Hospital Financing and the Development and Adoption of New Technologies

CHE Research Paper 26
Hospital Financing and the Development and Adoption of New Technologies

Marisa Miraldo

Centre for Health Economics
University of York
York YO10 5DD
Email: mafm100@york.ac.uk

April 2007
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Acknowledgements

This article has been strongly improved with comments by Hugh Gravelle, Matteo Galizzi, Michael Kuhn, Pau Olivela, Paula Gonzalez, Peter C. Smith and Xavier Martinez Giralt to whom I am grateful. This paper was written with financial support from the Sub-Programa Ciência e Tecnologia do 2o Quadro Comunitário de Apoio Ministério da Ciência e Tecnologia and Fundação Calouste Gulbenkian, both in Portugal.

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Centre for Health Economics
Alcuin College
University of York
York, UK
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Abstract

We study the influence of different reimbursement systems, namely Prospective Payment System, Cost Based Reimbursement System and Mixed Reimbursement System on the development and adoption of different technologies with an endogenous supply of these technologies. We focus our analysis on technology development and adoption under two models: private R&D and R&D within the hospital. One of the major findings is that the optimal reimbursement system is a pure Prospective Payment System or a Mixed Reimbursement System depending on the market structure.

Keywords: Prospective Payment System; Cost Based Reimbursement; R&D

JEL classification: I11, O33
1 Introduction

Technological progress has been identified as one of the major contributors to the rising health care expenditure (Newhouse (1992)) with changes in treatment accounting for most of the growth in spending on specific diseases (Cutler, McClellan and Remler (1998); Cutler et al (1998)). This contribution is a product of two processes: the development and the adoption of technologies, both of fundamental importance for the development of both health benefits and costs. Given the preponderance of technology on total health care expenditure it urges the study of mechanisms behind the development, adoption and diffusion of technology. One of the possible factors that might affect the technology market is the health regulatory policy. If the regulation on the health care market can stimulate innovation, this has profound implications for the health policy targets and the choice of instruments to pursue them.

Our goal in this paper is to build a theoretical setting which allows the analysis of the influence of prospective, cost based and mixed reimbursement on the development and adoption of new technologies with an endogenous supply of these technologies. Incorporating, both, the demand and supply side of the innovation market we can examine the full welfare effects of reimbursement policies. The main difference with the existing literature is that we endogeneise the technology supply.

The idea that different reimbursement systems lead to the adoption of different types of technology has been widely defended in the existing literature (Romeo, Wagner and Lee (1984); Gelijins and Rosenberg (1994) and Cutler and McClellan (1996)). Nonetheless, there has been some tendency to treat technology development as a black box. Indeed, although much of the existing literature targeted the effects of alternative policy instruments on technology innovation, the focus was constrained to the effects of policy on technology adoption and diffusion. In reality, this body of literature shows that the technology adopted by hospitals is sensitive to reimbursement policies but little attention has been paid on the externality of these policies on the supply side of the R&D process. While Cost Based Reimbursement is believed to create incentives for the provision of quality for any cost level, hospitals that are reimbursed through a Prospective Payment System (PPS) scheme focus on short-run cost savings rather than on treatment quality.

Even though these studies contribute to the study of the effect of health financing policies on technology adoption, we believe that the way in which the R&D sector is affected by these policies is an important issue and has not been explored by the existing literature.
Indeed, beyond such case studies, just a limited literature (partly) documents how recent changes in the financial flows taking place in the health care market are affecting the rate and pattern of innovation. Nevertheless, the above described studies seem to open the door to the conjecture on the relation between the R&D market and the underlying health care institutional set up, by allowing to further infer on the relation between the patterns of technological change and the health care financial flows.

The diffusion process of existing technologies may feed back into the R&D sector since the incentives to create new technologies depend on the propensity to apply them. If different reimbursement schemes create different demands for innovation then it must be the case that they also influence the R&D sector decisions. In fact, some early studies (Weisbrod (1991) and Palmeri (2001)) already point in this direction. Weisbrod (1991) states that fee for service insurance biases the innovation/adoption process toward higher quality but higher cost technologies. Palmeri (2001) describes an example of how payment systems can affect technological innovations. For cochlear implants, the Medicaid payment was below its average cost, making hospitals reducing the use of its supply.

In another study, McClellan and Kessler (2002) analyze the incentives behind technological change. Namely the author analyses to what extent the patterns of technological change are somehow correlated with the underlying incentive mechanisms that regulate the provision of health care. The factors identified as being responsible for technological change are mostly supply side incentives having as target hospitals.

Furthermore, another study by Weisbrod and LaMay (1999) show some evidence on the fact prospective payment system being favorable to costly technologies. By comparing the prices of 468 original DRG categories established in 1983 with the prices of the twenty categories added in the following decade, the authors conclude that after the implementation of the DRG system almost all the new DRGs, such as liver transplantation, were associated to higher payment levels.

Still on the same article, the authors present a study on the role of mixed signals from public policy and the future of health care R&D. The study focuses on the impact of the U.S. health insurance system on the incentives facing the R&D sector. By examining the Medicare DRG system, the authors state that even though such a regulatory system could, potentially, have a negative impact on the adoption of costly innovations as it happened for the cochlear implant technology for the deaf, there is still some evidence suggesting that quality enhancing innovations consisting of major medical advances will keep on being adopted and developed despite of, to some ex-
tent, its cost implications.

Behind these studies resides the belief that the survival of an innovation depends on whether it is perceived as worthwhile by the organizations that will directly determine whether it is adopted and the scale of use. If the innovation is to persist and expand in use, it must be the case that the market perceives it as being profitable to adopt and employ, meaning that, in our context, the hospital must view the treatment as efficacious.

From the R&D firm perspective, the two main elements ruling the expected profitability of a particular new technology are the size of the market and the price at which it will be sold at, meaning, in our context, the rate of use by the hospital and the reimbursement levels. Together these factors determine the revenue side of the market.

In other words, given that the development of new technologies is influenced by the potential demand for particular innovation, the preferences, rules and behavior of these various actors exert an important influence on not only on the path but also on the rate of development and adoption of new technologies.

Despite constituting a starting point to the study of the relation between health care and R&D markets, the studies above described lack of a theoretical base able to allow the generalization of such findings. Indeed, the only two theoretical papers linking reimbursement policies and development of new technologies are the ones by Goddeeris (1984) and Baumgardner (1991). The first author allows for an endogenous supply of this technologies. In both articles results show that there is a link between technology and the financial context in the health care sector. Indeed, the first author finds that insurance biases technological change in the direction of innovations that increase medical expenditure, whereas the second, by investigating the relationship between technical change, welfare and type of insurance contract, concludes that the value of a specific development in technology depends on the type of insurance contract.

Therefore, this paper presents two models, one where the R&D and the hospital are two separate agents and a second where the R&D process is done within the hospital. The former illustrates cases where the technology to be used in medical treatment represents a variation of a technology originally developed in another market (e.g. laser) while the latter illustrates those technologies that are specifically developed for the health market. Both models are solved for the technologies optimal quality and cost decreasing levels and for the decision on optimal reimbursement by a central planner.

The structure of the paper is as follows: in section 2 we describe briefly the common features of the model, in section 3 we study the private R&D
case, in the following section we analyze the model when R&D is carried out within the hospital and, finally, section 5 draws the conclusions.

2 The model

We study an economy with a continuum of identical patients of mass standardized to one.

The number of agents varies with the setting up of the model. In the model of private R&D (section 3) the R&D firm and the hospital are two separate agents. In this case the economy has three agents: the government, the R&D firm and the hospital. Given the reimbursement schedule decided by the government, the hospital decides on the level of quality to be provided and buys the technology from the R&D firm at a price $t$. While in the second model (section 4) R&D within the hospital, the economy has two agents: the hospital and the government. Also here the government decides on the optimal reimbursement to the hospital and the hospital decides on the technology to be developed.

Prior to technology development and adoption, treatment with quality $q_0 > 0$ may be provided at an original marginal cost of $e_0 > 0$ and this treatment is processed by the use of a single technology.

One can think about the development of a new technology as, on one hand, a product innovation and, on the other hand, a process innovation. Our technology covers both aspects. It is characterized by two parameters: $\eta$ and $\pi$. The first, $\eta$, is a treatment quality parameter that is composed by two elements, the existing quality level $q_0$ and the quality innovation parameter $q$ that represents the product innovation defined by $\eta = q_0 - q$. The second, $\pi$, a cost formula function of the status quo marginal cost $e_0$ and a cost decreasing parameter $e$ that represents the process innovation. Increasing $\eta$ decreases treatment quality and, increasing $e$, decreases treatment marginal cost.

Developing technology is assumed to involve quality $\frac{q^2}{\pi}$, "design" costs-$\frac{e^2}{\pi}$ and other technology production costs. For simplicity, we will assume that, as the design costs are so big when compared with the cost of producing technology, the latest are negligible and thus set to zero$^1$. The cost

$\frac{1}{\pi}$The cost specification might seem awkward in the sense that quality decreasing technology increase costs via $\frac{q^2}{\pi}$ (while also decreasing demand). Other, more plausible cost functions have been tried namely

$$C = c(q - q_0)(e - e_0) + \frac{q^2}{2} + \frac{e^2}{2}$$
associated with quality, \( \frac{q^2}{2} \), can be thought of as the costs inherent to the basic research aimed at deriving the fundamental knowledge behind the development of new technologies\(^2\).

Quality decreasing technologies might be justified by, for example, the need to ration demand. Such quality decreasing technology could be costly if it implied the need to use less productive inputs. Or we can interpret \( \frac{q^2}{2} \) as a cost arising from an altruistic concern about providing low quality.

We assume a continuum of individuals of mass standardized to one. Individuals are heterogeneous on their tastes represented by \( z \) uniformly distributed on the interval \( \left[-\frac{1}{2}, \frac{1}{2}\right] \). Patients’ utility from not consuming any medical care is assumed to be

\[ U = 0 \]

While the utility from receiving medical care is represented by,

\[ U = \overline{q} - p_c - \gamma |z - x| \]

With \( \overline{q} = q_0 - q \) standing for the level of quality of the treatment received; \( p_c \) for the price paid for medical care and \( \gamma |z - x| \) the disutility incurred from obtaining care at a provider which location \( x \) differs from the most preferred location \( z \). Without loss of generality we assume that the hospital is located at \( x = 0 \). Moreover, being a public health system patients receive treatment free of charge, i.e., \( p_c = 0 \).

Given patients’ preferences the demand faced by the hospital is given by

\[ D = \frac{2(q_0 - q)}{\gamma} = a (q_0 - q) \quad \text{with} \quad a = \frac{2}{\gamma} \]

With consumer surplus being,

\[ CS = \frac{(q_0 - q)^2}{\gamma} = \frac{a (q_0 - q)^2}{2} \]

The first order conditions from these other specifications are not tractable and therefore a closed form solution is not obtainable.

\(^2\)Note that the cost of decreasing quality is assumed the same as the one of increasing quality. Despite not being the mot realistic assumption it allows to have a smooth function and therefore avoid the kinks that would arise otherwise.
3 Private R&D

3.1 The Model

In this set-up we have three agents: one hospital, one R&D firm and the government. The hospital supplies treatment to patients and buys technology from the R&D firm at a price $t$. This price paid can be thought as a royalty, i.e. the hospital pays an amount $t$ for each utilization of technology. Alternatively, one can think that each patient requires one unit of technology. This situation can be illustrated with the example of drugs. In this case for each patient to be treated, the hospital will need to buy one pack of drugs at a price $t$.

The cost associated with quality, $\frac{q^2}{2}$, is borne by the hospital. The assumption that these costs are borne by the hospital, can be justified by the fact that the hospital is the agent with more information concerning the different diseases and the different treatments’ efficacy in treating those diseases. The design cost $\frac{2^2}{2}$ will be paid by the R&D firm.

As patients’ demand for treatment only depends on quality, the hospital decides on the quality level that will provide $q$.

The R&D firm decides on the price $-t$- and on the level of cost decreasing parameter $e$.

Finally, the government decides on the reimbursement scheme. As instruments the government will use $R$ (prospective payment system fee) and $r$ (cost based reimbursement rate).

Therefore, we are in the presence of a pure Prospective Payment System when $R > 0$ but $r = 0$. A pure Cost Based reimbursement system is characterized by $R = 0$, $r > 0$. Finally, a reimbursement scheme is classified as being mixed for $R > 0$, $r > 0$.

3.2 Timing

The game will be developed in three stages as described in the following diagram,
In the first stage the reimbursement system will be decided, that is, the
government, optimally, decides between the implementation of one of three
regimes: Cost Based Reimbursement System (CBR), Prospective Payment
System (PPS) and mixed system (MRS). We assume that quality is not
observable and therefore not reimbursed.

In a Cost Based Reimbursement system the hospital costs are fully or
partly reimbursed \textit{ex-post}. In this system, reimbursement is based on the
incurred costs. We assume that hospitals are reimbursed on its costs through
a reimbursement rate \( r \geq 0 \). For \( r < 1 \) the hospital is partly reimbursed on
its costs, \( r = 1 \) we are in the presence of full reimbursement and \( r > 1 \) could
be interpreted as a subsidy.

Under a prospective reimbursement system (PPS) the hospital payment
is determined \textit{ex ante} and the reimbursement is independent of the costs
that the hospital will incur when treatment is provided. Throughout the
paper, we will assume that the prospective reimbursement consists of a
per case payment, that is, the hospital is paid a fee \( R > 0 \) for each pa-
tient treated. This reimbursement could be thought as a Diagnostic Related
Groups System (DRG-system) where, for sake of simplicity, only one group
is considered for our analysis (patients are homogeneous on illness type as
well as on severity).

Finally a Mixed Reimbursement System (MRS) is just a combination of
PPS and CBR, i.e., a scheme where \( r > 0 \) and \( R > 0 \).

In the second stage the R&D firm will decide on the technology price to
charge to the hospital as well on the technology parameter \( e \) that will be
developed.

Finally, on the last stage, the hospital will decide on the quality \( q \) pro-
vided.

The model will be solved backwards.

\subsection*{3.3 The hospital}

For a patients’ treatment demand \( D = a\overline{q} \), the hospital profit function is as
follows,

\[ \Pi_H = Ra\overline{q} + (r - 1)(e_0 - e + t)a\overline{q} - \frac{q^2}{2} \tag{1} \]

With \( \overline{q} = q_0 - q \).

Being a profit maximizer agent the hospital problem is given by,
\[
\max_q \left( Raq + (r - 1)(e_0 - e + t)aq - \frac{q^2}{2} \right)
\]

s.t. \( q \geq 0 \)

Maximizing with respect to \( q \), the optimum is characterized by the first order condition,

\[-aR - (r - 1)(e_0 - e + t)a - q = 0\]

Solving the first order condition the optimum is given by\(^3\),

\[ q^* = a[(1 - r)(e_0 - e + t) - R] \tag{2} \]

Having that \( \overline{q} = q_0 - q \) then, the optimal quality level is given by

\[ \overline{q}^* = q_0 - a[(1 - r)(e_0 - e + t) - R] \]

Proceeding with some comparative statics,

\[ \frac{\partial q}{\partial t} \leq 0, \quad \frac{\partial q^*}{\partial t} < 0, \quad \frac{\partial q^*}{\partial R} > 0, \quad \frac{\partial q^*}{\partial r} > 0, \quad \frac{\partial q^*}{\partial e_0} < 0, \quad \frac{\partial q^*}{\partial q_0} > 0 \tag{3} \]

The quality of treatment supplied to patients, \( \overline{q} \), is increasing with the initial quality level, it is decreasing with the status quo marginal costs \( e_0 \). Also, as expected, it is decreasing with the technology price \( t \). Furthermore, it is always increasing in the reimbursement fee \( R \) and reimbursement rate \( r \). Indeed, by increasing quality, the hospital boosts demand and, consequently, revenues. The profitability of this demand increase will be higher the higher the marginal revenue for each patient, i.e., the reimbursement rate either \( R \) or \( r \) (or both). It will never be optimal for the hospital to demand technology that reduces completely the level of quality, i.e., \( q = q_0 \). Indeed, this would imply zero demand and, consequently, negative profits. Therefore, we can focus our analysis on a range of parameters such that \( q < q_0 \).

3.4 The R&D firm

The R&D firm will, through a profit maximizing problem and anticipating the hospital behavior, choose the level of \( e \) and the technology price \( t \) ensuring that the hospital makes non negative profits, that is, its objective will be defined by,

\(^{3}\text{Note that the second order conditions are always satisfied as } \frac{\partial^2 \Pi_H}{\partial q^2} = -1 < 0\)
Hospital financing and the development and adoption of new technologies

Proof. Denote $\Pi_H = Raq + (r - 1)(e_0 + q) - q^2/2$

We have that $\partial \Pi_H / \partial q = -(q_0 - q)(1 - r)$ meaning that for $\partial \Pi_H / \partial q \neq 0$ we need $r \neq 1$ and $q_0 \neq q$. Since $r \in [0, 1]$ we are left with the case $r < 1$.

Now we have that $\partial \Pi_H / \partial q|_{q=q_0} = -q_0 - a[R - (1 - r)(e_0 + z)]$ and for $z = 0$

$\implies \partial \Pi_H / \partial q|_{q=q_0} < 0$ implying that $q^* < q_0$. Recall $\partial \Pi_H / \partial q = -(q_0 - q)(1 - r)$ meaning that the hospital profit is decreasing in $z$. Therefore in order for the hospital to prefer buying technology than not buying, i.e. to ensure $\pi_H (e, t; e_0, q_0) \geq \pi_H (e_0, q_0)$ it is enough to ensure $z \leq 0 \implies t \leq e$. Q.E.D.

Solving the maximization problem we find two solution types depending on the level of the initial marginal cost $e_0$. The following propositions summarize these results. For high (initial) marginal cost of treatment,

**Proposition 1** For $t < e$ the hospital participation constraint for the hospital is satisfied.

**Proof.** Denote $z = t - e$. Then the hospital profit can be written as

$\Pi_H = Raq + (r - 1)(e_0 + z)aq - q^2/2$

We have that $\partial \Pi_H / \partial q = -(q_0 - q)(1 - r)$ meaning that for $\partial \Pi_H / \partial q \neq 0$ we need $r \neq 1$ and $q_0 \neq q$. Since $r \in [0, 1]$ we are left with the case $r < 1$.

Now we have that $\partial \Pi_H / \partial q|_{q=q_0} = -q_0 - a[R - (1 - r)(e_0 + z)]$ and for $z = 0$

$\implies \partial \Pi_H / \partial q|_{q=q_0} < 0$ implying that $q^* < q_0$. Recall $\partial \Pi_H / \partial q = -(q_0 - q)(1 - r)$ meaning that the hospital profit is decreasing in $z$. Therefore in order for the hospital to prefer buying technology than not buying, i.e. to ensure $\pi_H (e, t; e_0, q_0) \geq \pi_H (e_0, q_0)$ it is enough to ensure $z \leq 0 \implies t \leq e$. Q.E.D.

Solving the maximization problem we find two solution types depending on the level of the initial marginal cost $e_0$. The following propositions summarize these results. For high (initial) marginal cost of treatment,
increases quality and for \( e_0 = \frac{aR + q_0[a^2(1-r) - 1]}{a(1-r)} \) is quality neutral. This equilibrium is characterized by,

\[
e^* = \frac{a[R + (r - 1)e_0]a + q_0}{a^2(r - 1) + 2} > 0
\]
\[
t^* = \frac{a[(1 - r)e_0 - R] - q_0}{a[a^2(r - 1)^2 + 2(r - 1)]} > 0
\]
\[
q^* = \frac{a[(1 - r)e_0 - R] + q_0[1 + a^2(r - 1)]}{a^2(r - 1) + 2}
\]

**Proof.** Proof in Appendix

Intuitively, *ceteris paribus*, the higher the initial marginal costs \( e_0 \) the lower the demand for quality. Consequently, in order to make profits the R&D firm needs to supply technology that decreases costs and, consequently, as a second order effect, increases the quality increasing technology employed by the hospital.

Instead, when (initial) marginal costs of treatment are lower,

**Proposition 3** Let

\[
e_0 = \left[ q_0 + aR - \frac{a^2R + aq_0}{1 - r} \right], \quad r < 1
\]

then the constraint \( e \leq e_0 \) is binding and the technology developed decreases initial marginal costs while its impact on quality level is ambiguous. This technology is characterized by,

\[
t^* = \frac{aR + q_0}{2a(1 - r)}, \quad e^* = e_0, \quad q^* = \frac{1}{2}(q_0 - aR)
\]

**Proof.** Proof in Appendix

The quality of treatment will depend on the trade-off between the reimbursement level and the *status quo* marginal cost, i.e., the demand for quality varies positively with the reimbursement level and negatively with the initial marginal cost. Anticipating this reaction by the hospital, the R&D firm is better off by supplying cost decreasing quality such that the negative impact

\[1\]With the first condition ensuring \( e \leq e_0 \) is binding and that the participation constraint of the hospital is satisfied, while the second that the second order conditions for a maximum hold
of \(e_0\) is decreased and, therefore, the demand for quality simply depends on the reimbursement level and the technology price. For sufficiently high reimbursement fees the hospital will demand quality increasing technology, while, if the reimbursement fee is high enough, precisely for \(R > \frac{e_0}{a}\) then the technology adopted and developed will be quality increasing.

**Comparative Statics** We now proceed with a comparative statics analysis. According to the different optima found. The following proposition summarizes the comparative statics results.

**Proposition 4** For high initial marginal costs of treatment \(e_0\), the equilibrium is characterized by (4) and, therefore, the comparative statics analysis is described by,

\[
\begin{align*}
\frac{\partial t}{\partial R} &= \frac{1}{2(1-r) - a^2(r-1)^2} \\
\frac{\partial t}{\partial r} &= \frac{a^2(r-1)(e_0 + 2R) + 2R}{[a^2(r-1)^2 + 2(r-1)]^2} \\
\frac{\partial e}{\partial R} &= \frac{a^2}{a^2(r-1) + 2} \\
\frac{\partial e}{\partial r} &= \frac{a^2(2e_0 - a^2R - aq_0)}{[a^2(1-r) + 2]^2} \\
\frac{\partial q}{\partial R} &= \frac{a}{a^2(1-r) - 2} \\
\frac{\partial q}{\partial r} &= \frac{a(a^2R + aq_0 - 2e_0)}{[a^2(r-1) + 2]^2}
\end{align*}
\]

While for low initial marginal costs of treatment \(e_0\) the equilibrium is characterized by (5) and, therefore, the comparative statics analysis is described by,

\[
\begin{align*}
\frac{\partial t}{\partial R} &= \frac{1}{2(1-r)}, \quad \frac{\partial e}{\partial R} = \frac{\partial e}{\partial r} = \frac{\partial q}{\partial r} = 0 \\
\frac{\partial q}{\partial R} &= -\frac{a}{2}
\end{align*}
\]

Given the results above we can state the following,

**Corollary 5** For \(\forall e_0\) quality is strictly increasing with the reimbursement fee \(R\) and increasing in the reimbursement rate \(r\). Technology price is strictly
increasing in both reimbursement variables and marginal costs are decreasing in both reimbursement parameters. That is,

\[
\begin{align*}
\frac{\partial t}{\partial R} & > 0, \quad \frac{\partial t}{\partial r} > 0, \quad \frac{\partial q}{\partial R} < 0, \\
\frac{\partial q}{\partial r} & \leq 0, \quad \frac{\partial e}{\partial r} \geq 0, \quad \frac{\partial e}{\partial R} \geq 0.
\end{align*}
\]

**Proof.** For \( e_0 \geq a^2 \frac{R}{2} \) given that the conditions for the existence of this optimum require that \( e_0 \geq a^2 \frac{R+aq_0}{2} \) and \( a^2 < \frac{2}{1-r} \) \( ^5 \) we have that in (6) \( \frac{\partial t}{\partial R} > 0, \frac{\partial t}{\partial r} > 0, \frac{\partial e}{\partial r} > 0, \frac{\partial e}{\partial R} < 0, \frac{\partial q}{\partial r} < 0 < 0 \). For \( e_0 < a^2 \frac{R+aq_0}{2} \) technology developed will be characterized by (5), it then follows from (7) that \( \frac{\partial t}{\partial R} > 0, \frac{\partial e}{\partial R} > 0, \frac{\partial q}{\partial R} < 0, \frac{\partial q}{\partial r} < 0 \) Q.E.D. \( \blacksquare \)

### 3.5 Pure Prospective Payment System

We can now analyze the technology characteristics at the optimum as well as its price for a pure Prospective Payment system, i.e., for \( r = 0 \) and \( R > 0 \). Plugging \( r = 0 \), on (4) under a pure PPS, at the optimum, the technology developed and adopted will be cost decreasing but the effect on quality will depend on the reimbursement level. This optimum is characterized by,

\[
\begin{align*}
\hat{e}_{PPS}^* & = a \left[ R - e_0 \right] a + q_0 \\
\hat{t}_{PPS}^* & = a \left[ e_0 - R \right] - q_0 \\
\hat{q}_{PPS}^* & = a \left[ e_0 - R \right] + q_0 \left[ 1 - a^2 \right]
\end{align*}
\]

From Proposition 2 we have that \( R > e_0 - \frac{a^2}{a} \) and \( 2 - a^2 > 0 \). Hence, technology adopted and developed will decrease costs. Concerning the effect of technology on quality, it depends on the reimbursement fee \( R \), i.e., for a sufficiently high reimbursement fee, technology will be quality increasing.\( ^7 \)

Both, the cost decreasing parameter and the level of quality are increasing in the reimbursement fee.\( ^8 \) Nevertheless, increasing the level of reimbursement drives the technology price up.

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\( ^5 \)With the first condition arising from the constraint \( e \leq e_0 \) being slack and the second by concavity of the profit function

\( ^6 \)Note that \( \frac{\partial q}{\partial r} < 0 \Rightarrow \frac{\partial q}{\partial R} > 0 \) and \( \frac{\partial q}{\partial r} < 0 \Rightarrow \frac{\partial q}{\partial R} > 0 \)

\( ^7 \)This threshold is given by \( R > e_0 + q_0 \frac{1-a^2}{q} \)

\( ^8 \)Indeed, \( \frac{\partial q^*}{\partial R} < 0 \Rightarrow \frac{\partial q^*}{\partial r} > 0 \)
3.6 Pure Cost Based Reimbursement System

In a pure CBR we have that $R = 0$. Hence, plugging $R$ on (4), the optimum is characterized by:

For $e \in \left[ \frac{q_0}{2}, \frac{q_0}{a(1-r)} \right]$

\[
\begin{align*}
    t_{CBR} &= \frac{a(1-r)e_0 - q_0}{a \left[ a^2 (r-1)^2 + 2(r-1) \right]} \\
    e_{CBR} &= \frac{\left[(r-1)e_0\right] a + q_0}{a^2 (r-1) + 2} > 0 \\
    q_{CBR} &= \frac{a(1-r)e_0 + q_0 \left[1 + a^2 (r-1)\right]}{a^2 (r-1) + 2}
\end{align*}
\]

Technology, in equilibrium will decrease costs. The impact of technology on quality will depend on the reimbursement rate. For a sufficiently low reimbursement rate $r$ technology will decrease the level of quality. Otherwise, i.e. for high reimbursement rates, technology will increase the level of quality.\(^9\)

3.7 Some remarks

Before proceeding with the optimal reimbursement it is useful to make a comment on the possible values of the reimbursement rate $r$.

The existence of the optima found above require an $r < 1$. Nonetheless, one could ask what happens in the other possible values of $r$. For $r = 1$, in a mixed reimbursement system, the hospital demand for quality will not depend neither on the technology price nor on the cost decreasing parameter. Hence, the R&D firm will produce quality increasing technology but at an infinite price and this technology will not contribute for the decrease of the marginal cost of treatment. Indeed, for $r = 1$ the hospital problem becomes

\[
\max_q \Pi_H = Raq - \frac{q^2}{2}
\]

Solving the first order conditions the maximum is given by $q^* = -Ra$. Plugging into the R&D firm profit function and maximizing with respect to $t$ and $e$.

\(^9\)This threshold is given by $r = \frac{ae_0}{aq_0 + q_0}$
\[
\max_{t,e} \Pi_{R&D} = ta(q_0 + Ra) - \frac{e^2}{2}
\]

Differentiation with respect to \( t \) and \( e \) yields,
\[
\begin{align*}
\frac{\partial \Pi_{R&D}}{\partial t} &= a(q_0 + Ra) > 0 \\
\frac{\partial \Pi_{R&D}}{\partial e} &= -e < 0
\end{align*}
\]

Therefore, having that \( \frac{\partial \Pi_{R&D}}{\partial t} > 0 \) and \( \frac{\partial \Pi_{R&D}}{\partial e} < 0 \) in equilibrium \( t^* \rightarrow +\infty \) and \( e^* \rightarrow -\infty \).

Under this scenario, in a pure CBR \((R = 0)\) with full cost reimbursement, \( r = 1 \), one can see that, as the design costs are financial responsibility of the hospital, its best strategy is to supply no additional quality in order to avoid negative profits.

Instead, an \( r > 1 \) means that the hospital is reimbursed for more than the incurred marginal costs. This means that the government is left with the whole responsibility of these costs. In such a case, in a mixed reimbursement system and in a pure CBR, one can easily see that the hospital and the R&D firm strategies result in that the technology developed and adopted is traded, at the optimum, at an infinite price. Moreover, this technology will be infinitely quality increasing and will not contribute for the decrease of the marginal cost of treatment. In this case, our results match the existing literature. In a pure Cost Based Reimbursement System quality is provided but at very high costs. Furthermore, a pure Cost Based Reimbursement System provides more quality than a pure Prospective Payment System.

### 3.8 Optimal reimbursement

Finally, on the first stage, accounting for the hospital and the R&D firm behavior, the government will decide on the reimbursement variables: \( r \) and \( R \).

The government will then maximize an utilitarian social welfare function \( W \) by deciding on the reimbursement system to be implemented having as instruments the reimbursement variables \( r \) and \( R \),

\[
\max_{r,R} W = CS + \Pi_H + \Pi_{R&D} \\
-(1+\lambda)[RD + r(e_0 - e + t)D] \\
\text{s.t. } \Pi_{RD} \geq 0
\]

14
Where the first term $CS$ is patient’s surplus and is given by $a\frac{(q_0 - q)^2}{2}$, the second term $\Pi_H$ stands for the hospital profit and is given by $D[R - (1 - r)] - \frac{q^2}{2}$, $\Pi_{R&D} = tD - \frac{q^2}{2}$ is the R&D firm profit and $(1 + \lambda) RD + (1 + \lambda)t(q_0 - e + t)D$ is the government reimbursement to the hospital weighed by the cost of public funds $\lambda$.

**Proposition 6** For $e_0 < \min \left\{ \frac{\sqrt{2q_0(1+\lambda)}}{1+2\lambda-a}, \frac{aq_0(1+\lambda)}{1+4\lambda-a} \right\}$ and $\lambda \in \left[ \frac{a+1}{2}, \frac{1+a}{2} \right]$ the optimal reimbursement is characterized by a prospective payment system, and will be given by

\[
\begin{align*}
    r^* &= 0 \\
    R^* &= \frac{q_0(1 + a - 2\lambda)}{a(1 + 4\lambda - a)}
\end{align*}
\]

Implying that technology in the optimum is characterized by

\[
\begin{align*}
    q^* &= \frac{q_0(a - 3\lambda)}{a - 1 - 4\lambda} \\
    t^* &= \frac{q_0(1 + \lambda)}{(1 + 4\lambda - a)a} \\
    e^* &= e_0
\end{align*}
\]

For $e_0 \in \left[ \frac{\sqrt{2q_0(1+\lambda)}}{1+2\lambda-a}, \frac{aq_0(1+\lambda)}{1+2\lambda-a} \right]$, $\lambda > \frac{a+1}{2}$ and $2 - a^2 > 0$ the optimal reimbursement is characterized by a mixed payment system, in particular

\[
\begin{align*}
    r^* &= 1 - \frac{2q_0^2(1 + \lambda)^2}{e_0^2(a - 1 - 2\lambda)^2} \\
    R^* &= \frac{(a + 1)q_0}{(2\lambda + 1 - a)a}
\end{align*}
\]

10 Where $e_0 > \frac{\sqrt{2q_0(1+\lambda)}}{1+4\lambda-a}$ arises from $\Pi_{R&D} \geq 0$ binding, $e_0 < \frac{aq_0(1+\lambda)}{1+4\lambda-a}$ gives the parameters’ interval for which the second order conditions are satisfied as well as ensuring that $R \geq 0$

11 With $e_0 \geq \frac{\sqrt{2q_0(1+\lambda)}}{1+2\lambda-a}$ ensuring that $r \geq 0$, $e_0 < \frac{aq_0(1+\lambda)}{1+2\lambda-a}$ corresponding to the parameter values for which the corresponding R&D optimum is defined for, $\lambda \geq \frac{a+1}{2}$ can be obtained by solving for $\lambda R \geq 0$ and $t \geq 0$. Note that the second order conditions for a maximum are always satisfied.
Implying that technology in the optimum is characterized by

\[ q^* = \frac{(\lambda - a)q_0}{2\lambda + 1 - a} \]
\[ t^* = \frac{2(\lambda + 1 - a)e_0^2}{2q_0(1 + \lambda)a} \]
\[ e^* = e_0 \]

Finally, for \( e_0 \geq \frac{aq_0(1+\lambda)}{1+4\lambda-a-\lambda a^2} \) and \( \lambda > \frac{a+1}{2-a^2} \) the optimal reimbursement will be given by

\[ r^* = 0 \text{ (12)} \]
\[ R^* = \frac{(ae_0 - q_0)(a + a^2\lambda + 1 - 2\lambda)}{a(-1 - 4\lambda + 2a^2\lambda + a + a^2)} \]

Implying that technology in the optimum is characterized by

\[ e^* = \frac{a(1 + \lambda)(ae_0 - q_0)}{-1 - 4\lambda + 2a^2\lambda + a + a^2} \]
\[ q^* = \frac{q_0(2\lambda a^2 + a^2 + a - 3\lambda) - ae_0(1 + \lambda)}{-1 - 4\lambda + 2a^2\lambda + a + a^2} \]
\[ t^* = \frac{(1 + \lambda)(ae_0 - q_0)}{a(-1 - 4\lambda + 2a^2\lambda + a + a^2)} \]

**Proof.** Proof in Appendix

First notice that the welfare function is always increasing in \( e \), indeed

\[ \frac{dW}{de} = \frac{d\Pi_H}{de} + \frac{d\Pi_{R&D}}{de} + aq(2 + \lambda)r \]
\[ = \frac{d\Pi_{R&D}}{de} + aq(2 + \lambda)r \]

From the envelope theorem \( \frac{d\Pi_{R&D}}{de} = 0 \) in case 2 \( \frac{d\Pi_{R&D}}{de} > 0 \) in case 1 of the R&D problem, what implies that \( \frac{dW}{de} > 0 \) i.e. the social welfare is

\(^{12}\)The first condition ensures that the optimum from the R&D is valid. Note that for this range of parameters the participation constraint of the R&D firm is always satisfied. The second condition corresponds to the second order condition for a maximum.
always increasing in \( e \). Hence, it is always socially optimal to have \( e^* = e_0 \).

Moreover, notice that,

\[
\frac{dW}{dt} = \frac{d\Pi_H}{dt} + \frac{d\Pi_{R&D}}{dt} - a\bar{\Pi}(1 + \lambda)r
\]

By the envelope theorem we know that \( \frac{d\Pi_{R&D}}{dt} = 0 \), therefore,

\[
\frac{dW}{dt} = \frac{d\Pi_H}{dt} + \frac{d\Pi_{R&D}}{dt} - a\bar{\Pi}(1 + \lambda)r
\]

\[
= D(r - 1 - (1 + \lambda)r) < 0
\]

Consequently, the central planner will aim at inducing the minimum possible price, the highest possible cost containment as well as the optimal quality level through the use of the two instruments \( \{R, r\} \). In the second case (5) the R&D firm already chooses the socially optimal level of \( e \). The planner is then left with two instruments to induce the optimal quality and technology price. We further know that \( \frac{dt}{dr} > 0, \frac{dt}{dR} > 0 \) while \( \frac{dq}{dr} = 0 \) and \( \frac{dq}{dR} > 0 \). Therefore, optimal quality will be induced using \( R \) while technology price will be controlled via \( r \). Therefore, we would envisage that the optimal reimbursement system would be characterized by a pure prospective payment system with the reimbursement rate \( r \) set at its minimum, i.e. \( r = 0 \), and a positive fee, \( R > 0 \), in order to, respectively, induce the optimal price and the optimal quality. Nevertheless, the government must further ensure that the participation constraint of the R&D firm is satisfied.

While for \( e_0 < \min \left\{ \frac{\sqrt{2\eta_0(1+\lambda)}}{1+4\lambda-a}, \frac{a\theta(1+\lambda)}{1+4\lambda-a} \right\} \) a prospective scheme ensures non negative profits for the R&D firm, for higher status quo cost parameters, i.e. \( e_0 \in \left[ \frac{\sqrt{2\eta_0(1+\lambda)}}{1+2\lambda-a}, \frac{a\theta(1+\lambda)}{1+2\lambda-a} \right] \), under a fully prospective payment system the R&D firm would exit the market. In this case the planner must allow for a positive reimbursement rate \( r > 0 \) in order to ensure the R&D participation constraint is just satisfied. Therefore, for low \( e_0 \) a prospective payment system is optimal and is characterized by (10) while for intermediate cost parameters the optimal reimbursement is a mixed system. For higher cost parameters, i.e. \( e_0 \geq \frac{a\theta(1+\lambda)}{1+4\lambda-a-\lambda_0} \) the optimal \( e \) chosen by the R&D firm not necessarily corresponds to the socially optimal level (4). Therefore, the government has now less instruments \( \{R, r\} \) than variables to control \( \{t, q, e\} \), meaning that it will not be able to induce the first best for all variables (12).
4 R&D within the Hospital

4.1 The model

We will now consider the case where the R&D and the Hospital are vertically integrated, i.e., the R&D process is carried out by the hospital. The model has then only two agents: the government and the hospital. The model specifications remain as described in the section 2.

The government decides on the reimbursement scheme: \( R \) (Prospective Payment system fee) and \( r \) (Cost Based reimbursement rate), while the hospital decides on the technology to be developed and adopted.

4.2 Timing

The game will then be developed in a two stage game described in the following diagram,

In the first stage, the government decides on the optimal way to finance the hospital by (optimally) deciding on the reimbursement instruments \( R \) and \( r \) and on the second stage the hospital decides on the characteristics of the technology to be developed and adopted \((q, e)\).

As usual, the model will be solved backwards.

4.3 The hospital problem

The hospital objective function is thus:

\[
\max_{q, e} \pi_H = RD + (r - 1)(e_0 - e)a\bar{q} - \frac{q^2}{2} - \frac{e^2}{2}
\]

\[s.t. \quad e \leq e_0, \quad q_0 \geq q, \quad q_0 - q > e_0 - e\]

With \( D = a\bar{q} \).
Solving the first order conditions for $q$ and $e$ the optima are characterized by two cases described in the following sub sections. Depending on the value of the initial marginal costs of treatment, $e_0$, solutions will differ. For low (initial) marginal costs results are as described in case 1 while for high (initial) marginal costs results are described by case 2.

4.3.1 Case 1: $e = e_0$

**Proposition 7** For $e \leq e_0$ binding, i.e., for $e_0 \in [0, a (1 - r) [a R + q_0])$ we have that at the optimum the hospital will set

$$
e^* = e_0, \quad q^* = -aR$$  \hspace{2cm} (13)

**Proof.** Proof in Appendix ■

The condition states the parameters’ values for which $e \leq e_0$ binds. Note that second order conditions for a maximum are always satisfied.

In this case technology, at the optimum, will increase quality and decrease costs. Both, the quality level and the cost decreasing parameter do not vary with the reimbursement rate $r$. Moreover, the reimbursement fee $R$ has a positive impact on quality and a null impact on costs.

**Corollary 8** Under a pure PPS technology is cost decreasing and quality increasing

$$q^{PPS} = -aR, \quad e^{PPS} = e_0$$  \hspace{2cm} (14)

While for a pure CBR technology is quality neutral but cost decreasing and is characterized by,

$$e^{CBR} = e_0, \quad q^{CBR} = 0$$

**Proof.** The proof is obtained by substitution on the equilibrium $r = 0$ for a PPS and $R = 0$ in the CBR case. ■

Comparing the two reimbursement systems, Prospective Payment and Cost Based Reimbursement, in a pure PPS the level of quality is higher than under a pure Cost Based Reimbursement system. Nevertheless, both systems are equally efficient in cost control incentives\(^\text{13}\).

\(^{13}\)In a pure PPS we have that $r = 0$ and $R > 0$ hence the technology developed and adopted is characterized by:

$$q^{PPS} = -aR < 0, \quad e^{PPS} = e_0$$  

19
Indeed, a pure CBR does not reward quality investment for \( r \leq 1 \), while under a pure PPS increasing quality has a positive impact on profits via increased demand. Given that the initial marginal costs \( e_0 \) are sufficiently low (by definition of this case), under both systems it will be profitable to decrease them totally as the marginal benefit of reducing it (decreased marginal cost of treatment) exceeds the marginal cost (design cost).

### 4.3.2 Case 2: \( e \leq e_0 \)

**Proposition 9** For \( e_0 \in \left[ a |aR + q_0| (1 - r), \frac{q_0 (1-a(1-r)^2) + aR(1-a(r-1))}{1+a(1-r)(r-2)(1-r)a} \right] \) and 
\[
1 - a^2 (r - 1)^2 > 0 \quad 14
\]
in equilibrium technology supplied is characterized by 
\[
\begin{align*}
    e^* &= a (1 - r) \left[ \frac{aR + (r - 1) (ae_0 (r - 1) + q_0)}{1 - a^2 (r - 1)^2} \right] \\
    q^* &= a \left[ \frac{(1 - r) [q_0 + aq_0 (r - 1)] - R}{1 - a^2 (r - 1)^2} \right]
\end{align*}
\]

**Proof.** Proof in Appendix.

Even though the technology adopted is always cost decreasing its impact on the quality level will depend on the initial marginal costs. 

For \( e_0 \in \left( a |aR + q_0| (1 - r), aq_0 (1 - r) + \frac{R}{(1-r)} \right) \) the level of quality will be increased by technology. For \( e_0 \in \left( aq_0 (1 - r) + \frac{R}{(1-r)}, \frac{q_0}{a(1-r)} + \frac{R}{(1-r)} \right) \) technology is quality decreasing. On what concerns costs, technology is always cost decreasing.

Indeed, under a mixed reimbursement system, increasing quality affects profits through a rise in demand. If the (initial) marginal cost is considerably low, then the marginal revenue arising on increased demand exceeds the marginal cost and therefore it is profitable to invest in quality. The marginal cost of supplying an extra unit of quality is increasing in the marginal cost of

---

With quality level given by \( \overline{q} = q_0 - q \) we have that in the optimum \( \overline{q} = q_0 + Ra \).

In a pure CBR \( R = 0 \) and \( r > 0 \). Thus, the technology developed and adopted in this case is characterized by:

\[
q^{CBR} = 0, \quad e^{CBR} = e_0
\]

With \( a > 0, R > 0 \) and the quality level given by \( \overline{q} = q_0 - q \), the result follows \( \overline{q}^{PPS} > \overline{q}^{CBR} \) and \( e^{PPS} = e^{CBR} \).

---

14The first condition defines the range of parameters for which the constraint \( e \leq e_0 \) and is slack and for which \( q_0 - q > e_0 - e \) holds while the second condition ensures that the second order conditions for a maximum are satisfied.
treatment and consequently on the initial marginal cost \( e_0 \), therefore higher \( e_0 \) reduce the profitability of investing in quality.

It is now useful to analyze the optima of a pure Prospective Payment System and of a pure Cost Based Reimbursement System.

**Proposition 10** A pure Cost Based Reimbursement system leads to the development and adoption of both quality and cost decreasing technologies. While in a pure PPS, technology, at the optimum, will be cost decreasing. Its impact on quality level depends on the reimbursement fee \( R \). For highly enough reimbursement levels technology is quality increasing.

**Corollary 11** Comparing the two systems, the effectiveness in cost control depends on the reimbursement schedule. In what concerns quality, for a sufficiently high reimbursement fee, i.e. \( R > e_0 - aq_0 \), technology developed under a PPS increases quality while within a CBR system it decreases the level of quality. For low reimbursement fees both systems decrease quality. The relative magnitude of this effect will depend on the relation between the two reimbursement instruments - \( r \) and \( R \).

**Proof.** In a pure CBR system we have that, at the optimum, the hospital will set \((q, e)\) such that,

\[
q_{CBR} = a \frac{(r - 1)e_0 + aq_0(r - 1)^2}{a^2(r - 1)^2 - 1}, \quad e_{CBR} = a \frac{(r - 1)^2e_0 + a(r - 1)q_0}{a^2(r - 1)^2 - 1}
\]

As, by concavity, \( a(r - 1)^2 - 1 < 0 \) a \( e_0 \in \left[ \frac{q_0}{a(1 - r)} + \frac{R}{1 - r}, a(1 - r)[aR + q_0] \right] \) implies \( q_{CBR} > 0 \) and \( e_{CBR} < 0 \). Under a PPS \( r = 0 \). Hence, the optimum is given by,

\[
q_{PPS} = a \frac{e_0 - R - aq_0}{1 - a^2}, \quad e_{PPS} = a \frac{aR + ae_0 - q_0}{1 - a^2}
\]

Furthermore, under a pure PPS we have that, by concavity \( 1 - a^2 > 0 \). Moreover, this optimum is defined for \( e_0 \in ]a^2R + aq_0, aq_0 + R[ \). Hence, we have that \( R > e_0 - aq_0 \Rightarrow q^* < 0 \), technology increases the quality level. While for \( R < e_0 - aq_0 \Rightarrow q^* > 0 \), technology is quality decreasing. Concerning \( e \) we have that \( e_0 < aq_0 + R \Rightarrow e^* > 0 \) Q.E.D.

Summarizing the results described under the previous two cases, for a mixed reimbursement system, for low levels of the initial marginal cost \( e_0 \)

\[ ^{15} \text{Condition that states } q \leq q_0 \text{ and } e \leq e_0 \]
technology is cost decreasing and quality increasing, moreover technology completely offsets the marginal cost of treatment \( e = e_0 \). For intermediate values of \( e_0 \), technology decreases costs and increases quality, but in this case it is no longer profitable to completely offset the marginal cost of treatment. Finally, for higher levels of \( e_0 \), technology decreases both quality and costs. Consequently, results show that, in a sense, the priority is to achieve low treatment costs, and, just then, there is investment on quality improvements.

4.4 Optimal reimbursement

Finally, given the hospital behavior the government will decide on the reimbursement variables. The government objective will be to decide on the reimbursement policy by, optimally, choosing the reimbursement instruments in order to maximize social welfare \( W \),

\[
\max_{r,R} W = CS + \Pi_H - (1 + \lambda) \left[ Ra_\theta + r(e_0 - e)a_\theta \right]
\]

Where the first term \( CS \) is patients’ surplus and is given by \( \frac{a(q_0 - q)^2}{2} \) the second term \( \Pi_H \) stands for the hospital profit and is given by \( \left[ R - (1 - r)(e_0 - e) \right] a_\theta - \frac{q^2}{2} - \frac{e^2}{2} \) and \( (1 + \lambda) \left[ Ra_\theta + r(e_0 - e)a_\theta \right] \) is the government reimbursement to the hospital weighted by the cost of public funds \( \lambda \).

**Proposition 12** For \( e_0 < \frac{aq_0(1+\lambda)}{2a+1-a} \), \( 1 + 2\lambda - a > 0 \) \(^{16}\) the optimal reimbursement system is characterized by a pure prospective payment system given by\(^{17}\)

\[
\begin{align*}
r^* &= 0 \\
R^* &= \frac{(a-\lambda)q_0}{a(2\lambda + 1 - a)}
\end{align*}
\]

**Proof.** Proof in Appendix ■

Therefore, technology is characterized by

\[
\begin{align*}
e^* &= e_0 \\
q^* &= \frac{q_0(a-\lambda)}{a - 1 - 2\lambda}
\end{align*}
\]

\(^{16}\)First condition ensures that the conditions for which the hospital optimum exists hold. The second condition is a second order necessary condition for a maximum.

\(^{17}\)Note that none of the variables chosen by the hospital depend on \( r \)
**Proposition 13** For \( e_0 \geq \frac{aq_0(1+\lambda)}{2a+1-a-\lambda a^2}, \ \lambda > \frac{a+a^2-1}{(1-a^2)^2} \) the optimal reimbursement system is characterized by a prospective payment system given by\(^{19}\)

\[
R^* = \frac{(a + a^2 \lambda - \lambda)(ae_0 - q_0)}{a(a + a^2 + 2\lambda a^2 - 1 - 2\lambda)}
\]

\[
r^* = 0
\]

**Proof.** Proof in Appendix

Therefore technology is characterized by

\[
e^* = \frac{(1 + \lambda)(ae_0 - q_0)}{a + a^2 + 2\lambda a^2 - 1 - 2\lambda}
\]

\[
q^* = \frac{(a^2 + a + 2a^2 \lambda - \lambda)q_0 - e_0a(1 + \lambda)}{a + a^2 + 2\lambda a^2 - 1 - 2\lambda}
\]

Given that,

\[
\frac{dW}{de} = \frac{d\Pi_H}{de} + a\sigma(1 + \lambda)r
\]

From the envelope theorem \( \frac{d\Pi_H}{de} = 0 \) in case 2 and \( \frac{d\Pi_H}{de} > 0 \) in case 1 of the hospital problem, what implies that \( \frac{dW}{de} > 0 \) i.e. the social welfare is increasing in \( e \). Hence, it is always socially optimal to have \( e^* = e_0 \). Notice that in this case we have that the planner has two instruments and two variables to control for and therefore, in principle, it should be feasible to induce the first best levels of cost containment and quality. In the first case (14) of the hospital problem the optimal level of \( e \) is already chosen. Therefore, the two financial instruments can be used to implement the optimal quality. Given that \( \frac{dq}{dr} = 0, \frac{dq}{dr} > 0 \) the optimum will be characterized by a prospective payment scheme. In the second case (15) the planner has to induce the optimal levels of both \( e \) and \( q \). For these range of parameters the first best would be implemented with a scheme where \( R > 0 \) but \( r < 0 \), meaning that the hospital would be reimbursed on a proportion higher than his total costs. Nevertheless, such feature is not contemplated in our model as we constrain the reimbursement fee to be non negative meaning that the best the planner can do is implement a prospective payment system where \( r^* = 0 \).

\(^{18}\)First condition ensures that the conditions for which the hospital optimum exists hold.

The second condition is a second order necessary condition for a maximum.

\(^{19}\)Note that none of the variables chosen by the hospital depend on \( r \)
5 Conclusions

Previous literature on the impact of reimbursement systems on quality and on cost decreasing efforts has mostly concluded that, while retrospective reimbursement encourages quality but lacks sensitivity towards cost containment, PPS encourages cost efficiency but has perverse effects on quality improvement. Nevertheless, we have shown that, within the described set up, these results may not hold.

We focus our analysis on technology development and adoption under two set-ups: private R&D and R&D within the hospital.

In the first best, for low values of the status quo marginal cost technology not only increases quality but also drives marginal costs to zero. For intermediate values of the status quo marginal cost technology is cost decreasing and quality increasing. Finally, for high marginal costs, technology still decreases the marginal costs to zero but also decreases quality.

In the former set up results depend on the value of the reimbursement rate \( r \). We have been able to show that, for \( r < 1 \), under a mixed reimbursement system there is space for the development and adoption of cost decreasing/quality increasing technologies. By first treating the reimbursement as exogenous, we have shown that under both reimbursement systems, Cost Based Reimbursement System and Prospective Payment System, technology developed and adopted is cost decreasing. In what concerns quality, under a MRS, for sufficiently high reimbursement fees technology developed and adopted will increase quality. Under a PPS, results remain qualitatively the same. For a sufficiently high reimbursement fee, technology increases the initial level of quality. Nonetheless, the higher the reimbursement fee the more expensive technology will be. Under CBR for medium initial marginal costs the impact of technology on quality depends on the reimbursement rate \( r \), for a sufficiently high reimbursement rate technology increases the level of quality. However, for sufficiently low initial marginal costs, technology decreases quality. Moreover, as this quality level does not depend on the reimbursement variable, the latter can not be used as a quality level regulatory instrument.

For \( r = 1 \), in a mixed reimbursement system, the hospital demand for quality will not depend neither on the technology price nor on the cost decreasing parameter \( t \). Hence, the R&D firm will produce quality increasing technology but at an infinite price and this technology will not contribute for the decrease of the marginal cost of treatment. Under this scenario, in a pure CBR (\( R = 0 \)) with full cost reimbursement, \( r = 1 \), one can see that, as the design costs are financial responsibility of the hospital, its best strategy
is to supply no quality in order to avoid negative profits.

Instead, for a reimbursement rate \( r > 1 \), our results match the existing literature. In a pure Cost Based Reimbursement System quality is provided but at very high costs. Furthermore, a pure Cost Based Reimbursement System provides more quality than a pure Prospective Payment System. Finally, going one step further and endogeneising the reimbursement decision stage, we have also been able to show that depending on the costs of treatment pre technology adoption the optimal reimbursement scheme is either fully prospective or mixed.

In the latter case, when the R&D is carried out within the hospital, for high initial marginal costs, a pure prospective payment system leads to the adoption of cost decreasing technologies. The impact of technology on quality will depend on the level of the reimbursement fee (for high reimbursement fees technology will increase quality). On the other hand, under a pure Cost Based Reimbursement system, after technology development and adoption quality will be higher but also the marginal costs.

For low initial marginal costs, PPS is efficient in cost control and quality improvement. Indeed, technology adopted is cost decreasing and quality increasing. On the other hand, under CBR the type of technology developed and adopted has no impact on quality even though it decreases costs.

Comparing the two reimbursement systems, we may conclude that, if the reimbursement rate \( r \) is less than unity then a pure Prospective payment system provides more incentives for the development of quality increasing/cost decreasing technologies. For an \( r \) greater than unity we found that, in what concerns costs savings, a pure Prospective Payment System is more efficient.

Concerning quality, we have been able to show that, for a sufficiently high prospective reimbursement fee \( R \), the technologies developed under a pure prospective payment system provide more quality than the ones developed under a pure Cost Based Reimbursement system. Finally, by endogeneising the reimbursement policy, we found that, it is optimal for the government to reimburse the hospital on a prospective or mixed basis.

We use a simple setup, allowing us to obtain clear-cut results and to highlight the effects driving the technology choices under each financing scheme and R&D sector. Nevertheless, the model could be extended in a number of directions, enriching the set of results. One could have considered asymmetric information and heterogeneous patients. With heterogeneity in patients’ severity, the treatment costs are not contractible, making it profitable for the hospital to misreport the costs incurred. In this case, the technology developed and adopted could assume different characteristics from the ones that arise from our optima.
Another possible extension to our model would be the differentiation between services which quality is perfectly perceived by the patients from services where patients are not enough informed to detect its quality. For example, a patient might be sensitive to the type of technology used on his treatment but then not aware of the right number of sessions needed before discharge. In such a context one can expect that the hospital would only supply quality in the type of treatment where the patients can detect quality.

A third aspect is that we consider that patients’ demand for treatment is sensitive to quality. In the real world this is not always true. For instance we have that in emergency treatment, even if this assumption holds, in the end the demand that the hospital faces is not affected by this sensitivity.

Finally, one could also introduce competition between hospitals and, maybe more crucial, in the R&D sector.
A Appendix

A.1 Proof of propositions 2 and 3

The R&D problem is given by,

$$\max_{e,t} taq - \frac{e^2}{2}$$

s.t. \(e \leq e_0, \ t \leq e\)

\(q^* = a[(1 - r)(e_0 - e + t) - R]\)

The constraint \(t \leq e\) will be controled ex-post, i.e. we solve for the is then given by,

$$L = taq^*(e,t), e,t - \frac{e^2}{2} - \Psi(e - e_0)$$

For \(\Psi > 0\) solving the first order conditions we have that

$$t^* = \frac{q_0 + aR}{2a(1 - r)}, \ e^* = e_0$$

$$\Psi^* = \frac{aq_0 + a^2R}{2} - e_0$$

Therefore the complementary slackness conditions implies that \(e_0 < \frac{aq_0 + a^2R}{2}\).

The second order conditions for a maximum are satisfied. Indeed, the bordered Hessian is given by

$$H = \begin{bmatrix}
0 & \frac{\partial g(e)}{\partial e} & \frac{\partial g(e)}{\partial t} \\
\frac{\partial g(e)}{\partial e} & \frac{\partial^2 L}{\partial e^2} & \frac{\partial^2 L}{\partial e \partial t} \\
\frac{\partial g(e)}{\partial t} & \frac{\partial^2 L}{\partial t \partial e} & \frac{\partial^2 L}{\partial t^2}
\end{bmatrix}$$

Where \(g(e) \equiv e \leq e_0\). We still need to check that the sign of the determinant of the bordered Hessian is the same as \((-1)^n = 1 > 0\) where \(n\) states the number of variables. Computing the determinant it can be easily shown that

$$|H| = -\frac{\partial^2 L}{\partial e^2} = 2a^2(1 - r) > 0 \iff r < 1.$$ Therefore the sufficient condition for a maximum is ensured if and only if \(r < 1\).

The constraint \(q_0 - q > e_0 - e\) is always satisfied, indeed \(\frac{q_0 + aR}{2} > 0\).

Finally, the constraint \(t^* \leq e^* \implies e_0 \geq \frac{q_0 + aR}{2a(1 - r)}\).
On the other hand suppose that $\Psi = 0$: in this case solving the first order conditions we find that in equilibrium

$$
t^\ast = \frac{a \left[(1 - r) e_0 - R\right] - q_0}{a \left[a^2 \left(r - 1\right)^2 + 2 \left(r - 1\right)\right]}
$$

$$
e^\ast = \frac{a \left[R + (r - 1) e_0\right] a + q_0}{a^2 \left(r - 1\right)^2 + 2}
$$

It is easy to show that the determinant of the Hessian for this case is given by,

$$
|H| = a^2 \left(1 - r\right) \left(2 - a^2 \left(r - 1\right)\right)
$$

Therefore the sufficient condition for a maximum is ensured if and only if $2 - a^2 \left(r - 1\right) > 0$. The complementary slackness condition requires that $\frac{\partial \Psi}{\partial e} = \frac{2e_0 - aq_0 - a^2 R}{2 - a^2 \left(r - 1\right)} \geq 0$. Given that the second order conditions for a maximum require $2 - a^2 \left(r - 1\right)$, the complementary slackness condition is then satisfied as long as $e_0 \geq \frac{aq_0 + a^2 R}{2}$.

To check that $q^* \leq q_0$ holds. Plugging $q^*$ and solving $q^* - q_0 = 0$ for $e_0$ it follows that the condition is satisfied as long as $e_0 \leq \frac{q_0}{a \left(1 - r\right)} + \frac{R}{1 - r}$. Moreover, $q_0 - q > e_0 - e \implies e_0 < \frac{(aR + q_0)(1 + a)}{2 + a \left(1 - r\right)}$. Meaning that this equilibrium is defined for $e_0 < \min \left\{ \frac{q_0}{a \left(1 - r\right)} + \frac{R}{1 - r}, \frac{(aR + q_0)(1 + a)}{2 + a \left(1 - r\right)} \right\}$.

Checking the constraint $t^\ast \leq e^\ast$ can be written as

$$
a \frac{[R + (r - 1) e_0] a + q_0}{a^2 \left(r - 1\right)^2 + 2} \geq \frac{a \left[(1 - r) e_0 - R\right] - q_0}{a \left[a^2 \left(r - 1\right)^2 + 2 \left(r - 1\right)\right]}.
$$

implying $\frac{(1 - a^2 \left(1 - r\right))}{a \left(1 - r\right)^2 \left(2 - a^2 \left(1 - r\right)\right)} \left(q_0 + aR - ae_0 \left(1 - r\right)\right) \geq 0$

We know that from SOCs $2 - a^2 \left(r - 1\right) > 0$ moreover $r \in [0, 1]$ therefore

$$
\frac{(1 - a^2 \left(1 - r\right))}{a \left(1 - r\right)^2 \left(2 - a^2 \left(1 - r\right)\right)} \left(q_0 + aR - ae_0 \left(1 - r\right)\right) \geq 0
$$

implies that

$$
(1 - a^2 \left(1 - r\right)) \left(q_0 + aR - ae_0 \left(1 - r\right)\right) \leq 0
$$

Suppose for now that $1 - a^2 \left(1 - r\right) > 0$ then it must be that

$$
q_0 + aR - ae_0 \left(1 - r\right) \leq 0 \implies e_0 \geq \frac{q_0 + aR}{1 - r}
$$

28
Recall that the complementary slackness condition required $e_0 \geq \frac{aq_0 + a^2 R}{2}$. Therefore these conditions can be written as $e_0 \geq \max \left\{ \frac{q_0 + aR}{1-r}, \frac{aq_0 + a^2 R}{2} \right\}$. It is easy to see that for $2 - a^2 (r - 1) > 0$ the max is the second term.

Now if $1 - a^2 (1 - r) < 0$ then it must be that $e_0 \leq \frac{q_0 + aR}{1-r}$. Given that the complementary slackness condition required $e_0 \geq \frac{aq_0 + a^2 R}{2}$ and $q^* \leq q_0 \implies e_0 \leq \frac{q_0}{a(1-r)} + \frac{R}{1-r}$ then the optimum is valid for $e_0 \leq \min \left\{ \frac{q_0}{a(1-r)} + \frac{R}{1-r}, \frac{aq_0 + a^2 R}{2} \right\}$.

A.2 Proof of propositions 7 and 8

Notice that the constraint $q_0 \geq q$ will never bind otherwise the hospital will make negative profits. Therefore, the hospital problem can be written as following optimization problem,

$$
\max_{q,e} \mathcal{L} = Raq + (r-1)(e_0 - e)aq - \frac{q^2}{2} - \frac{e^2}{2} - \Psi (e - e_0)
$$

For $\Psi > 0$ solving the first order conditions we have that

$$
q^* = -aR, \quad e^* = e_0
$$

$$
\Psi^* = c_0 - a[aR + q_0](1-r)
$$

Therefore the complementary slackness conditions implies that

$$
e_0 < a[aR + q_0](1-r)
$$

The second order conditions for a maximum are satisfied. Indeed, the bordered Hessian is given by

$$
\mathcal{H} = \begin{bmatrix}
0 & \frac{\partial g(e)}{\partial q} & \frac{\partial g(e)}{\partial e_0}

\frac{\partial g(e)}{\partial q^2} & \frac{\partial^2 g(e)}{\partial q^2} & \frac{\partial^2 g(e)}{\partial q \partial e_0}

\frac{\partial g(e)}{\partial e_0^2} & \frac{\partial^2 g(e)}{\partial e_0^2} & \frac{\partial^2 g(e)}{\partial e_0 \partial e_0^2}
\end{bmatrix} = \begin{bmatrix}
0 & 0 & 1

0 & -1 & (r-1)a

1 & (r-1)a & -1
\end{bmatrix}
$$

Where $g(e) \equiv e \leq e_0$. We still need to check that the sign of the determinant of the bordered Hessian is the same as $(-1)^n = 1 > 0$ where $n$ states the
number of variables. Computing the determinant it can be easily shown that $\det \mathbf{F} = 1 > 0$ and therefore the sufficient condition for a maximum is ensured.

Finally $q_0 - q^* > e_0 - e^*$ is always satisfied indeed $q_0 + aR > 0$.

On the other hand suppose that $\Psi = 0$: in this case solving the first order conditions we find that in equilibrium

$$e^* = a(1 - r) \left[ \frac{aR + (r - 1)(ae_0 (r - 1) + q_0)}{1 - a^2 (r - 1)^2} \right]$$

$$q^* = a \left[ \frac{(1 - r) [e_0 + aq_0 (r - 1)] - R}{1 - a^2 (r - 1)^2} \right]$$

The complementary slackness condition requires that $\frac{\partial L}{\partial \Psi} > 0$ implying that this equilibrium is valid as long as $e_0 \geq a [aR + q_0] (1 - r)$. Computing the second order conditions it can be easily shown that this is a maximum for $1 - a^2 (r - 1)^2 > 0$. Finally $q_0 - q^* > e_0 - e^* \implies e_0 < \frac{q_0(1-a(1-r)^2)+aR(1-a(r-1))}{1+a(1-r)(r-2)(1-r)a}$.

### A.3 Proof of Proposition 6

The optimization problem of the government is given by

$$\max_{R, r} W = CS + \Pi_H + \Pi_{R&D}$$

$$- (1 + \lambda) [RD + r(e_0 - e + t)D]$$

s.t. $\pi_{RD} \geq 0, r \geq 0$

The Lagrangian for this problem is given by

$$\mathcal{L} = W - \Upsilon (\pi_{RD}) - \Phi (-r)$$

With $\{\Upsilon, \Phi\}$ being the Lagrangian multipliers.

For the first solution on the R&D problem we have that it can easily be proved that if $\Upsilon > 0$ then $\Phi = 0$ and for $\Upsilon = 0$ the $\Phi > 0$. So we are left with two cases.

For $\{\Upsilon > 0, \Phi = 0\}$ solving $\frac{\partial \mathcal{L}}{\partial R}, \frac{\partial \mathcal{L}}{\partial r}, \frac{\partial \mathcal{L}}{\partial \Upsilon}$ for $\{R, r, \Upsilon\}$ we have that

$$r^* = 1 - \frac{2q_0^2 (1 + \lambda)^2}{e_0^2 (a - 1 - 2\lambda)^2} \quad (16)$$

$$R^* = \frac{(a + 1) q_0}{(2\lambda + 1 - a) a}$$

$$\Upsilon^* = \lambda \quad (17)$$
We further know that from the R&D stage the R&D equilibrium was defined for

\[ e_0 < \frac{aq_0 + a^2 R}{2} \implies e_0 < \frac{aq_0 (1 + \lambda)}{2\lambda + 1 - a} \]

\[ r < 1 \implies 1 - 2a + a^2 + 4\lambda - 4a\lambda + 4\lambda^2 > 0 \]

The second order conditions require that the determinant of bordered Hessian being positive. With the bordered Hessian given by

\[
\mathcal{H} = \begin{bmatrix}
0 & \frac{\partial \pi_{RD}}{\partial L} & \frac{\partial \pi_{RD}}{\partial \lambda}
\end{bmatrix}
\begin{bmatrix}
\frac{\partial^2 \pi_{RD}}{\partial r^2} & \frac{\partial^2 \pi_{RD}}{\partial r \partial L} & \frac{\partial^2 \pi_{RD}}{\partial r \partial \lambda}
\end{bmatrix}
\]

It follows that \(|\mathcal{H}| = \frac{64}{a^2} (1+4\lambda-2a+a^2+a^2)(a-1-2\lambda)^3 e_0^8\). For \(|\mathcal{H}| > 0\) we need to impose \(-(1 + 4\lambda - 2a + 4\lambda^2 + a^2 - 4a\lambda)(a - 1 - 2\lambda)^3 e_0^8 > 0\). Notice that \(R > 0 \implies \lambda > \frac{a-1}{2}\). Therefore it follows that \(|\mathcal{H}| > 0 \iff 1 - 2a + a^2 + 4\lambda - 4a\lambda + 4\lambda^2 > 0 \implies \lambda > \frac{a+a^2-a}{2(2-a)^2}\). Combining these two conditions on \(\lambda\) we have that \(\lambda > \max \left\{ \frac{a+a^2-a}{2(2-a)^2}, \frac{a-1}{2} \right\} = \frac{a-1}{2}\).

Finally, the complementary slackness condition requires \(\frac{\partial C}{\partial r} > 0\) implying that \(e_0 \geq \frac{\sqrt{q_0 (1+\lambda)}}{1+2\lambda-a}\).

For \(\Upsilon = 0\) computing \(\left\{ \frac{\partial C}{\partial r}, \frac{\partial C}{\partial L}, \frac{\partial C}{\partial \lambda} \right\}\) it can be easily seen that \(\frac{\partial C}{\partial r} < 0\). Solving the system of two equations \(\frac{\partial C}{\partial r}\) for \(R\) we have that

\[ R^* = \frac{q_0 (1 + a - 2\lambda)}{a (1 + 4\lambda - a)} \]

\[ r^* = 0 \]

The complementary slackness condition requires that \(\frac{\partial C}{\partial \Upsilon} > 0\) implying that this equilibrium is valid as long as \(e_0 < \frac{\sqrt{q_0 (1+\lambda)}}{1+2\lambda-a}\). Computing the second order conditions it can be easily shown that this is a maximum for \(1 - a^2 (r - 1)^2 > 0\). To check the second order conditions are satisfied we need to study the sign of the determinant bordered Hessian. Given that \(|\mathcal{H}| = a^2 (4\lambda + 1 - a)\) and we have two variables and just one binding constraint we need \(|\mathcal{H}| > 0\) that is true as long as \(4\lambda + 1 - a > 0\). Notice that \(R > 0 \implies 1 + a - 2\lambda > 0 \implies \lambda < \frac{1+a}{2}\).

Finally we know that the R&D solution exits for

\[ e_0 < \frac{aq_0 + a^2 R}{2} \implies e_0 < \frac{aq_0 (1 + \lambda)}{4\lambda + 1 - a} \]
and for \( r < 1 \) that is always satisfied.

For the second solution of the R&D problem following the same process as above we find that for \( \{ \Upsilon = 0, \Phi > 0 \} \)

\[
\begin{align*}
\text{r}^* &= 0 \\
\text{R}^* &= \frac{(ae_0 - q_0)(a + a^2\lambda + 1 - 2\lambda)}{a(-1 - 4\lambda + 2a^2\lambda + a + a^2)}
\end{align*}
\]

Second order condition requires that \( |\mathcal{H}| = a^2 \frac{1+4\lambda-a-a^2-2\lambda a^2}{(a^2-2)^2} > 0 \implies 1 + 4\lambda - a - a^2 - 2\lambda a^2 > 0 \implies \lambda > \frac{a + a^2 - 1}{2(2-a^2)}. \) We further know that from the R&D stage the R&D equilibrium was defined for

\[
2 + a^2 (r - 1) > 0 \implies a \leq \sqrt{2}
\]

\[
e_0 \geq \frac{a e_0 + a^2 R}{2} \implies \begin{cases} 1 + 4\lambda - a - \lambda a^2 > 0 \\ e_0 \geq \frac{(1+\lambda)ae_0}{1+4\lambda-a-\lambda a^2} \end{cases}
\]

Notice that from the two conditions \( 1 + 4\lambda - a - a^2 - 2\lambda a^2 > 0 \) and \( 1 + 4\lambda - a - \lambda a^2 > 0 \), the first is more restrictive and therefore if verified then the second is automatically verified too. Moreover, \( t > 0 \implies e_0 < ae_0 \) therefore for \( R > 0 \) we need to impose that \( \lambda > \frac{a + 1}{2-a^2}. \) These three conditions imply that \( \lambda > \max \left\{ \frac{a+1}{2-a^2}, \frac{a+a^2-1}{2(2-a^2)} \right\} = \frac{a+1}{2-a^2}. \)

The remaining possible combinations of the Lagrangian multipliers are not feasible. Indeed, consider for instance the case \( \Upsilon > 0, \Phi = 0 \) solving the first order conditions we find that \( r^* = 1 - \frac{2}{a^2}, r > 0 \iff a > \sqrt{2}. \) However from the R&D stage we know that the R&D equilibrium is defined for \( 2 - a^2 (1 - r) > 0. \) Plugging \( r^* \) we have that the condition requires \( 0 > 0 \) that is obviously a contradiction.

### A.4 Proof of Propositions 11 and 12

The optimization problem of the government is given by

\[
\begin{align*}
\max_{r,R} W &= CS + \Pi_H \\
&\quad - (1 + \lambda) [RD + r(e_0 - e)D] \\
\text{s.t. } &\quad r \geq 0
\end{align*}
\]

With \( \Phi \) being the Lagrangian multiplier.

For the first solution on the R&D problem we have that variables do not depend on \( r. \) As funds are costly then in equilibrium \( r^* = 0. \) So we are left with an optimization problem in one variable.
Solving the first order condition with respect to $R$ we have that the optimal solution is characterized by

$$r^* = 0$$
$$R^* = \frac{(a - \lambda) q_0}{(2\lambda + 1 - a) a}$$

The second order conditions require that $\frac{\partial^2 W}{\partial R^2} < 0 \iff 1 + 2\lambda - a > 0$. We further know that from the R&D stage the R&D equilibrium was defined for

$$e_0 < a [q_0 + aR] (1 - r) \implies e_0 < \frac{aq_0 (1 + \lambda)}{2\lambda + 1 - a}$$

For the second solution of the R&D problem following the same process as above we find that the first order condition for $r$ is always negative and therefore $r^* = 0$. Solving the first order condition of $R$ we find that

$$R^* = \frac{(ae_0 - q_0) (a + a^2\lambda - \lambda)}{a (-1 - 2\lambda + 2a^2\lambda + a + a^2)}$$

Second order condition requires that $|\mathcal{H}| = -a^2 \frac{(-1 + a^2 + 2\lambda a^2 - 2\lambda + a)}{(-1 + a^2)^2} > 0 \implies 2 + a^2 (r - 1) > 0 \implies a \leq \sqrt{2}$

$$e_0 \geq a [q_0 + aR] (1 - r) \implies e_0 \geq \frac{aq_0 (1 + \lambda)}{2\lambda + 1 - a - \lambda a^2}$$

References


