Donaldson progressivity index.[11] We also note that:

$$H_x = T_{wn}(x) - T_{rn}(x) \tag{9}$$

Both of the schedules $T_{wn}(x)$ and $T_{rn}(x)$ are illustrated for Canada in 1990 in Section 4 ahead.

An additional attractive feature of our construction is that, using the index H (or H_1), HI can be tracked down to its sources, namely to particular demographic subgroups and/or income

[10] In general terms, the Blackorby and Donaldson (1994) progressivity index is best defined through the Atkinson index. With C defined as in (2), the Atkinson (1970) index of inequality for **x** is $I(e) = C/\mu = 1 - \xi/\mu$. Let the Atkinson indices before and after tax be I^b and I^a. The Blackorby and Donaldson (1984) progressivity index is then $\Pi = [I^b - I^a] / [1 - I^b]$. Greater detail on the links between these indices can be found in the appendix.

[11] There is symmetry here, for the Blackorby-Donaldson index effectively measures the vertical stance of an income tax relative to an *equal-welfare flat tax*, whilst redistributive effect measures it relative to an *equal-revenue flat tax*. Let MLD_x be the mean logarithmic deviation of post-tax incomes at a pre-tax income x. For inequality aversion e=1, H_1 and H_{LR} are closely related: $H_{LR} = \Sigma p_x MLD_x$ and $H_1 = 1 - \Sigma p_x \mu_x^a \exp\{-MLD_x\}/\mu^a$.

brackets. Suppose the population is partitioned into demographic subgroups Γ_k, k = 1,2,3... (as distinct from the existing partition into equals groups Ω_x, $x \in \mathbf{R}$). We can decompose HI into "between and within group" contributions, both overall and for any tranche of pre-tax income recipients:

Theorem 3

Overall HI can be decomposed as

$$H = \Sigma_k\, q_k H_k + H^* \tag{10}$$

where $q_k = |\Gamma_k|/N$ is the proportion of the overall population who are in group Γ_k, H_k is HI within group Γ_k and H^* is HI arising from the effect of the tax between the Γ_k, k = 1,2,3... Furthermore, for any pre-tax income level z, the aggregate of local HI from x=0 to x=z, call it H^z, can be decomposed as:

$$H^z = \Sigma_k\, q_k H^z_k + H^{z*} \tag{11}$$

in which the constituent terms also express the aggregates of local HI up to x=z.

See the Appendix for the formal definitions of H_k, H^*, H_k^z and H^{z*}. We use these decompositions to investigate in Section 4 the sources of HI, and of changes in HI, in Canada.

3. Implementation: statistical and modelling issues

If, as will generally be the case, one's sample micro-data is drawn from an (approximately) continuous joint population distribution of individual incomes x^a and x^b, the sample probability of observing exact equals is virtually nil. This is the 'identification of equals' problem already referred to. However, using recent statistical advances, we can estimate non-parametrically and consistently the continuous population distribution of x^a and x^b using the empirical joint distribution of the two variables, and integrate over that estimated distribution to yield consistent estimates of the HI indices H and H_1 and progressivity indices Π and Π_{wn}.

To be more precise, a consistent estimator $\hat{f}(x_a|x)$ of the conditional density function for x^a (given pre-tax income x) can be used to generate natural consistent estimators of ξ^a_x, μ^a_x, $T_{wn}(x)$, $T_{rn}(x)$, and H_x by integration. From the definition of the EDE income for equals group Ω_x, we will have, for instance, that a natural estimator $\hat{\xi}^a_x$ for ξ^a_x is given by:

$$U_e(\hat{\xi}_x{}^a) = \int_{\Omega x} U_e(x^a)\, \hat{f}(x^a|x)dx^a \tag{12}$$

Similarly, a natural estimator for μ^a_x is given by:

$$\hat{\mu}^a_x = \int_{\Omega x} x^a\, \hat{f}(x^a|x)\, dx^a \tag{13}$$

By (9), this yields:

$$\hat{H}_x = \hat{\mu}^a_x - \hat{\xi}_x{}^a \tag{14}$$

If $\hat{f}(x^a|x)$ is continuous over x, these estimators will also be continuous across x. Integrating $\hat{H}_x$ over x will give an estimator of the overall cost of HI:

$$\hat{H} = \int_o^\infty \hat{H}_x\, \hat{f}(x)dx \tag{15}$$

where $\hat{f}(x)$ is the estimator of the implied marginal density function for pre-tax income. Alternatively, if income x is estimated to be at percentile p, so that $p=\int_o^x f(z)dz$, then we may write $\hat{H}_x=\hat{H}(p)$ and:

$$\hat{H} = \int_o^1 \hat{H}(p)dp \tag{16}$$

In the application to follow, we use a non-parametric kernel estimation procedure, with Gaussian kernel and bandwidth chosen to minimise the mean integrated square error in measuring the shape of a wide range of possible population densities.[12] For a univariate distribution, we use the relatively robust bandwidth defined as:

$$h = 0.9An^{-0.2} \text{ where } A = \mathit{min}\ \{\text{standard deviation, interquartile range}/1.34\} \tag{17}$$

For the bivariate applications, we follow the related suggestions of Silverman (1986, p.87). We expect our results to vary somewhat with the precise choice of the bandwidth. Were we to know the true form of the population distribution, we would be able to remove the uncertainty concerning the "correct" value of the bandwidths to use, but, knowing the true distribution, we would then of course not need to estimate it. The larger the bandwidths, the smoother the estimates of the densities and of the cost of HI along percentiles of pre-tax incomes become, but the less representative of local variations in HI these estimates also become. To check the sensitivity of our estimates to bandwidth selection, we also assess our results using bandwidths 20% larger and 20% lower than the above "standard" or reference ones, as well as by using an "adaptive kernel", which allows the bandwidth of the Gaussian kernel to vary smoothly from one region of a distribution to another according to the density of the regions (see Silverman (1986), section 5.3)). As we shall see, these checks change our results very little, since our estimates of post-tax income dispersion are locally very smooth. Finally, whatever our particular choice of a class of asymptotically vanishing bandwidths, we can be confident that distributions estimated with the kernel method tend asymptotically to the true ones if the latter are continuous.[13]

The kernel approach thus enables us to 'reconstruct' indices of local HI, of the cost of inequality and of EDE income as functions of pre-tax income x. This is a statistically preferable and more sophisticated procedure to the use of discrete banding into 'close equals groups', which has been the practice in some previous literature on the measurement of classical HI.[14] In the Lambert and Ramos (1997a) approach, for instance, a viewing window is effectively passed across the pre-tax income range, of arbitrarily chosen fixed width and with no overlapping of windows. For the purposes of empirical implementation, an additional normative principle is added to the pure classical HI principle, that "near equals" should be treated "near equally" (which for Lambert and Ramos means without vertical redistribution within these discrete groupings). For the kernel method, it is a statistical assumption that is added for empirical implementability: the HI process generating unequals from exact pre-tax equals is supposed to evolve smoothly across the income distribution. Neighbouring information is first used to compute levels of expected post-tax incomes; neighbouring information on the dispersion of post-tax incomes around these expected values is then used to help infer the shape of $f(x^a|x)$. As the

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[12] See Silverman (1986), p.48. Another approach would be to choose the bandwidth parameter to minimise the square error in measuring H.

[13] See Silverman (1986), pp.71-72.

[14] Berliant and Strauss (1985), Aronson et al. (1994) and Lambert and Ramos (1997a) do this explicitly. Such an approach could also be devised for the indices and decompositions expounded in this paper: see Lambert (1995).

window passes smoothly along the pre-tax income scale, greater weight is ascribed to observations near the middle of the window. The kernel approach thus separates the vertical effect first, and then estimates the randomness of post-tax incomes (conditional on a level of pre-tax income) by using neighbouring information. The statistical arbitrariness of the close equals groupings is replaced by the choice of kernel bandwidths possessing known statistical properties, providing smooth estimates with automatic convergence to the true population values under weak regularity conditions.

There remains the issue of how many and which pre-tax and post-tax income points should be used to compute the estimates and approximate the integrals defining $\hat{H}$. The answer depends on the numerical accuracy which we desire in the computation of our point estimates. Our own choice has been to compute $\hat{H}_x$ and $\hat{H}(p)$ at each point observed in the data. For ease of programming and for computational speed, we did not estimate or simulate a full non-parametric distribution for $\hat{f}(x^a|x)$, but estimated instead its variance and drew simulations under a conditional normality assumption. This seemed sufficiently accurate given the statistical sampling variability of our estimates (see for instance Tables 2 and 3 below).

4. Canada, 1981 to 1994

The Canadian Survey of Consumer Finances (SCF) provides sample micro-data annually on pre-tax and pre-benefit family incomes, provincial and federal personal income taxes, and cash transfers received from the provincial and federal governments.[15] We use the 1981, 1985, 1990 and 1994 data sets, which each comprise between 36,000 and 46,000 observations[16]. 1994 is the latest year for which we could obtain SCF data. We reach back to 1981 since the distribution and redistribution of income has been subjected to important disturbances since the beginning of the 1980's. Canada witnessed a severe recession between 1981 and 1983, followed by a significant recovery with relatively high growth rates until the end of 1988, and with the beginning of another recession thereafter -- whose effects on incomes and employment can be felt to this day. To this were added important labour market, demographic and technological changes. The last decade was also the decade of major tax reforms; in Canada, taxation was particularly altered by

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[15] Market income, which stands here for pre-tax and pre-benefit income, includes wages and salaries, self-employment income, private pensions and investment income. Transfers include Federal and Québec family and youth allowances, Child Tax Credits, Old Age Security Pensions and Guaranteed Income Supplement, Canada/Québec Pension Plan Benefits, Unemployment Insurance Benefits, Social Assistance Benefits and provincial income supplements, various tax credits and grants to individuals, veterans' pensions, pensions to widows, and workers' compensation. Taxes include both federal and provincial income taxes. Net income is market income minus taxes plus transfers.

[16] The survey methodology remained essentially the same during that period, although the information and the variables gathered on taxes paid and benefits received evolved with the changes in the tax and transfer system.

the 1987 revision of personal income taxation, which decreased the number of tax brackets, trimmed the top marginal tax rates, replaced a number of tax allowances by tax credits, broadened the tax base, and aimed, generally, to improve the perceived "fairness" of the tax system. The social security system (including the unemployment insurance, public pension, and provincial social assistance schemes) also evolved significantly with important changes in public policy and in the socio-demographic environment. To adjust the data for heterogeneity in the size and the composition of families, we first equivalised all income and tax and transfer variables using the OECD equivalence scale. We also removed families reporting negative market or net incomes[17]. For expositional convenience, we have normalized market and net incomes by their means for Figure 2 and all subsequent Figures and for all Tables.

Table 1 shows some illustrative estimates from our 1981 and 1994 samples at various percentiles of market incomes. Under each column, we find kernel estimates of the expected net incomes conditional on a percentile of market income as well as estimates of the conditional densities (in parentheses), both for the total population ("total") and for three separate population groups. Group 1 comprises households for which at least one of the members is aged 60 or over; group 2 is made of households whose members are all aged between 18 and 60; group 3 includes younger households (all members below 60) with at least one child. We note that expected conditional incomes are highest for group 1, regardless of the percentiles, but that overall expected net income for group 1 lies below that of group 2 since the density of group 1 members falls sharply with the levels of market incomes. The proportion of group 3 members (households with children) has declined from 60% of the population in 1981 to only 48% in 1994.

The kernel estimate of the joint density function for market and net equivalent incomes is shown in Figure 3. All of the information we use to estimate our indices is essentially summarised in that Figure. In particular, the densities of net incomes conditional on market incomes are easily seen by cutting the Figure alongside the net income axis at given values of market incomes. These conditional densities give, *inter alia*, estimates of ξ^a_x, $T_{wn}(x)$ and H_x. If there were no HI in the tax and transfer system, the joint density would be positive only above a single line, showing a deterministic relationship between market and net income. The flatter the conditional density of net income (given a level of market income), the greater the HI at that market income level. Alternatively, the more unequal are net incomes conditional on a particular level of market income, the more HI there is.[18]

To indicate the cost of HI at different market income levels, we display in Figure 4 the levels of revenue-neutral taxation, $T_{rn}(x)$, and the differences, $H_x = T_{wn}(x)-T_{rn}(x)$, between welfare-neutral and revenue-neutral taxation (scale on the right vertical axis) for 1994 Canada and for two

[17] This procedure deleted a very small number of observations (generally less than 0.3% of the original sample).

[18] This suggests that a test of 'conditional HE dominance' of one joint distribution over another might be constructed, in a manner analogous to the well-known tests of Lorenz dominance for the measurement of inequality. This is a topic for future attention.

values of the inequality aversion parameter. The size of $T_{rn}(x)$ in Figure 4 corresponds to the distance between predicted net incomes μ^a_x and market incomes. $T_{rn}(x)$ is negative for values of x up to about 95% of *per capita* market income, and varies in size from -45% to 125% of it. $T_{wn}(x)-T_{rn}(x)$ is positive everywhere, ranging between 10% and 1% of *per capita* market income for e=0.75 and between 2% and 0.2% for e=0.25. This excess of welfare-neutral over revenue-neutral tax is larger at lower values of market income and is higher for a greater degree of inequality aversion (see appendix).

Figure 5 displays the cost of HI, H(p), at different percentiles of the Canadian income distribution in 1981, 1985, 1990 and 1994 for e=0.75 and as a proportion of *per capita* net income (recall (16)). H(p) is mostly decreasing with p. The most costly occurrences of HI are within the bottom quintile of the population. HI is generally greater in 1990 than in 1981, except at the bottom and top percentiles, and very similar in 1985 and in 1990. It is, however, much larger in 1994 than in any of the previous three other years. Removing HI welfare-neutrally for low values of p would yield in the order of 4% of *per capita* net income in additional tax revenue for 1981, 1985 and 1990, rising to above 10% of average income for 1994.

An increased revenue share of the government in the economy and increased benefit targetting and selectivity between 1981 and 1994 makes the rise of HI between these two years not wholly surprising. Beach and Slotsve (1996), for instance, find an increase in the effective Canadian tax rates at all quintiles between 1981 and 1992, with the largest increase reached at the top quintile. In a similar vein, Smith (1995) notes an increase in the ratio of fiscal revenues to GDP from 36.9% in 1980 to 42% in 1993, another hint of the increased tax and benefit presence of the government in the distribution and redistribution of income. Smith (1995) also reports increased tax and benefit progressivity between these years, which he attributes to a widened tax base and a greater targetting of tax relief (e.g., replacement of tax deductions by tax credits) and of transfers Duclos (1998) finds that reranking by the tax and transfer system has increased by about 50% between 1981 and 1990 for a wide range of ethical parameters (1994 figures were not computed); unsurprisingly, this was associated with a significantly greater level of redistribution in 1990 than in 1981, which nevertheless compensated for a significantly greater level of market income inequality in 1990 than in 1981[19].

For greater numerical precision of the estimates of Figure 5, and for an indication of the statistical reliability of the estimates, Table 2 shows the point estimates of the cost of HI at selected quantiles of market incomes along with 95% confidence intervals obtained by bootstrapping the 1981 and 1994 samples[20]. The cost of HI is very precisely estimated, with the

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[19] We come back to further plausible explanations for the HI rise later in the text.

[20] This was done by drawing with replacement from the 1981 and 1994 Surveys of Consumer Finance 200 random samples of sizes equal to the Survey samples, and redoing for each of these 200 samples all of the calculations leading to the point estimates. Increasing the number of the random samples from 200 to 1000 made no noticeable difference to the estimated confidence intervals.

95% confidence intervals never exceeding a region of ±1% of the point estimates. Whatever the percentiles of market incomes, the cost of HI is roughly three times as large in 1994 as in 1981.

The areas under the curves H(p) in Figure 5 give the overall money-metric costs of HI (recall (16)). These values are shown in Table 3 as percentages of *per capita* net income, under the heading H_1, along with estimates of the Blackorby and Donaldson (1984) progressivity indices Π for the actual tax system and $\theta\Pi_{wn}$ for a welfare equivalent horizontally equitable tax. 95% confidence intervals are also shown. Compared to a flat tax, the Canadian tax and benefit system, when made horizontally equitable, would yield between 4.1% and 7.6% more *per capita* income for e=0.25 and between 25.1% and 48.8% for e=0.75. Π in 1994 is significantly greater than in all other years, by more than 18% of *per capita* income when e=0.75. HI reduces the overall redistributive performance of the Canadian fiscal system by between 0.3% and 0.8% of *per capita* income for e=0.25 and by between 1.0% and 3.2% for e=0.75. Again, sampling error margins are relatively small. Table 4 indicates that these estimates are not very sensitive to the choice of the kernel bandwidths. Varying the bandwidth by ±20% changes the estimates of the cost of HI for e=0.75 by ±2% for 1981 and by less than ±1% for 1994. The use of an adaptive kernel has a somewhat larger effect on the estimated cost of HI, but does not upset the results very much[21].

Table 5 reports results on the sensitivity of our estimates to the choice of an equivalence scale, and on between and within-group components, with the groups as previously defined for Table 1. The equivalence scale choice can be important since it defines the scale along which equals are identified. We use three different equivalence scales: the OECD one, the one computed from Statistics Canada's Low Income Cut-Offs (LICO), and the Cutler and Katz (1992) double-parameter equivalence scale defined as $(N_A+kN_K)^s$, where N_A and N_K indicate respectively the number of adults and children in the household, and where k and s are parameters lying between 0 and 1 and encompass the importance of adults and children in the computation of adult-equivalents. For each scale, we have computed estimates of: the cost of inequality of a welfare-neutral flat tax (C^f), the cost of inequality (C^a) after the actual tax and benefit system[22], the overall cost of HI (H), the sum of HI within groups ($\Sigma q_k H_k$), and HI between groups (H^*). HI reduces the equity performance of the tax system by between 2.26% and 4.55% of *per capita* net income depending on the equivalence scale selected; in the absence of HI, C^a would fall by the value of the index H. HI is highest when a *per capita* equivalence scale is chosen (s=1 and k=1), and lowest when no account is taken of family size (s=0). Hence, the 1994 Canadian fiscal system would be deemed the least horizontally inequitable if we only looked at total household income and were not concerned with accounting for family size and composition. For all scales, within-group HI is deemed significantly more important (about four or five times more important)

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[21] We also checked how alternative bandwidths influence the estimated value of H(p) across percentiles of market incomes (such as in Figure 5). The estimated curves are almost undistinguishable one from another, even at low values of p.

[22] The difference between C^f and C^a gives the index Π of progressivity.

than between-group HI[23]. Hence, HI is by no means due solely to inequities *across* socio-demographic groups[24]; it seems rather to stem mostly from unequal treatment within socio-demographically homogeneous households.

Figure 6 decomposes into more detail the 1994 cost of HI, both between and within the three socio-economic groups considered, and across the distribution of market incomes. Whatever the percentile of market income, it is the sum of HI *within* groups that accounts by far for the largest part of H(p). As p increases from 0 to 1, it is first HI among older households (group 1) that is the greatest (but then rapidly decreasing), and then that among childless younger households (group 2) and younger households with children (group 3). Comparing 1994 with 1981 (which we do not show here to save space), we find that HI within groups (and most importantly HI within group 1 for lower values of p) increased significantly during that period; between group HI stayed roughly the same. Thus, the tax and transfer treatment of the elderly poor would seem to be an important factor in the significant rise in HI between 1981 and 1994.

Apart from changes in the structure of the tax and transfer system, differences in the socio-demographic structure and in the distribution of incomes can also contribute to explain differences in HI across time (or across societies). For instance, in the light of the important within-group HI noted above among older households, the increase in the proportion of older individuals in our samples across years (a rise from 16% in 1981, to 16.8% in 1990, and to 19% in 1994) would in itself tend to lead towards increased overall HI. Further, it is well-known that the distribution of income for low percentiles has worsened significantly during the Canadian recession of the early 1990's[25]. Since the HI indices we use here are rather sensitive to the variability in low incomes, this worsening of the distribution of income at low percentiles -- even if keeping unchanged the basic inequities in the structure of the tax and benefit system -- would lead to an increase in the estimated HI.

To throw greater light on the evolution and on the sources of HI between 1981 and 1994, Figures 7, 8 and 9 show kernel estimates of the standard deviation of taxes and benefits across quantiles of market income for 1981, 1990 and 1994. Taxes and benefits are lumped into groups: income taxes (provincial and federal), family and various benefits (youth allowances, child tax

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[23] This result is of course conditional on the level of group disaggregation; the greater the number of groups considered, the more we would expect between-group HI to be important.

[24] Hicks (1997) points out that there important intergenerational transfers across age groups, which could account for a significant part of the between-group HI found here. For instance, she finds large positive net transfers for the old and lower positive ones for the young, with net tax contributors lying somewhere in the middle (the majority of Group 2's households with children would come from these net contributors).

[25] The National Council of Welfare (1996) reports for instance an increase in the national poverty headcount of 14.6% to 16.6% between 1990 and 1994.

credits and other income and tax credits from government sources), old age transfers[26], and social assistance and unemployment insurance (UI) benefits. The net tax burden is shown as "Taxes minus benefits". Old age transfers are by far the most variable for low percentiles and for all years shown. As mentioned above, this is significant because the indices of inequality we consider are particularly sensitive to the variability in low incomes. The greater variability of Old age transfers remains for percentile values up to between 0.6 and 0.85, at which point income taxes become the most volatile. Thus, HI would apparently be caused mostly by age-conditional transfers for a large range of market income percentiles, and by the income tax system for the richest 15% to 40% of the population. This greater variability of old age transfers is not surprising since they also appear to be the most redistributive (see Howard et al. (1994) and Duclos (1998)). The variability in old age transfers has increased slightly between 1981 and 1990, but considerably between 1990 and 1994. In 1990, the standard deviation of old transfers exceeded 25% of average market income for the bottom 7% of the population, but in 1994 it was so for the bottom 16% of the population. That this variability has much increased between 1981 and 1994 is in line with the fact that old age transfers represent 4.5% of average income in our 1981 sample, but rise to 7.7% of average market income in the 1994 sample.

Social assistance and UI benefits, and Family and various benefits, follow basically the same trend across percentiles and across years as old age transfers but with a standard deviation respectively 30% and 50% lower. The increase in the impact of social assistance and UI on the variability of net income is compatible with the increase in their proportion of market incomes, which rose from 3.2% in 1981 to 4.6% in 1990 and 5.7% in 1994. Since social assistance programmes differ across provinces, their rising size relative to market incomes is a clear source of increased HI. Poschmann (1998) also notes that the actual structure of UI benefits, which grants more generous benefits in the Maritime provinces (due to different labour market conditions) for a similar level of market incomes, may favour inter-provincial HI.

There is also a slight increase in the variability of income taxes between 1981 and 1990 for market incomes below the median, but no change for higher percentiles. Keeping in mind the increased share of government revenues between these years, this result nevertheless is in line with the objectives of the 1987 reform. Besides arising from the increased tax burden on families, the increased variability of taxes for low percentiles can be due, *inter alia*, to greater fiscal decentralisation in favour of the provinces (see Bird (1995)), which differ in their provincial tax structures. Bird (1995) also points out that the introduction of tax credits increased the targetting of tax relief and ensured a greater take-up than typically seen for transfers, but it has also increased the complexity and irregularities of the tax system. There is little change in the variability of taxes between 1990 and 1994. From Figures 7, 8 and 9, it is also clear that transfers

[26] These transfers include Old Age Security Pensions, Guaranteed Income Supplements, and Canada/Québec Pension Plan Benefits. These benefits serve both as a form of social insurance against retirement (Canada/Québec Pension Plan Benefits) and as universal or means-tested sources of social assistance (Guaranteed Income Supplements stand as a form of means-tested social assistance for those above retirement age).

are a greater source of HI than taxes, and this also explains why it is the change in the variability of benefits which drives the change in HI between 1981 and 1990 and between 1990 and 1994. Again, this is in line with the much greater role of benefits in the redistribution of income (see *e.g.* Beach and Slotsve (1996) and Duclos (1998)).

5. Overview and conclusion

There is a need among tax policymakers, as well as tax designers and administrators, for meaningful summary indicators of HI to guide the process of reform. This paper offers a systematic and normatively sound approach, according to which HI is measured by the amount an inequality-averse SDM would pay to have it removed, both in dollars and as a percentage of *per capita* income. Being money-metric, the indicators can be used, for example, to determine ethically if the dollar increase in efficiency (or fall in inequality) exerted by some government policy is worth the dollar cost of HI which this policy may cause. The degree of inequality aversion must be specified by the analyst, and this offers the opportunity to test robustness of conclusions using sensitivity analysis - of itself a new development in the HI measurement literature[27]. The methodology also shows that HI can be seen as 'loss of performance': a new decomposition of Blackorby and Donaldson's (1984) index of tax progressivity demonstrates this.

We have discussed the implementation difficulties arising from the identification problem, and have proposed statistically attractive procedures to estimate HI, both locally and globally, and to compare the HI characteristics of alternative tax and transfer systems. Our illustrative application showed that HI in the Canadian tax and transfer system did not change much between 1981 and 1990, but significantly increased between 1990 and 1994. Most of the HI stems from the treatment of the bottom 20% of the market income population, to a large extent attributable to old age transfers and to HI within older households. Overall HI is responsible for the loss of around 10% of vertical equity, and amounts to 3% of *per capita* net income in 1994 for an inequality aversion parameter equal to 0.75. Our estimates are quite robust to sampling variability, to the choice of kernel bandwidth, and to the choice of equivalence scale.

The methodology has wide applicability, both as a means to investigate the performance and the equity of the tax and transfer system in itself and in comparisons between countries or levels of government, and over time. Is it socially less costly, in foregone tax dollars, for government to collect its revenue using a range of taxes, or by engaging a single tax instrument

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[27] This degree of inequality aversion could in principle differ for progressivity and HI, but then the connection in Theorem 2 would be altered. For instance, a "minimal state" SDM could be insensitive to the exercise of vertical equity, and to levels of vertical inequality, but ethically very sensitive to violations of horizontal inequity by the state. If progressivity is evaluated using one value of e, and HI another, then the term H_1 in Theorem 2 would need supplementing by a value representing "additional HI dislike" in order to capture the SDM's perceived true cost of HI.

like the expenditure tax? How socially costly is the HI (and inequality) introduced by private sector rules and markets (e.g., by gender or racial discrimination)? Our measures can inform topical questions such as this. Indeed, the technology we have described is capable of extracting from any scattergram of normatively significant variables, such as that in Figure 2, a horizontal-vertical characterization of the underlying data generating process. It may be of interest in biometrics, for example in characterizing biodiversity (Polasky and Solow (1995)), as well as in economics.

Appendix

With C defined as in (2), the Atkinson (1970) index of inequality for **x** is:

$$I(e) = C/\mu = 1 - \xi/\mu \qquad \text{(A1)}$$

Let the Atkinson indices before and after tax be I^b and I^a. The Blackorby and Donaldson (1984) progressivity index is:

$$\Pi = [I^b - I^a] / [1 - I^b] \qquad \text{(A2)}$$

Let μ^b and ξ^b be the mean and EDE income levels overall before tax, and let μ^a and ξ^a be those after tax. Define g and γ by:

$$(1\text{-}g)\mu^b = \mu^a \ , \quad (1\text{-}\gamma)\xi^b = \xi^a \qquad \text{(A3)}$$

These are the rates of the equal-yield flat tax and equal-welfare flat tax respectively.[28] The cost-of-inequality measures for $\mathbf{x^a}$ and $\mathbf{x^f} = (1\text{-}\gamma)\mathbf{x^b}$ are:

$$C^a = \mu^a - \xi^a \ , \quad C^f = (1\text{-}\gamma).\mu^b - \xi^a \qquad \text{(A4)}$$

It follows using (A3) that Π can be written:

$$\Pi = [C^f - C^a] / \mu^a \qquad \text{(A5)}$$

If C^* is the cost of the inequality after application of $T_{wn}(\bullet)$, then corresponding to (A5) we have:

$$\Pi_{wn} = (C^f - C^*)/\mu^* \qquad \text{(A6)}$$

where μ^* is mean income after application of $T_{wn}(\bullet)$.[29]

The proof of Theorem 1 follows from the choice of weighting scheme. The income saving which would come from eliminating post-tax inequality with social indifference in Ω_x is H_x *per capita*, and therefore the income saving overall, or additional tax revenue generated, is $\Sigma_x N_x. H_x$

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[28] Duclos (1995a) defines a performance index τ for the tax system, which captures the distinction between g and γ. Specifically, τ is the proportional surcharge on post-tax incomes $\mathbf{x^a}$ which would reduce post-tax welfare to that after revenue-neutral flat tax: $(1\text{-}\tau)\xi^a = (1\text{-}g)\xi^b$. It follows from (A3) that $(1\text{-}\tau) = (1\text{-}g) / (1\text{-}\gamma)$.

[29] The Blackorby-Donaldson index for a given tax system thus measures the proportion of after-tax income the SDM would pay to convert a flat tax system with the same welfare into the given one; this is positive if the given tax is progressive; the more progressive, the more he would pay. Also, the more inequality-averse the SDM is, the more he would pay. Duclos (1995a,1997) demonstrates this, in respect of his performance index τ, which is related to Π by $\Pi = \tau/(1\text{-}\tau)$. (Compare $\Pi = [g - \gamma] / [1 - g]$, which comes by substituting in (A2) from (A1) and (A3), with the relationship between τ, g and γ shown in the previous footnote).

$/ N = \Sigma_x\, p_x.\, H_x = H$ *per capita*.

For the proof of Theorem 2, note that if inequality of post-tax income were eliminated with social indifference, by moving from $\mathbf{x^a}$ to the distribution in which everybody got ξ^a, the income saving *per capita* would be C^a; whilst if we moved to perfect equality from the distribution after application of $T_{wn}(\bullet)$, in which people in Ω_x get $\xi^a{}_x\ \forall x$, the income saving would be C^*. Equating the overall income saving with the sum of those arising *(i)* from application of $T_{wn}(\bullet)$ and *(ii)* from subsequent equalization, we have:

$$C^a = H + C^* \tag{A7}$$

This can readily be validated formally.[30] Now subtract each side of (A7) from C^f, and divide by μ^a. Theorem 2 follows from (A5) and (A6), with $\theta = \mu^* / \mu^a = \Sigma_x\, p_x.\ \xi^a{}_x\ / \Sigma_x\, p_x.\ \mu^a{}_x$ which is less than 1 because $\xi^a{}_x < \mu^a{}_x$, $\forall x$.

For the proof of Theorem 3, let $\theta_{k,x} = |\Omega_x \cap \Gamma_k|/N$ be the proportion of people in the overall population who have pre-tax income x and are in group k, so that $\Sigma_k\ \theta_{k,x} = p_x$ and $\Sigma_x\ \theta_{k,x} = q_k$. A similar formula to (A7) can be derived for the post-tax income distribution within Ω_x, to explain how the cost of inequality H_x is made up across the demographic subgroups Γ_k, k = 1,2,3....:

$$H_x = \mu^a{}_x - \xi^a{}_x = \Sigma_k\ \{\theta_{k,x}/p_x\}(\mu^a{}_{k,x} - \xi^a{}_{k,x}) + C_x^* \tag{A8}$$

with $C^*_x = \mu^*_x - \xi^a_x$ and obvious adaptations of the notation. Multiplying through in (A8) by p_x, and summing over all values of x, we have $H = \Sigma_k\ q_k\, H_k + H^*$, where $H_k = \Sigma_k\ \theta_{k,x}/q_k(\mu^a{}_{k,x} - \xi^a{}_{k,x})$ is HI in demographic group Γ_k and $H^* = \Sigma_x\, p_x\, C_x^*$ is HI arising from the effect of the tax between these groups. If we take instead a partial sum in (A8), up to say x = z, we obtain similarly $H^z = \Sigma_k\ q_k\, H^z{}_k + H^{z*}$, in which the terms express the aggregates of local HI up to x=z.

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[30] Use the formulae for C^a and H in (A4) and (4); note that $C^* = \mu^* - \xi^a$, where $\mu^* = \Sigma_x\, p_x.\ \xi^a{}_x$; and use $\mu^a = \Sigma_x\ p_x.\ \mu^a{}_x$. This is a particular case of the general between-and-within-groups decomposition of Blackorby *et al.* (1981). See also Blewitt (1982).

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Table 1

Expected net incomes* at quantiles of market incomes
(densities of net incomes conditional on a quantile of market incomes)

p	1981				1994			
	Total	Groups			Total	Groups		
		1	2	3		1	2	3
0.05	0.39 (1.00)	0.50 (0.47)	0.31 (0.15)	0.28 (0.38)	0.40 (1.00)	0.51 (0.43)	0.29 (0.26)	0.35 (0.31)
0.25	0.62 (1.00)	0.78 (0.17)	0.62 (0.14)	0.59 (0.69)	0.54 (1.00)	0.74 (0.27)	0.44 (0.27)	0.49 (0.46)
0.50	0.85 (1.00)	1.02 (0.10)	0.88 (0.15)	0.82 (0.75)	0.85 (1.00)	1.09 (0.13)	0.84 (0.27)	0.80 (0.60)
0.75	1.24 (1.00)	1.41 (0.10)	1.27 (0.29)	1.20 (0.62)	1.26 (1.00)	1.50 (0.09)	1.28 (0.37)	1.20 (0.54)
0.95	1.91 (1.00)	2.08 (0.09)	1.91 (0.56)	1.87 (0.35)	2.02 (1.00)	2.24 (0.08)	2.04 (0.60)	1.93 (0.33)
Overall group net income	1.00	0.95	1.31	0.89	1.00	0.90	1.19	0.91
Overall group density	1.00	0.16	0.24	0.60	1.00	0.19	0.33	0.48

* The net incomes are shown as proportions of the overall mean of net incomes.

Group 1: older households. Group 2: younger households without children. Group 3: younger households with children.

Table 2

Cost of HI at selected quantiles of market incomes

Point estimates
[95% confidence intervals]

p	**e = 0.25**		**e = 0.75**	
	1981	**1994**	**1981**	**1994**
0.1	0.82 [0.82 - 0.83]	2.24 [2.23 - 2.25]	2.62 [2.60 - 2.64]	10.38 [10.35 -10.42]
0.3	0.25 [0.25 - 0.25]	0.91 [0.90 - 0.91]	0.75 [0.75 - 0.75]	2.83 [2.82 - 2.85]
0.5	0.17 [0.17 - 0,17]	0.35 [0.35 - 0.36]	0.51 [0.51 - 0.51]	1.07 [1.07 - 1.08]
0.7	0.16 [0.16 - 0.16]	0.23 [0.24 - 0.24]	0.47 [0.47 - 0.48]	0.71 [0.70 - 0.71]
0.9	0.14 [0.14 - 0.14]	0.18 [0.18 - 0.18]	0.43 [0.42 - 0.43]	0.53 [0.53 - 0.54]

Table 3

Progressivity and horizontal inequity

Point estimates
[95% confidence intervals]

	e = 0.25			e = 0.75		
	Π	$\theta\Pi_{wn}$	H_1	Π	$\theta\Pi_{wn}$	H_1
1981	3.83 [3.83-3.84]	4.14 [4.14-4.15]	0.31 [0.31-0.31]	25.08 [25.06-25.10]	26.05 [26.04-26.07]	0.97 [0.96-0.98]
1985	4.68 [4.68-4.68]	5.07 [5.06-5.07]	0.39 [0.39-0.39]	30.31 [30.29-30.34]	31.54 [31.52-31.56]	1.23 [1.22-1.24]
1990	4.85 [4.85-4.85]	5.22 [5.22-5.22]	0.37 [0.37-0.37]	28.41 [28.38-28.43]	29.56 [29.55-29.58]	1.16 [1.15-1.17]
1994	6.83 [6.82-6.84]	7.62 [7.62-7.63]	0.80 [0.79-0.80]	48.83 [48.77-48.89]	52.02 [51.96-52.07]	3.19 [3.17-3.21]

Table 4

Cost of total HI with alternative kernel bandwidths and with adaptive kernel (e = 0.75)

	1981	1994
Standard kernel	0.97	3.19
20% lower standard	0.98	3.21
20% higher standard	0.95	3.17
Adaptive kernel	1.02	3.23

Table 5

Split of total HI within and between groups*
(1994; e = 0.75)

Equivalence Scales	C^f	C^a	**H**	$\Sigma q_k H_k$	H^*
OECD	64.56	15.73	3.19	2.52	0.67
LICO	62.69	14.42	2.85	2.19	0.66
Cutler and Katz:					
s = 0	63.65	15.08	2.26	1.80	0.46
k = 1, s = 1	65.30	19.09	4.55	3.59	0.96
k = 1, s = 0.5	63.55	14.19	2.44	1.86	0.58
k = 0.5, s = 0.5	63.37	13.81	2.31	1.81	0.50
k = 0.5, s = 1	64.80	16.18	3.51	2.94	0.57

Notes: C^f is the cost of inequality of a flat tax that is welfare equivalent to the actual tax system
C^a is the cost of inequality of the net income distribution
H is the cost of HI
$\Sigma q_k H_k$ is the sum of within-group HI
H^* is between-group HI

* The groups are as in Table 1.

Figure 1
The cost of inequality

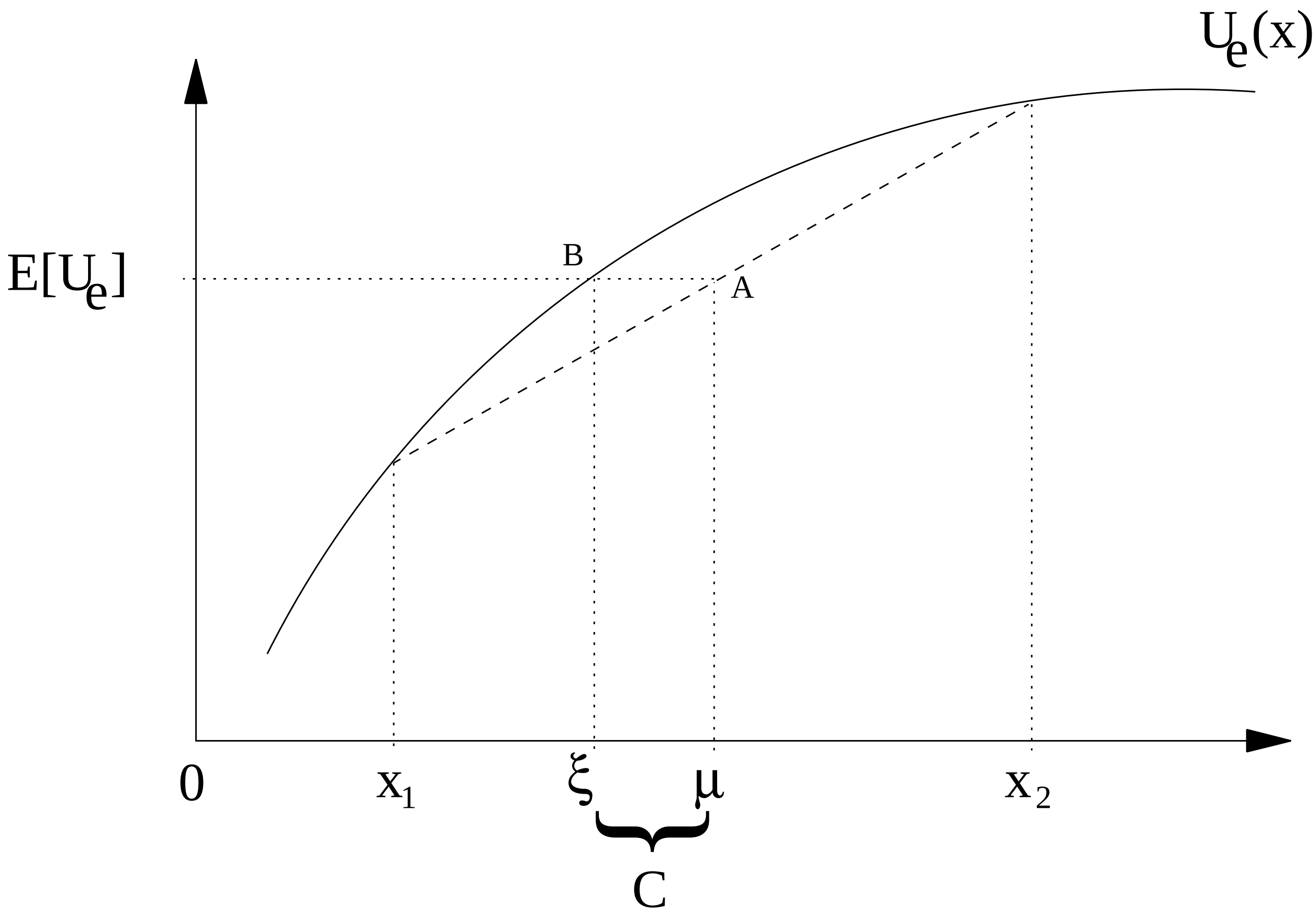


Figure 2: Scatter Plot of Market and Net Income, Canada 1994

(All incomes are shown as multiples of the relevant means)

Figure 3

Joint density of Market and Net Incomes, Canada 1994

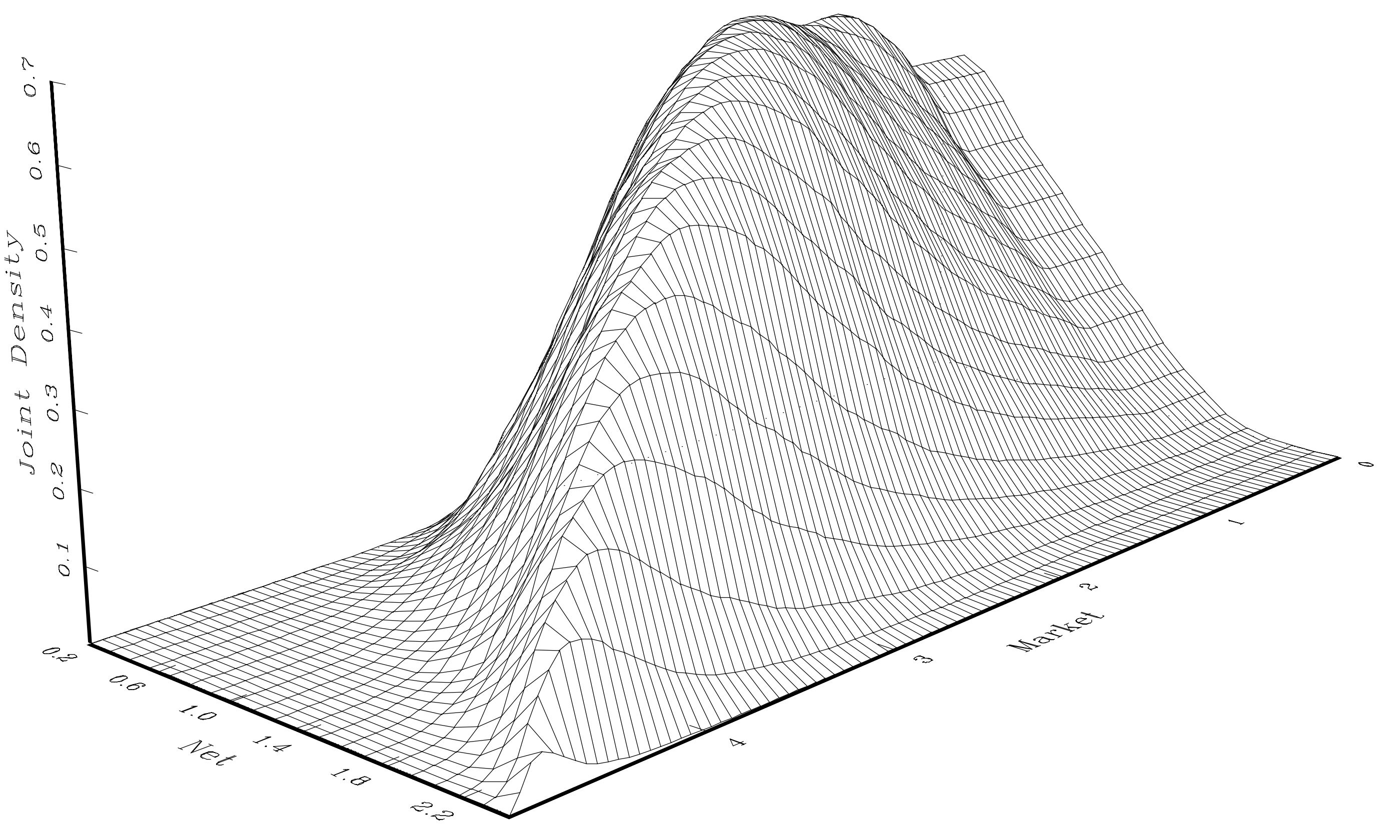


Figure 4

Revenue-neutral and welfare-neutral taxation

Canada 1994

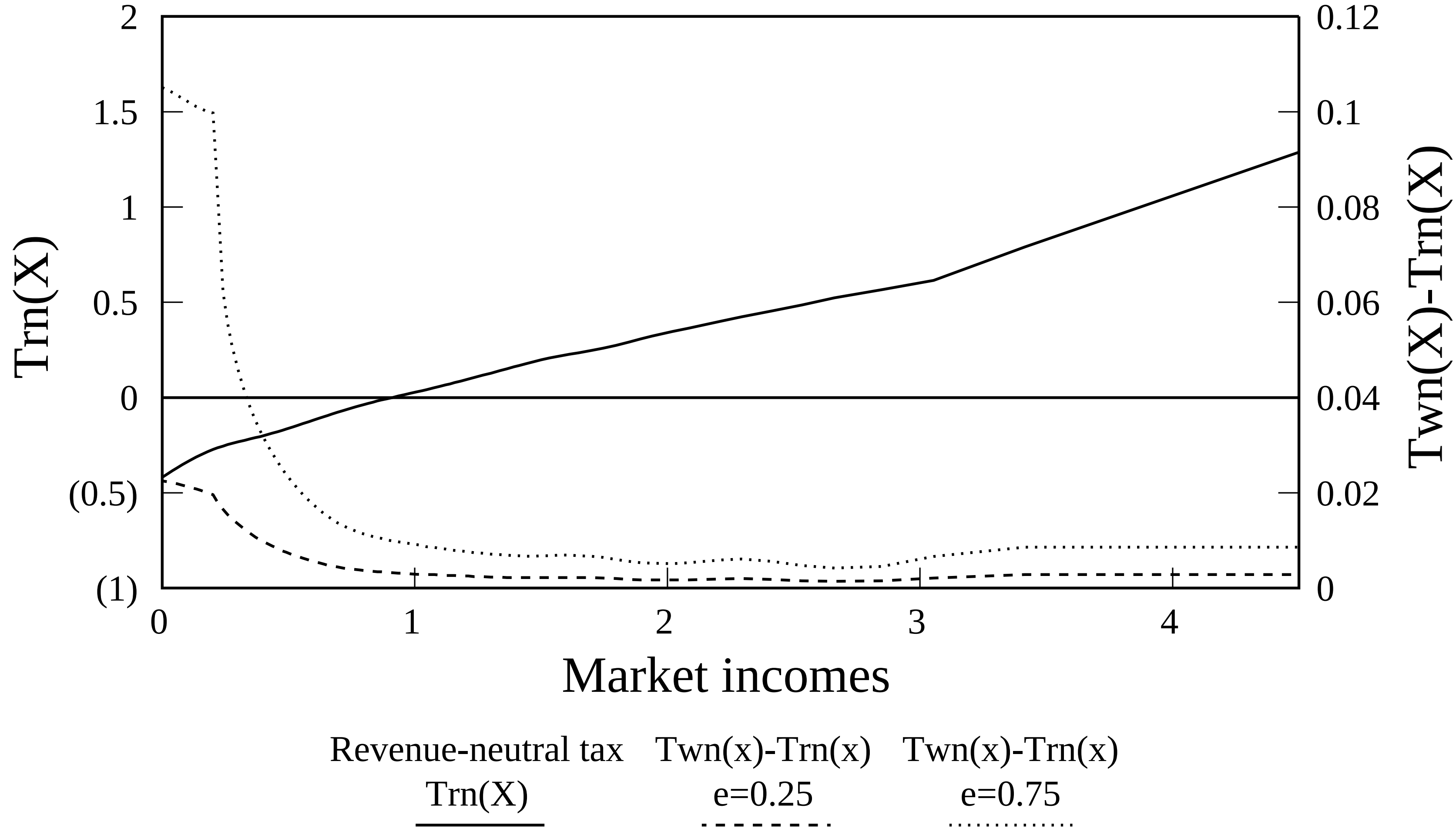
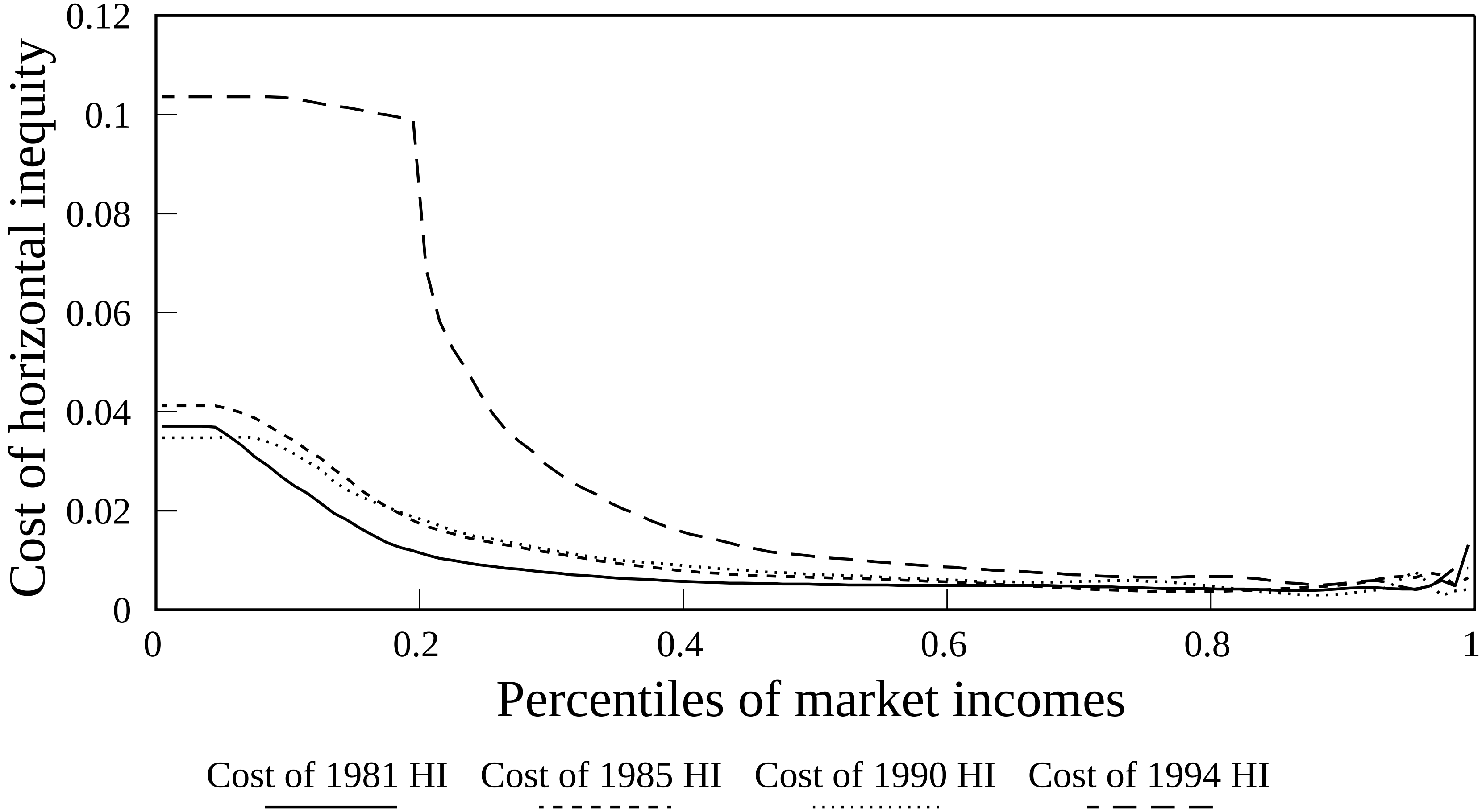


Figure 5

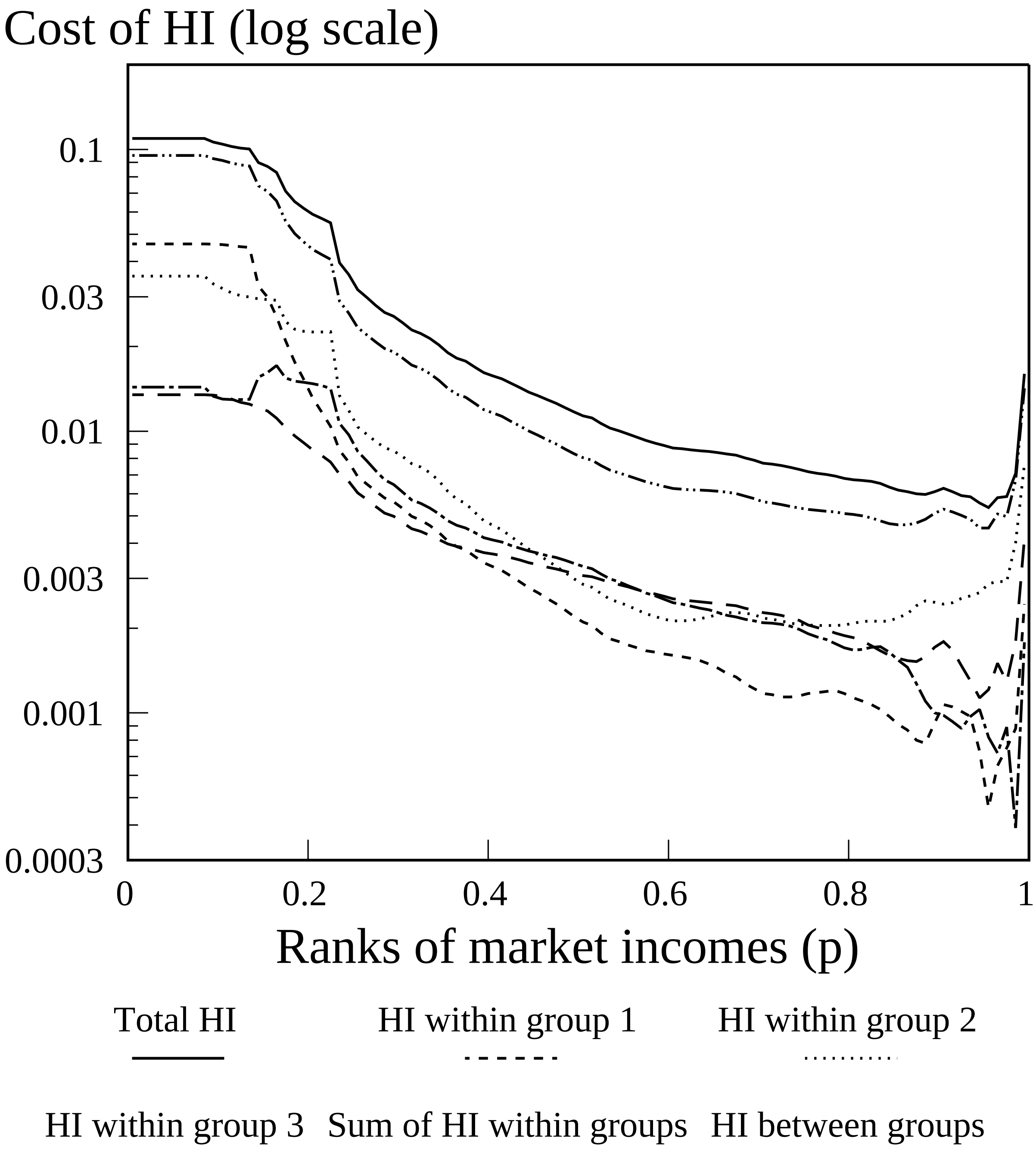
Cost of horizontal inequity
1981, 1985, 1990 and 1994



epsilon=0.75

Figure 6

Decomposition of cost of total HI at different quantiles of market incomes (1994)



epsilon=0.75

Figure 7

Standard deviation of taxes and benefits at quantiles of market incomes (1981)

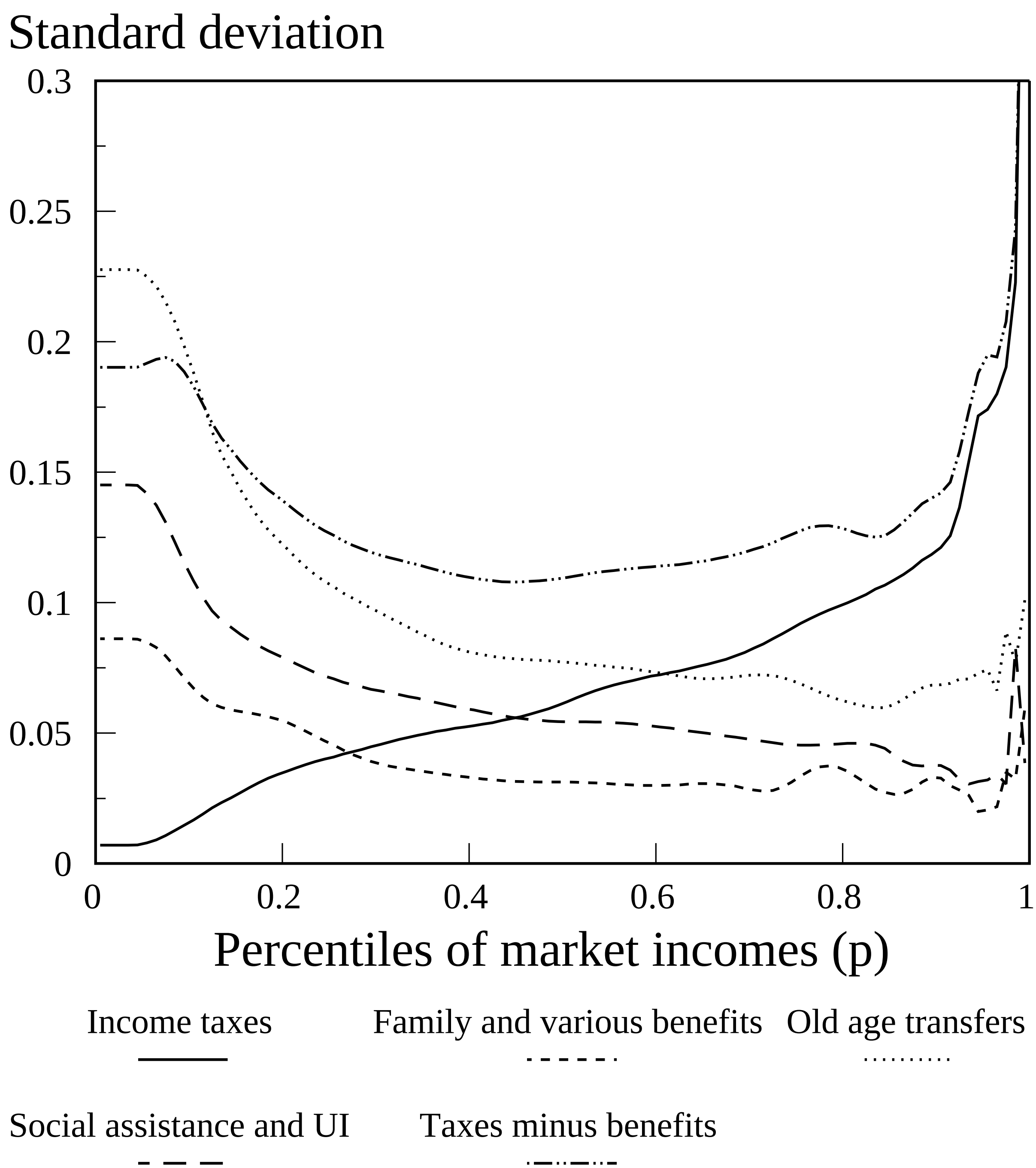


Figure 8

Standard deviation of taxes and benefits at quantiles of market incomes (1990)

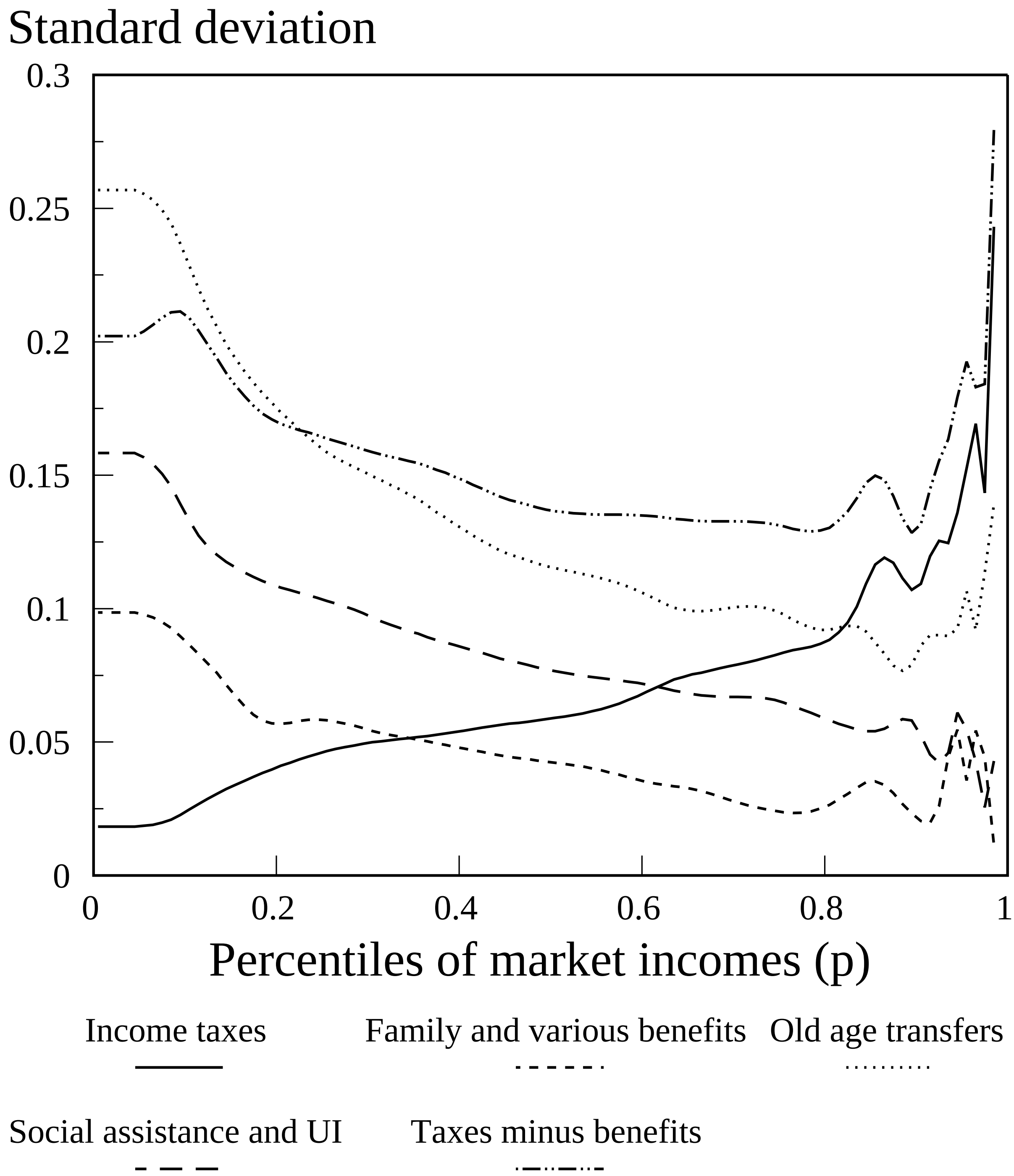


Figure 9

Standard deviation of taxes and benefits at quantiles of market incomes (1994)