Price Regulation of Pluralistic Markets Subject to Provider Collusion

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Abstract

We analyse incentives for collusive behaviour when heterogeneous providers are faced with regulated prices under two forms of yardstick competition, namely discriminatory and uniform schemes. Providers are heterogeneous in the degree to which their interests correspond to those of the regulator, with close correspondence labelled altruism. Deviation of interests may arise as a result of de-nationalisation or when private providers enter predominantly public markets.

We assess how provider strategies and incentives to collude relate to provider characteristics and across different market structures. We differentiate between “pure” markets with either only self-interested providers or with only altruistic providers and “pluralistic” markets with a mix of provider type.

We find that the incentive for collusion under a discriminatory scheme increases in the degree to which markets are self-interested whereas under a uniform scheme the likelihood increases in the degree of provider homogeneity. Providers’ choice of cost also depends on the yardstick scheme and market structure. In general, costs are higher under the uniform scheme, reflecting its weaker incentives. In a pluralistic market under the discriminatory scheme each provider’s choice of cost is decreasing in the degree of the other provider’s altruism, so a self-interested provider will operate at a lower cost than an altruistic provider. Under the uniform scheme providers always choose to operate at the same cost. The prospect of defection serves to moderate the chosen level of operating cost.

Keywords: Price regulation; yardstick competition; collusion; altruism.

JEL classification: I1, I18, L33
1. Introduction

Many countries have introduced some form of yardstick competition in order to regulate prices in contexts where providers face limited competitive pressure. Examples are the maximum price limits each water company may charge its customers in the UK (Ofwat, 1993); price caps imposed by the Federal Energy Regulatory Commission to hold down the wholesale price of natural gas and electricity in interstate commerce in the US (US Department of Energy, 2002); postal tariffs determined by independent regulators in countries such as Germany, the Netherlands and the UK (NERA, 2004); and prospective payment system (PPS) that have been introduced to pay for health care services in many countries (Schreyögg et al., 2006; Ma, 1994).

The fundamental idea behind yardstick competition is that the price (or price cap) faced by each provider is dependent on the actions of all the other providers (Schleifer, 1985; Laffont and Tirole, 1993). According to Schleifer’s *discriminatory rule*, the price each provider faces is based on the costs of all other providers in the industry but not its own. This creates strong incentives for cost control: each provider’s cost reducing effort will not be detrimental to the price it faces. Incentives are weaker when a *uniform* pricing rule is applied, under which all providers face common prices or price caps.

A potential drawback with yardstick competition is that providers have an incentive to collude on higher costs, first because they can get a higher price for their services and, second, because they can exert less cost reducing effort, thereby benefiting from slack (Wilson, 1989).

In contexts where there is a large number of providers, this is unlikely to be problematic, mainly because the cost of collusion rises (Pope, 1989). But there is greater potential for collusive behaviour in contexts where there is a limited number of providers. This is likely for utilities, rail or postal services. But it can arise in health care, for instance because specialist services (like bone marrow or lung transplantation) are concentrated among a handful of providers or in places such as Northern Ireland or Iceland, which are considering introducing PPS arrangements despite there being fewer than five hospitals in each country.

The incentive to collude with other providers will depend on the objectives of the provider, particularly the extent to which their objectives correspond with those of the price-setting regulator. We use the terms “*altruistic*” to describe providers that have objectives closely related to those of the regulator and “*self-interested*” to describe providers whose interests are more divergent from those of the regulator (Rose-Ackerman, 1996; Bozeman, 1984; Rainey et al. 1976). If providers differ in their degree of altruism, they may behave quite differently in response to financial incentives (Aas, 1995). Divergence among providers may arise in situations where greater plurality of provision is being encouraged. For example, traditionally public (National Health Service) systems such as England, France, Portugal and Italy are encouraging more private sector organisations to enter the health care market (Oliveira and Pinto, 2002; Aballea et al 2006; Levaggi, 2007; Pollock and Godden 2008). Similarly many countries have de-nationalised many services, either wholly or in part. Public providers may have a strong sense of mission, aiming to maximize the well-being of the people they serve (Wilson 1989), just as the regulator would like. But private providers are also accountable to their shareholders, with an interest in profit making. This implies that they have a weaker sense of “public” service mission, and might have objectives that are less closely aligned to those of the regulator (Newhouse, 1970; Hansmann, 1980; Glaeser and Schleifer, 2001).

There are a number of works that have addressed the issue of collusion under yardstick competition (Boardman et al 1986; Tangeras, 2002; Chong and Huet, 2006). Our paper is particularly close to Potters et. al. (2004). The authors present an adapted version of Schleifer’s model (Schleifer, 1985) and test it experimentally in order to explore collusion incentives under different yardstick competition schemes. However the existing literature assumes homogeneous providers. The aim of the paper is to analyze by means of a theoretical model how incentives to collude relate to provider characteristics (altruistic or self-interested) and across different market structures. We differentiate between “pure” markets with either only self-interested providers or with only altruistic providers and “pluralistic” markets with a mix of provider type. For each yardstick scheme, we analyse the choice of cost when providers do not co-operate and when they collude, and we consider incentives to defect from the collusive agreement.

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1 In the context of this paper we define market structure very narrowly as asymmetry in provider altruism.
When a *discriminatory* yardstick rule is considered, we find that an industry populated by self-interested providers is more prone to collusion than a mixed market which in turn is more prone to collusion than an industry served by altruistic providers. When we consider a *uniform* yardstick rule we find that pure markets are more prone to collusion than mixed markets.

Providers’ choice of cost also depends on the yardstick scheme and market structure. In general, costs are higher under the uniform scheme, reflecting its weaker incentives. In a pluralistic market under the discriminatory scheme each provider’s choice of cost is decreasing in the degree of the other provider’s altruism, so a self-interested provider will operate at a lower cost than an altruistic provider. Under the uniform scheme providers always choose to operate at the same cost. The possibility of defection serves to moderate the chosen level of operating cost.

The paper is organized as follows. Section 2 introduces the main assumptions of the model, and considers collusive behaviour under a discriminatory yardstick competition model. Section 3 presents the results under the uniform yardstick regulatory rule. Section 4 summarizes the main results of the paper and section 5 draws the main conclusions.

### 2. The model

Consider a market with three types of agent: consumers, providers and a regulatory authority. We consider two providers $i$ with $i = 1, 2$ each with its own population of consumers defined geographically, so that each provider is a local monopolist facing a downward-sloping demand curve $q(p_i)$ with $p_i$ being the price paid by consumers for each unit of service $q$.

In some contexts, such as for postal services or utilities, consumers may face the full or a partially subsidised price. In the health care context, where services are often free at the point of consumption, we can assume that providers compete on the basis of quality (Pope, 1989). Quality is loosely defined, but might include low waiting times and good amenities, all of which are cost increasing, the implication being that demand is decreasing in the cost of quality.

Under yardstick competition, the regulator establishes a payment that gives the providers incentives to reduce costs. We will start the model by considering a *discriminatory* yardstick, whereby each provider faces a regulated price set beforehand equal to the average (say) of the marginal costs of all the other providers in the market except from its own. We assume that costs are observed by the regulator.

The regulator sets a cap $\hat{p}$ on the price that each provider can charge. Note that this restriction will bind in equilibrium (i.e. $\frac{\partial \Pi(p_i)}{\partial p_i} > 0$). If not, there would be no need for regulation.

The main objectives of a regulation policy are to promote technical efficiency and allocative efficiency by simulating the outcomes of competitive markets (Laffont and Tirole, 1993). When providers enjoy a degree of monopoly power, they can provide a lower volume of output than they would in a competitive situation and, thereby, secure higher prices. This causes welfare loss. Moreover, monopoly firms lack incentives to be cost efficient, thus undermining technical efficiency.

The utility of provider $i - U_i$ is a function of the regulated price $\hat{p}$, the marginal costs $c_i$ and the altruism level $\alpha_i$. We assume that altruistic and self-interested providers are distinguished by their degree to which they are concerned about consumer surplus,

$$CS = \int_{p}^{x} q(x) dx$$  \hspace{1cm} (1)

This is graphically represented by the area under the demand curve for their services, above their price. Recall that consumer surplus is decreasing in the unit price of the service, so that the greater the
degree of altruism, the greater the utility the providers derive from lower prices.

We further assume that the provider cares about consumer welfare to some proportion \( \alpha_i \) with \( i = 1, 2 \). Without loss of generality we assume that provider 2 is more altruistic than provider 1, i.e. \( \alpha_1 < \alpha_2 \).

Provider’s benefit from slack, \( S(c_i) \). This element captures the utility from avoiding cost reducing effort (Bradford and Craycraft, 1996; Pope, 1989). The benefit of slack \( S(c_i) \) is an increasing function of cost at a decreasing rate \( (S'(c_i) > 0, S''(c_i) < 0) \).

Thus the utility of each provider is given by the sum of net revenues, the benefit from slack, and the utility the provider derives from increased consumer welfare,

\[
U(\hat{p}_i, c_i, \alpha_i) = (\hat{p}_i - c_i)q_i(\hat{p}_i) + S(c_i) + \alpha_i \int_{\hat{p}_i}^{\infty} q(x)d(x)
\]

(2)

2.1 The first best

For comparison purposes we first develop a benchmark. Consider a first best scenario by which the regulator can decide on both the price and the cost of each service. In each local market the optimum is then characterized by the pair \( \{p_i, c_i\} \) that maximizes social welfare \( W(.) \) given by the sum of consumer surplus and the provider’s utility, i.e.:

\[
W(p_i, c_i) = (1 + \alpha_i) \int_{p_i}^{\infty} q(x)d(x) + (p_i - c_i)q(p_i) + S(c_i)
\]

(3)

with \( i = 1, 2 \).

Maximizing welfare with respect to price and cost, the social optimum\(^2\) is then given by the first order conditions with respect to the price,

\[
(p_i - c_i)q'(p_i) = \alpha_i q(p_i)
\]

(4)

and with respect to the cost,

\[
S'(c_i) = q(p_i)
\]

(5)

According to (4) the optimal price should be such that the marginal net revenues due to an increase in the price equal the change in consumer surplus weighted by the altruistic parameter \( \alpha_i \). Correspondingly (5) entails that the provider’s marginal benefit from slack should be equal to the effect of increased costs on revenues. From (4), the socially optimum price rule can be written as:

\[
p^* = c^* + \alpha \frac{q(p^*)}{q'(p^*)}
\]

Given that \( q(p^*)/q'(p^*) \) is a negative term, then for \( \alpha \in [0,1] \) the first best price is lower than the marginal cost.

\(^2\)Social optimum solved in Appendix
For \( \alpha_i = 0 \), i.e. in the case of purely self-interested providers, the first best is such that price equals the marginal cost:

\[
p^* = c^*
\]

### 2.2 The provider’s problem

We will analyse two types of game. First we start by describing a setting in which providers strategically chose the cost level in a one shot game. In section 2.3 we characterize a repeated game.

In each one shot game, each provider \( i \) wishes to maximize utility, achieved by choosing the cost \( c_i \) given the price rule to which the regulator will commit. Provider \( i \)'s problem is given by,

\[
\max_{c_i} U_i(p_i, c_i) = (p_i - c_i)q_i(p_i) + S(c_i) + \alpha_i \int q(x)d(x)
\]

(6)

The first order condition with respect to cost \( \partial U_i / \partial c_i = 0 \), is given by

\[
(\frac{\partial \hat{p}_i}{\partial c_i} - 1)q(\hat{p}_i) + (\hat{p}_i - c_i)\frac{\partial q(\hat{p}_i)}{\partial \hat{p}_i} \frac{\partial \hat{p}_i}{\partial c_i} + S'(c_i) - \alpha_i q(\hat{p}_i) \frac{\partial \hat{p}_i}{\partial c_i} = 0
\]

Since we are considering a two-agent model, the yardstick rule is such that provider \( i \) faces a price per service that is equal to the other provider's marginal cost in providing the same service, i.e. \( \hat{p}_i = c_{-i} \).

**Non cooperative solution**

When the two providers act non-cooperatively the first order condition is given by,

\[
\frac{\partial}{\partial c_i} U_i = -q(c_{-i}) + S'(c_i) = 0 \quad \forall \alpha_i \in [0,1], i = \{1,2\}
\]

(7)

Note that, firstly, despite differing in the degree of altruism, providers' non cooperative choices are symmetric, i.e. \( c_{1}^{nc} = c_{2}^{nc} = c^{nc} \) (where the superscript \( nc \) indicates the non cooperative solution).

Take the more altruistic provider (\( i = 2 \)), which affords greater weight to consumer surplus. The price this provider faces depends on the costs of the other provider, implying that the consumer surplus has less influence on its own choice of costs. The opposite rationale holds for the more self-interested provider. The implication is that the first order condition (7) is analogous to that found in the social optimum (5).

Comparing (7) with (5), we can see the two providers optimally choose a cost that is equal to the social optimum, i.e. \( c_{1}^{nc} = c_{2}^{nc} = c^* \). This result arises for the simple reason that, while a provider's cost reduction leads to a reduced price faced by the other provider, it does not adversely affect its own price. This arrangement gives both providers strong incentives to operate at a socially optimal cost level. These results apply to any market structure irrespective of the degree of altruism.

\[
\begin{align*}
\frac{dc_{i}^{nc}}{d\alpha_i} &= -\frac{\partial^2 V_i}{\partial c_i \partial \alpha_i} = 0 \\
\frac{dc_{-i}^{nc}}{d\alpha_{-i}} &= -\frac{\partial^2 V_i}{\partial c_i \partial \alpha_{-i}} = 0
\end{align*}
\]

(8)
The achievement of the socially optimal cost is not related to any altruistic feature of the objective function. It is simply the result of the utility maximizing behaviour of each provider.

**Cooperative solution**

Still on a one shot game, the next step is to allow the providers to collude on costs. The advantage of collusion is that the providers can avoid “competing” against each other in lowering their production costs. Collusion allows providers to limit their cost reducing effort while receiving a higher price for their services. Offsetting these benefits, there are the negative effects resulting from lower demand as well as reduced consumer surplus (which affects utility in proportion of \( \alpha \)). Thus, the final outcome will depend on the balance of these effects.

If there is collusion, providers maximize their joint utilities \( JU \) by optimally choosing the cost of production:

\[
\max_{c_1, c_2} JU = \sum_i (c_{i-} - c_i) q(c_{i-}) + S(c_i) + \alpha \int_{c_{i-}}^{c_i} q(x) d(x)
\]

The choice of cost for each provider is given by,

\[
\frac{\partial JU}{\partial c_i} = -q(c_{i-}) + S'(c_i) + (c_i - c_{i-}) q'(c_i) + (1 - \alpha_{i-}) q(c_i) = 0 \quad i = 1, 2 \tag{9}
\]

Note that the decisions are symmetric apart from the differences between providers’ altruistic levels, namely \(- \alpha_2 q(c_1) \) and \(- \alpha_1 q(c_2)\). This implies that the costs in one provider decrease in relation to the level of altruism displayed by the other. Thus for \( \alpha_1 < \alpha_2 \)

\[
c_1^c < c_2^c
\]

where the superscript \( c \) indicates the cooperative solution.

As before, the more altruistic provider cannot influence the consumer surplus it produces as this depends solely on the cost chosen by the other, less altruistic, provider. The situation under this discriminatory yardstick regime is akin to the two providers swapping their roles. Indeed, even though provider 2 is more altruistic than provider 1, a situation of pure collusion is such that provider 1 is most cost responsive in order to reflect the impact of costs on consumer surplus. Comparing the collusive with the non-cooperative behaviour rule, for a given \( c_2^c \), we note that the cooperative strategy and the non-cooperative best response of provider 1, as given respectively by (9) and (7), differ in the term,

\[
(1 - \alpha_2) q(c_1^c) + (c_1^c - c_2^c) q'(c_1^c) > 0
\]

The first term is the net effect that provider 1’s cost directly has on provider 2’s revenues. The second term is the effect of a unit of provider 1’s cost on the joint surplus as determined through the demand function. As we may note, eventually the impact is positive because \( c_1^c < c_2^c \). Thus we can conclude that, for a given \( c_2^c \), the cooperative strategy of the more self-interested provider is that it will operate at a higher cost than if there was no collusion.

The same comparison for the more altruistic provider 2 leads us to an ambiguous conclusion. In fact we find that, for a given \( c_1^c \), although the optimal decision is still the product of the same two effects as before, these now have opposite signs. Nevertheless, we are able to say something more by considering a particular case. Let us consider the case by which the market is served by a purely self-interested provider \( (\alpha_1 = 0) \) and an altruistic provider \( (\alpha_2 = 0) \) and compare this situation to one in which there are two purely self-interested providers \( (\alpha_1 = 0) \). Analysing (9) evaluated in a mixed
market we obtain:

Provider 1:  
\[ -q(c_2) + S' (c_1) + (c_1 - c_2)q_i (c_1) + (1 - \alpha_2)q(c_1) = 0 \]  
(10)

Provider 2:  
\[ -q(c_1) + S' (c_2) + (c_2 - c_1)q_i (c_2) + q(c_2) = 0 \]  
(11)

and comparing with the optimal decisions arising in a market comprising only purely self-interested providers:

Provider 1:  
\[ -q(c_2) + S' (c_1) + (c_1 - c_2)q_i (c_1) + q(c_1) = 0 \]

Provider 2:  
\[ -q(c_1) + S' (c_2) + (c_2 - c_1)q_i (c_2) + q(c_2) = 0 \]

Providers’ cooperative solutions in the different market structures differ in the altruistic component, 
\(-\alpha_2 q(c_1)\) in (10). It follows that, for a given \(c_2\), the altruistic component makes provider 1 choose a lower cost in mixed markets.

Given the same line of argument, comparing (10) and (11) with the cooperative strategies in a market comprising altruistic providers (\(\alpha_i \neq 0\)):

Provider 1:  
\[ -q(c_2) + S' (c_1) + (c_1 - c_2)q_i (c_1) + (1 - \alpha_2)q(c_1) = 0 \]

Provider 2:  
\[ -q(c_1) + S' (c_2) + (c_2 - c_1)q_i (c_2) + (1 - \alpha_i)q(c_2) = 0 \]

We note that the difference between these conditions is the altruistic term \(-\alpha_i q(c_2)\) missing in equation (11). Therefore, it follows that, for a given \(c_1\), provider 2 in a mixed market operates at a higher cost than it would if the market was served solely by altruistic providers.

Thus, with the exception of providers that are completely altruistic, when there is collusion providers will always choose to operate at a higher cost than in the social optimum. This holds irrespective of the market structure.

Finally, we provide comparative statics,

\[ \frac{dc_i^c}{d\alpha_i} = -\frac{\partial^3 JP / \partial c_i^c \partial \alpha_i}{\partial^2 JP / \partial c_i^c \partial \alpha_i} = 0 \]

\[ \frac{dc_i^c}{d\alpha_{-i}} = -\frac{\partial^3 JP / \partial c_i^c \partial \alpha_{-i}}{\partial^2 JP / \partial c_i^c \partial \alpha_{-i}} \]

\[ \Rightarrow -\frac{q(c_i^c)}{S' (c_i^c) + (2 - \alpha_{-i})q_i (c_i^c) + (c_i^c - c_{-i}^c)q_i (c_i^c)} < 0 \]

This demonstrates that the optimal cost of provider \(i\) when collusion takes place is independent of its own altruism level and is decreasing in the degree of the other’s altruism.

Given that the consumer surplus depends on the regulated price and given that the regulatory scheme sets \(\hat{p}_i = c_{-i}\), the maximization of the joint utilities \(-JU\) is such that provider \(i\)’s choice will affect provider \(-i\)’s consumer surplus. It follows that provider \(i\) makes a decision on costs bearing in mind the altruism level of the other provider.

**Defection solution**

We will now analyse the incentives to deviate from the collusive agreement. Consider provider \(i\). If this provider defects from the collusive agreement, then it will revert to behaving accordingly to the best response function as in (7) but evaluated at \(c_{-i} = c_{-i}^a\), with the optimal defection cost \(c_i^d\) (where the
superscript \( d \) indicates the defection solution) satisfying

\[-q(c^d_i) + S'(c^d_i) = 0\]

It can be shown that \( c^nc_i < c^d_i < c^c_i \). Intuitively, provider \( i \) chooses a cost \( c^d_i < c^c_i \), given the choice of the other, and it would still face a higher price and therefore increase its surplus. The provider’s decision is based on the maximization of its own utility and the first order condition will coincide with the non cooperative first order condition. However the defection level will differ from the non cooperative level as provider \(-i\) is still playing the cooperative solution. Therefore, \( i \)'s defection cost has to be higher than the optimal non cooperative choice.

Note that the defection cost is independent of the degree of altruism,

\[
\frac{dc^d_i}{d\alpha_i} = -\frac{\partial^2 V_i / \partial c^d_i \partial \alpha_i}{\partial^2 V_i / \partial^3 c^d_i} = 0
\]

\[
\frac{dc^d_i}{d\alpha_{-i}} = -\frac{\partial^2 V_i / \partial c^d_i \partial \alpha_{-i}}{\partial^2 V_i / \partial^3 c^d_i} = 0
\]

(13)

Recall that by defecting from the collusive agreement the provider can revert simply maximizing its own utility\(^3\) just as would happen in a non cooperative scenario. But we have already found that the optimal strategy in non cooperation is to choose the socially optimum cost independently of any altruistic concern.

To sum up we have shown that in a one shot game defection is always profitable and, consequently, collusion is never sustainable. Therefore the one shot Nash equilibrium is non cooperative. This result is consistent to the findings of the existent literature (Tirole, 1988).

2.3 Repeated game: Incentives to collude

Let us consider a repeated game in which the providers can play grim trigger strategies (Friedman, 1971). At the beginning of each period the two providers agree on a cost \( \bar{c}_i \). But if one of them defects in some period \( t \), choosing a cost level \( c_i \neq \bar{c}_i \), then in \( t+1 \) the other provider reverts to play his best response to defection from that point onwards. This is a typical "trigger strategy", whereby if a provider deviates from the collusive agreement all providers revert to the one shot Nash equilibrium from thereon. Therefore, in deciding whether to stick to the collusive agreement, a provider compares the stream of profits of cooperating \( U^c \) \( / (1-\delta) \) with the stream of utilities obtained by deviating \( U^d + \delta U^{nc} \) \( / (1-\delta) \), where \( \delta = 1/(1+r) \). Here \( \delta \in [0,1] \) denotes the discount rate (with \( r \in [0,\infty] \)), \( U^{nc} \) is the equilibrium payoff the provider receives in the non-cooperative scenario, \( U^c \) is the payoff gained in collusion and \( U^d \) is the payoff obtained in defection. Therefore, we can define the threshold discount rate \( \bar{r} \) as the maximum discount rate that can support the collusive equilibrium by

\[
r = \frac{U^c - U^{nc}}{U^d - U^c}
\]

The higher the rate the greater is the incentive to collude.

The threshold discount rate that can support collusion in a mixed market is given by,

\(^3\) Disregarding the impact of its decision on the other provider’s utility
\[ \frac{(c^*_i - c^i)q(c^*_i) + S(c^*_i) - S(c^i) + \alpha \left( \int_{c^i}^{\infty} q(x)d(x) - \int_{c^i}^{\infty} q(x)d(x) \right)}{(c^*_i - c^d)q(c^*_i) + S(c^d) - S(c^i)} \]

with \( \alpha \in [0,1] \).

We note the denominator in (14) does not depend on the level of altruism. The numerator shows the difference between the collusive payoff and the non cooperative payoff. If providers switch to the co-operative strategy, the altruistic provider gains less than the self-interested provider. This is because collusion reduces the amount of consumer surplus. This reduction is captured by the final bracketed term in the numerator, the size the reduction conditional upon the value of \( \alpha \). In fact, it is easy to see that\(^4\), in the case of purely altruistic providers, i.e. for \( \alpha = 1 \), joint utility maximization yields the first best solution. It is as if there no incentives to collude. Indeed note that \( c^*_i = c^* \Rightarrow r_i = 0 \).

For the special case in which providers have the same degree of altruism the rate turns out to be,

\[ \frac{S(c^*_i) - S(c^i) + \alpha \left( \int_{c^i}^{\infty} q(x)d(x) - \int_{c^i}^{\infty} q(x)d(x) \right)}{(c^*_i - c^d)q(c^*_i) + S(c^d) - S(c^i)} \]

We will now proceed with some comparative static analysis to study how provider \( i \)'s behaviour changes with degree of altruism of provider \( -i \). Our aim is to assess the change in behaviour of any provider when passing either from a mixed market to a pure market or vice versa.

Let us consider, first, the case of provider 1, the more self-interested provider, by differentiating (14) with respect to \( \alpha_2 \),

\[ \frac{\partial r_1}{\partial \alpha_2} = \frac{\partial U^d_1 / \partial \alpha_2 - \partial U^{nc}_1 / \partial \alpha_2}{U^d_1 - U^{nc}_1} \]

Note that the denominator is always positive. Therefore, the sign of (14) will depend solely on the sign of the numerator. Given (8), (12) and (13), it can be shown that \( \partial U^*_1 / \partial \alpha_2 < 0 \), \( \partial U^{nc}_1 / \partial \alpha_2 = 0 \) and \( \partial U^d_1 / \partial \alpha_2 = 0 \). Moreover, the difference \( U^*_1 - U^{nc}_1 \) is positive; otherwise there would be no incentive to collude\(^6\). We have also shown that \( U^d_1 - U^*_1 \) is positive with providers attaining higher payoffs from defecting than by cooperating in a one shot game.

It then follows that \( \frac{\partial r_1^d / \partial \alpha_2 < 0 }{\partial r_1^d / \partial \alpha_2 < 0 } \), i.e., as provider 2's degree of altruism diminishes, provider 1's rate increases. In other words, the rate of the more self-interested provider, when it shares a market with a more altruistic provider, is lower than the analogous rate in a market served by two purely self-interested providers. Thus results show that the incentive to collude is higher in a market covered only by self-interested providers than in a mixed market.

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\(^4\) See Appendix for illustration of results
\(^5\) In fact the non-cooperative payoff is equal to the collusive one
\(^6\) Recall that we have shown that the cooperative payoff is higher than the non cooperative payoff
Analogously for provider 2, given (8), (12) and (13), it can be shown that $\partial U^c_2 / \partial \alpha_1 < 0$, $\partial U^{nc}_2 / \partial \alpha_1 = 0$, $\partial U^d_2 / \partial \alpha_1 = 0$, $(U^d_2 - U^c_2) > 0$ and $(U^{nc}_2 - U^c_2) > 0$. Therefore it follows that $\frac{\partial r_2}{\partial \alpha_i} < 0$, i.e., as provider 1’s degree of altruism increases, provider 2’s rate diminishes.

Thus, the results show that the rate (incentive to collude) to a more altruistic provider when interacting with a more self-interested provider is higher than the rate in a market served by two altruistic providers. As such, plurality renders markets to become “average” regarding incentives for providers to collude. In summary, the incentive for collusion is stronger in a market served by self-interested providers than in a mixed market. In turn, the incentive is stronger in mixed market than in one served by altruistic providers. Note that, while the first result is in line with the existing literature that has shown that asymmetries between providers are an obstacle to collusion (see for e.g. Scherer, 1970; Barla, 2000; Compte and Ray, 2002), the second result suggests that the incentive for collusion is dependent on the nature of provider heterogeneity.

3. Uniform yardstick competition

So far we have considered a yardstick competition regulatory environment by which each provider is paid according to the cost performance of its competitors. An alternative scheme is one in which the price or price cap is uniform across providers. This arrangement typifies the prospective payment systems that have been implemented to pay for health services in many countries (Street et al., 2007; Schreyögg et al., 2006). In such settings the price is based on the observed costs of all providers in the market, which might be summarised at the average, i.e.

$$\hat{p}_i = \frac{1}{n} \sum_{i=1}^{n} c_i$$

As the number of providers increases, the weight of each provider’s cost on the industry average progressively decreases, and therefore this (uniform) system approaches the previous (discriminatory) one.

Reverting again to our two-provider model, we find that the non cooperative one shot equilibrium cost $c^{ncU}_i$ (with $i=1, 2$ and the superscript $U$ indicating the uniform scheme) given by the following decision rule

$$\frac{\partial U_i}{\partial c_i} = -\frac{1}{2} q \left( \frac{c_i + c_{-i}}{2} \right) + \frac{1}{4} (c_{-i} - c_i) \partial q(.) / \partial c_i + S'(c_i) - \frac{\alpha_i}{2} q \left( \frac{c_i + c_{-i}}{2} \right) = 0$$

(17)

is higher than the first best, $c^{ncU}_i > c^*$. This is true for any market structure considered. The reason is that the effort exerted by the provider in reducing costs negatively affects its own yardstick, thus the provider itself has an incentive to maintain costs above the socially optimal level. Moreover, in a mixed market structure, the more altruistic provider chooses a lower cost than the more self-interested provider does. Recall that the discriminatory regime affords providers strong incentives to attain the social optimum and this result is independent of the level of altruism. The uniform yardstick, instead, involves greater efficiency losses confirming it as a lower powered incentive scheme. That said, the uniform rule allows the altruism of the provider to counteract the lack of efficiency properties. That is, if the provider is altruistic the efficiency losses are partially offset by the provider’s concern about consumer surplus.

If there is collusion the providers’ one shot optimal decision rules are symmetric in either market structure.
\[
\frac{\partial JU}{\partial c_i} = S (c_i) - \frac{\alpha_i}{2} q(c_i + c_{-i}) - \frac{\alpha_{-i}}{2} q(c_i + c_{-i}) = 0
\]  

(18)

and the optimal decisions are above the socially optimum, \( c_{i}^{c, U} > c^* \).

Note that the situation is akin to that under discriminatory rule, where providers’ costs are higher than the social optimum. This is because under either yardstick rule it pays to agree on a higher cost in order to secure a higher price from the regulator. By comparing (9) with (18), we notice that the marginal effects of cost on the joint surplus, brought in (9) by the demand and the price, drop off the decision rule (18) because of the symmetry that characterizes the uniform scheme. As under the discriminatory rule, the provider’s decision under the uniform regime accounts for the benefit from slack as well as for the effect of its strategy regarding the consumer surplus realised by the other provider. However, unlike the discriminatory scheme, the provider internalizes the effect that its cost strategy has on its own consumer surplus.

Comparing costs in pure markets we observe that, \( c_{i}^{c, U} > c_{i}^{nc, U} \) i.e. costs are higher when collusion occurs. But results in mixed markets are ambiguous as can be seen by comparing (17) with (18). In (17) the first two terms represent the effects of the increased cost on the provider’s revenues, respectively through the positive effect on the price and through the negative effect on demand. We note that, when it comes to the maximization of joint utilities \(-JU\) the previous effects on both providers’ revenues offset each other. Thus providers’ optimal decisions account for the benefit from slack and the effect that the other’s cost strategy has on its own consumer surplus. In contrast, the collusive solution shown in (18) is such that each provider accounts for the effect of its strategy on the consumer surplus achieved by the other provider. Given that these effects move in different directions, the equilibrium cooperative cost will depend on their magnitude.

Note that in the special case where one of the providers is purely altruistic (i.e. \( \alpha_2 = 1 \)), the cost when providers collude is lower than when they do not co-operate.

It follows that, even though we can rank the costs in a pure market \( -c_{i}^{nc, U} < c_{i}^{d, U} < c_{i}^{c, U} \) - results are inconclusive about the effect of defection on costs in a mixed market. Under either yardstick rule there is some scope to defect from the agreed solution. In fact, if providers operate according to their non cooperative optimal decision rule, choosing a cost lower than when they collude, they can still make profit and benefit from slack.

Therefore when analysing the incentives to collude we can conclude that in a pure market, the uniform rule is less prone to collusive behaviour (see Appendix for rate comparison). This is because providers have an incentive to operate at a higher cost than the social optimum, so the pay-off from collusion is lower under this scheme than under the discriminatory one.

We now consider how provider \( i \) ’s behaviour changes with degree of altruism of provider \( -i \) under the uniform yardstick scheme: \( \partial r_{-i}^{U} / \partial \alpha_{-i} \)

\[
\frac{\partial r_{-i}^{U}}{\partial \alpha_{-i}} = \left[ \frac{\partial U_{i}^{d} / \partial \alpha_{-i} - \partial U_{i}^{nc} / \partial \alpha_{-i}}{U_{i}^{d} - U_{i}^{nc}} \right] \left[ \frac{\partial U_{i}^{d} / \partial \alpha_{-i} - \partial U_{i}^{nc} / \partial \alpha_{-i}}{U_{i}^{d} - U_{i}^{nc}} \right] \left( U_{i}^{c} - U_{i}^{nc} \right)
\]

(19)

Given that the denominator in (19) is always positive, we need to determine the sign of numerator. Take the first element (\( \partial U_{i}^{d} / \partial \alpha_{-i} - \partial U_{i}^{nc} / \partial \alpha_{-i} \)). It is possible to show that,
\[
\frac{\partial U_i^c}{\partial \alpha_{-i}} = \frac{\partial c_{-i}^c}{\partial \alpha_{-i}} \left[ \frac{\alpha_{-i} - \alpha_i}{2} q(.) \right] 
\]

where \( \frac{\partial c_{-i}^c}{\partial \alpha_{-i}} < 0 \) and the term in brackets is the difference between the cost effects on the consumer surplus of the two providers. Furthermore, we found that,

\[
\frac{\partial U_i^{nc}}{\partial \alpha_{-i}} = \frac{\partial c_{-i}^{nc}}{\partial \alpha_{-i}} \left[ (1-\alpha_i)q(.) + \left( p^{nc}_i - c^{nc}_{-i} \right) \frac{\partial q(.)}{\partial c_i} \right] 
\]

where \( \frac{\partial c_{-i}^{nc}}{\partial \alpha_{-i}} < 0 \). The term in brackets comprises two effects: the net effect of costs on consumer surplus and the effect of costs on revenues.

Take the self-interested provider, so that \( i = 1 \). It follows that \( \frac{\partial U_1^c}{\partial \alpha_2} < 0 \) and \( \frac{\partial U_1^{nc}}{\partial \alpha_2} < 0 \). That is an increase in the level of altruism of the other provider reduces both the collusive and the non-cooperative payoff under a uniform scheme. Recall that under the discriminatory scheme there was no reduction in the non-cooperative payoff because the optimal solution was independent of the degree of altruism, reflecting its status as a more powerful incentive scheme.

Returning to the uniform scheme, the sign of (19) is indeterminate and will depend on the magnitude of the effects (20) and (21) that change the relative profitability from collusion. That said term in brackets in (21) is larger than the one in (20) because the effect of the self-interested provider’s costs on its revenues is positive. It follows that \( \left| \frac{\partial U_i^{nc}}{\partial \alpha_i} \right| > \left| \frac{\partial U_i^c}{\partial \alpha_i} \right| \).

Following the same line of reasoning, consider now the third element in the numerator of (19), \( \frac{\partial U_i^d}{\partial \alpha_{-i}} - \frac{\partial U_i^c}{\partial \alpha_{-i}} \). It is easy to show that,

\[
\frac{\partial U_i^d}{\partial \alpha_{-i}} = \frac{\partial c_{-i}^d}{\partial \alpha_{-i}} \left[ (1-\alpha_i)q(.) + \left( p^d - c^d_{-i} \right) \frac{\partial q(.)}{\partial c_i} \right] 
\]

Even though the sign of (22) is ambiguous, it is possible to show that \( \left| \frac{\partial U_i^d}{\partial \alpha_i} \right| > \left| \frac{\partial U_i^c}{\partial \alpha_i} \right| \), because the effect of defection on revenues is negative. Given the inequalities above, we obtain a rate change that is negatively correlated to the change in altruism, \( \frac{\partial r_i^U}{\partial \alpha_2} < 0 \). This is the same result as in the discriminatory scheme. Hence we conclude that as provider 2’s degree of altruism diminishes provider 1’s rate increases. Therefore we can conclude that the incentive to collude for a self-interested provider is higher in a market served only by self-interested providers than in a mixed market.

Following the same line of arguments for the altruistic provider \( i = 2 \), we find that \( \frac{\partial U_2^c}{\partial \alpha_i} > 0 \). Intuitively, when an altruistic provider shifts from a mixed market to one with more altruistic providers its equilibrium collusive payoff will increase.

Even though the sign of both \( \frac{\partial U_2^{nc}}{\partial \alpha_i} \) and \( \frac{\partial U_2^d}{\partial \alpha_i} \) is ambiguous, by comparing the magnitude of the effects it is possible to show that \( \left| \frac{\partial U_2^{nc}}{\partial \alpha_i} \right| > \left| \frac{\partial U_2^c}{\partial \alpha_i} \right| \) and \( \left| \frac{\partial U_2^d}{\partial \alpha_i} \right| < \left| \frac{\partial U_2^c}{\partial \alpha_i} \right| \).

These inequalities entail a positive change in rate, \( \frac{\partial r_i^U}{\partial \alpha_i} > 0 \), i.e. in a mixed market as provider 1’s degree of altruism increases, the incentive for the altruistic provider to collude increases. Note that the last result contrasts to that obtained under the discriminatory scheme where the incentive for collusion is stronger in a market served by self-interested providers and weaker in one served by altruistic providers. Under the uniform rule, increasing plurality of the market reduces the incentive for collusion.
4. Summary of results

In the previous sections we have shown that under a discriminatory yardstick regulatory rule the incentive to collude increases in the degree to which markets are self-interested. In fact, a market served by self-interested providers is more prone to collusion than a mixed market and, in turn, providers in a mixed market are more likely to collude than providers in an altruistic market. As such increasing the plurality of provision will not necessarily always lead to a market more susceptible to collusion. It will all depend on the composition of the original market.

Under the uniform rule, instead, results show that increasing the plurality of providers decreases incentives to collude irrespective of whether the mixed market replaces a previously self interested or altruistic market.

We compare costs to the first best and results are summarised in table 1.

<table>
<thead>
<tr>
<th>Table 1. Summary of results</th>
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</thead>
<tbody>
<tr>
<td>Yardstick scheme</td>
</tr>
<tr>
<td>Discriminatory</td>
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<tr>
<td>Uniform</td>
</tr>
</tbody>
</table>

Under a discriminatory scheme when there is no collusion providers will choose to operate at a cost equal to the first best. This is due to the strong incentives embedded in the discriminatory form of yardstick competition whereby the provider’s cost reducing effort does not adversely affect the price it faces. When there is collusion providers will operate at a higher cost than the first best. Each provider’s choice of cost is decreasing in the degree of the other provider’s altruism, so a self-interested provider will operate at a lower cost than an altruistic provider in a pluralistic market. Given that one provider sticks to the collusion agreement, the other provider faces the incentive to defect, that is to choose a lower operating cost.

Under a uniform yardstick scheme with no collusion providers choose to operate at a higher cost than the first best, with costs increasing in the degree of the provider’s self-interest. The choice to operate at a cost above the first best reflects the weaker incentives associated with this regime, where a provider’s cost reducing effort has adverse consequences for the price it faces. Providers in a pluralistic market choose to operate at the same cost, which again is above the first best but it is unclear whether this cost higher or lower than chosen when providers do not collude. Providers operating in pure markets will collude on a higher cost under a uniform scheme than they would have under the discriminatory scheme and in the absence of collusion. As to the incentive for defection, results for the mixed market are unclear about the cost ranking. Intuitively, and as demonstrated by the results for the pure markets, under a uniform scheme providers face the same incentive to defect from the collusive agreement as they do under the discriminatory scheme. The gains from defection, though, are smaller under the uniform scheme.

5. Conclusion

A potential drawback with yardstick competition regulation is that it might be susceptible to collusion, because by colluding on higher costs, providers may be able to secure a higher price for their services. We suggest that the incentive will depend both on the particular form of yardstick regulation and on the degree to which provider objectives correspond to those of the regulator.
There are a number of works that have addressed the issue of collusion under yardstick competition (Tangeras, 2002; Chong and Huet 2006; Potters et al. 2004). The unifying element of the existing literature is that it assumes homogeneous providers. The paper that lies the closest to ours is Potters et al. (2004). The authors adapt Schleifer’s model (Schleifer, 1985) to explore their hypothesis that the incentive to collude tacitly depends on the particular form of yardstick competition that is applied.

We generalize their analysis by allowing for i) provider heterogeneity in their degree of altruism and ii) by considering different market structures. Thus, the contribution of our research is that we relax the assumption of homogeneity among providers to analyse how collusion incentives relate to plurality of provision across different providers (altruistic vs. self-interested) and across different health care structures (pure markets vs. mixed markets).

We believe that by relaxing the assumption of provider homogeneity we can capture a fundamental change in the provision of public services where greater plurality is being encouraged. For example, traditionally public health systems such as England, France, Portugal and Italy are encouraging more private sector organisations to enter the health care market. Similarly many countries have de-nationalised many other public services, either wholly or in part. The important qualitatively different results obtained by our framework indicate that market structure should be considered when designing yardstick competition arrangements.

We find that the incentive for collusion depends on both the nature of the yardstick scheme and on the composition of the market. Under a uniform scheme, the incentive for collusion is decreasing in the degree of provider heterogeneity. This means that pure markets served either by only self-interested providers or by only altruistic providers are both more susceptible to collusion than a pluralistic market.

There is some intuition to this result. Indeed it is easier to coordinate on a particular cost for providers that share similar objectives and, therefore, it is more likely that these providers will stick to an agreement. This result is consistent with the recognition that the more providers differ in their cost functions, the less likely they will engage in maximization of joint profits (Scherer, 1970) and with the vast literature that has shown that asymmetries between providers are an obstacle to collusion (see for e.g. Barla, 2000; Compte and Ray, 2002). More generally, our work relates to a wider stream of theoretical literature about industrial organisation. In recent years, this literature has explored the role of different sources of provider asymmetries on collusive behaviour: Rothschild (1999) focussed on cost asymmetry, Compte and Ray (2002) on capacity heterogeneity, Kühn (2004) on product variety, Vasconcelos (2005) on capacity/costs, and Davies and Oczak (2008) on market shares. Although the specifics of these models vary, the whole literature seems to cast a general message that asymmetries reduce the likelihood of collusion.

Our study considers how asymmetry affects providers’ incentives to collude, with asymmetry defined as the degree to which provider objectives correspond to those of the regulator. Under a discriminatory scheme results are somewhat striking. Indeed, in contrast to the uniform scenario, the incentive to collude is no longer dictated by provider symmetry. In fact we have been able to show that an industry populated by self-interested providers is more prone to collusion than a mixed market which in turn is more prone to collusion than an industry served by altruistic providers.

This “unexpected” result can also be explained intuitively. Even if there are two homogeneous altruistic providers in the market, they will not gain from collusive behaviour. Nevertheless, whenever a self-interested provider enters the market, the high gain from collusion might overcome the drawbacks of heterogeneity and encourage the altruistic provider to collude. In a sense, it is as if the imbalance imposed by providers’ asymmetry that otherwise could prevent collusive behaviour is offset by the (asymmetric) regulatory rule.

Our analysis demonstrates that it is important to consider the composition of the market when designing yardstick competition arrangements. The incentives to collude depend not only on the type of yardstick scheme but also on the extent to which providers share similar objectives. With pluralistic markets being encouraged in many countries and sectors of the economy it is increasingly important that provider heterogeneity is taken into account when designing regulatory policies.
References


A Appendix

A1 First Best Solution

The solution to the following optimization program

\[
\max_{p_i,c_i} W(c_i,p_i) = (1 + \alpha_i) \int q(x)d(x) + (p_i - c_i)q_i(p_i) + S(c_i)
\]

must satisfy the following first order conditions:

\[
\frac{\partial W_i}{\partial p_i} = -q_i(p_i) - \alpha q_i(p_i) + q_i(p_i) + (p - c)q(p) + S(c) = 0
\]

\[\Rightarrow p_i = c_i + \alpha_i \frac{q_i(p_i)}{q_i(p_i)} \tag{A1.1}\]

\[
\frac{\partial W_i}{\partial c_i} = -q_i(p_i) + S'(c_i) = 0
\]

\[\Rightarrow S'(c_i) = q(p_i) \tag{A1.2}\]

So that \( p^* \) and \( c^* \) denote the optimal level of price and cost that solve the system of equations defined by (A1.1) and (A1.2).

A2 Discriminatory rule - Non Cooperative Provider’s Solution

Given the utility function being maximized,

\[
\max_{c_i} U(\hat{p}_i,c_i) = (\hat{p}_i - c_i)q(\hat{p}_i) + S(c_i) + \alpha_i \int q(x)d(x)
\]

The first order condition with respect to cost \( c_i \) is given by,

\[
\frac{\partial U_i}{\partial c_i} = (\frac{\partial \hat{p}_i}{\partial c_i} - 1)q(\hat{p}_i) + (\hat{p}_i - c_i) \frac{\partial q(\hat{p}_i)}{\partial \hat{p}_i} \frac{\partial \hat{p}_i}{\partial c_i} + S'(c_i) - \alpha q(\hat{p}_i) \frac{\partial \hat{p}_i}{\partial c_i} = 0 \tag{A2.1}
\]

According to the regulatory rule \( \hat{p}_i = c_{-i} \). Therefore, the first order condition (A2.1) when the two providers have the same level of altruism \( \alpha_1 = \alpha_2 = \alpha \) so that, in equilibrium by symmetry, \( c_{-i} = c_i = c \), is given by,

\[
\frac{\partial}{\partial c_i} U_i = 0 \Rightarrow -q(c) + S'(c) = 0 \tag{A2.2}
\]
Evaluating (A2.2) at $c = c^*$ and given (A1.2) we get the same optimal condition as the first best, so that the optimal decision of the non cooperative cost $c^{nc}$, is equal to the socially optimum for both ownership types, $c^{nc} = c^*$. 

Note that the optimal cost does not depend on the level of altruism, indeed $dc/d\alpha = 0$

### A3 Collusive Provider’s Solution

The two providers maximize their joint profits by optimally choosing $c_i$,

$$\max_{c_i} JU = \sum_{i=1}^{2} \left[ (\hat{p}_i - c_i)q(\hat{p}_i) + S(c_i) + \alpha \int_{\hat{p}_i}^{\infty} q(x)dx \right]$$

Thus,

$$\max_{c_1, c_2} JU = \sum_{i} \left[ (c_i - c_i)q(c_i) + S(c_i) + \alpha \int_{c_i}^{\infty} q(x)dx \right]$$

When the providers have the same level of altruism, i.e. when $\alpha_1 = \alpha_2 = \alpha$, so that, in equilibrium by symmetry, $c_i = c$, the first order condition with respect to cost is given by,

$$\frac{\partial JU}{\partial c} = S'(c) - \alpha q(c) = 0 \quad (A3.1)$$

By evaluating (A3.1) at $c = c^*$ and comparing with (A1.2) it follows that, for $\alpha \in [0, 1]$, 

$$S'(c^*) - \alpha q(c^*) > 0$$

i.e. the providers have the incentive to increase the cost above the socially optimum. Moreover as found above $c^{nc} = c^*$ therefore the collusive cost, $c^c$, is higher than the optimal non cooperative cost, $c^{nc}$.

Proceeding with some comparative static analysis,

$$dc^c/d\alpha = \frac{-q(c)}{S'(c) - \alpha q(c)} < 0$$

i.e., the higher is the providers’ altruism level the lower is the collusive cost.

In the special case of the purely altruistic provider, i.e. for $\alpha = 1$, by evaluating (A3.1) at $c = c^*$ and given (A1.2) we obtain the regulator’s solution

$$S'(c^*) - q(c^*) = 0$$

We have that the collusive cost is equal to the social optimum; this means that the purely altruistic provider has no incentive to collude.
A4 Uniform rule – Non cooperative solution

The regulatory rule in two-provider markets is given by \( \hat{p}_i = \frac{1}{2}(c_i + c_{-i}) \).

Therefore, the first order condition (A2.1) when the two providers have the same level of altruism \( \alpha_1 = \alpha_2 = \alpha \) so that, in equilibrium by symmetry, \( c_{-i} = c_i = c \), is given by,

\[
\frac{\partial U}{\partial c_i} = S'(c) - \frac{(1 + \alpha)}{2} q(c) = 0
\] (A4.1)

By evaluating (A4.1) at \( c = c^* \) and comparing with (A1.2) it follows that, for \( \alpha \in [0, 1] \)

\[
S'(c) - \frac{(1 + \alpha)}{2} q(c) > 0
\]

i.e. the providers have the incentive to increase the cost above the socially optimum.

Proceeding with some comparative static analysis,

\[
\frac{dc^{nc,U}}{d\alpha} = -\frac{1}{2} \frac{q(c)}{S''(c) - \frac{(1 + \alpha)}{2} q'(c)} < 0
\]

i.e., the higher is the providers’ altruism level the lower is the non cooperative cost.

In the special case of the purely altruistic provider, i.e. for \( \alpha = 1 \), by evaluating (A4.1) at \( c = c^* \) and given (A1.2) we obtain the regulator’s solution

\[
S'(c^*) - q(c^*) = 0
\]

A5 Collusive Provider’s Solution

The maximization of joint utilities under the uniform scheme is given by,

\[
\max_{c_i, c_{-i}} JU = \sum_i \left( \frac{c_{-i} - c_i}{2} \right) q(c_i + c_{-i}) + S(c_i) + \alpha \int_{c_i}^{c_{-i}} q(x) dx
\]

Each provider maximizes the joint utilities by optimally choosing \( c_i \).

When the providers have the same level of altruism, i.e. when \( \alpha_1 = \alpha_2 = \alpha \) so that, in equilibrium by symmetry, \( c_{-i} = c_i = c \), the first order condition with respect to cost is given by,

\[
\frac{\partial JU}{\partial c} = S'(c) - \alpha q(c) = 0
\] (A5.1)

The analysis made for the equation (A3.1) holds for (A5.1) as well.
A6 The rate in non-mixed markets

The rate under discriminatory rule is given by,

$$r_{\text{nonmix}}^D = \frac{S(c^c) - S(c^*) + \alpha \left( \int_{c^c}^{c^*} q(x) \, dx - \int_{c^*}^{\infty} q(x) \, dx \right)}{(c^c - c^d)q(c^c) + S(c^d) - S(c^c)}$$

(A6.1)

Recall that the numerator of the rates shows the difference between the cooperative and the non-cooperative payoffs.

The rate under the uniform rule is given instead by,

$$r_{\text{nonmix}}^U = \frac{S(c^{c,U}) - S(c^{nc,U}) + \alpha \left( \int_{c^{c,U}}^{c^{nc,U}} q(x) \, dx - \int_{c^{nc,U}}^{\infty} q(x) \, dx \right)}{S(c^{d,U}) - S(c^{c,U}) + \left( \frac{c^{d,U} - c^{c,U}}{2} \right) q(c^{nc,U}) + \alpha \left( \int_{c^{d,U}}^{\infty} q(x) \, dx - \int_{c^{c,U}}^{\infty} q(x) \, dx \right)}$$

(A6.2)

By comparing (A6.1) and (A6.2), knowing that $c^* < c^{nc,U} < c^{c,U}$, it can be shown that the numerator of $r_{\text{nonmix}}^D$ is bigger than that of $r_{\text{nonmix}}^U$. Regarding the denominator, instead, we should consider that the defection payoff under the uniform scheme is generally smaller than the defection payoff under discriminatory scheme. The reason is that reducing the cost has a negative influence on the price-cap in the uniform scheme. Moreover, we have to consider also the increase in the consumer surplus that comes along with defective behaviour. This shrinks the difference between the two denominators, allowing us to consider the numerator difference as dominating. Thus we may conclude that $r_{\text{nonmix}}^D > r_{\text{nonmix}}^U$, that is we can expect more collusive behaviour under a discriminatory than under a uniform yardstick scheme.