Reference pricing versus Co-Payment in the Pharmaceutical industry: price, quality and market coverage: Technical Appendix

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A Co-payment Reimbursement

We first investigate the case where the expenses in pharmaceuticals are reimbursed through a co-payment system: patients are reimbursed a fraction $0 \leq \alpha \leq 1$ of drug prices.

A.1 Second Stage: the Price Game

In this stage firms compete simultaneously in prices. With p_i the drug price of firm i, and D_i the demand faced by firm i, the duopolists profit functions π_i are given by

$$\pi_i = p_i D_i - \frac{q_i^2}{2} \qquad i = 1, 2 \tag{1}$$

As mentioned before, as the demand function is kinked, firms' profit functions are segmented. Thus, given the demand function, if

$$0 \leq p_1 \leq p_2 + \frac{q_1 - q_2}{(1-\alpha)} + \frac{x_1 - x_2}{(1-\alpha)} \tag{2}$$

The firm will be a monopolist and the profit function is given by,

$$\pi_1 = p_1 (z_4 - z_1) - \frac{q_1^2}{2}$$

Otherwise, if

$$p_2 + \frac{q_1 - q_2}{(1-\alpha)} + \frac{x_1 - x_2}{(1-\alpha)} \leq p_1 \leq \frac{q_1 + q_2 + x_1 - x_2 - (1-\alpha) p_2 + 2k}{1-\alpha} \tag{3}$$

the market structure will be competitive and the firm 1 profit is given by,

$$\pi_1 = p_1 (\overline{z} - z_1) - \frac{q_1^2}{2}$$

Finally, if

$$\text{if} \quad \frac{q_1+q_2+x_1-x_2-(1-\alpha)\,p_2+2k}{1-\alpha} \leq p_1 \leq \frac{k+q_1+x_1}{1-\alpha} \tag{4}$$

the market structure will be characterized by local monopolies and firm 1 profit function is given by,

$$\pi_1 = p_1\,(z_3 - z_1) - \frac{q_1^2}{2}$$

In fact, for $0 \leq p_1 \leq p_2 + \frac{q_1-q_2}{(1-\alpha)} + \frac{x_1-x_2}{(1-\alpha)}$ firm one faces the whole demand earning monopolist profits.

For $p_i \in \left[p_j + \frac{x_1-x_2+qi-q_j}{(1-\alpha)}, \frac{q_i^{rp}+q_j+x_1-x_2-(1-\alpha)p_j+2k}{1-\alpha}\right] (i,j = 1,2,\ i \neq j)$ firms compete for the marginal consumer z located in the centre of the market. This case is the one denoted as the *competitive scenario.*

Finally, for $p_i \in \left[\frac{q_i^{rp}+q_j+x_1-x_2-(1-\alpha)p_j+2k}{1-\alpha}, \frac{k+q_1}{1-\alpha}\right]$ $i,j = 1,2,\ i \neq j$ firms do not compete for consumers forming *local monopolies*, that is, are monopolists in a demand subset.

Analogously for firm 2,

$$\pi_2 = p_2\,(z_4 - z_1) - \frac{q_2^2}{2}$$

$$if \quad 0 \leq p_2 \leq p_1 + \frac{q_2-q_1}{(1-\alpha)} + \frac{x_1-x_2}{(1-\alpha)} \tag{5}$$

$$\pi_2 = p_2\,(z_4 - z) - \frac{q_2^2}{2}$$

$$if \quad p_1 + \frac{q_2-q_1}{(1-\alpha)} + \frac{x_1-x_2}{(1-\alpha)} \leq p_2 \leq \frac{q_2+q_1+x_1-x_2-(1-\alpha)\,p_1+2k}{1-\alpha} \tag{6}$$

$$\pi_2 = p_2\,(z_4 - z_2) - \frac{q_2^2}{2}$$

$$if \quad \frac{q_2+q_1+x_1-x_2-(1-\alpha)\,p_1+2k}{1-\alpha} 0 \leq p_2 \leq \frac{k+q_2+x_2}{1-\alpha} \tag{7}$$

We will now look for the Nash Equilibria in pure strategies (NE) of the simultaneous moves price game played by the two firms in the last stage of the overall game.

A price p_i such that $0 \leq p_i \leq p_j + \frac{q_i^{rp}-q_j}{(1-\alpha)} + \frac{x_i-x_j}{(1-\alpha)}$ $i,j = 1,2$ and $i \neq j$, can never constitute a pure strategies Nash Equilibrium of the price subgame. The proof consists of a standard undercutting argument. Within this price range one of the firms will be a monopolists and the second firm would be out of the market, earning zero profits. The latter will always have incentives to undercut on the monopolist price strategy in order to gain the whole demand.

Having ruled out the monopolist case as a candidate Nash equilibrium in the price subgame, we will then focus on the two polar cases: *competitive scenario,* either with partial or full coverage, and the *local monopolists scenario.* Moreover, in asymmetric locations, we will only study the case for which $1 - x_1 - x_2 > 0$. The other case, $1 - x_1 - x_2 < 0$ is symmetric and therefore results qualitatively the same.

Maximizing profits with respect to prices and solving the first order conditions, the Nash Equilibrium in the price game for these two cases is summarized in the propositions that follow,

Proposition 1 *For* $p_i \in \left[\frac{q_i^{rp}+q_j+x_i-x_j-(1-\alpha)p_j+2k}{1-\alpha}, \frac{k+q_i^{rp}}{1-\alpha}\right]$ *with* $i, j = 1, 2$ *and* $i \neq j$ *the market is characterized by two local monopolists and the Nash Equilibrium in the price stage is given by*[1]*,*

$$p_i^{*lm} = \frac{k + q_i^{rp}}{2(1-\alpha)} \qquad i = 1, 2 \tag{8}$$

For $p_i \in \left[\frac{q_i^{rp}+q_j+x_1-x_2-(1-\alpha)p_j+2k}{1-\alpha}, \frac{k+q_i^{rp}}{1-\alpha}\right]$ with $i, j = 1, 2$ and $i \neq j$[2] firms do not compete for the marginal consumer. There are consumers in the centre of the market that are better off by not buying any of the drugs. Hence, firms behave like local monopolists.

Notice, however, that if, for some parameters' configuration, $\frac{k+q_i^{rp}}{2(1-\alpha)}$ does not fall in the interval $\left[\frac{q_i^{rp}+q_j+x_1-x_2-(1-\alpha)p_j+2k}{1-\alpha}, \frac{k+q_i^{rp}}{1-\alpha}\right]$, then the local monopolist Nash equilibrium can not exist in the price subgame.

In such a case, having ruled out the existence of a NE where just one firm covers the whole market, a Nash Equilibrium of the price subgame, if any, needs to be in the last, competitive scenario.

The latter occurs whenever $p_i \in \left[p_j + \frac{q_i^{rp}-q_j}{(1-\alpha)} + \frac{x_1-x_2}{(1-\alpha)}, \frac{q_i^{rp}+q_j+x_1-x_2-(1-\alpha)p_j+2k}{1-\alpha}\right]$, firms profit functions being $\pi_1 = p_1(z - z_1) - \frac{q_1^2}{2}$ and $\pi_2 = p_2(z_4 - z) - \frac{q_2^2}{2}$.

Proposition 2 *For* $p_i \in \left[p_j + \frac{q_i^{rp}-q_j}{(1-\alpha)} + \frac{x_1-x_2}{(1-\alpha)}, \frac{q_i^{rp}+q_j+x_1-x_2-(1-\alpha)p_j+2k}{1-\alpha}\right]$[3] *the market is competitive and the Nash Equilibrium in the price stage is* [4]

$$\begin{aligned} p_1^{c*} &= \frac{7(x_1 - x_2) + 3q_2 - 17q_1 - 14k}{35(\alpha - 1)} \\ p_2^{c*} &= \frac{7(x_1 - x_2) + 3q_1 - 17q_2 - 14k}{35(\alpha - 1)} \end{aligned} \tag{9}$$

[1] Second order conditions are always satisfied. Indeed, $\frac{\partial^2 \pi_i}{\partial p_i^2} = -4 + 4\alpha < 0$.

[2] From market structure conditions

[3] This is a market structure condition. For reservation prices that satisfy this condition the market will be competitive

[4] Second order conditions always satisfied indeed, $\frac{\partial^2 \pi_i}{\partial p_i^2} = -3 + 3\alpha < 0$.

A.2 First Stage: the Quality Game

Plugging the above found NE prices for each scenario in the relative range of the firms' profit functions, and maximizing with respect to qualities, we obtain the optimal quality levels for the given prices. Substituting back these optimal qualities in the Nash Equilibrium prices, we are then able to fully characterize the subgame perfect NE of the two-stage quality-then-price game. The sub game perfect Nash Equilibria will depend on the co-payment rate. Given that the second order conditions are satisfied for $\alpha \in [0, 0.29]$ the analysis will be done within this range. More precisely, we will have two sets of results one for $\alpha \in [0, 0.16]$ and other for $\alpha \in [0.16, 0.29]$.

Therefore for $\alpha \in [0, 0.16]$ under a competitive scenario two types of equilibrium arise: an equilibrium with full market coverage and an asymmetric equilibrium with partial market coverage. Note that an equilibrium with partial market coverage will never arise. The SPNE is characterized on the proposition that follows.

Proposition 3 *Under a competitive scenario, for* $k \in [\max\{k_{14c}, k_{ii4c}, k_{1i4c}\}, \min\{k_{19c}, k_{2i4c}, k_{15c}\}]$ *and under condition* Ω_i *or condition* Ω_{4ii}*, the SPNE is characterized by*

$$q_1^* = \frac{1960\alpha x_1 + 490\alpha x_2 + 501k - 1470\alpha k - 583x_1 - 82x_2}{(1295\alpha - 326)} \tag{10}$$

$$q_2^* = \frac{980\alpha x_1 + 245\alpha x_2 - 2172k - 735\alpha k + 1366x_1 - 806x_2}{3\,(1295\alpha - 326)} \tag{11}$$

$$p_1^* = \frac{665\alpha x_1 + 490\alpha x_2 + 175k - 175\alpha k - 257x_1 - 82x_2}{(1-\alpha)\,(1295\alpha - 326)}$$

$$p_2^* = \frac{-805\alpha x_1 + 770\alpha x_2 - 1575k + 1575\alpha k + 1009x_1 - 566x_2}{3\,(1-\alpha)\,(1295\alpha - 326)} \tag{12}$$

For $k \in [k_{2c}, k_{3c}]$ *and under condition* Ω[5]*, the market is fully covered and the subgame perfect Nash equilibrium qualities and prices are*[6]

$$q_1^* = \frac{5x_1 + 2x_2 - 1 - 3k}{3} \tag{13}$$

$$q_2^* = \frac{6 - 3k - 2x_1 - 5x_2}{3} \tag{14}$$

$$p_1^* = \frac{2x_1 + 2x_2 - 1}{3\,(1-\alpha)}$$

$$p_2^* = \frac{3 - 2x_1 - 2x_2}{3\,(1-\alpha)} \tag{15}$$

Proof. For the co-payment the analysis is analogous to the reference pricing case. The second order conditions on the price game are always satisfied for

[5] Equilibrium holding for $x_1 \in \left[x_2 - \frac{1}{2}, \frac{1}{2}\right]$ and $x_2 \in \left[\frac{1}{2}, 1\right]$

[6] Second order conditions satisfied for $\frac{\partial \pi_i^2}{\partial q_i^2} = \frac{1225\alpha - 35}{1225(1-\alpha)} < 0 \Longrightarrow \alpha < 0.29$

$\alpha \in [0,1]$. Nevertheless, in the quality stage the second order conditions are satisfied for $\alpha < 0.29$. In this interval, the optima will vary depending on whether $\alpha \in [0,0.16]$ or $\alpha \in [0.16,0.29]$. Consequently, the analysis will be developed for the two ranges of the co-payment separately. Lets consider now the case $\alpha \in [0,0.16]$ .The profit function of firm 1 and firm 2 are given by

$$\begin{aligned}
\pi_1 &= p_1\left(\overline{z}-z_1\right)-\frac{q_1^2}{2} \\
\pi_2 &= p_2\left(z_4-\overline{z}\right)-\frac{q_2^2}{2}
\end{aligned}$$

The conditions that define the competitive scenario are i.e.,[7]

$$p_2+\frac{q_1-q_2}{1-\alpha}+\frac{x_1-x_2}{1-\alpha} \leq p_1 \leq \frac{(q_1+q_2)}{1-\alpha}+\frac{x_1-x_2}{1-\alpha}-p_2+2k \tag{16}$$

$$p_1+\frac{q_2-q_1}{1-\alpha}+\frac{x_1-x_2}{1-\alpha} \leq p_2 \leq \frac{q_2+q_1}{1-\alpha}+\frac{x_1-x_2}{1-\alpha}-p_1+2k \tag{17}$$

These conditions can be written as

$$p_1-\frac{q_1+q_2}{1-\alpha}-\frac{x_1-x_2}{1-\alpha}+p_2-\frac{2k}{1-\alpha} \leq 0 \tag{C1}$$

$$p_2+\frac{q_1-q_2}{1-\alpha}+\frac{x_1-x_2}{1-\alpha}-p_1 \leq 0 \tag{C2}$$

$$p_2-\frac{q_2+q_1}{1-\alpha}-\frac{x_1-x_2}{1-\alpha}+p_1-\frac{2k}{1-\alpha} \leq 0 \tag{C3}$$

$$p_1+\frac{q_2-q_1}{1-\alpha}+\frac{x_1-x_2}{1-\alpha}-p_2 \leq 0 \tag{C4}$$

Note that $C1 = C3$. Depending on the parameters the SPNE will differ. We will then study four cases. Case 1.1: full market coverage, i.e. $\lambda > 0, \Psi > 0$. Case 1.2: symmetric partial market coverage, i.e., $\lambda = \Psi = 0$. Case 1.3: asymmetric partial market coverage with $\lambda = 0$ and $\Psi > 0$.Case 1.4: asymmetric partial market coverage with $\lambda > 0$ and $\Psi = 0$. In Case 1.1: Full Market Coverage solving the first order conditions we find that the SPNE is given by

$$\begin{aligned}
q_1^* &= \frac{5x_1+2x_2-1-3k}{3} \\
q_2^* &= \frac{6-3k-2x_1-5x_2}{3} \\
p_1^* &= \frac{2x_1+2x_2-1}{3\left(1-\alpha\right)} \\
p_2^* &= \frac{3-2x_1-2x_2}{3\left(1-\alpha\right)}
\end{aligned} \tag{18}$$

[7] As these conditions apply to all cases defined under a competitive scenario we will throughout the analysis make reference to them

And the Lagrangian multipliers by

$$\begin{aligned}
\lambda &= \frac{119x_2 - 406x_1 + 525k(1-\alpha) + 595x_2\alpha + 1120x_1\alpha - 39 - 420\alpha}{315(1-\alpha)} \\
\Psi &= \frac{-119x_1 + 406x_2 + 525k(1-\alpha) - 595x_1\alpha - 1120x_2\alpha - 326 + 1295\alpha}{315(1-\alpha)}
\end{aligned}$$

Therefore we have,

$$\begin{aligned}
q_1^* &> 0 \Rightarrow k < k_{3c} \\
q_2^* &> 0 \Rightarrow k < k_{4c} \\
p_1^* &> 0 \Rightarrow x_1 + x_2 > \frac{1}{2} \\
p_2^* &> 0 \Rightarrow x_1 + x_2 < \frac{3}{2} \\
\lambda &> 0 \Rightarrow k > k_{1c} \\
\Psi &> 0 \Rightarrow k > k_{2c}
\end{aligned}$$

With k_{3c} the instant utility from treatment that solves $q_1^* = 0$, k_4 the instant utility from treatment that solves $q_3^* = 0$, k_{1c} the instant utility from treatment that solves $\lambda = 0$ and k_{2c} the instant utility from treatment that solves $\Psi = 0$. Therefore a maximum exists for

$$k \in [\max\{k_{1c}, k_{2c}\}, \min\{k_{3c}, k_{4c}\}]$$

It is straightforward to check that,

$$\begin{aligned}
\max\{k_{1c}, k_{2c}\} &= k_{2c} \\
\min\{k_{3c}, k_{4c}\} &= k_{3c}
\end{aligned}$$

Thus an equilibrium exists for $k \in [k_{2c}, k_{3c}]$. Checking that $[k_{2c}, k_{3c}]$ is non empty

$$k_{2c} - k_{3c} < 0 \Leftrightarrow x_1 + x_2 > \frac{\alpha - 167/490}{\alpha - 252/490}$$

The market structure conditions hold for the set of conditions Ω with Ω defined by,

$$\Omega = \begin{cases} x_1 \in \left[x_2 - \frac{1}{2}, \frac{1}{2}\right] \\ x_2 \in \left[\frac{1}{2}, 1\right] \end{cases}$$

Therefore, summarizing, an equilibrium with full market coverage described by (35) exists for $k \in [k_{2c}, k_{3c}]$ and if Ω holds.

Case 1.2 Symmetric Partial Market Coverage. Solving the first order conditions we find that the SPNE is given by,

$$\begin{aligned}
q_i^* &= \frac{51(x_1 - x_2 - 2k)}{175\alpha - 73} \qquad (19) \\
p_i^* &= \frac{35(x_1 - x_2 - 2k)}{175\alpha - 73}
\end{aligned}$$

And plugging q_i^* and p_i^* into the first order conditions on the Lagrangian multipliers we have that,

$$\begin{aligned}\frac{\partial L_1}{\partial \lambda} &= \frac{16x_2 - 89x_1 + 105k(1-\alpha) + 140\alpha x_1 + 35\alpha x_2}{175c - 73} \\ \frac{\partial L_2}{\partial \Psi} &= \frac{16x_1 - 89x_2 - 105k(1-\alpha) + 140\alpha x_2 + 35\alpha x_1 + 73}{175c - 73}\end{aligned}$$

By SOCs $175c - 73 < 0$, therefore $q_i^* > 0$ and $p_i^* > 0$ for $i = 1, 2$. Moreover we need to control whether $\frac{\partial L_1}{\partial \lambda} > 0$ and $\frac{\partial L_2}{\partial \Psi} > 0$,

$$\begin{aligned}\frac{\partial L_1}{\partial \lambda} &> 0 \Rightarrow k < k_{6c} \\ \frac{\partial L_2}{\partial \Psi} &> 0 \Rightarrow k < k_{7c}\end{aligned}$$

With k_{6c} the instant utility from treatment that solves $\frac{\partial L_1}{\partial \lambda} = 0$ and k_{7c} the instant utility from treatment that solves $\Psi = 0$. Therefore a maximum exists for

$$k < \min\{k_{6c}, k_{7c}\}$$

Studying the difference between these two thresholds, i.e., $k_{6c} - k_{7c}$ we have that,

$$k_{6c} - k_{7c} = \frac{(1 - x_1 - x_2)(73 - 175\alpha)}{105(\alpha - 1)}$$

and, since $1 > x_1 + x_2$ and, by SOCs, $73 - 175\alpha > 0$, for $\alpha \in [0, 1]$ we have that $k_{6c} < k_{7c}$. Hence, an equilibrium exists for $k < k_{6c}$. The market structure conditions hold for$k < \frac{x_1 - x_2}{2}$.As, by assumption, $x_1 < x_2$ for $k > 0$ this condition never holds, therefore there is no equilibrium with symmetric partial market coverage.

In case Case 1.3 proceeding in an analogous way as in the previous case we have that equilibrium in qualities, in this case, is characterized by,

$$\begin{aligned}q_1^* &= \frac{1225\alpha + 560 - 2172k - 1366x_2 + 806x_1 - 980x_2\alpha - 245\alpha x_1 - 735k\alpha}{3(1295\alpha - 326)} \\ q_2^* &= \frac{2450\alpha - 665 + 501k + 583x_2 + 82x_1 - 1960x_2\alpha - 490\alpha x_1 - 1470k\alpha}{(1295\alpha - 326)} \\ \Psi &= \frac{1}{9(\alpha - 1)(1295\alpha - 326)}\big(-2993 - 656x_1 + 3649x_2 + 4305k + 25060\alpha \\ &\quad 2485\alpha x_1 - 30030k\alpha - 27545\alpha x_2 + 34300x_2\alpha^2 - 42875\alpha^2 + 8575\alpha^2 x_1 + \\ &\quad +25725k\alpha^2\big) \\ \lambda &= 0\end{aligned}$$

And the equilibrium in prices,

$$\begin{aligned}p_1^* &= \frac{-805x_2\alpha + 770\alpha x_1 - 1575k\alpha + 35\alpha - 443 + 1575k + 1009x_2 - 566x_1}{3(1295\alpha - 326)(-1 + \alpha)} \\ p_2^* &= \frac{665x_2\alpha + 490\alpha x_1 + 175k\alpha - 1155\alpha + 339 - 175k - 257x_2 - 82x_1}{(1295\alpha - 326)(-1 + \alpha)}\end{aligned}$$

The conditions that need to be satisfied are given by,

$$\begin{aligned}
q_1^* &> 0 \Rightarrow k > k_{9c} \\
q_2^* &> 0 \Rightarrow k > k_{10c} \\
p_1^* &> 0 \Rightarrow k > k_{11c} \\
p_2^* &> 0 \Rightarrow k < k_{12c} \\
\frac{\partial L_1}{\partial \lambda} &> 0 \Rightarrow k > k_{8c} \\
\Psi &> 0 \Rightarrow k < k_{13c}
\end{aligned}$$

with k_8, $k_9, k_{10}, k_{11}, k_{12}$ and k_{13} the instant utility from treatment thresholds that solve, respectively, $\lambda = 0$, $q_1^* = 0$, $q_2^* = 0$, $p_1^* = 0$, $p_2^* = 0$ and $\Psi = 0$ .Therefore, a maximum exists for

$$k \in [\max \{k_{8c}, k_{9c}, k_{10c}, k_{11c}\}, \min \{k_{12c}, k_{13c}\}]$$

After checking the differences between all the thresholds, and we conclude that

$$\begin{aligned}
\max \{k_{8c}, k_{9c}, k_{10c}, k_{11c}\} &= k_{10c} \\
\min \{k_{12c}, k_{13c}\} &= k_{13c}
\end{aligned}$$

Thus, an equilibrium exists for $k \in [k_{10c}, k_{13c}]$. However, by checking that $[k_{10c}, k_{13c}]$ is non empty, it turns out that,

$$k_{10c} - k_{13c} > 0$$

Therefore this contradicts the condition needed for $[k_{10c}, k_{13c}]$ being non-empty , i.e. $k_{13c} > k_{10c}$ and consequently no equilibrium with asymmetric partial market coverage (with the right hand side of the market fully covered) exists.

Finally for Case 1.4 Asymmetric Partial Market Coverage, proceeding in an analogous way as in the previous case we have that equilibrium in this case is characterized by,

$$\begin{aligned}
q_1^* &= \frac{1960\alpha x_1 + 490\alpha x_2 + 501k - 583x_1 - 82x_2 - 1470\alpha k}{1295\alpha - 236} \\
q_2^* &= \frac{980\alpha x_1 + 245\alpha x_2 - 2172k + 1366x_1 - 806x_2 - 735\alpha k}{3(1295\alpha - 236)} \\
\lambda &= \frac{1}{9(1-\alpha)(1295\alpha - 326)} (3649x_1 - 656x_2 - 4305k + 30030\alpha k - 27545\alpha x_1 + \\
&\quad +2485\alpha x_2 - 25725\alpha^2 k + 34300\alpha^2 x_1 + 8575\alpha^2 x_2) \\
\Psi &= 0
\end{aligned}$$

And the equilibrium in prices,

$$\begin{aligned}
p_1^* &= \frac{-805x_2\alpha + 770\alpha x_1 - 1575k\alpha + 35\alpha - 443 + 1575k + 1009x_2 - 566x_1}{3(1295\alpha - 326)(-1+\alpha)} \\
p_2^* &= \frac{665x_2\alpha + 490\alpha x_1 + 175k\alpha - 1155\alpha + 339 - 175k - 257x_2 - 82x_1}{(1295\alpha - 326)(-1+\alpha)}
\end{aligned}$$

Checking the conditions that need to hold in equilibrium, returns a set of thresholds in k,

$$\begin{aligned} q_1^* &> 0 \Rightarrow k < k_{15c} \\ q_2^* &> 0 \Rightarrow k > k_{16c} \\ p_1^* &> 0 \Rightarrow k < k_{17c} \\ p_2^* &> 0 \Rightarrow k > k_{18c} \\ \lambda &> 0 \Rightarrow k > k_{14c} \\ \frac{\partial L_2}{\partial \Psi} &> 0 \Rightarrow k < k_{19c} \end{aligned}$$

with k_{14c}, k_{15c}, k_{16c}, k_{17c}, k_{18c} and k_{19c} the instant utility from treatment thresholds that solve, respectively, $\lambda = 0$, $q_1^* = 0$, $q_2^* = 0$, $p_1^* = 0$, $p_2^* = 0$ and $\frac{\partial L_2}{\partial \Psi} = 0$. Therefore, a maximum exists for

$$k \in [\max\{k_{14c}, k_{16c}, k_{18c}\}, \min\{k_{15c}, k_{17c}, k_{19c}\}]$$

Analyzing the differences between the thresholds $k_{14c}, k_{16c}, k_{18c}$ and the differences between $k_{15c}, k_{17c}, k_{19c}$ we find that,

$$\begin{aligned} \max\{k_{14c}, k_{16c}, k_{18c}\} &= k_{14c} \\ \min\{k_{15c}, k_{17c}, k_{19c}\} &= \begin{cases} k_{15} & \text{for } x_1 + x_2 < \frac{\alpha - 167/490}{\alpha - 18/35} \\ k_{19} & \text{for } x_1 + x_2 > \frac{\alpha - 167/490}{\alpha - 18/35} \end{cases} \end{aligned}$$

so that an equilibrium exists for $k \in [k_{14c}, \min\{k_{15c}, k_{19c}\}]$. Checking that

$$[k_{14c}, \min\{k_{15c}, k_{19c}\}]$$

is non empty by computing the differences between k_{14c} and $\{k_{15c}, k_{19c}\}$ we find that,

$$\begin{aligned} k_{14c} - k_{15c} &< 0 \\ k_{14c} - k_{19c} &< 0 \end{aligned}$$

Therefore, $[k_{14c}, \min\{k_{15c}, k_{19c}\}]$ is non empty. Finally, we still need to check that the market structure conditions are satisfied, i.e.,

$$\begin{aligned} k &> \max\{k_{1i4c}, k_{ii4c}\} \\ k &< k_{2i4c} \end{aligned}$$

Where k_{ii4c}, k_{1i4c} and k_{2i4c} stand for the thresholds that solve, respectively, $C1 = C3 = 0, C2 = 0$ and $C4 = 0$. Therefore, we need to check if the intersection of the two ranges of k defined by $[k_{14c}, \min\{k_{15c}, k_{19c}\}]$ and $[\max\{k_{1i4c}, k_{ii4c}\}, k_{2i4c}]$ is non-empty. Comparing the thresholds we can conclude that, for $x_1 + x_2 < \frac{\alpha - \frac{167}{490}}{\alpha - \frac{18}{35}}$, the thresholds in k for which the equilibrium exists are described by the set of conditions Ω_{4i} with Ω_{4i} given by,

$$\Omega_{4i} = \begin{cases} k \in [k_{14c}, \min\{k_{2i4c}, k_{15c}\}] & \text{for} \quad x_2 > 2x_1, \quad x_2 < 3x_1 \\ k \in [k_{ii4c}, \min\{k_{2i4c}, k_{15c}\}] & \text{for} \quad x_2 > 2x_1, \quad x_2 > 3x_1 \\ k \in [k_{14c}, \min\{k_{2i4c}, k_{15c}\}] & \text{for} \quad x_2 < 2x_1 \end{cases}$$

For $x_1 + x_2 > \frac{\alpha - \frac{167}{490}}{\alpha - \frac{18}{35}}$ the thresholds in k for which the equilibrium is exists are described by the set of conditions Ω_{4ii} with Ω_{4ii} given by,

$$\Omega_{4ii} = \begin{cases} k \in [k_{ii4c}, k_{19c}] \text{ for } x_2 > \max\left\{\frac{1}{2}, 3x_1\right\}, \quad x_1 > x_2 - \frac{1}{2} \\ k \in [k_{ii4c}, k_{2i4c}] \text{ for } x_2 \in \left[3x_1, \frac{1}{2}\right], \quad x_1 > x_2 - \frac{1}{2} \\ k \in [k_{14c}, k_{19c}] \text{ for } x_2 \in \left[\frac{1}{2}, 3x_1\right], \quad x_1 > x_2 - \frac{1}{2} \\ k \in [k_{14c}, k_{2i4c}] \text{ for } x_2 < \min\left\{\frac{1}{2}, 3x_1\right\}, \quad x_1 > x_2 - \frac{1}{2} \end{cases}$$

where k_{ii4c}, k_{1i4c} and k_{2i4c} stand for the thresholds that solve, respectively, $C1 = C3 = 0, C2 = 0$ and $C4 = 0$. On the other hand for $x_1 < x_2 - \frac{1}{2}$, there exists no equilibrium. Summarizing the results, an asymmetric equilibrium with partial market coverage exists either for $x_1 + x_2 < \frac{\alpha - \frac{167}{490}}{\alpha - \frac{18}{35}}$ under Ω_{4i} or for $x_1 + x_2 > \frac{\alpha - \frac{167}{490}}{\alpha - \frac{18}{35}}$ when Ω_{4ii} is satisfied. By ordering the thresholds of the instant utility from treatment k for which the equilibrium with full market coverage (in 2.1) with the equilibrium with partial market coverage (in 2.4) we find that $k_{3c} > k_{2c} > \max\{k_{19c}, k_{2i4c}, k_{15}c\}$. So that the equilibria never overlap on the same interval of parameters and the equilibrium with full market coverage (2.1) is defined for higher values of k than the equilibrium with partial market coverage (2.4). While the local monopolies equilibrium only arises for much lower values of k then the ones necessary for both equilibrium in the competitive case ■

For low reservation prices, the market will be served by two local monopolists and the SPNE is described in the proposition that follows.

Proposition 4 *For sufficiently low reservation prices firms behave as local monopolists and the SPNE will depend on the level of the instant utility from treatment. For $k < 2x_1 - \overline{Q}$ and $k < x_2 - x_1 - \overline{Q}$ the market is partly covered with non buyers on both extremes of the market and the SPNE is characterized by,*

$$\begin{aligned} q_1^* &= q_2^* = \overline{Q} \\ p_1^* &= p_2^* = \frac{k + \overline{Q}}{2(1-\alpha)} \end{aligned}$$

For $k \in \left[2x_1 - \overline{Q}, 2 - 2x_2 - \overline{Q}\right]$ and $k < 2x_2 - 4x_1 - \overline{Q}$[8] the market is partly covered but all consumers located on the left extreme of the market are covered while on the extreme right of the market there exist consumers not buying any drug. The SPNE is characterized by,

$$\begin{aligned} q_1^* &= 2x_1 - k \\ q_2^* &= \overline{Q} \\ p_1^* &= \frac{x_1}{1-\alpha} \\ p_2^* &= \frac{k + \overline{Q}}{2(1-\alpha)} \end{aligned}$$

[8] These two conditions are compatible for $x_1 \leq \frac{x_2}{3}$

Finally, for $k > 2 - 2x_2 - \overline{Q}$ and $x_1 \leq x_2 - \frac{1}{2}$ the market is partly covered with consumers located around the centre of the market being the only non buyers. The SPNE is given by,

$$\begin{aligned} q_1^* &= 2x_1 - k \\ q_2^* &= 2 - 2x_2 - k \\ p_1^* &= \frac{x_1}{1-\alpha} \\ p_2^* &= \frac{1-x_2}{1-\alpha} \end{aligned}$$

Proof. Local Monopolies. Under Co-payment the conditions that define a local monopolies market structure are given by

$$p_1 - \frac{k + q_1 + x_1}{1-\alpha} \leq 0 \tag{C1}$$

$$\frac{q_1 + q_2 + x_1 - x_2}{1-\alpha} - p_2 + \frac{2k}{1-\alpha} - p_1 \leq 0 \tag{C2}$$

Proceeding in an analogous way as in the reference pricing regime, for the co-payment scenario we will have that, for $k < 2x_1 - \overline{Q}$,

$$\begin{aligned} q_1^* &= q_2^* = \overline{Q} \\ p_1^* &= p_2^* = \frac{k + \overline{Q}}{2(1-\alpha)} \end{aligned} \tag{20}$$

Market structure conditions satisfied for

$$k < k_{ip}$$

Where k_{ip} is the instant utility from treatment that solves $C1 = 0$ and is given by $k_{ip} = x_2 - x_1 - \overline{Q}$. Therefore an equilibrium exists for $k < \min\left\{2x_1 - \overline{Q}, k_{ip}\right\}$ For $k \in \left[2x_1 - \overline{Q}, 2 - 2x_2 - \overline{Q}\right]$

$$\begin{aligned} q_1^* &= 2x_1 - k \\ q_2^* &= \overline{Q} \\ p_1^* &= \frac{x_1}{(1-\alpha)} \\ p_2^* &= \frac{k + \overline{Q}}{2(1-\alpha)} \end{aligned} \tag{21}$$

Market structure conditions satisfied for

$$k < 2x_2 - 4x_1 - \overline{Q}$$

That is compatible with $k \in \left[2x_1 - \overline{Q}, 2 - 2x_2 - \overline{Q}\right]$ as long as $x_1 < \frac{x_2}{3}$. Finally for $k > 2 - 2x_2 - \overline{Q}$

$$\begin{aligned} q_1^* &= 2x_1 - k \\ q_2^* &= 2 - 2x_2 - k \\ p_1^* &= \frac{x_1}{(1-\alpha)} \\ p_2^* &= \frac{1-x_2}{(1-\alpha)} \end{aligned} \tag{22}$$

Market structure conditions satisfied for

$$x_1 \leq x_2 - \frac{1}{2}$$

■

For low reservation prices ($k \in [\max\{k_{14c}, k_{ii4c}, k_{1i4c}\}, \min\{k_{19c}, k_{2i4c}, k_{15c}\}]$) the market is partly covered but the neighborhood of firm 1 is fully covered. Comparing firms' prices and qualities we have

$$\begin{aligned} \Delta p^* &= p_1^* - p_2^* = \frac{20\left(105k\left(1-\alpha\right) + x_1\left(140\alpha - 89\right) + x_2\left(35\alpha + 16\right)\right)}{3\left(1-\alpha\right)\left(1295\alpha - 326\right)} \\ \Delta q^* &= q_1^* - q_2^* = \frac{35\left(105k\left(1-\alpha\right) + x_1\left(140\alpha - 89\right) + x_2\left(35\alpha + 16\right)\right)}{3\left(1295\alpha - 326\right)} \end{aligned} \tag{23}$$

And the market coverage is given by,

$$M^c = \frac{7\left(x_1\left(17 + 85\alpha\right) - 75k\left(1-\alpha\right) - x_2\left(58 - 160\alpha\right)\right)}{\left(1295\alpha - 326\right)} \tag{24}$$

For higher reservation prices, i.e. $k \in [k_2, k_3]$, the market is (endogenously) *fully covered ($M^c = 1$)* and, by standard comparative statics analysis, it immediately follows that

$$\begin{aligned} \frac{\partial q_i^*}{\partial \alpha} &= 0, \qquad \frac{\partial p_i^*}{\partial \alpha} > 0 \\ \frac{\partial q_i^*}{\partial k} &< 0, \qquad \frac{\partial p_i^*}{\partial k} = 0 \end{aligned}$$

the effect of the reimbursement rate on quality is null, but is positive on equilibrium prices. Furthermore, the instant utility from treatment has a negative effect on quality but a nil effect on prices.

Moreover, optimal firms' prices and qualities might differ. These differences are a function of both locations and the reimbursement variable α:

$$\begin{aligned} \Delta p^* &= p_1^* - p_2^* = \frac{4\left(x_1 + x_2 - 1\right)}{3\left(1-\alpha\right)} \\ \Delta q^* &= q_1^* - q_2^* = \frac{7}{3}\left(x_1 + x_2 - 1\right) \end{aligned} \tag{25}$$

Analyzing these quality and price gaps, between firms, we have that $\frac{\partial(p_1^*-p_2^*)}{\partial\alpha} = \frac{(1-x_1+x_2)(3+\alpha)}{(\alpha-1)^3}$ and $\frac{\partial(q_1^*-q_2^*)}{\partial\alpha} = \frac{2(x_1+x_2-1)}{(\alpha-1)^2}$. Hence, for $x_1+x_2 < 1$ (> 1) the drug produced by firm 1 is less (more) expensive and has lower (higher) quality than the drug produced by firm 2. Moreover, the price gap is decreasing (increasing) in the reimbursement variable α.

When the market is served by two local monopolists, for low reservation prices, i.e., $k < 2x_1 - \overline{Q}$, firms pricing and quality strategies are equal and the market is partly covered with consumers on both sides of the market not consuming any of the drugs. The market coverage is given by $M^c = 2k + 2\overline{Q}$.

For $k \in \left[2x_1 - \overline{Q}, 2 - 2x_2 - \overline{Q}\right]$, the price and quality gaps are given by,

$$\begin{aligned}
\Delta p^* &= p_1^* - p_2^* = \frac{2x_1 - \overline{Q} - k}{(1-\alpha)} \\
\Delta q^* &= q_1^* - q_2^* = 2x_1 - 2\overline{Q} - k
\end{aligned}$$

and the market coverage is given by $M^c = 2k + 2\overline{Q} + 2x_1$.

As this equilibrium exists for $k \in \left[2x_1 - \overline{Q}, 2 - 2x_2 - \overline{Q}\right]$ firm 1 sets a lower price and quality than firm 2, i.e., $\Delta p^* < 0$ and $\Delta q^* < 0$. Indeed, for $x_1+x_2 < 1$, firm 2 has a locational advantage and thus higher market power allowing for higher prices.

Finally, also for $k > 2 - 2x_2 - \overline{Q}$ drugs' prices and qualities differ among firms. Indeed,

$$\begin{aligned}
\Delta p^* &= p_1^* - p_2^* = \frac{x_1 + x_2 - 1}{(1-\alpha)} \\
\Delta q^* &= q_1^* - q_2^* = 2\left(x_1 + x_2 - 1\right)
\end{aligned}$$

For $x_1 + x_2 < 1$ (> 1) drug 1 is sold at a lower (higher) price and quality than drug 2, i.e., $\Delta p^* < 0$ (> 0) and $\Delta q^* < 0$ (> 0). Market coverage is given by $M^c = 2 - 2x_2 + 2x_1$.

We will now describe the results for the remaining range of co-payment rates, i.e. for $\alpha \in [0.16, 0.29]$.

For higher co-payment rates, i.e. $\alpha \in [0.16, 0.29]$, the above described competitive equilibria (with asymmetric partial market coverage and full market coverage) will still hold (even though the range of the instant utility from treatment for which they exist will differ) but the local monopolies equilibria will no longer exist[9]. Additionally, the existence of an equilibrium with partial market coverage will depend on the relation between firms locations, namely on whether $x_1 > \frac{x_2}{3}$ or $x_1 \leq \frac{x_2}{3}$ holds. Therefore, the SPNE in this case is given by,

[9] Note that for some parameter configurations we could have that by increasing the reservation prices k the market structure would switch from competitive to local monopolies. Nevertheless, allowing for this possibility would lead to further sub-cases that would not bring further insight on the qualitative results despite of complicating even further the analysis. Therefore we have restrained the analysis from these cases and focus on the range of parameters for which they will not arise.

Proposition 5 *For* $x_1 < \frac{x_2}{3}$ *and for* $k \in [0, k_{6c}]$ *the market is asymmetrically partially covered and the SPNE is characterized by*

$$q_1^* = \frac{1960\alpha x_1 + 490\alpha x_2 + 501k - 1470\alpha k - 583x_1 - 82x_2}{(1295\alpha - 326)} \quad (26)$$
$$q_2^* = \frac{980\alpha x_1 + 245\alpha x_2 - 2172k - 735\alpha k + 1366x_1 - 806x_2}{3\,(1295\alpha - 326)} \quad (27)$$
$$p_1^* = \frac{665\alpha x_1 + 490\alpha x_2 + 175k - 175\alpha k - 257x_1 - 82x_2}{(1 - \alpha)\,(1295\alpha - 326)}$$
$$p_2^* = \frac{-805\alpha x_1 + 770\alpha x_2 - 1575k + 1575\alpha k + 1009x_1 - 566x_2}{3\,(1 - \alpha)\,(1295\alpha - 326)} \quad (28)$$

For $k \in [k_{1c}, k_{3c}]$ *and under condition* Ω_1 *the market is fully covered and the SPNE is given by,*

$$q_1^* = \frac{5x_1 + 2x_2 - 1 - 3k}{3} \quad (29)$$
$$q_2^* = \frac{6 - 3k - 2x_1 - 5x_2}{3} \quad (30)$$
$$p_1^* = \frac{2x_1 + 2x_2 - 1}{3\,(1 - \alpha)}$$
$$p_2^* = \frac{3 - 2x_1 - 2x_2}{3\,(1 - \alpha)} \quad (31)$$

Hence, for such range of locations we have two separate equilibria each arising within a specific interval of the instant utility from treatment.

On the contrary we will now describe a situation where multiple equilibria can arise.

Proposition 6 *For* $\frac{x_2}{3} < x_1 < 0.46x_2$*, for* $k \in [0, k_{6c}]$ *we have multiple equilibria, one with symmetric and another with asymmetric partial market coverage. The sub game perfect Nash equilibria is given by,*

$$q_i^* = \frac{51\,(x_1 - x_2 - 2k)}{175c - 73} \quad (32)$$
$$p_i^* = \frac{35\,(x_1 - x_2 - 2k)}{175c - 73}$$

while the SPNE is given by (26). Within the same range of locations, $\frac{x_2}{3} < x_1 < 0.46x_2$*, but for* $k \in [k_{1c}, k_{3c}]$ *instead, there still exists a SPNE with full market coverage characterized by (29).*

Proof. Proof Follows below ■

Finally, results remain qualitatively the same for $x_1 > 0.46x_2$ with the only prominent difference that there is an interval of (low) values of k within which only the equilibrium with partial market coverage exists.

Proposition 7 *For $x_1 > 0.46x_2$ and $k \in [0, \min\{k_{ii4c}, k_{1i4c}\}]$ there is a unique SPNE characterized by partial coverage (32). For $k \in [\max\{k_{ii4c}, k_{1i4c}\}, k_6c]$ there are multiple equilibria one with symmetric and another asymmetric partial market coverage, respectively characterized by (32) and (26). Finally for $k \in [k_{1c}, k_{3c}]$ the market is fully covered and the SPNE is given by (29).*

Proof. Proof Follows below ■

Analyzing the results described in the propositions, for $x_1 \le \frac{x_2}{3}$ and $k \in [0, k_{6c}]$ the market is partially and asymmetrically covered and the price and quality gaps are given by (23) and the market coverage by (24). For $k \in [k_{1c}, k_{3c}]$ and Ω_1 the market is endogenously fully covered ($M = 1$) and the price and quality gaps are given by (25).

For $x_1 > \frac{x_2}{3}$ a new equilibrium exists (under the conditions specified in propositions 6 and 7). When this equilibrium holds firms pricing and quality strategies are equal, implying null quality and price gaps, i.e. $\Delta p^* = \Delta q^* = 0$. The market is partially covered and the number of consumers buying a drug is given by,

$$M^c = \frac{105\,(2k + x_2 - x_1)\,(1-\alpha)}{73 - 175\alpha} < 1$$

Now follows the proof of these three last propositions.

Proof. Recall from proof of proposition 20 that the second order conditions on the price game are always satisfied for $\alpha \in [0, 1]$. Nevertheless, in the quality stage the second order conditions are satisfied for $\alpha < 0.29$. In this interval, the optima will vary depending on whether $\alpha \in [0, 0.16]$ or $\alpha \in [0.16, 0.29]$. Having previously solved the case for which.$\alpha \in [0, 0.16]$, lets now consider the case $\alpha \in [0.16, 0.29]$.The profit function of firm 1 and firm 2 are given by

$$\begin{aligned}
\pi_1 &= p_1\left(\overline{z} - z_1\right) - \frac{q_1^2}{2} \\
\pi_2 &= p_2\left(z_4 - \overline{z}\right) - \frac{q_2^2}{2}
\end{aligned}$$

For $\alpha \in [0.16, 0.29]$ the conditions that define the competitive scenario are the same as the ones defined under Case 1, i.e.,

$$p_2 + \frac{q_1 - q_2}{1-\alpha} + \frac{x_1 - x_2}{1-\alpha} \le p_1 \le \frac{(q_1 + q_2)}{1-\alpha} + \frac{x_1 - x_2}{1-\alpha} - p_2 + 2k \quad (33)$$

$$p_1 + \frac{q_2 - q_1}{1-\alpha} + \frac{x_1 - x_2}{1-\alpha} \le p_2 \le \frac{q_2 + q_1}{1-\alpha} + \frac{x_1 - x_2}{1-\alpha} - p_1 + 2k \quad (34)$$

These conditions can be written as

$$p_1 - \frac{q_1 + q_2}{1-\alpha} - \frac{x_1 - x_2}{1-\alpha} + p_2 - \frac{2k}{1-\alpha} \le 0 \quad \text{(C1)}$$

$$p_2 + \frac{q_1 - q_2}{1-\alpha} + \frac{x_1 - x_2}{1-\alpha} - p_1 \le 0 \quad \text{(C2)}$$

$$p_2 - \frac{q_2 + q_1}{1-\alpha} - \frac{x_1 - x_2}{1-\alpha} + p_1 - \frac{2k}{1-\alpha} \le 0 \quad \text{(C3)}$$

$$p_1 + \frac{q_2 - q_1}{1-\alpha} + \frac{x_1 - x_2}{1-\alpha} - p_2 \le 0 \quad \text{(C4)}$$

Note that $C1 = C3$.

Case 2.1 Full Market Coverage. Solving the first order conditions we find that the SPNE is given by

$$q_1^* = \frac{5x_1 + 2x_2 - 1 - 3k}{3} \tag{35}$$

$$q_2^* = \frac{6 - 3k - 2x_1 - 5x_2}{3} \tag{36}$$

$$p_1^* = \frac{2x_1 + 2x_2 - 1}{3(1-\alpha)}$$

$$p_2^* = \frac{3 - 2x_1 - 2x_2}{3(1-\alpha)} \tag{37}$$

And the Lagrangian multipliers by

$$\lambda = \frac{119x_2 - 406x_1 + 525k(1-\alpha) + 595x_2\alpha + 1120x_1\alpha - 39 - 420\alpha}{315(1-\alpha)}$$

$$\Psi = \frac{-119x_1 + 406x_2 + 525k(1-\alpha) - 595x_1\alpha - 1120x_2\alpha - 326 + 1295\alpha}{315(1-\alpha)}$$

Therefore, checking the conditions for which that must hold in equilibrium, we have that,

$$\begin{aligned} q_1^* &> 0 \Rightarrow k < k_{3c} \\ q_2^* &> 0 \Rightarrow k < k_{4c} \\ p_1^* &> 0 \Rightarrow x_1 + x_2 > \frac{1}{2} \\ p_2^* &> 0 \Rightarrow x_1 + x_2 < \frac{3}{2} \\ \lambda &> 0 \Rightarrow k > k_{1c} \\ \Psi &> 0 \Rightarrow k > k_{2c} \end{aligned}$$

With k_{3c} the instant utility from treatment that solves $q_1^* = 0$, k_{4c} the instant utility from treatment that solves $q_2^* = 0$, k_1 the instant utility from treatment that solves $\lambda = 0$ and k_{2c} the instant utility from treatment that solves $\Psi = 0$. Therefore a maximum exists for,

$$k \in [\max\{k_{c1}, k_{2c}\}, \min\{k_{3c}, k_{4c}\}]$$

Analyzing the differences between $\{k_{c1}, k_{2c}\}$ and $\{k_{3c}, k_{4c}\}$ we find that,

$$\begin{aligned} \max\{k_{1c}, k_{2c}\} &= k_{1c} \\ \min\{k_{3c}, k_{4c}\} &= k_{3c} \end{aligned}$$

Hence, an equilibrium with full market coverage exists for $k \in [k_{1c}, k_{3c}]$. The market structure conditions hold for the set of conditions Ω_1 defined as,

$$\Omega_1 = \begin{cases} x_1 \in \left[x_2 - \frac{1}{2}, \frac{1}{2}\right] \\ x_2 \in \left[\frac{1}{2}, 1\right] \\ x_1 + x_2 > \frac{1}{2} \end{cases}$$

Case 2.2 Symmetric Partial Market Coverage. Solving the first order conditions we find that the SPNE is given by,

$$\begin{aligned} q_i^* &= \frac{51\left(x_1 - x_2 - 2k\right)}{175\alpha - 73} \\ p_i^* &= \frac{35\left(x_1 - x_2 - 2k\right)}{175\alpha - 73} \end{aligned} \tag{38}$$

And, by plugging q_i^* and p_i^* into the first order conditions on the Lagrangian multipliers we have that,

$$\begin{aligned} \frac{\partial L_1}{\partial \lambda} &= \frac{16x_2 - 89x_1 + 105k\left(1-\alpha\right) + 140\alpha x_1 + 35\alpha x_2}{175\alpha - 73} \\ \frac{\partial L_2}{\partial \Psi} &= \frac{16x_1 - 89x_2 - 105k\left(1-\alpha\right) + 140\alpha x_2 + 35\alpha x_1 + 73}{175\alpha - 73} \end{aligned}$$

By SOCs $175\alpha - 73 < 0$, therefore $q_i^* > 0$ and $p_i^* > 0$ for $i = 1, 2$. Moreover we need to control whether $\frac{\partial L_1}{\partial \lambda} > 0$ and $\frac{\partial L_2}{\partial \Psi} > 0$,

$$\begin{aligned} \frac{\partial L_1}{\partial \lambda} &> 0 \Rightarrow k < k_{6c} \\ \frac{\partial L_2}{\partial \Psi} &> 0 \Rightarrow k < k_{7c} \end{aligned}$$

Where k_{6c} standing for the instant utility from treatment that solves $\frac{\partial L_1}{\partial \lambda} = 0$ and k_{7c} the instant utility from treatment that solves $\Psi = 0$. Therefore, a maximum exists for $k < \min\left\{k_{6c}, k_{7c}\right\}$. Computing the difference between the two thresholds we find that,

$$k_{6c} - k_{7c} = \frac{\left(1 - x_1 - x_2\right)\left(73 - 175\alpha\right)}{105(\alpha - 1)}$$

and as $1 > x_1 + x_2$ and, by SOCs $73 - 175\alpha > 0$, for $\alpha \in [0.16, 0.29]$ we have that $k_{6c} < k_{7c}$. Hence, an equilibrium exists for $k \in [0, k_{6c}]$. This range is non empty if and only if $k_{6c} > 0$, what implies that $x_1 > \frac{x_2}{3}$. The market structure conditions hold for

$$k > \frac{x_1 - x_2}{2}$$

As, by assumption, $x_1 < x_2$ for $k > 0$ this condition always holds and an equilibrium with partial market coverage exists for $k \in [0, k_{6c}]$ and for $x_1 > \frac{x_2}{3}$.

Case 2.3 Asymmetric partial market coverage. Proceeding in an analogous way as in the previous case we have that equilibrium in qualities, in this case, is characterized by,

$$
\begin{aligned}
q_1^* &= \frac{1225\alpha + 560 - 2172k - 1366x_2 + 806x_1 - 980x_2\alpha - 245\alpha x_1 - 735k\alpha}{3\,(1295\alpha - 326)} \\
q_2^* &= \frac{2450\alpha - 665 + 501k + 583x_2 + 82x_1 - 1960x_2\alpha - 490\alpha x_1 - 1470k\alpha}{(1295\alpha - 326)} \\
\Psi &= \frac{1}{9\,(\alpha - 1)\,(1295\alpha - 326)}\big(-2993 - 656x_1 + 3649x_2 + 4305k + 25060\alpha \\
&\quad +2485\alpha x_1 - 30030k\alpha - 27545\alpha x_2 + 34300x_2\alpha^2 - 42875\alpha^2 + 8575\alpha^2 x_1 + \\
&\quad +25725k\alpha^2\big) \\
\lambda &= 0
\end{aligned}
$$

And the equilibrium in prices,

$$
\begin{aligned}
p_1^* &= \frac{-805x_2\alpha + 770\alpha x_1 - 1575k\alpha + 35\alpha - 443 + 1575k + 1009x_2 - 566x_1}{3\,(1295\alpha - 326)\,(-1 + \alpha)} \\
p_2^* &= \frac{665x_2\alpha + 490\alpha x_1 + 175k\alpha - 1155\alpha + 339 - 175k - 257x_2 - 82x_1}{(1295\alpha - 326)\,(-1 + \alpha)}
\end{aligned}
$$

The conditions that need to be satisfied are given by,

$$
\begin{aligned}
q_1^* &> 0 \Rightarrow k > k_{9c} \\
q_2^* &> 0 \Rightarrow k > k_{10c} \\
p_1^* &> 0 \Rightarrow k > k_{11c} \\
p_2^* &> 0 \Rightarrow k < k_{12c} \\
\frac{\partial L_1}{\partial \lambda} &> 0 \Rightarrow k > k_{8c} \\
\Psi &> 0 \Rightarrow k < k_{13c}
\end{aligned}
$$

Where $k_{8c}, k_{9c}, k_{10c}, k_{11c}, k_{12c}$ and k_{13c} the instant utility from treatment thresholds that solve, respectively, $\lambda = 0$, $q_1^* = 0$, $q_2^* = 0$, $p_1^* = 0$, $p_2^* = 0$ and $\Psi = 0$. Therefore, a maximum exists for

$$k \in [\max\{k_{8c}, k_{9c}, k_{10c}, k_{11c}\}, \min\{k_{12c}, k_{13c}\}]$$

Computing the differences between the thresholds we find that,

$$
\begin{aligned}
\max\{k_{8c}, k_{9c}, k_{10c}, k_{11c}\} &= k_{10c} \\
\min\{k_{12c}, k_{13c}\} &= k_{13c}
\end{aligned}
$$

Thus, an equilibrium exists for $k \in [k_{10c}, k_{13c}]$. However, we find that

$$k_{10c} - k_{13c} > 0$$

what contradicts the condition $k_{13c} > k_{10c}$ required for non-emptiness of $k \in [k_{10c}, k_{13c}]$. Consequently no equilibrium exists where the market is asymmetrically partially covered with all consumers on the neighborhood of firm 2 buying the drug.

Case 2.4 Asymmetric Partial Market Coverage. Proceeding in an analogous way as in the previous case we have that equilibrium in this case is characterized by,

$$
\begin{aligned}
q_1^* &= \frac{1960\alpha x_1 + 490\alpha x_2 + 501k - 583x_1 - 82x_2 - 1470\alpha k}{1295\alpha - 236} \\
q_2^* &= \frac{980\alpha x_1 + 245\alpha x_2 - 2172k + 1366x_1 - 806x_2 - 735\alpha k}{3\,(1295\alpha - 236)} \\
\lambda &= \frac{1}{9\,(1-\alpha)\,(1295\alpha - 326)}(3649x_1 - 656x_2 - 4305k + 30030\alpha k - \\
&\quad -27545\alpha x_1 + 2485\alpha x_2 - 25725\alpha^2 k + 34300\alpha^2 x_1 + 8575\alpha^2 x_2) \\
\Psi &= 0
\end{aligned}
\tag{39}
$$

And the equilibrium in prices,

$$
\begin{aligned}
p_1^* &= \frac{-805x_2\alpha + 770\alpha x_1 - 1575k\alpha + 35\alpha - 443 + 1575k + 1009x_2 - 566x_1}{3\,(1295\alpha - 326)\,(-1+\alpha)} \\
p_2^* &= \frac{665x_2\alpha + 490\alpha x_1 + 175k\alpha - 1155\alpha + 339 - 175k - 257x_2 - 82x_1}{(1295\alpha - 326)\,(-1+\alpha)}
\end{aligned}
$$

Checking the conditions for strictly positive quality and price levels and the conditions on the Lagrangian multipliers we find the thresholds on k that will define the range of k for which an equilibrium with asymmetric partial market coverage is defined. These conditions are given by,

$$
\begin{aligned}
q_1^* &> 0 \Rightarrow k < k_{15c} \\
q_2^* &> 0 \Rightarrow k > k_{16c} \\
p_1^* &> 0 \Rightarrow k < k_{17c} \\
p_2^* &> 0 \Rightarrow k > k_{18c} \\
\lambda &> 0 \Rightarrow k > k_{14c} \\
\frac{\partial L_2}{\partial \Psi} &> 0 \Rightarrow k < k_{19c}
\end{aligned}
$$

With $k_{14c}, k_{15c}, k_{16c}, k_{17c}, k_{18c}$ and k_{19c} the instant utility from treatment thresholds that solve, respectively, $\lambda = 0$, $q_1^* = 0$, $q_2^* = 0$, $p_1^* = 0$, $p_2^* = 0$ and $\frac{\partial L_2}{\partial \Psi} = 0$. Therefore, a maximum exists for,

$$k \in [\max\{k_{14c}, k_{15c}, k_{17c}, k_{19c}\}, \min\{k_{16c}, k_{18c}\}]$$

Computing the differences between the thresholds

$$\{k_{14c}, k_{15c}, k_{17c}, k_{19c}\}$$

and

$$\{k_{16c}, k_{18c}\}$$

it follows immediately that,

$$\begin{aligned}\max\{k_{14c}, k_{15c}, k_{17c}, k_{19c}\} &= k_{14c} \\ \min\{k_{16c}, k_{18c}\} &= k_{16c}\end{aligned}$$

An equilibrium, then, exists for $k \in [k_{16c}, k_{14c}]$. Indeed, it is straightforward to check that $[k_{16c}, k_{14c}]$ is non empty as,

$$k_{14c} - k_{16c} > 0$$

Finally, we need to control that the condition $k \in [k_{16c}, k_{14c}]$ satisfies the market structure conditions. Market structure conditions are satisfied if and only if,

$$\begin{aligned}k &> \max\{k_{1i4c}, k_{ii4c}\} \\ k &< k_{2i4c}\end{aligned}$$

Where k_{ii4c}, k_{1i4c} and k_{2i4c} stand for the thresholds that solve, respectively, $C1 = C3 = 0, C2 = 0$ and $C4 = 0$. Therefore, the equilibrium satisfies the market structure conditions for

$$\begin{aligned}k &> \max\{k_{16c}, k_{1i4c}, k_{ii4c}\} \\ k &< \min\{k_{14c}, k_{2i4c}\}\end{aligned}$$

Checking that $k_{14c} - k_{2i4c} < 0$, it follows that the $\min\{k_{14c}, k_{2i4c}\} = k_{14c}$. Nevertheless, the $\max\{k_{16c}, k_{1i4c}, k_{ii4c}\}$ will depend on firms locations x_1 and x_2. In particular, by checking whether the thresholds k_{16c}, k_{1i4c} and k_{ii4c} are positive we find that,

$$\begin{aligned}k_{1i4c} &< 0 \Longleftrightarrow x_1 < 0.46x_2 \\ k_{ii4c} &< 0 \Longleftrightarrow x_1 < 0.46x_2 \\ k_{16c} &< 0 \Longleftrightarrow x_1 < 0.46x_2\end{aligned}$$

Thus, for $x_1 < 0.46x_2$ an equilibrium with asymmetric partial market coverage with exists for

$$k \in [0, k_{14c}]$$

Instead, for $x_1 > 0.46x_2$ since $k_{16c} < \{k_{1i4c}, k_{ii4c}\}$ we will have two cases arising on which of the thresholds k_{16c}, k_{1i4c} and k_{ii4c} is the maximum. For $0.46x_2 < x_1 < \frac{x_2}{2}$, computing the difference $k_{1i4c} - k_{ii4c}$ we find that,

$$\max\{k_{1i4c}, k_{ii4c}\} = k_{ii4c}$$

Therefore the equilibrium exists for

$$k \in [k_{ii4c}, k_{14c}]$$

While for $x_1 > \frac{x_2}{2}$

$$\max\{k_{16c}, k_{1i4c}, k_{ii4c}\} = k_{1i4c}$$

And, therefore the equilibrium exists for

$$k \in [k_{1i4c}, k_{14c}]$$

Hence an equilibrium with asymmetric partial coverage exists for any $k \in [0, k_{14c}]$ with $x_1 < 0.46x_2$ and for any $k \in [\max\{k_{1i4c}, k_{ii4c}\}, k_{14c}]$ with $x_1 > 0.16x_2$. Summarizing the 4 sub-cases 2.1, 2.2, 2.3 and 2.4 and ordering the thresholds that define the range within which each equilibrium exists (by computing the differences between the relevant thresholds) we can state that for $\alpha \in [0.16, 0.29]$ we find that $k_{3c} > k_{1c} > k_{6c}$ so that the equilibrium with full market coverage in 2.1 is always arising for higher values of the instant utility from treatment k than the ones necessary for the existence of the equilibrium with partial market coverage in 2.4. Moreover, we also find that $k_{6c} = k_{14c}$ so that the upper bounds of the intervals within which the equilibria with both asymmetric and symmetric partial market coverage exist (2.4 and 2.2) are, in fact, coinciding, so that they can overlap for a range of k while 2.1 is always defined for k strictly higher that this upper bound $k_{6c} = k_{14c}$. The above ranges of k (for which 2.2 and 2.4 exist) coincide perfectly as long as $\frac{x_2}{3} < x_1 < 0.46x_2$. Note that as $k > 0$ and given that the equilibrium of local monopolies exists for lower reservation prices than under a competitive scenario, since a competitive equilibrium always exists for all $k > 0$, there exists no equilibrium for a local monopolist ■

B Reference Pricing System

In this section we address the analysis of the effects of reference pricing on firms quality and price strategies. The model structure follows closely the one used in the previous section, only differing in the reimbursement system.

Expenses in pharmaceuticals are reimbursed through a reference pricing system: patients are reimbursed a lump sum amount p_r independently of the drug bought.

B.1 Second Stage: the Price Game

In this stage firms compete simultaneously in prices. With p_i the drug price of firm i and D_i the demand faced by firm i, the duopolists profit functions π_i are given by

$$\pi_i = p_i D_i - \frac{q_i^2}{2} \qquad i = 1, 2 \tag{40}$$

Given (**??**) and (**??**), the market structure and profit function for firm 1 are described as,

$$\text{if } \; 0 \leq p_1 \leq p_2 + q_1 - q_2 + x_1 - x_2 \tag{41}$$

Firm 1 will be a *monopolist* and the profit function is given by,

$$\pi_1 = p_1 (z_4 - z_1) - \frac{q_1^2}{2}$$

Instead, if

$$p_2 + q_1 - q_2 + x_1 - x_2 \leq p_1 \leq q_1 + q_2 + x_1 - x_2 - p_2 + 2(k + p_r) \tag{42}$$

we will be under a *competitive scenario* and firm 1 profit function is given by,

$$\pi_1 = p_1 (z - z_1) - \frac{q_1^2}{2}$$

Finally if

$$\text{if} \quad q_1 + q_2 + x_1 - x_2 - p_2 + 2(k + p_r) \leq p_1 \leq k + q_1 + p_r + x_1 \tag{43}$$

the market will be served by two *local monopolies* and firm 1 profit function is given by,

$$\pi_1 = p_1 (z_3 - z_1) - \frac{q_1^2}{2}$$

Analogously for firm 2, the profit function is

$$\pi_2 = p_2 (z_4 - z_1) - \frac{q_2^2}{2}$$

$$\text{if} \quad 0 \leq p_2 \leq p_1 + q_2 - q_1 + x_1 - x_2 \tag{44}$$

$$\pi_2 = p_2 (z_4 - z) - \frac{q_2^2}{2}$$

$$\text{if} \quad p_1 + q_2 - q_1 + x_1 - x_2 \leq p_2 \leq q_2 + q_1 + x_1 - x_2 - p_1 + 2(k + p_r) \tag{45}$$

$$\pi_2 = p_2 (z_4 - z_2) - \frac{q_2^2}{2}$$

$$\text{if} \quad q_2 + q_1 + x_1 - x_2 - p_1 + 2(k + p_r) \leq p_2 \leq k + q_2 + p_r + x_2 \tag{46}$$

We now look for the pure-strategies subgame perfect Nash Equilibria of the two stages quality-then-price game. As usual, by backward induction, we first describe the Nash equilibria of the simultaneous moves price game in the second stage.

Maximizing profits with respect to prices and solving the system of first order conditions, the Nash Equilibria in the price game will be analyzed under each of the three different market structures: monopoly, competitive equilibria and local monopolists.

Once again, for

$$\begin{aligned}
p_1 &\in [0, p_2 + q_1 - q_2 + x_1 - x_2] \\
p_2 &\in [0, p_1 + q_2 - q_1 + x_1 - x_2]
\end{aligned}$$

no Nash Equilibrium in the price game can ever exist. In fact, within this price range one of the firms will be a monopolist and the other would be out of the market. The latter will always have incentives to pick up a different strategy in order to improve profits.

Hence we will focus on the two polar cases: competitive scenario and the local monopolists scenario.

Proposition 8 *Under a competitive market, the Nash Equilibrium in the price stage is*[10]

$$\begin{aligned}
p_1^{rp*} &= \frac{2(p_r + k) + x_2 - x_1}{5} + \frac{17q_2 - 3q_1}{35} \\
p_2^{rp*} &= \frac{2(p_r + k) + x_2 - x_1}{5} + \frac{17q_1 - 3q_2}{35}
\end{aligned} \tag{47}$$

Proof. Under a competitive scenario, i.e., for $p_2 + q_1 - q_2 + x_1 - x_2 \leq p_1 \leq q_1 + q_2 + x_1 - x_2 - p_2 + 2(k + p_r)$ and $p_1 + q_2 - q_1 + x_1 - x_2 \leq p_2 \leq q_2 + q_1 + x_1 - x_2 - p_1 + 2(k + p_r)$ firms maximization problem characterized by,

$$\begin{aligned}
\max_{p_1} \pi_1 &= p_1 (z - z_1) - \frac{q_1^2}{2} \\
\max_{p_2} \pi_2 &= p_2 (z_4 - z) - \frac{q_2^2}{2}
\end{aligned}$$

Maximizing profits with respect to prices the first order conditions are given by,

$$\begin{aligned}
\frac{1}{2}p_2 - 3p_1 - \frac{x_1 - x_2}{2} + \frac{3q_1 - q_2}{2} + p_r + k &= 0 \\
\frac{1}{2}p_1 - 3p_2 - \frac{x_1 - x_2}{2} + \frac{3q_2 - q_1}{2} + p_r + k &= 0
\end{aligned}$$

And the second order conditions always satisfied and given by,

$$\frac{\partial^2 \pi_i}{\partial p_i^2} = -3 < 0 \quad \text{for } i = 1, 2$$

Solving the first order conditions the equilibrium in prices is given by,

$$\begin{aligned}
p_1^{rp*} &= \frac{2(p_r + k) + x_2 - x_1}{5} + \frac{17q_2 - 3q_1}{35} \\
p_2^{rp*} &= \frac{2(p_r + k) + x_2 - x_1}{5} + \frac{17q_1 - 3q_2}{35}
\end{aligned} \tag{48}$$

[10] Second order conditions always satisfied as $\frac{\partial^2 \pi_i}{\partial p_i^2} = -3 < 0$.

Plugging this expressions in the profit functions we obtain firms' objective functions in the quality stage,

$$\begin{aligned}
\max_{q_1} \pi_1 \left(p_1^*, p_2^*; q_1, q_2; ...\right) &= p_1^* \left(z(.) - z_1(.)\right) - \frac{q_1^2}{2} \\
s.t.\ \ p_1^* &= \frac{2\left(p_r + k\right) + x_2 - x_1}{5} + \frac{17q_2 - 3q_1}{35}, \\
p_2^* &= \frac{2\left(p_r + k\right) + x_2 - x_1}{5} + \frac{17q_1 - 3q_2}{35}, \quad z_1 \geq 0, \quad z_4 \leq 1 \\
\max_{q_2} \pi_2 \left(p_1^*, p_2^*; q_1, q_2; ...\right) &= p_2^* \left(z_4(.) - z(.)\right) - \frac{q_2^2}{2} \\
s.t.\ \ p_1^* &= \frac{2\left(p_r + k\right) + x_2 - x_1}{5} + \frac{17q_2 - 3q_1}{35} \\
p_2^* &= \frac{2\left(p_r + k\right) + x_2 - x_1}{5} + \frac{17q_1 - 3q_2}{35}, \quad z_1 \geq 0, \quad z_4 \leq 1
\end{aligned}$$

Writing the Lagrangian functions

$$\begin{aligned}
L_1 &= \pi_1 \left(p_1^*, p_2^*; q_1, q_2; ...\right) - \lambda\left(-z_1\right) - \Psi\left(z_4 - 1\right) \\
L_2 &= \pi_2 \left(p_1^*, p_2^*; q_1, q_2; ...\right) - \lambda\left(-z_1\right) - \Psi\left(z_4 - 1\right)
\end{aligned}$$

where λ and Ψ stand for the Lagrangian multipliers of the constraints $z_1 \geq 0$ and $z_4 \leq 1$ respectively. Note that even though z_1 only depends on firm's 1 pricing strategy and z_4 only on firm 2 pricing strategy, since p_1 and p_2 are functions of both q_1 and $q_2$the constraints $z_1 \geq 0$ and $z_4 \leq 1$ need to be imposed on both firms' optimization problems. Maximizing L_i w.r.t. λ, Ψ, q_1, q_2 the optimum must satisfy the following conditions,

$$\frac{\partial L_1}{\partial q_1} = 0 \tag{49}$$

$$\frac{\partial L_2}{\partial q_2} = 0 \tag{50}$$

$$\frac{\partial L_1}{\partial \lambda} \geq 0 \tag{51}$$

$$\frac{\partial L_2}{\partial \Psi} \geq 0 \tag{52}$$

We will, then, have four possible cases: Case 1: $\lambda > 0$, $\Psi > 0$ Case 2: $\lambda = \Psi = 0$, Case 3: $\lambda = 0$, $\Psi > 0$ and Case 4: $\lambda > 0$, $\Psi = 0$. It is easy to show that case 3 and case 4 will not hold simultaneously. Indeed these cases arise due to asymmetric locations and their existence depends on the nature of the asymmetry between firms' locations, i.e., on whether $x_1 + x_2 \lesseqgtr 1$. Without loss of generality, we will assume that if firms are asymmetrically located $(x_1 + x_2 \neq 1)$ it will be the case that $x_1 + x_2 < 1$. Therefore, there exists no equilibrium for which $\lambda = 0$, $\Psi > 0$ i.e. in Case 3. Solving the cases,

Case 1 Full Market Coverage Solving the system (49), (50), (51) and (52) for λ, Ψ, q_1 and q_2 we obtain,

$$\begin{aligned} q_1^* &= \frac{5}{3}x_1 + \frac{2}{3}x_2 - \frac{1}{3} - (p_r + k) \\ q_2^* &= 2 - k - p_r - \frac{5}{3}x_2 - \frac{2}{3}x_1 \\ \lambda &= \frac{5}{3}(k + p_r) + \frac{17x_2 - 58x_1}{45} - \frac{13}{105} \\ \Psi &= \frac{5}{3}(k + p_r) - \frac{17x_1 - 58x_2}{45} - \frac{326}{315} \end{aligned} \tag{53}$$

Plugging into (48) the equilibrium in prices are given by,

$$\begin{aligned} p_1^* &= \frac{2(x_1 + x_2) - 1}{3} \\ p_2^* &= \frac{3 - 2(x_1 + x_2)}{3} \end{aligned}$$

For second order conditions satisfied, the above expressions constitute a maximum if and only if $\Psi > 0$ and $\lambda > 0$. Therefore, equilibrium will hold for the following conditions

$$\begin{aligned} \frac{5}{3}(k + p_r) + \frac{17x_2 - 58x_1}{45} - \frac{13}{105} &> 0 \\ \frac{5}{3}(k + p_r) - \frac{17x_1 - 58x_2}{45} - \frac{326}{315} &> 0 \end{aligned}$$

Writing with respect to k,

$$\begin{aligned} k &> \frac{13}{175} - \frac{17x_2 - 58x_1}{75} - p_r \\ k &> \frac{326}{525} + \frac{17x_1 - 58x_2}{75} - p_r \end{aligned}$$

Additionally we need to write down the conditions for which

$$\begin{aligned} q_i^* &> 0 \\ p_i^* &> 0 \end{aligned}$$

for $i = 1, 2$. Writing the first two conditions with respect to k, the following inequalities must be satisfied,

$$\begin{aligned} k &< \frac{2x_2 + 5x_1}{3} - p_r - \frac{1}{3} \\ k &< 2 - \frac{5x_2 + 2x_1}{3} - p_r \end{aligned}$$

On what concerns the conditions on prices,

$$\begin{aligned} p_1^* &> 0 \Leftrightarrow x_1 + x_2 > \frac{1}{2} \\ p_2^* &> 0 \Leftrightarrow x_1 + x_2 < \frac{3}{2} \end{aligned}$$

Therefore since the second condition is always true for $x_1 + x_2 < 1$ we simply need to impose that $x_1 + x_2 > \frac{1}{2}$. Define k_{3p} the instant utility from treatment that solves $q_1^* = 0$, k_{2p} the instant utility from treatment that solves $q_2^* = 0$, k_{1p} the instant utility from treatment that solves $\lambda = 0$ and k_{2p} the instant utility from treatment that solves $\Psi = 0$. The equilibrium described above exists for

$$\begin{aligned} k &< \min\{k_{3p}, k_{4p}\} \\ k &> \max\{k_{1p}, k_{2p}\} \end{aligned}$$

With,

$$\begin{aligned} k_{1p} &= \frac{13}{175} - \frac{17x_2 - 58x_1}{75} - p_r \\ k_{2p} &= \frac{326}{525} + \frac{17x_1 - 58x_2}{75} - p_r \\ k_{3p} &= \frac{2x_2 + 5x_1}{3} - p_r - \frac{1}{3} \\ k_{4p} &= 2 - \frac{5x_2 + 2x_1}{3} - p_r \end{aligned}$$

Since we need to order these thresholds in order to define the range for which the equilibrium holds we need to compute the difference between them,

$$\begin{aligned} k_{1p} - k_{2p} &= -\frac{41}{75}(1 - x_1 - x_2) \\ k_{3p} - k_{4p} &= -\frac{7}{3}(1 - x_1 - x_2) \end{aligned}$$

for $1 > x_1 + x_2$, $k_{1p} < k_{2p}$ and $k_{3p} < k_{4p}$. Therefore, an equilibrium then exists for $k \in [k_{2p}, k_{3p}]$. For $[k_{2p}, k_{3p}]$ non-empty we need to have that $k_{2p} < k_{3p}$. Computing the difference $k_{2p} - k_{3p}$ we find that,

$$k_{2p} - k_{3p} = -\frac{36}{25}(x_1 + x_2) + \frac{167}{175}$$

$k_{2p} < k_{3p}$ is true for

$$x_1 + x_2 > 0.66$$

Finally, since we are in a competitive scenario we still need to check that the equilibria found does satisfy the (market structure) conditions for which firms' profit functions are defined, i.e.,[11]

$$\begin{aligned} p_2 + q_1 - q_2 + x_1 - x_2 &\leq p_1 \leq q_1 + q_2 + x_1 - x_2 - p_2 + 2(k + p_r) \quad (54) \\ p_1 + q_2 - q_1 + x_1 - x_2 &\leq p_2 \leq q_2 + q_1 + x_1 - x_2 - p_1 + 2(k + p_r) \quad (55) \end{aligned}$$

[11] As these conditions apply to all cases defined under a competitive scenario we will throughout the analysis make reference to them.

These conditions can be written as,

$$
\begin{aligned}
p_1 - q_1 - q_2 - x_1 + x_2 + p_2 - 2(k + p_r) &\leq 0 \quad &\text{(C1)}\\
p_2 + q_1 - q_2 + x_1 - x_2 - p_1 &\leq 0 \quad &\text{(C2)}\\
p_2 - q_2 - q_1 - x_1 + x_2 + p_1 - 2(k + p_r) &\leq 0 \quad &\text{(C3)}\\
p_1 + q_2 - q_1 + x_1 - x_2 - p_2 &\leq 0 \quad &\text{(C4)}
\end{aligned}
$$

Note that $C1 = C3$. Plugging in the expressions that characterize the equilibrium, we have that, (C1), (C2), (C3) and (C4) hold for,

$$x_1 \in \left[x_2 - \frac{1}{2}, \frac{1}{2}\right]$$

Let Φ be a set of constraints defined as

$$\Phi = \begin{cases} x_1 \in \left[x_2 - \frac{1}{2}, \frac{1}{2}\right] \\ x_1 + x_2 > 0.66 \end{cases}$$

we can conclude that an equilibrium with full market coverage exists for $x_1 \in \left[x_2 - \frac{1}{2}, \frac{1}{2}\right]$, $x_1 + x_2 > 0.66$ and $k \in [k_{2p}, k_{3p}]$.

Case 2 Symmetric Partial Market Coverage Proceeding in an analogous way as in the previous case we have that equilibrium in this case is characterized by

$$
\begin{aligned}
q_i^* &= \frac{51\,[2k + 2p_r + x_2 - x]}{73} \qquad (56)\\
p_i^* &= \frac{35\,[2k + 2p_r + x_2 - x_1]}{73}
\end{aligned}
$$

We have that as, by definition, $x_2 > x_1$, $q_i^* > 0$ and $p_i^* > 0$ for $i = 1, 2$. Moreover, we have that

$$
\begin{aligned}
\frac{\partial L_1}{\partial \lambda} &> 0 \Rightarrow k < k_{11p}\\
\frac{\partial L_2}{\partial \Psi} &> 0 \Rightarrow k < k_{12p}
\end{aligned}
$$

With k_{11p} the instant utility parameter that solves $\frac{\partial L_1}{\partial \lambda} = 0$ and k_{12p} the instant utility parameter that solves $\Psi = 0$. Therefore a maximum exists for

$$k < \min\{k_{11p}, k_{12p}\}$$

As it turns out that,

$$k_{11p} - k_{12p} = -\frac{73}{105}(1 - x_1 - x_2) < 0$$

a maximum exists as long as $k < k_{11p}$. Checking the market structure conditions we have that a competitive market structure holds for

$$k > k_{ii2p}$$

With k_{ii2p} the instant utility parameter that solves $C4 = 0$. Therefore, an equilibrium exists for

$$k \in [k_{ii2p}, k_{11p}]$$

and $[k_{ii2p}, k_{11p}]$ is non empty for $x_1 > \frac{x_2}{3}$. Therefore an equilibrium with partial market coverage exists for $k \in [k_{ii2p}, k_{11p}]$ and $x_1 > \frac{x_2}{3}$.

Case 3 Asymmetric Partial Market Coverage. Proceeding in an analogous way as in the previous case we have that equilibrium in this qualities is characterized by,

$$\begin{aligned}
q_1^* &= \frac{683x_2 - 403x_1}{489} - \frac{280}{489} + \frac{362}{163}(p_r + k) \\
q_2^* &= \frac{665}{326} - \frac{501}{326}(p_r + k) - \frac{583}{326}x_2 - \frac{41}{163}x_1 \\
\lambda &= 0 \\
\Psi &= \frac{1435}{978}(k + p_r) + \frac{3649}{2934}x_2 - \frac{328}{1467}x_1 - \frac{2993}{2934}
\end{aligned}$$

And the equilibrium in prices,

$$\begin{aligned}
p_1^* &= \frac{1009}{978}x_2 - \frac{283}{489}x_1 - \frac{443}{978} + \frac{525}{326}(p_r + k) \\
p_2^* &= \frac{339}{326} - \frac{175}{326}(p_r + k) - \frac{257}{326}x_2 - \frac{41}{163}x_1
\end{aligned}$$

The conditions that need to be verified are,

$$\begin{aligned}
q_1^* &> 0 \Rightarrow k > k_{14p} \\
q_2^* &> 0 \Rightarrow k < k_{15p} \\
p_1^* &> 0 \Rightarrow k > k_{16p} \\
p_2^* &> 0 \Rightarrow k < k_{17p} \\
\frac{\partial L_1}{\partial \lambda} &> 0 \Rightarrow k > k_{13p} \\
\Psi &> 0 \Rightarrow k < k_{18p}
\end{aligned}$$

Where k_{13p}, $k_{14p}, k_{15p}, k_{16p}, k_{17p}$ and k_{18p} the reservation prices that solve, respectively, $\lambda = 0$, $q_1^* = 0$, $q_2^* = 0$, $p_1^* = 0$, $p_2^* = 0$ and $\Psi = 0$. Therefore, a maximum exists for,

$$k \in [\max\{k_{13p}, k_{14p}, k_{16p}\}, \min\{k_{15p}, k_{17p}, k_{18p}\}]$$

By computing the differences between the thresholds

$$\{k_{13p}, k_{14p}, k_{16p}\}$$

and

$$\{k_{15p}, k_{17p}, k_{18p}\}$$

we find that,

$$\begin{aligned}\max\{k_{13p},k_{14p},k_{16p}\} &= k_{13p}\\ \min\{k_{15p},k_{17p},k_{18p}\} &= k_{18p}\end{aligned}$$

An equilibrium exists for $k \in [k_{13p}, k_{18p}]$. Checking that $[k_{13p}, k_{18p}]$ is non empty by computing the difference between k_{13p} and k_{18p} we find that

$$k_{13p} - k_{18p} > 0$$

Therefore the condition for $[k_{13p}, k_{18p}]$ non empty, i.e. $k_{18p} > k_{13p}$ is not verified and consequently there exists equilibrium with asymmetric partial market coverage for which all consumers on the neighborhood of firm 2 buy a drug.

Case 4 Asymmetric Partial Market Coverage. Finally, again proceeding in an analogous way as in the previous cases, we have that the SPNE, in this case is characterized by,

$$\begin{aligned}q_1^* &= \frac{583}{326}x_1 + \frac{41}{326}x_2 - \frac{501}{326}(p_r+k)\\ q_2^* &= \frac{403}{489}x_2 - \frac{683}{489}x_1 + \frac{362}{163}(p_r+k)\\ p_1^* &= \frac{257}{326}x_1 + \frac{41}{163}x_2 - \frac{175}{326}(p_r+k)\\ p_2^* &= \frac{283}{489}x_2 - \frac{1009}{978}x_1 + \frac{525}{163}(p_r+k)\end{aligned} \tag{57}$$

And is valid for,

$$\begin{aligned}q_1^* &> 0 \Rightarrow k < k_{20p}\\ q_2^* &> 0 \Rightarrow k > k_{21p}\\ p_1^* &> 0 \Rightarrow k < k_{22p}\\ p_2^* &> 0 \Rightarrow k > k_{23p}\\ \lambda &> 0 \Rightarrow k > k_{19p}\\ \frac{\partial L_2}{\partial \Psi} &> 0 \Rightarrow k < k_{24p}\end{aligned}$$

With k_{19p}, $k_{20p}, k_{21p}, k_{22p}, k_{23p}$ and k_{24p} the instant utility parameter thresholds that solve, respectively, $\lambda = 0$, $q_1^* = 0$, $q_2^* = 0$, $p_1^* = 0$, $p_2^* = 0$ and $\frac{\partial L_2}{\partial \Psi} = 0$. Therefore, a maximum exists for

$$k \in [\max\{k_{19p}, k_{21p}, k_{23p}\}, \min\{k_{20p}, k_{22p}, k_{24p}\}]$$

Computing the differences between the thresholds

$$\{k_{19p}, k_{21p}, k_{23p}\}$$

and

$$\{k_{20p}, k_{22p}, k_{24p}\}$$

we find that,

$$\begin{aligned} \max\{k_{19p}, k_{21p}, k_{23p}\} &= k_{19p} \\ \min\{k_{20p}, k_{22p}, k_{24p}\} &= \begin{cases} k_{20p} & \text{for } x_1 + x_2 < 0.66 \\ k_{24p} & \text{for } x_1 + x_2 > 0.66 \end{cases} \end{aligned}$$

An equilibrium exists for $k \in [k_{19p}, \min\{k_{20p}, k_{24p}\}]$. Checking that

$$[k_{19p}, \min\{k_{20p}, k_{24p}\}]$$

is non empty we find that,

$$\begin{aligned} k_{19p} - k_{20p} &< 0 \\ k_{19p} - k_{24p} &< 0 \end{aligned}$$

Finally we need to check that the market structure conditions are satisfied. By computing the differences between the thresholds k_{19p}, k_{20p}, k_{24p}, k_{2i4p}, k_{ii4p} and k_{1i4p} we can further state that for $x_1 + x_2 < 0.66$ an equilibrium with asymmetric partial market coverage exists if the set of constraints Ξ_i is satisfied, with Ξ_{ii} defined by

$$\Xi_i = \begin{cases} k \in [k_{19p}, k_{2i4p}] & \text{for } x_2 \in [2x_1, 3x_1] \\ k \in [k_{ii4p}, k_{20p}] & \text{for } x_2 > \max\{2x_1, 3x_1\} \\ k \in [k_{19p}, k_{1i4p}] & \text{for } x_2 \in [1.39x_1, 2x_1] \\ k \in [k_{19p}, k_{20p}] & \text{for } x_2 < \min\{2x_1, 1.39x_1\} \end{cases}$$

For $x_1 + x_2 > 0.66$ an equilibrium with asymmetric partial market coverage exists if the set of constraints Ξ_{ii} is satisfied, with Ξ_{ii} defined by

$$\Xi_{ii} = \begin{cases} k \in [k_{19p}, k_{2i4p}] & \text{for } x_2 < \min\left\{2x_1, \frac{1}{2}\right\}, \quad x_1 < \frac{1}{2} \\ k \in [k_{19p}, k_{24p}] & \text{for } x_2 \in \left[\frac{1}{2}, 2x_1\right], \quad x_1 < \frac{1}{2} \end{cases}$$

Where k_{ii4p}, k_{2i4p} and k_{1i4p} stand for the reservation prices that solve, respectively, $C1 = C3 = 0, C4 = 0$ and $C2 = 0$. For $x_2 > 2x_1$ and for $\left\{x_2 < 2x_1, x_1 > \frac{1}{2}\right\}$ there exists no equilibrium with asymmetric partial market coverage. Hence if $k \in [\max\{k_{19p}, k_{ii4p}\}, \min\{k_{2i4p}, k_{20p}, k_{1i4p}, k_{24p}\}]$ and under conditions Ξ_i and Ξ_{ii} there exists an equilibrium with asymmetric partial market coverage. Analyzing further and ordering the thresholds we also find that $k_{ii2p} < k_{11p} \equiv \max\{k_{19p}, k_{ii4p}\} < \min\{k_{2i4p}, k_{20p}, k_{1i4p}, k_{24p}\} \equiv k_{2p} < k_{3p}$ so that not only the three equilibria never overlap but also they jointly cover the whole range of parameters ■

In this case we have that prices are increasing in the reference price and in the instant utility parameter.

Proposition 9 *Within the local monopolists scenario* $p_i \in [p_j + q_i^{rp} - q_j + x_1 - x_2, q_i^{rp} + q_j + x_1 - x_2 - p_j] + 2(k + p_r)$ *with* $i, j = 1, 2$ *and* $j \neq i$ *the symmetric Nash Equilibrium in the price stage is*[12]

$$p_i^{lm} = \frac{k + p_r + q_i^{rp}}{2} \tag{58}$$

[12] Second order conditions always satisfied indeed, $\frac{\partial^2 \pi_i}{\partial^2 p_i} = -4 < 0$.

Proof. For firm 1 the problem will be characterized by,

$$\max\ \pi_1 = p_1 (z_3 - z_1) - \frac{q_1^2}{2} \tag{59}$$

With π_1 defined for $q_1 + q_2 + x_1 - x_2 - p_2 + 2(k + p_r) \le p_1 \le k + q_1 + p_r$. Therefore the market structure conditions are given by

$$\begin{aligned} p_1 - k - q_1 - p_r - x_1 &\le 0 & \text{(C1)} \\ q_1 + q_2 + x_1 - x_2 - p_2 + 2(k + p_r) - p_1 &\le 0 & \text{(C2)} \end{aligned}$$

Maximizing with respect to prices the first order condition is given by,

$$2k - 4p_1 + 2p_r + 2q_1 = 0$$

while the second order condition by,

$$-4 < 0$$

Solving with respect to p_1, at the optimum we have,

$$p_1^* = \frac{q_1 + k + p_r}{2} \tag{60}$$

■

Again, it can be noticed that both the instant utility parameter and the reference price have a positive effect on the price level.

For $p_i \in \left[\begin{array}{c} p_j + q_i^{rp} - q_j + x_1 - x_2, \\ q_i^{rp} + q_j + x_1 - x_2 - p_j + 2(k + p_r) \end{array} \right]$ firm 1 and firm 2 do not compete for the marginal consumer. There are consumers in the centre of the market that are better off by not buying any of the drugs. Hence, firms behave like local monopolists. If p_i^m does not fall in that interval, then the local monopolist equilibrium does not exist, and the only price game Nash equilibrium is the one under the competitive scenario.

B.2 First Stage: the Quality Game

Plugging the above found NE prices for each scenario into the relative range of the firms' profit functions, and maximizing with respect to qualities, we obtain the optimal quality levels for the given prices. Substituting back these optimal qualities in the Nash Equilibrium prices, we are then able to fully characterize the subgame perfect NE of the two-stage quality-then-price game.

Proposition 10 *Under the competitive scenario, if $k \in [k_{ii2p}, k_{11p}]$ and for $x_1 > \frac{x_2}{3}$ the market is partially covered, and the subgame perfect Nash equilibrium prices and qualities are*[13]

$$\begin{aligned} q_i^* &= \frac{51\,[2k + 2p_r + x_2 - x]}{73} \\ p_i^* &= \frac{35\,[2k + 2p_r + x_2 - x_2]}{73} \end{aligned} \tag{61}$$

[13] Equilibrium valid for $x_1 \ge \frac{x_2}{3}$ and second order conditions always satisfied as $\frac{\partial^2 \pi_i}{\partial q_i^2} = -\frac{358}{1225} < 0$.

for $i = 1, 2$. Still, under the competitive scenario, if

$$k \in [\max\{k_{19p}, k_{ii4p}\}, \min\{k_{2i4p}, k_{20p}, k_{1i4p}, k_{24p}\}]$$

and under condition Ξ_i or Ξ_{ii} the market is partially covered but fully covered in the left extreme of the preferences line, and the subgame Nash equilibrium prices and qualities are described by the corner solution

$$\begin{aligned}
q_1^* &= \frac{583}{326}x_1 + \frac{41}{326}x_2 - \frac{501}{326}(p_r + k) \\
q_2^* &= \frac{403}{489}x_2 - \frac{683}{489}x_1 + \frac{362}{163}(p_r + k) \\
p_1^* &= \frac{257}{326}x_1 + \frac{41}{163}x_2 - \frac{175}{326}(p_r + k) \\
p_2^* &= \frac{283}{489}x_2 - \frac{1009}{978}x_1 + \frac{525}{163}(p_r + k)
\end{aligned} \tag{62}$$

Finally, for $k \in [k_{2p}, k_{3p}]$ and under condition Φ the market is (endogenously) fully covered and the SPNE is characterized by,

$$\begin{aligned}
q_1^* &= \frac{5}{3}x_1 + \frac{2}{3}x_2 - \frac{1}{3} - (p_r + k) \\
q_2^* &= 2 - k - p_r - \frac{5}{3}x_2 - \frac{2}{3}x_1 \\
p_1^* &= \frac{2(x_1 + x_2) - 1}{3} \\
p_2^* &= \frac{3 - 2(x_1 + x_2)}{3}
\end{aligned} \tag{63}$$

Proof. Under a competitive scenario, i.e., for $p_2 + q_1 - q_2 + x_1 - x_2 \leq p_1 \leq q_1 + q_2 + x_1 - x_2 - p_2 + 2(k + p_r)$ and $p_1 + q_2 - q_1 + x_1 - x_2 \leq p_2 \leq q_2 + q_1 + x_1 - x_2 - p_1 + 2(k + p_r)$ firms maximization problem characterized by,

$$\begin{aligned}
\max_{p_1} \pi_1 &= p_1(z - z_1) - \frac{q_1^2}{2} \\
\max_{p_2} \pi_2 &= p_2(z_4 - z) - \frac{q_2^2}{2}
\end{aligned}$$

Maximizing profits with respect to prices the first order conditions are given by,

$$\begin{aligned}
\frac{1}{2}p_2 - 3p_1 - \frac{x_1 - x_2}{2} + \frac{3q_1 - q_2}{2} + p_r + k &= 0 \\
\frac{1}{2}p_1 - 3p_2 - \frac{x_1 - x_2}{2} + \frac{3q_2 - q_1}{2} + p_r + k &= 0
\end{aligned}$$

And the second order conditions always satisfied and given by,

$$\frac{\partial^2 \pi_i}{\partial p_i^2} = -3 < 0 \quad \text{for } i = 1, 2$$

Solving the first order conditions the equilibrium in prices is given by,

$$\begin{aligned} p_1^{rp*} &= \frac{2(p_r+k)+x_2-x_1}{5}+\frac{17q_2-3q_1}{35} \\ p_2^{rp*} &= \frac{2(p_r+k)+x_2-x_1}{5}+\frac{17q_1-3q_2}{35} \end{aligned} \tag{64}$$

Plugging this expressions in the profit functions we obtain firms' objective functions in the quality stage,

$$\begin{aligned} \max_{q_1} \pi_1(p_1^*, p_2^*; q_1, q_2; ...) &= p_1^*(z(.)-z_1(.))-\frac{q_1^2}{2} \\ s.t.\ \ p_1^* &= \frac{2(p_r+k)+x_2-x_1}{5}+\frac{17q_2-3q_1}{35}, \\ p_2^* &= \frac{2(p_r+k)+x_2-x_1}{5}+\frac{17q_1-3q_2}{35}, \quad z_1 \geq 0, \quad z_4 \leq 1 \\ \max_{q_2} \pi_2(p_1^*, p_2^*; q_1, q_2; ...) &= p_2^*(z_4(.)-z(.))-\frac{q_2^2}{2} \\ s.t.\ \ p_1^* &= \frac{2(p_r+k)+x_2-x_1}{5}+\frac{17q_2-3q_1}{35} \\ p_2^* &= \frac{2(p_r+k)+x_2-x_1}{5}+\frac{17q_1-3q_2}{35}, \quad z_1 \geq 0, \quad z_4 \leq 1 \end{aligned}$$

Writing the Lagrangian functions

$$\begin{aligned} L_1 &= \pi_1(p_1^*, p_2^*; q_1, q_2; ...)-\lambda(-z_1)-\Psi(z_4-1) \\ L_2 &= \pi_2(p_1^*, p_2^*; q_1, q_2; ...)-\lambda(-z_1)-\Psi(z_4-1) \end{aligned}$$

where λ and Ψ stand for the Lagrangian multipliers of the constraints $z_1 \geq 0$ and $z_4 \leq 1$ respectively. Note that even though z_1 only depends on firm's 1 pricing strategy and z_4 only on firm 2 pricing strategy, since p_1 and p_2 are functions of both q_1 and $q_2$the constraints $z_1 \geq 0$ and $z_4 \leq 1$ need to be imposed on both firms' optimization problems. Maximizing L_i w.r.t. λ, Ψ, q_1, q_2 the optimum must satisfy the following conditions,

$$\frac{\partial L_1}{\partial q_1} = 0 \tag{65}$$

$$\frac{\partial L_2}{\partial q_2} = 0 \tag{66}$$

$$\frac{\partial L_1}{\partial \lambda} \geq 0 \tag{67}$$

$$\frac{\partial L_2}{\partial \Psi} \geq 0 \tag{68}$$

We will, then, have four possible cases: Case 1: $\lambda > 0$, $\Psi > 0$ Case 2: $\lambda = \Psi = 0$, Case 3: $\lambda = 0$, $\Psi > 0$ and Case 4: $\lambda > 0$, $\Psi = 0$. It is easy to show that case 3 and case 4 will not hold simultaneously. Indeed these cases arise due to asymmetric locations and their existence depends on the nature of the asymmetry between firms' locations, i.e., on whether $x_1 + x_2 \lesseqgtr 1$. Without loss of generality, we will assume that if firms are asymmetrically located ($x_1 + x_2 \neq 1$) it will be the case that $x_1 + x_2 < 1$. Therefore, there exists no equilibrium for which $\lambda = 0$, $\Psi > 0$ i.e. in Case 3. Solving the cases,

Case 1 Full Market Coverage Solving the system (49), (50), (51) and (52) for λ, Ψ, q_1 and q_2 we obtain,

$$\begin{aligned}
q_1^* &= \frac{5}{3}x_1 + \frac{2}{3}x_2 - \frac{1}{3} - (p_r + k) \\
q_2^* &= 2 - k - p_r - \frac{5}{3}x_2 - \frac{2}{3}x_1 \\
\lambda &= \frac{5}{3}(k + p_r) + \frac{17x_2 - 58x_1}{45} - \frac{13}{105} \\
\Psi &= \frac{5}{3}(k + p_r) - \frac{17x_1 - 58x_2}{45} - \frac{326}{315}
\end{aligned}$$

Plugging into (48) the equilibrium in prices are given by,

$$\begin{aligned}
p_1^* &= \frac{2(x_1 + x_2) - 1}{3} \\
p_2^* &= \frac{3 - 2(x_1 + x_2)}{3}
\end{aligned}$$

For second order conditions satisfied, the above expressions constitute a maximum if and only if $\Psi > 0$ and $\lambda > 0$. Therefore, equilibrium will hold for the following conditions

$$\begin{aligned}
\frac{5}{3}(k + p_r) + \frac{17x_2 - 58x_1}{45} - \frac{13}{105} &> 0 \\
\frac{5}{3}(k + p_r) - \frac{17x_1 - 58x_2}{45} - \frac{326}{315} &> 0
\end{aligned}$$

Writing with respect to k,

$$\begin{aligned}
k &> \frac{13}{175} - \frac{17x_2 - 58x_1}{75} - p_r \\
k &> \frac{326}{525} + \frac{17x_1 - 58x_2}{75} - p_r
\end{aligned}$$

Additionally we need to write down the conditions for which

$$\begin{aligned}
q_i^* &> 0 \\
p_i^* &> 0
\end{aligned}$$

for $i = 1, 2$. Writing the first two conditions with respect to k, the following inequalities must be satisfied,

$$\begin{aligned}
k &< \frac{2x_2 + 5x_1}{3} - p_r - \frac{1}{3} \\
k &< 2 - \frac{5x_2 + 2x_1}{3} - p_r
\end{aligned}$$

On what concerns the conditions on prices,

$$\begin{aligned} p_1^* &> 0 \Leftrightarrow x_1 + x_2 > \frac{1}{2} \\ p_2^* &> 0 \Leftrightarrow x_1 + x_2 < \frac{3}{2} \end{aligned}$$

Therefore since the second condition is always true for $x_1 + x_2 < 1$ we simply need to impose that $x_1 + x_2 > \frac{1}{2}$. Define k_{3p} the instant utility parameter that solves $q_1^* = 0$, k_{2p} the instant utility parameter that solves $q_2^* = 0$, k_{1p} the instant utility parameter that solves $\lambda = 0$ and k_{2p} the instant utility parameter that solves $\Psi = 0$. The equilibrium described above exists for

$$\begin{aligned} k &< \min\{k_{3p}, k_{4p}\} \\ k &> \max\{k_{1p}, k_{2p}\} \end{aligned}$$

With,

$$\begin{aligned} k_{1p} &= \frac{13}{175} - \frac{17x_2 - 58x_1}{75} - p_r \\ k_{2p} &= \frac{326}{525} + \frac{17x_1 - 58x_2}{75} - p_r \\ k_{3p} &= \frac{2x_2 + 5x_1}{3} - p_r - \frac{1}{3} \\ k_{4p} &= 2 - \frac{5x_2 + 2x_1}{3} - p_r \end{aligned}$$

Since we need to order these thresholds in order to define the range for which the equilibrium holds we need to compute the difference between them,

$$\begin{aligned} k_{1p} - k_{2p} &= -\frac{41}{75}(1 - x_1 - x_2) \\ k_{3p} - k_{4p} &= -\frac{7}{3}(1 - x_1 - x_2) \end{aligned}$$

for $1 > x_1 + x_2$, $k_{1p} < k_{2p}$ and $k_{3p} < k_{4p}$. Therefore, an equilibrium then exists for $k \in [k_{2p}, k_{3p}]$. For $[k_{2p}, k_{3p}]$ non-empty we need to have that $k_{2p} < k_{3p}$. Computing the difference $k_{2p} - k_{3p}$ we find that,

$$k_{2p} - k_{3p} = -\frac{36}{25}(x_1 + x_2) + \frac{167}{175}$$

$k_{2p} < k_{3p}$ is true for

$$x_1 + x_2 > 0.66$$

Finally, since we are in a competitive scenario we still need to check that the equilibria found does satisfy the (market structure) conditions for which firms' profit functions are defined, i.e.,[14]

$$\begin{aligned} p_2 + q_1 - q_2 + x_1 - x_2 &\leq p_1 \leq q_1 + q_2 + x_1 - x_2 - p_2 + 2(k + p_r) \quad (69) \\ p_1 + q_2 - q_1 + x_1 - x_2 &\leq p_2 \leq q_2 + q_1 + x_1 - x_2 - p_1 + 2(k + p_r) \quad (70) \end{aligned}$$

[14] As these conditions apply to all cases defined under a competitive scenario we will throughout the analysis make reference to them.

These conditions can be written as,

$$p_1 - q_1 - q_2 - x_1 + x_2 + p_2 - 2(k + p_r) \leq 0 \tag{C1}$$
$$p_2 + q_1 - q_2 + x_1 - x_2 - p_1 \leq 0 \tag{C2}$$
$$p_2 - q_2 - q_1 - x_1 + x_2 + p_1 - 2(k + p_r) \leq 0 \tag{C3}$$
$$p_1 + q_2 - q_1 + x_1 - x_2 - p_2 \leq 0 \tag{C4}$$

Note that $C1 = C3$. Plugging in the expressions that characterize the equilibrium, we have that, (C1), (C2), (C3) and (C4) hold for,

$$x_1 \in \left[x_2 - \frac{1}{2}, \frac{1}{2}\right]$$

Let Φ be a set of constraints defined as

$$\Phi = \begin{cases} x_1 \in \left[x_2 - \frac{1}{2}, \frac{1}{2}\right] \\ x_1 + x_2 > 0.66 \end{cases}$$

we can conclude that an equilibrium with full market coverage exists for $x_1 \in \left[x_2 - \frac{1}{2}, \frac{1}{2}\right]$, $x_1 + x_2 > 0.66$ and $k \in [k_{2p}, k_{3p}]$.

Case 2 Symmetric Partial Market Coverage Proceeding in an analogous way as in the previous case we have that equilibrium in this case is characterized by

$$\begin{aligned} q_i^* &= \frac{51\left[2k + 2p_r + x_2 - x_2\right]}{73} \qquad (71) \\ p_i^* &= \frac{35\left[2k + 2p_r + x_2 - x_1\right]}{73} \end{aligned}$$

We have that as, by definition, $x_2 > x_1$, $q_i^* > 0$ and $p_i^* > 0$ for $i = 1, 2$. Moreover, we have that

$$\begin{aligned} \frac{\partial L_1}{\partial \lambda} &> 0 \Rightarrow k < k_{11p} \\ \frac{\partial L_2}{\partial \Psi} &> 0 \Rightarrow k < k_{12p} \end{aligned}$$

With k_{11p} the instant utility parameter that solves $\frac{\partial L_1}{\partial \lambda} = 0$ and k_{12p} the instant utility parameter that solves $\Psi = 0$. Therefore a maximum exists for

$$k < \min\{k_{11p}, k_{12p}\}$$

As it turns out that,

$$k_{11p} - k_{12p} = -\frac{73}{105}(1 - x_1 - x_2) < 0$$

a maximum exists as long as $k < k_{11p}$. Checking the market structure conditions we have that a competitive market structure holds for

$$k > k_{ii2p}$$

With k_{ii2p} the instant utility parameter that solves $C4 = 0$. Therefore, an equilibrium exists for

$$k \in [k_{ii2p}, k_{11p}]$$

and $[k_{ii2p}, k_{11p}]$ is non empty for $x_1 > \frac{x_2}{3}$. Therefore an equilibrium with symmetric partial market coverage exists for $k \in [k_{ii2p}, k_{11p}]$ and $x_1 > \frac{x_2}{3}$.

Case 3 Asymmetric Partial Market Coverage. Proceeding in an analogous way as in the previous case we have that equilibrium in this qualities is characterized by,

$$\begin{aligned}
q_1^* &= \frac{683x_2 - 403x_1}{489} - \frac{280}{489} + \frac{362}{163}(p_r + k) \\
q_2^* &= \frac{665}{326} - \frac{501}{326}(p_r + k) - \frac{583}{326}x_2 - \frac{41}{163}x_1 \\
\lambda &= 0 \\
\Psi &= \frac{1435}{978}(k + p_r) + \frac{3649}{2934}x_2 - \frac{328}{1467}x_1 - \frac{2993}{2934}
\end{aligned}$$

And the equilibrium in prices,

$$\begin{aligned}
p_1^* &= \frac{1009}{978}x_2 - \frac{283}{489}x_1 - \frac{443}{978} + \frac{525}{326}(p_r + k) \\
p_2^* &= \frac{339}{326} - \frac{175}{326}(p_r + k) - \frac{257}{326}x_2 - \frac{41}{163}x_1
\end{aligned}$$

The conditions that need to be verified are,

$$\begin{aligned}
q_1^* &> 0 \Rightarrow k > k_{14p} \\
q_2^* &> 0 \Rightarrow k < k_{15p} \\
p_1^* &> 0 \Rightarrow k > k_{16p} \\
p_2^* &> 0 \Rightarrow k < k_{17p} \\
\frac{\partial L_1}{\partial \lambda} &> 0 \Rightarrow k > k_{13p} \\
\Psi &> 0 \Rightarrow k < k_{18p}
\end{aligned}$$

Where k_{13p}, $k_{14p}, k_{15p}, k_{16p}, k_{17p}$ and k_{18p} the reservation prices that solve, respectively, $\lambda = 0$, $q_1^* = 0$, $q_2^* = 0$, $p_1^* = 0$, $p_2^* = 0$ and $\Psi = 0$. Therefore, a maximum exists for,

$$k \in [\max\{k_{13p}, k_{14p}, k_{16p}\}, \min\{k_{15p}, k_{17p}, k_{18p}\}]$$

By computing the differences between the thresholds

$$\{k_{13p}, k_{14p}, k_{16p}\}$$

and

$$\{k_{15p}, k_{17p}, k_{18p}\}$$

we find that,

$$\max\{k_{13p}, k_{14p}, k_{16p}\} = k_{13p}$$
$$\min\{k_{15p}, k_{17p}, k_{18p}\} = k_{18p}$$

An equilibrium exists for $k \in [k_{13p}, k_{18p}]$. Checking that $[k_{13p}, k_{18p}]$ is non empty by computing the difference between k_{13p} and k_{18p} we find that

$$k_{13p} - k_{18p} > 0$$

Therefore the condition for $[k_{13p}, k_{18p}]$ non empty, i.e. $k_{18p} > k_{13p}$ is not verified and consequently there exists no equilibrium with asymmetric partial market coverage for which all consumers on the neighborhood of firm 2 buy a drug.

Case 4 Asymmetric Partial Market Coverage. Finally, again proceeding in an analogous way as in the previous cases, we have that the SPNE, in this case is characterized by,

$$\begin{aligned} q_1^* &= \frac{583}{326}x_1 + \frac{41}{326}x_2 - \frac{501}{326}(p_r + k) \\ q_2^* &= \frac{403}{489}x_2 - \frac{683}{489}x_1 + \frac{362}{163}(p_r + k) \\ p_1^* &= \frac{257}{326}x_1 + \frac{41}{163}x_2 - \frac{175}{326}(p_r + k) \\ p_2^* &= \frac{283}{489}x_2 - \frac{1009}{978}x_1 + \frac{525}{163}(p_r + k) \end{aligned} \tag{72}$$

And is valid for,

$$\begin{aligned} q_1^* &> 0 \Rightarrow k < k_{20p} \\ q_2^* &> 0 \Rightarrow k > k_{21p} \\ p_1^* &> 0 \Rightarrow k < k_{22p} \\ p_2^* &> 0 \Rightarrow k > k_{23p} \\ \lambda &> 0 \Rightarrow k > k_{19p} \\ \frac{\partial L_2}{\partial \Psi} &> 0 \Rightarrow k < k_{24p} \end{aligned}$$

With $k_{19p}, k_{20p}, k_{21p}, k_{22p}, k_{23p}$ and k_{24p} the instant utility parameter thresholds that solve, respectively, $\lambda = 0$, $q_1^* = 0$, $q_2^* = 0$, $p_1^* = 0$, $p_2^* = 0$ and $\frac{\partial L_2}{\partial \Psi} = 0$. Therefore, a maximum exists for

$$k \in [\max\{k_{19p}, k_{21p}, k_{23p}\}, \min\{k_{20p}, k_{22p}, k_{24p}\}]$$

Computing the differences between the thresholds

$$\{k_{19p}, k_{21p}, k_{23p}\} \ \ and \{k_{20p}, k_{22p}, k_{24p}\}$$

we find that,

$$\begin{aligned} \max\{k_{19p}, k_{21p}, k_{23p}\} &= k_{19p} \\ \min\{k_{20p}, k_{22p}, k_{24p}\} &= \begin{cases} k_{20p} & \text{for } x_1 + x_2 < 0.66 \\ k_{24p} & \text{for } x_1 + x_2 > 0.66 \end{cases} \end{aligned}$$

An equilibrium exists for $k \in [k_{19p}, \min\{k_{20p}, k_{24p}\}]$. Checking that

$$[k_{19p}, \min\{k_{20p}, k_{24p}\}]$$

is non empty we find that,

$$\begin{aligned} k_{19p} - k_{20p} &< 0 \\ k_{19p} - k_{24p} &< 0 \end{aligned}$$

Finally we need to check that the market structure conditions are satisfied. By computing the differences between the thresholds k_{19p}, k_{20p}, k_{24p}, k_{2i4p}, k_{ii4p} and k_{1i4p} we can further state that for $x_1 + x_2 < 0.66$ an equilibrium with asymmetric partial market coverage exists if the set of constraints Ξ_i is satisfied, with Ξ_{ii} defined by

$$\Xi_i = \begin{cases} k \in [k_{19p}, k_{2i4p}] & \text{for } \; x_2 \in [2x_1, 3x_1] \\ k \in [k_{ii4p}, k_{20p}] & \text{for } \; x_2 > \max\{2x_1, 3x_1\} \\ k \in [k_{19p}, k_{1i4p}] & \text{for } \; x_2 \in [1.39x_1, 2x_1] \\ k \in [k_{19p}, k_{20p}] & \text{for } \; x_2 < \min\{2x_1, 1.39x_1\} \end{cases}$$

For $x_1 + x_2 > 0.66$ an equilibrium with asymmetric partial market coverage exists if the set of constraints Ξ_{ii} is satisfied, with Ξ_{ii} defined by

$$\Xi_{ii} = \begin{cases} k \in [k_{19p}, k_{2i4p}] & \text{for } \; x_2 < \min\left\{2x_1, \frac{1}{2}\right\}, \quad x_1 < \frac{1}{2} \\ k \in [k_{19p}, k_{24p}] & \text{for } \; x_2 \in \left[\frac{1}{2}, 2x_1\right], \quad x_1 < \frac{1}{2} \end{cases}$$

Where k_{ii4p}, k_{2i4p} and k_{1i4p} stand for the reservation prices that solve, respectively, $C1 = C3 = 0, C4 = 0$ and $C2 = 0$. For $x_2 > 2x_1$ and for $\left\{x_2 < 2x_1, x_1 > \frac{1}{2}\right\}$ there exists no equilibrium with asymmetric partial market coverage. Hence if

$$k \in [\max\{k_{19p}, k_{ii4p}\}, \min\{k_{2i4p}, k_{20p}, k_{1i4p}, k_{24p}\}]$$

and under conditions Ξ_i and Ξ_{ii} there exists an equilibrium with asymmetric partial market coverage. Analyzing further and ordering the thresholds we also find that

$$k_{ii2p} < k_{11p} \equiv \max\{k_{19p}, k_{ii4p}\} < \min\{k_{2i4p}, k_{20p}, k_{1i4p}, k_{24p}\} \equiv k_{2p} < k_{3p}$$

so that not only the three equilibria never overlap but also they jointly cover the whole range of parameters ■

For low reservation prices the market will be served by two *local monopolies* and the SPNE will depend on the state of art of quality, i.e. $\overline{Q}$.

Proposition 11 *For sufficiently low reservation prices the market is served by two local monopolists. For $k < 2x_1 - k - p_r$ the market is symmetrically partly covered and the SPNE is characterized by,*

$$\begin{aligned} q_1^* &= q_2^* = \overline{Q} \\ p_1^* &= p_2^* = \frac{k + \overline{Q} + p_r}{2} \end{aligned}$$

For

$$k \in \left[2x_1 - \overline{Q} - p_r, 2 - 2x_2 - \overline{Q} - p_r\right]$$

the market is asymmetrically partly covered and the SPNE is given by,

$$\begin{aligned} q_1^* &= 2x_1 - k - p_r \\ q_2^* &= \overline{Q} \\ p_1^* &= x_1 \\ p_2^* &= \frac{k + \overline{Q} + p_r}{2} \end{aligned}$$

Finally, for $k > 2 - 2x_2 - \overline{Q} - p_r$ *the market is symmetrically partly covered but just consumers located on the centre of the market do not buy any of the drugs. The SPNE is given by,*

$$\begin{aligned} q_1^* &= 2x_1 - k - p_r \\ q_2^* &= 2 - 2x_2 - k - p_r \\ p_1^* &= x_1 \\ p_2^* &= 1 - x_2 \end{aligned}$$

Proof. For firm 1 the problem will be characterized by,

$$\max \ \pi_1 = p_1 (z_3 - z_1) - \frac{q_1^2}{2} \tag{73}$$

With π_1 defined for $q_1 + q_2 + x_1 - x_2 - p_2 + 2(k + p_r) \leq p_1 \leq k + q_1 + p_r$. Therefore the market structure conditions are given by

$$\begin{aligned} p_1 - k - q_1 - p_r - x_1 &\leq 0 && \text{(C1)} \\ q_1 + q_2 + x_1 - x_2 - p_2 + 2(k + p_r) - p_1 &\leq 0 && \text{(C2)} \end{aligned}$$

Maximizing with respect to prices the first order condition is given by,

$$2k - 4p_1 + 2p_r + 2q_1 = 0$$

while the second order condition by,

$$-4 < 0$$

Solving with respect to p_1, at the optimum we have,

$$p_1^* = \frac{q_1 + k + p_r}{2} \tag{74}$$

On the quality stage, firm one problem will be characterized by,

$$\max_{q_1} \ L_1 = \pi_1 - \lambda(-z_1) - \Psi(z_4 - 1) \tag{75}$$

Plugging (74) on (75) and maximizing w.r.t. quality – q_1 – the first order conditions is given by,

$$k + p_r - \lambda > 0$$

Therefore, we will have two cases depending on the value of λ. For $\lambda = 0$ the first order condition is always positive therefore the SPNE will be given by the following corner solution,

$$\begin{aligned} q_1^* &= \overline{Q} \\ p_1^* &= \frac{k + p_r + \overline{Q}}{2} \end{aligned}$$

and holds as a maximum for $k < 2x_1 - \overline{Q} - p_r$. Indeed,

$$\frac{\partial L_1}{\partial \lambda} > 0 \Rightarrow k < 2x_1 - \overline{Q} - p_r$$

Otherwise for $\lambda > 0$ the SPNE is given by,

$$\begin{aligned} q_1^* &= 2x_1 - k \\ p_1^* &= x_1 \end{aligned}$$

and holds as a maximum for,

$$\lambda > 0 \Rightarrow k > 2x_1 - \overline{Q} - p_r$$

Analogously for firm 2, we have that for $k < 2 - 2x_2 - \overline{Q} - p_r$ the SPNE is given by,

$$\begin{aligned} q_2^* &= \overline{Q} \\ p_2^* &= \frac{k + p_r + \overline{Q}}{2} \end{aligned}$$

Otherwise, for $k > 2 - 2x_2 - \overline{Q} - p_r$ it is given by

$$\begin{aligned} q_2^* &= 2 - 2x_2 - k \\ p_2^* &= 1 - x_2 \end{aligned}$$

It is now useful to combine the two firms' SPNE and order them according to k. Notice that, also here, given that $x_1 + x_2 < 1$ we can never have the case for which $z_1 \geq 0$ is slack and $z_4 \leq 1$ binds (case analogous to case 3 in the competitive scenario). Indeed, $k > 2 - 2x_2 - \overline{Q} - p_r$ and $k < 2x_1 - \overline{Q} - p_r$ cannot hold simultaneously. Therefore, we have that for $k < 2x_1 - \overline{Q} - p_r$,

$$\begin{aligned} q_1^* &= q_2^* = \overline{Q} \\ p_1^* &= p_2^* = \frac{k + p_r + \overline{Q}}{2} \end{aligned} \tag{76}$$

Market structure conditions are always satisfied. For

$$k \in \left[2x_1 - \overline{Q} - p_r, 2 - 2x_2 - \overline{Q} - p_r\right]$$

we have that,

$$
\begin{aligned}
q_1^* &= 2x_1 - k - p_r \\
q_2^* &= \overline{Q} \\
p_1^* &= x_1 \\
p_2^* &= \frac{k + p_r + \overline{Q}}{2}
\end{aligned} \tag{77}
$$

Market structure conditions are satisfied for,

$$k < 2x_2 - 4x_1 - \overline{Q} - p_r$$

Which is compatible with $k \in \left[2x_1 - \overline{Q} - p_r, 2 - 2x_2 - \overline{Q} - p_r\right]$ as long as $x_1 < \frac{x_2}{3}$. Finally, for $k > 2 - 2x_2 - \overline{Q} - p_r$ we have that,

$$
\begin{aligned}
q_1^* &= 2x_1 - k - p_r \\
q_2^* &= 2 - 2x_2 - k - p_r \\
p_1^* &= x_1 \\
p_2^* &= 1 - x_2
\end{aligned} \tag{78}
$$

While the market structure conditions satisfied for,

$$x_1 \leq x_2 - \frac{1}{2}$$

■

For $k \in [k_{ii2p}, k_{11p}]$, the level of market coverage under a competitive market with partial coverage is given by

$$M_{RP}^{pc} = \frac{105}{72} \left[2\left(k + p_r\right) + x_2 - x_1\right] \tag{79}$$

Comparing the firms pricing strategies we have,

$$
\begin{aligned}
\Delta q^* &= q_1^* - q_2^* = 0 \\
\Delta p^* &= p_1^* - p_2^* = 0
\end{aligned} \tag{80}
$$

Drugs are sold at the same price and have the same quality.

One can see that under a competitive market with partial coverage prices and qualities are increasing in the reference price and in the instant utility parameter. However, under a competitive scenario with full market coverage, quality is decreasing with the reservation and reference price while prices depend neither on reservation nor on the reference price. In a sense, in terms of utility and therefore demand, quality has the same impact as both the reference and reservation prices. Once the market is fully covered, an increase in the reference price and/or reservation prices does not further increase demand (as the market is already fully covered). It nevertheless allows the firm to (profitably) decrease the quality of the drug supplied, extracting (the extra) surplus from the consumers.

For $k \in [k_{11p}, k_{2p}]$ the market coverage is given by the following expression,

$$M_{RP}^{pc} = \frac{203}{163}x_2 - \frac{119}{326}x_1 + \frac{525}{326}(k + p_r) < 1 \tag{81}$$

Consumers on the left side of the market all consume a drug (drug 1) while on the right hand side of the market there are consumers that do not buy any of the drugs. This result arises from the nature of the location asymmetry between firms. Indeed, with $x_1 + x_2 < 1$ firm 2 has a locational advantage relatively to firm 1, conferring her higher market power and therefore allowing higher drug 2 prices (relatively to firm 1).

Comparing drugs' prices and quality,

$$\begin{aligned} \Delta q^* &= q_1^* - q_2^* = \frac{3115}{978}x_1 - \frac{280}{489}x_2 - \frac{1225}{326}(k + p_r) \\ \Delta p^* &= p_1^* - p_2^* = \frac{890}{489}x_1 - \frac{160}{489}x_2 - \frac{350}{163}(k + p_r) \end{aligned} \tag{82}$$

Still on a competitive market structure for $k \in [k_2, k_3]$ the market is fully covered ($M_{RP}^{pc} = 1$). Comparing drugs' prices and qualities

$$\begin{aligned} \Delta q^* &= q_1^* - q_2^* = \frac{7}{3}(x_1 + x_2 - 1) \\ \Delta p^* &= p_1^* - p_2^* = \frac{4}{3}(x_1 + x_2 - 1) \end{aligned} \tag{83}$$

When the market is fully covered, for a competitive market structure, firms' equilibrium strategies might differ. While under a co-payment reimbursement these differences are functions of both locations and reimbursement rate, under reference pricing they are a function of locations only. Only when firms are located symmetrically, $x_1 + x_2 = 1$, are drugs prices and qualities the same in equilibrium. However, this no longer holds for asymmetric locations. In particular, if $x_1 + x_2 > 1$ (< 1) drug 1 has higher (lower) quality but also higher (lower) price than drug 2. The reason is quite intuitive. For asymmetric locations one of the firms serves a larger neighborhood and, therefore, has a privileged position that allows it to sell its drug at higher price and quality.

Concerning local monopolies, by definition of this market structure, the market is always partly covered, as, at least, consumers located in between the two firms do not buy any of the drugs. Nevertheless, the market coverage increases with the instant utility parameter.

For $k < 2x_1 - k - p_r$ market coverage is given by

$$M^{lm} = 2k + 2\overline{Q} + 2p_r < 1$$

Quality and price gaps are given by,

$$\begin{aligned}\Delta q^* &= q_1^* - q_2^* = 0 \\ \Delta p^* &= p_1^* - p_2^* = 0\end{aligned} \tag{84}$$

For low reservation prices firms pricing and quality strategies are the same. Indeed, for such low reservation prices even with asymmetric locations the sub market faced by each firm has the same structure in the sense that their distance to the ends of the market is sufficiently big to both firms in order to restrain them from choosing qualities and prices that would allow all consumers located at the ends of the market to consume.

For $k \in \left[2x_1 - \overline{Q} - p_r, 2 - 2x_2 - \overline{Q} - p_r\right]$ market coverage is given by

$$M^{lm} = 2x_1 + k + \overline{Q} + p_r < 1$$

with consumers located towards the right hand side of the market not buying any of the drugs. In this case the relative locations of both firms allow firm 1 to profitably set prices and qualities that allow it to capture all the demand located at the left end of the market, while the same does not happen to firm 2.

The quality and price gaps are then given by,

$$\begin{aligned}\Delta q^* &= q_1^* - q_2^* = -k - p_r + 2x_1 - \overline{Q} < 0 \\ \Delta p^* &= p_1^* - p_2^* = x_1 - \frac{k + \overline{Q} + p_r}{2} < 0\end{aligned} \tag{85}$$

Drug 1 is sold at a lower price, but also lower quality with respect to drug 2.

Finally, for $k > 2 - 2x_2 - k - p_r$ market coverage is given by

$$M^{lm} = 2x_1 - 2x_2 + 2$$

In this case, the only consumers that opted out from the market are (some of the) consumers located between the two firms while all the others, including the individuals located towards the ends of the market, always buy one of the drugs.

In this case the quality and price gaps are given by,

$$\begin{aligned}\Delta q^* &= q_1^* - q_2^* = 2x_1 + 2x_2 - 2 < 0 \\ \Delta p^* &= p_1^* - p_2^* = x_1 + x_2 - 1 < 0\end{aligned} \tag{86}$$

Also here, for the locational advantage of firm 2 mentioned before, firm 1 will price at a lower level and supply less quality than firm 2.

C Exogenous Full Market Coverage

In this section we study a special case where demand is inelastic, in that consumers' instant utility parameter is so high that they are always willing to buy some of the drugs. This scenario corresponds to medical conditions in which consumers obtain very high health benefits from taking a drug, or in which patients suffer very hard health consequences when deprived from any drug consumption.

Investigating this scenario emphasizes the role of competition between the two firms and underlines the effects of reimbursement policies on firms' strategies. In the following, we first describe the case of *co-payment reimbursement*, and then the one of reference pricing.

C.1 Co-payment System

The general model adopted above will be just specified by imposing exogenous full market coverage:

$$\begin{aligned} z_1 &= 0 \\ z_4 &= 1 \end{aligned} \tag{87}$$

This implies the following demands,

$$\begin{aligned} D_1 &= \overline{z} \\ D_2 &= 1 - \overline{z} \end{aligned} \tag{88}$$

which do not depend on the instant utility parameter level k, with

$$\overline{z} = \frac{(1-\alpha)(p_2 - p_1) + (x_1 + x_2) + q_1 - q_2}{2}$$

The impact of the reimbursement rate α on firms' demand depends, qualitatively and quantitatively, on firms pricing strategies

$$\frac{\partial D_i}{\partial \alpha} = \frac{p_i - p_j}{2} \qquad i, j = 1, 2 \text{ and } i \neq j.$$

As, by the full market coverage assumption, all individuals buy one unit of the differentiated product, the reimbursement rate only affects the allocation of consumers between drugs.

Concerning the impact of pricing strategies on firms' demand, from

$$\begin{aligned} \frac{\partial D_i}{\partial p_i} &= -\frac{(1-\alpha)}{2} \quad i = 1, 2 \\ \frac{\partial D_i}{\partial p_j} &= \frac{(1-\alpha)}{2} \quad i, j = 1, 2,\ i \neq j, \end{aligned}$$

it can be seen that a firm demand is a decreasing function of its own price and increasing in the competitor price. The size of these effects is softened by α.

As, for k sufficiently high, all consumers buy a drug from one of the two firms, from (88), firms profit functions with the co-payment reimbursement are

$$\begin{aligned}
\pi_1 &= p_1\left(\frac{(1-\alpha)(p_2-p_1)+(x_1+x_2)+q_1-q_2}{2}\right)-\frac{q_1^2}{2} \\
\pi_2 &= p_2\left(\frac{2-(1-\alpha)(p_2-p_1)-(x_1+x_2)-q_1+q_2}{2}\right)-\frac{q_2^2}{2} \qquad (89)
\end{aligned}$$

Again, firms maximize their profits in a two-stage game, by first deciding quality strategies and then prices. The equilibrium is summarized the following Proposition.

Proposition 12 *Under a co-payment reimbursement system the subgame perfect Nash Equilibrium prices and qualities are*[15]

$$\begin{aligned}
p_1^* &= \frac{6\alpha-4-3(x_1+x_2)(1-\alpha)}{(1-\alpha)(9\alpha-7)} \qquad (90) \\
p_2^* &= \frac{12\alpha-10+3(x_1+x_2)(1-\alpha)}{(1-\alpha)(9\alpha-7)} \\
q_1^* &= \frac{6\alpha-4-3(x_1+x_2)(1-\alpha)}{3(1-\alpha)(9\alpha-7)} \\
q_2^* &= \frac{12\alpha-10+3(x_1+x_2)(1-\alpha)}{3(1-\alpha)(9\alpha-7)}
\end{aligned}$$

Proof. Under exogenous full market coverage $z_1 = 0$ and $z_4 = 1$. Therefore firms profits are given by,

$$\begin{aligned}
\pi_1 &= p_1 z - \frac{q_1^2}{2} \qquad (91) \\
\pi_2 &= p_2 z - \frac{q_2^2}{2}
\end{aligned}$$

Under co-payment, in the last stage, maximizing profits with respect to prices, the first order conditions are given by,

$$\begin{aligned}
\frac{\partial\pi_1}{\partial p_1} &= \frac{p_2(1-\alpha)}{2}-p_1(1-\alpha)+\frac{x_1+x_2+q_1-q_2}{2}=0 \\
\frac{\partial\pi_2}{\partial p_2} &= 1+\frac{p_1(1-\alpha)}{2}-p_2(1-\alpha)-\frac{x_1+x_2+q_1-q_2}{2}=0
\end{aligned}$$

The second order conditions given by,

$$\frac{\partial^2\pi_1}{\partial p_1^2}=\frac{\partial^2\pi_2}{\partial p_2^2}=\alpha-1$$

[15] Second order conditions in the price stage satisfied for $\alpha \in [0,1]$ and in the quality stage for $\alpha < \frac{8}{9}$

are always satisfied. Plugging these equilibrium prices on the profit function and maximizing with respect to qualities, in the first stage the equilibrium in qualities is characterized by the following first order conditions,

$$\begin{aligned}\frac{\partial \pi_1}{\partial q_1} &= \frac{2 - 8q_1 - q_2 + x_1 + x_2 + 9q_1\alpha}{9(1-\alpha)} = 0 \\ \frac{\partial \pi_2}{\partial q_2} &= \frac{4 - q_1 - 8q_2 - x_1 - x_2 + 9q_2\alpha}{9(1-\alpha)} = 0\end{aligned}$$

Second order conditions satisfied for,

$$\frac{\partial^2 \pi_1}{\partial q_1^2} = \frac{\partial^2 \pi_2}{\partial q_2^2} = \frac{9\alpha - 8}{9(1-\alpha)}$$

which are satisfied for $\alpha < \frac{8}{9}$. Consequently the SPNE is given by,

$$\begin{aligned}p_1^* &= \frac{6\alpha - 4 - 3(x_1 + x_2)(1-\alpha)}{(1-\alpha)(9\alpha - 7)} \qquad (92) \\ p_2^* &= \frac{12\alpha - 10 + 3(x_1 + x_2)(1-\alpha)}{(1-\alpha)(9\alpha - 7)} \\ q_1^* &= \frac{6\alpha - 4 - 3(x_1 + x_2)(1-\alpha)}{3(1-\alpha)(9\alpha - 7)} \\ q_2^* &= \frac{12\alpha - 10 + 3(x_1 + x_2)(1-\alpha)}{3(1-\alpha)(9\alpha - 7)}\end{aligned}$$

■

It follows immediately that the reimbursement rate α has a positive effect on equilibrium prices and quality. Indeed, proceeding with comparative statics analysis we have that,

$$\begin{aligned}\frac{\partial p_1}{\partial \alpha} &= \frac{6 + 3(x_1 + x_2) - (16 - 18\alpha)(6\alpha - 4 - 3(x_1 + x_2)(1-\alpha))}{[(1-\alpha)(9\alpha - 7)]^2} \\ \frac{\partial q_1}{\partial \alpha} &= \frac{6 + 3(x_1 + x_2) - (16 - 18\alpha)(6\alpha - 4 - 3(x_1 + x_2)(1-\alpha))}{[3(1-\alpha)(9\alpha - 7)]^2} \\ \frac{\partial p_2}{\partial \alpha} &= \frac{12 - 3(x_1 + x_2) - (16 - 18\alpha)(12\alpha - 10 + 3(x_1 + x_2)(1-\alpha))}{[(1-\alpha)(9\alpha - 7)]^2} \\ \frac{\partial q_2}{\partial \alpha} &= \frac{12 - 3(x_1 + x_2) - (16 - 18\alpha)(12\alpha - 10 + 3(x_1 + x_2)(1-\alpha))}{[3(1-\alpha)(9\alpha - 7)]^2}\end{aligned}$$

Since $q_i^{rp} > 0$ and $p_i > 0$ require that $6\alpha - 4 - 3(x_1 + x_2)(1-\alpha) < 0$ and $12\alpha - 10 + 3(x_1 + x_2)(1-\alpha) < 0$, and since by SOCs $16 - 18\alpha > 0$ then it immediately follows that $\frac{\partial p_i}{\partial \alpha} > 0, \frac{\partial q_i^{rp}}{\partial \alpha} > 0$ for $i = 1, 2$.

Equilibrium price and quality differences are functions of both locations and reimbursement rate α, indeed,

$$\Delta p_C = p_1^* - p_2^* = \frac{6(1 - x_1 - x_2)}{(9\alpha - 7)} \tag{93}$$
$$\Delta q_C = q_1^* - q_2^* = \frac{2(1 - x_1 - x_2)}{(9\alpha - 7)}$$

Moreover the drug supplied by drug 1 will be sold at a lower price and lower quality, i.e., $\Delta p_C < 0$ and $\Delta q_C < 0$.[16] This result arises from the nature of the asymmetry on locations that we have assumed, i.e., $1 > x_1 + x_2$.

Reference Pricing We now describe the model with exogenous full market coverage under a *reference pricing* policy. Demands are given by $D_1 = \overline{z}$ and $D_2 = 1 - \overline{z}$, with $\overline{z} = \frac{(p_2 - p_1) + (x_1 + x_2) + q_1 - q_2}{2}$

From these demands, firms' profit functions follow:

$$\pi_1 = p_1 \left(\frac{p_2 - p_1 + (x_1 + x_2) + q_1 - q_2}{2} \right) - \frac{q_1^2}{2} \tag{94}$$
$$\pi_2 = p_2 \left(1 - \frac{p_2 - p_1 + (x_1 + x_2) + q_1 - q_2}{2} \right) - \frac{q_2^2}{2}$$

A crucial aspect to be noticed is that, under reference pricing, the demand functions are affected neither by the instant utility parameter k nor by the reference price p_r. Therefore, firms' strategies will be independent from both of these variables.

This result is clearly due to the joint outcome of two hypotheses in force. First, by assuming that the market is fully covered, reference pricing can not have any impact on consumers' choice on whether to buy, or not, some of the differentiated products. Secondly, as the reference pricing is a lump sum reimbursement, it can not affect the distribution of consumers between firms.

Furthermore, firm's demand depends positively on the competitor price and decreases in its own price.

Proposition 13 *Under the reference pricing system the subgame perfect Nash*

[16] Note that for $q_i > 0$ and $p_i > 0$ for $i = 1, 2$ the numerators of the equilibrium prices and qualities in (90) can not be simultaneously (i.e. for both firms) positive. Therefore, for negative numerators, the denominators must be negative for strictly positive equilibrium qualities and prices, implying that $\alpha < \frac{7}{9}$. Consequently, for $1 > x_1 + x_2$, $\Delta p_C < 0$ and $\Delta q_C < 0$

Equilibrium prices and qualities is[17]

$$
\begin{aligned}
p_1^* &= \frac{3\left(x_1+x_2\right)+4}{7} \qquad (95)\\
q_1^* &= \frac{3\left(x_1+x_2\right)+4}{21}\\
p_2^* &= \frac{10-3\left(x_1+x_2\right)}{7}\\
q_2^* &= \frac{10-3\left(x_1+x_2\right)}{21}
\end{aligned}
$$

Proof. Under exogenous full market coverage $z_1 = 0$ and $z_4 = 1$. Therefore firms profits are given by,

$$
\begin{aligned}
\pi_1 &= p_1 z - \frac{q_1^2}{2} \qquad (96)\\
\pi_2 &= p_2 z - \frac{q_2^2}{2}
\end{aligned}
$$

Under reference pricing the first order conditions in prices are given by,

$$
\begin{aligned}
\frac{\partial \pi_1}{\partial p_1} &= \frac{p_2}{2} - p_1 + \frac{x_1+x_2+q_1-q_2}{2} = 0\\
\frac{\partial \pi_2}{\partial p_2} &= 1 + \frac{p_1}{2} - p_2 - \frac{x_1+x_2+q_1-q_2}{2} = 0
\end{aligned}
$$

While the second order conditions for a global maximum are always satisfied and given by

$$\frac{\partial^2 \pi_1}{\partial p_1^2} = \frac{\partial^2 \pi_2}{\partial p_2^2} = -1 < 0$$

Plugging these equilibrium prices on the profit function and maximizing with respect to qualities, in the first stage the equilibrium in qualities is characterized by the following first order conditions

$$
\begin{aligned}
\frac{\partial \pi_1}{\partial q_1} &= \frac{2-8q_1-q_2+x_1+x_2}{9} = 0\\
\frac{\partial \pi_2}{\partial q_2} &= \frac{4-q_1-8q_2-x_1-x_2}{9} = 0
\end{aligned}
$$

While the second order conditions for a global maximum are always satisfied and given by,

$$\frac{\partial^2 \pi_1}{\partial p_1^2} = \frac{\partial^2 \pi_2}{\partial p_2^2} = -\frac{8}{9} < 0$$

[17] Second order conditions always verified

Consequently the SPNE is given by,

$$\begin{aligned} p_1^* &= \frac{3(x_1+x_2)+4}{7} \\ q_1^* &= \frac{3(x_1+x_2)+4}{21} \\ p_2^* &= \frac{10-3(x_1+x_2)}{7} \\ q_2^* &= \frac{10-3(x_1+x_2)}{21} \end{aligned} \tag{97}$$

■

It can be seen that, under reference pricing, price and quality differences depend only on firms' locations.

$$\begin{aligned} \Delta p_{RP} &= p_1^* - p_2^* = \frac{6(x_1+x_2-1)}{7} \\ \Delta q_{RP} &= q_1^* - q_2^* = \frac{6(x_1+x_2-1)}{21} \end{aligned} \tag{98}$$

Once again, for $x_1 + x_2 > 1$ (< 1) drug 1 (2) is sold at a higher (lower) price and at a higher (lower) quality than drug 2 (1). When the instant utility parameter is high enough, consumers will always buy the differentiated product. This sort of demand rigidity softens competitive pressure on firms, which no longer need to compete for consumers at the edges of the market.

While, with partial market coverage, the reference price has an impact on both demand and profits by reinforcing the effect of the instant utility parameter, in the fully covered market case, the effect of the instant utility parameter is so overwhelming that the reference price has no marginal effect.

In other words, in the former case, for a given k, the level of p_r can affect profits by increasing demand. Conversely, in the latter case, demand is already at its maximum, so that p_r has no influence on it. In fact, equilibrium prices and qualities do not depend on its level.

On the other hand, the co-payment rate α has an impact on competition between firms for consumers located towards the centre, namely for the marginal consumer z.

It is easy to see that, in this case, reference pricing is nested in the co-payment system. Indeed, we have that whenever $\alpha \to 0$, $p_i^c \to p_i^{RP}$: in other words, the reference pricing system is equivalent, in terms of prices and qualities, to a system where there is no reimbursement. The only role of reference pricing is acting as "reimbursement ceiling" for the third party payer. Therefore, contrary to co-payment rate α, reference price can not be used as a regulatory instrument for the determination of prices, qualities or for market coverage.

Finally, by comparing the price and quality gaps across firms, we observe that the relation between price and quality gaps under the two different reimbursement systems depends not only on firms locations but also on the reimbursement variable α.

$$\Delta p_C - \Delta p_{RP} = \frac{54\alpha\,(x_1 + x_2 - 1)}{7\,(9\alpha - 7)}$$
$$\Delta q_C - \Delta q_{RP} = \frac{18\alpha\,(x_1 + x_2 - 1)}{7\,(9\alpha - 7)}$$

Interestingly, the difference in the gaps between the two reimbursement systems is not the same for prices and qualities level, $\Delta p_C - \Delta p_{RP} > \Delta q_C - \Delta q_{RP}$.

D Reference Pricing vs Co-payment: the case of symmetric locations

We will now compare prices, qualities and market coverage of the two reimbursement systems, for all the above described scenarios assuming symmetric locations, i.e., $x_1 + x_2 = 1$[18]. In order to proceed with the comparisons, under endogenous market coverage, we need to order the equilibria for all values of the instant utility from treatment k.

D.1 Competitive Market Structure

Since the sub-game perfect Nash equilibrium under a co-payment depends on the level of the co-payment rate the comparison analysis will be done for both cases separately. Therefore, for $\alpha \in [0, 0.16]$ comparing the two reimbursement systems leads to the results described in the following proposition.

Proposition 14 *A co-payment system leads to higher prices and quality level than a reference pricing system and at least the same, if not higher, market coverage. More precisely, for low and medium reference prices, market coverage is equal under the two reimbursement systems for high preferences parameter and is higher under co-payment for low preferences parameter. Instead, for high reference price levels both systems lead to full market coverage.*

Note that under these parameters' configurations expenditure in pharmaceuticals is always higher under co-payment but also quality is. Moreover, for low preferences parameter, this policy performs better than reference pricing in terms of access to care.

Instead, for $\alpha \in [0.16, 0.29]$ the comparisons (in quality, prices and market coverage) between a co-payment regime and a reference pricing will depend on the reference pricing level and on the instant utility from treatment .

Proposition 15 *For low reference price levels, i.e.* $p_r < p_{r2}$, *the equilibria under reference pricing are described by (53) and (71) while under co-payment*

[18] The analysis remains the same as previously stated. Results can be easily derived by substituiting $x_2 = 1 - x_1$ in the results and conditions found above.

by (35). Therefore, quality and prices are always higher under a co-payment regime. Concerning market coverage, for $p_r < p_{r13}$ market coverage is higher under co-payment while for $p_r \in [p_{r13}, p_{r2}]$ in a reference pricing system there are more consumers buying a drug[19]*.*

While it is clear that for $p_r < p_{r13}$ expenditure is higher under co-payment for higher reference prices results are ambiguous. Nevertheless, for $p_r < p_{r13}$, even though expenditure in pharmaceuticals is higher for the co-payment system relatively to a reference pricing system, this policy ensures higher market coverage and consequently is superior in terms of access to care. These results are specific to the range of parameters defined in the proposition . Indeed, as we will show in the following propositions, results are very sensitive to changes in both reimbursement instruments and preferences parameter. For example, in proposition 33 for low reference and preferences parameter quality is higher and pharmaceutical expenditure is clearly lower under co-payment than under reference pricing (due to lower prices and lower market coverage). Nevertheless, note that lower public expenditure, in this case, is achieved through not only lower prices but also lower market coverage. While the former might be desirable from a welfare perspective, the latter might jeopardize public policies targeted at tackling inequalities on access to care.

Additionally, for higher preferences parameter we observe that co-payment performances in terms of quality is weaken and becomes lower relatively to the reference pricing policy.

Proposition 16 *For medium reference price levels, i.e. $p_r \in [p_{r2}, p_{r7}]$, results are ambiguous.*

For low treatment instant utilities, i.e. $k \in [k_{ii2p}, k_{2p}]$, the SPNE under a co-payment regime is characterized by (19) and under reference pricing by (71). Under both systems the market is partly covered but the market coverage is lower under a co-payment. For low treatment instant utilities, i.e. $k \in [k_{ii2p}, k_e]$,co-payment system leads to higher quality and lower prices than a reference pricing system. For medium-low instant utility parameter (k) levels, i.e., $k \in [k_e, k_{2p}]$results are reversed, i.e., under a co-payment system drugs have a lower quality and higher prices than under a reference pricing system. For $k \in [k_{2p}, k_{6c}]$the SPNE under a co-payment regime is characterized by (19) and under reference pricing by (53). While under reference pricing the market is fully covered, under a co-payment policy there are consumers that opt-out from the market. The relation between prices and quality between the two regimes is again ambiguous and depends on the instant utility level.

For $k \in [k_{2p}, k_g]$ prices and quality are higher under co-payment. While, for $k \in [k_g, k_{6c}]$ under a co-payment system drugs are still sold at higher prices than under reference pricing, but have also lower quality.

Finally, for $k \in [k_{6c}, k_{3p}]$the market is fully covered under both regimes, and the SPNE is characterized by (35) and (53) for the co-payment and reference

[19] The conditions on the reference price are obtained by subtracting the equilibrium values of co-payment and reference pricing.

pricing respectively. For this range of treatment instant utilities, a co-payment system allows higher quality but also higher prices than a reference pricing policy. Also here expenditure in pharmaceuticals depends on the reimbursement instruments and instant utility.

Proposition 17 *For medium-high reference price levels, $p_r \in [p_{r7}, p_{r3}]$[20], the SPNE will depend on the instant utility.*

For low treatment instant utilities, i.e. $k \in [k_{ii2p}, k_{2p}]$ the SPNE under reference pricing is given by (71) while under co-payment by (19). The market is partly covered under both policies and the market coverage is higher under reference pricing. Prices are lower and quality higher under a co-payment regime.

For $k \in [k_{2p}, k_{6c}]$ the market is still fully covered under a reference pricing system but under a co-payment regime there are consumers not buying a drug. For low treatment instant utilities, i.e., $k \in [k_{2p}, k_g]$ quality is higher under a co-payment system and prices are lower. For medium treatment instant utilities, i.e., $k \in [k_g, k_h]$ prices are still lower under co-payment than under reference pricing but also quality is. For high treatment instant utilities, i.e., $k \in [k_h, k_{6c}]$ a co-payment system leads to higher prices and lower quality than a reference pricing system.

Still for the same range of reference pricing, for $k \in [k_{6c}, k_{3p}]$ the market is fully covered under both regimes, and quality and prices are higher under co-payment.

Proposition 18 *For high reference price levels, i.e., $p_r > p_{r_3}$, both reimbursement systems lead to partial coverage. Under a co-payment system drugs are sold at higher quality and lower prices than under reference pricing.*

Finally, this last proposition clearly describes a scenario where no only co-payment ensures lower pharmaceutical expenditure and higher quality but also full access to drugs.

It now follows the proofs of the propositions stated above.

Proof. Given the equilibria defined and ranked in k in sections 6.1 and 6.2 we can now rank in k the equilibria of both reimbursement regimes. We, therefore, need to compare the different reservation prices that define the equilibria under both regimes. Note that given that the equilibrium under co-payment depends on whether the reimbursement rate α is lower or higher than 0.16, we need to make the comparisons for both cases, i.e. for $\alpha \in [0, 0.16]$ and for $\alpha \in [0.16, 0.29]$

- (a) $\alpha \in [0, 0.16]$

The ranking of the different ks will depend on the level of reference pricing, p_r. Substituting $x_2 = 1 - x_1$ on the equilibria found on appendix 1 we have that

[20] The conditions on the reference price are obtained by subtracting the equilibrium values of co-payment and reference pricing.

in a scenario with reference pricing, For $k \in [k_{ii2p}, k_{2p}]$ the SPNE is given by,

$$\begin{aligned} q_i^* &= \frac{51\left[2k + 2p_r + 1 - 2x_1\right]}{73} \\ p_i^* &= \frac{35\left[2k + 2p_r + 1 - 2x_1\right]}{73} \end{aligned} \tag{99}$$

and the market structure conditions are satisfied for, $x_1 > \frac{1}{4}$ For $k > k_{2p}$ the SPNE is given by,

$$\begin{aligned} q_1^* &= q_2^* = x_1 + \frac{1}{3} - (p_r + k) \\ p_1^* &= p_2^* = \frac{1}{3} \end{aligned} \tag{100}$$

and the market structure conditions are satisfied for

$$x_1 \in \left[\frac{1}{4}, \frac{1}{2}\right]$$

Under co-payment reimbursement in a competitive market structure the SPNE is characterized by$q$

$$q_1^* = q_2^* = \frac{3x_1 - 3k + 1}{3}, \qquad p_1^* = p_2^* = \frac{1}{3(1-\alpha)} \tag{101}$$

This equilibrium exists for $k \in [k_{2c}, k_{3c}]$ and the market structure conditions are satisfied for

$$x_1 > \frac{1}{4}$$

In order to proceed with the comparisons of the prices, qualities and market coverage arising under each reimbursement system we need to define three zones for which the equilibria under co-payment and under reference pricing exist. Ordering the reservation prices thresholds we have that, for

$$\begin{aligned} p_r &< p_{r1} \Rightarrow k_{2c} < k_{ii2p} < k_{2p} < k_{3p} < k_{3c} \\ p_r &\in [p_{r1}, p_{r2}] \Rightarrow k_{iip} < k_{2c} < k_{2p} < k_{3p} < k_{3c} \\ p_r &\in [p_{r2}, p_{r3}] \Rightarrow k_{iip} < k_{2p} < k_{2c} < k_{3p} < k_{3c} \\ p_r &> p_{r3} \Rightarrow k_{iip} < k_{2p} < k_{3p} < k_{2c} < k_{3c} \end{aligned}$$

With,

$$\begin{aligned} p_{r1} &= \frac{73 + 29\alpha}{210(1-\alpha)} - \frac{292x_1}{210(1-\alpha)} \\ p_{r2} &= \frac{17\alpha}{35(1-\alpha)} \\ p_{r3} &= \frac{17}{35(1-\alpha)} \end{aligned}$$

Therefore for $p_r < p_{r1}$ we have that for $k \in [k_{2c}, k_{ii2p}]$ no equilibrium exists under reference pricing and the equilibria under co-payment is characterized by full market coverage and given by (101). For $k \in [k_{ii2p}, k_{2p}]$ under a reference pricing system the equilibrium is characterized by (99) and the market is partly covered while in a co-payment the equilibrium is characterized by (101) and the market is endogenously fully covered. In this area we have that,

$$\begin{aligned} p_i^c &< p_i^{rp} \quad \text{for } k > kd \\ p_i^c &> p_i^{rp} \quad \text{for } k < kd \\ q_i^c &< q_i^{rp} \quad \text{for } k > kc \\ q_i^c &> q_i^{rp} \quad \text{for } k < kc \end{aligned}$$

Where kd and kc stand for, respectively, the reservation prices that solve $p_i^c - p_i^{rp} = 0$ and $q_i^c - q_i^{rp} = 0$. For $p_r < p_{r1}$ it is easy to show that $kc < kd$. Therefore $k > kd$ and $k > kc$ can never hold simultaneously. Moreover as these two equilibria coexist for $k \in [k_{ii2p}, k_{2p}]$ and given that $kc > k2p$ and $kd > k2p$, it must be the case that $k < kd$ and $k < kc$. Therefore,

$$\begin{aligned} p_i^c &> p_i^{rp} \\ q_i^c &> q_i^{rp} \end{aligned}$$

On what concerns market coverage we have that $M^c > M^{p_r}$. For $k \in [k_{2p}, k_{3p}]$ under a reference pricing system the equilibria is characterized by (100) and the market is (endogenously) fully covered while in a co-payment the equilibria is characterized by (101) and the market is (endogenously) fully covered. In this area we have that

$$\begin{aligned} q_i^c - q_i^{rp} &= p_r > 0 \\ p_i^c - p_i^{p_r} &= \frac{\alpha}{3(1-\alpha)} > 0 \\ M^c - M^{p_r} &= 0 \end{aligned}$$

for $i = 1, 2$. Finally for $k > k_{3p}$ there exists no equilibrium under reference pricing while under co-payment the equilibrium is characterized by (101) and the market is (endogenously) fully covered. Analogously for $p_r \in [p_{r1}, p_{r2}]$we have that for $k \in [k_{ii2p}, k_{2c}]$ no equilibrium exists under co-payment and the equilibria under reference pricing is characterized by partial market coverage and given by (99). For $k \in [k_{2c}, k_{2p}]$ under a reference pricing system the equilibria is characterized by (99) and the market is partly covered while in a co-payment the equilibria is characterized by (101) and the market is endogenously fully covered. Also here,

$$\begin{aligned} p_i^c &< p_i^{rp} \quad \text{for } k > kd \\ p_i^c &> p_i^{rp} \quad \text{for } k < kd \\ q_i^c &< q_i^{rp} \quad \text{for } k > kc \\ q_i^c &> q_i^{rp} \quad \text{for } k < kc \end{aligned}$$

Where kd and kc stand for, respectively, the reservation prices that solve $p_i^c - p_i^{rp} = 0$ and $q_i^c - q_i^{rp} = 0$. For $p_r < p_{r1}$ it is easy to show that $kc < kd$. Therefore $k > kd$ and $k > kc$ can never hold simultaneously. Moreover as these two equilibria coexist for $k \in [k_{ii2p}, k_{2p}]$ and given that $kc > k2p$ and $kd > k2p$, it must be the case that $k < kd$ and $k < kc$. Therefore,

$$\begin{aligned} p_i^c &> p_i^{rp} \\ q_i^c &> q_i^{rp} \end{aligned}$$

On what concerns market coverage we have that $M^c > M^{p_r}$. For $k \in [k_{2p}, k_{3p}]$ under both reference pricing and co-payment the market is fully covered and the equilibria are characterized by (101) and (100) respectively. In this area we have that,

$$\begin{aligned} q_i^c - q_i^{rp} &= p_r > 0 \\ p_i^c - p &= \frac{\alpha}{3(1-\alpha)} > 0 \\ M^c - M^{p_r} &= 0 \end{aligned}$$

Finally for $k > k_{3p}$ there exists no equilibrium under reference pricing while under co-payment the equilibrium is characterized by (100) and (101) and the market is (endogenously) fully covered.

For $p_r \in [p_{r2}, p_{r3}]$we have that for $k \in [k_{ii2p}, k_{2p}]$ no equilibrium exists under co-payment while under reference pricing the market is partly covered and the equilibrium is characterized (99). For $k \in [k_{2p}, k_{2c}]$ no equilibrium exists under co-payment while under reference pricing the market is (endogenously) fully covered and the equilibrium is characterized (100). For $k \in [k_{2c}, k_{3p}]$ under both reference pricing and co-payment the market is (endogenously) fully covered and the equilibria is characterized by (100) and (101) respectively. In this area we have that

$$\begin{aligned} q_i^c - q_i^{rp} &= p_r > 0 \\ p_i^c - p &= \frac{\alpha}{3(1-\alpha)} > 0 \\ M^c - M^{p_r} &= 0 \end{aligned}$$

Finally for $k > k_{3p}$ there exists no equilibrium under reference pricing while under co-payment the equilibrium is characterized by (101) and the market is (endogenously) fully covered. Finally for $p_r > p_{r3}$ there exists no interval in k for which the equilibrium under reference pricing coexists with an equilibrium under co-payment.

- (b) $\alpha \in [0.16, 0.29]$

Proceeding in the same way as in case (a), under symmetric locations the asymmetric equilibrium with partial market coverage (39) will no longer exist. Indeed, recall that this equilibrium existed for $k \in [k_{1i4c}, k_{14c}]$ as with symmetric locations $k_{1i4c} = k_{14c}$ then the interval $[k_{1i4c}, k_{14c}]$ is empty. Therefore we are left with the equilibria with both symmetric partial and full market coverage for both reimbursement systems. Under co-payment these equilibria are defined, respectively by (38) and (35). Plugging $x_2 = 1 - x_1$ into the expressions that define the equilibria and the conditions for which they hold and the thresholds in k that define these conditions we find that $k_{6c} = k_{1c}$ and for $k \in [0, k_{1c}]$ the SPNE is characterized by partial market coverage and given by,

$$\begin{aligned} q_i^* &= \frac{51\,(2k+1-2x_1)}{73-175\alpha} \\ p_i^* &= \frac{35\,(2k+1-2x_1)}{73-175\alpha} \end{aligned} \tag{102}$$

while for $k \in [k_{1c}, k_{3c}]$ the SPNE is characterized by full market coverage and given by,

$$\begin{aligned} q_i^* &= \frac{3x_1-3k+1}{3}, \\ p_i^* &= p_2^* = \frac{1}{3\,(1-\alpha)} \end{aligned} \tag{103}$$

Both equilibria are valid for $x_1 > \frac{1}{4}$. Analogously, under reference pricing, with symmetric locations $k_{24p} = k_{19p} = k_{1p} = k_{2p}$ and for $k \in [k_{ii2p}, k_{2p}]$ the market is partly covered and the SPNE is characterized by,

$$\begin{aligned} q_i^* &= \frac{51\,[2k+2p_r+1-2x_1]}{73} \\ p_i^* &= \frac{35\,[2k+2p_r+1-2x_1]}{73} \end{aligned} \tag{104}$$

While for $k \in [k_{2p}, k_{3p}]$ the market is fully covered and the SPNE is characterized by

$$\begin{aligned} q_i^* &= x_1 + \frac{1}{3} - (p_r + k) \\ p_i^* &= \frac{1}{3} \end{aligned}$$

In order to proceed with the comparisons of the prices, qualities and market coverage arising under each reimbursement system we need to define three zones for which the equilibria under co-payment and under reference pricing exist. Ordering the reservation prices thresholds we have that, for

$$\begin{aligned} p_r &< p_{r1} \Rightarrow k_{6c} < k_{ii2p} < k_{2p} < k_{3p} < k_{3c} \\ p_r &\in [p_{r1}, p_{r2}] \Rightarrow k_{iip} < k_{6c} < k_{2p} < k_{3p} < k_{3c} \\ p_r &\in [p_{r2}, p_{r3}] \Rightarrow k_{iip} < k_{2p} < k_{6c} < k_{3p} < k_{3c} \\ p_r &> p_{r3} \Rightarrow k_{iip} < k_{2p} < k_{6c} < k_{3p} < k_{3c} \end{aligned}$$

With,

$$
\begin{aligned}
p_{r1} &= \frac{73+29\alpha}{210\,(1-\alpha)} - \frac{292x_1}{210\,(1-\alpha)} \\
p_{r2} &= \frac{17\alpha}{35\,(1-\alpha)} \\
p_{r3} &= \frac{17}{35\,(1-\alpha)}
\end{aligned}
$$

Therefore for $p_r < p_{r1}$ we have that for $k \in [0, k_{6c}]$ no equilibrium exists under reference pricing and the equilibria under co-payment is characterized by partial market coverage and given by (102). For $k \in [k_{6c}, k_{2iip}]$ no equilibrium exists under reference pricing and the equilibria under co-payment is characterized by full market coverage and given by (101). For $k \in [k_{ii2p}, k_{2p}]$ under a reference pricing system the equilibrium is characterized by (99) and the market is partly covered while in a co-payment the equilibrium is characterized by (101) and the market is endogenously fully covered. In this area we have that,

$$
\begin{aligned}
p_i^c &> p_i^{rp} \\
q_i^c &> q_i^{rp}
\end{aligned}
$$

Indeed computing the differences $q_i^c - q_i^{rp}$ and $p_i^c - p_i^{rp}$ we find that

$$
\begin{aligned}
p_i^c &> p_i^{rp} \quad \text{for } k > k_d \\
q_i^c &> q_i^{rp} \quad \text{for } k < k_c
\end{aligned}
$$

Where k_c and k_d are the reservation prices that solve respectively $q_i^c - q_i^{rp} = 0$ and $p_i^c - p_i^{rp} = 0$. Comparing k_c and k_d we find that

$$
\begin{aligned}
k_c &> k_d \quad \text{for } p_r > p_{r4} \\
k_c &< k_d \quad \text{for } p_r < p_{r4}
\end{aligned}
$$

Where p_{r4} stands for the reservation price that solves $k_c - k_d = 0$. However, since $p_{r4} > p_{r1}$ and this case is defined by $p_r < p_{r1}$ it can never be that $p_r > p_{r4.}$ It, then., follows $k_c < k_d$. Therefore we are left with three possibilities

$$
\begin{aligned}
k &< k_c \\
k &\in [k_c, k_d] \\
k &> k_d
\end{aligned}
$$

However, since $k_c > k_{2p}$ we are only left with $k < k_c$ and $k < k_d$ Therefore we can state that $p_i^c < p_i^{rp}$ and $q_i^c > q_i^{rp}$. On what concerns market coverage we have that $M^c > M^{p_r}$. For $k \in [k_{2p}, k_{3p}]$ under a reference pricing system the equilibria is characterized by (100) and the market is (endogenously) fully covered while in a co-payment the equilibria is characterized by (101) and the market is (endogenously) fully covered. In this area we have that

$$\begin{aligned} q_i^c - q_i^{rp} &= p_r > 0 \\ p_i^c - p_i^{rp} &= \frac{\alpha}{3(1-\alpha)} > 0 \\ M^c - M^{p_r} &= 0 \end{aligned}$$

for $i = 1, 2$. Finally for $k > k_{3p}$ there exists no equilibrium under reference pricing while under co-payment the equilibrium is characterized by (101) and the market is (endogenously) fully covered. Analogously, for $p_r \in [p_{r1}, p_{r2}]$ we have that for $k < k_{ii2p}$ no equilibrium exists under reference pricing and the equilibria under co-payment is characterized by partial market coverage and given by (102). For $k \in [k_{ii2p}, k_{6c}]$ under a reference pricing system the equilibria is characterized by (99) and the market is partly covered while in a co-payment the equilibria is characterized by (102) and the market partly covered covered. Also here,

$$\begin{aligned} p_i^c &> p_i^{rp} \\ q_i^c &> q_i^{rp} \end{aligned}$$

The proof is analogous to the previous proofs. On what concerns market coverage calculating the difference between the market coverage under co-payment and under reference pricing we find that,

$$M^c - M^{rp} > 0 \text{ if } p_r < p_{r13} (= \alpha q_i^{*c})$$

Where p_{r13} is the reference price p_r that solves $M^c - M^{rp} = 0$. As for $k \in [k_{ii2p}, k_{6c}]$ $p_{r13} \in [p_{r1}, p_{r2}]$ we can have both cases, i.e. for $p_r > p_{r13}$ the market coverage is higher under reference pricing and *vice versa*. For $k \in [k_{6c}, k_{2p}]$ under a reference pricing system the equilibria is characterized by (99) and the market is partly covered while in a co-payment the equilibria is characterized by (101) and the market is endogenously fully covered. We will then have,

$$\begin{aligned} p_i^c &> p_i^{rp} \\ q_i^c &> q_i^{rp} \end{aligned}$$

Indeed, recall that $k_c > k_d$ if and only if $p_r > p_{r4}$. Computing the differences between p_{r4} and $\{p_{r1}, p_{r2}\}$ we find that $p_{r4} \in [p_{r1}, p_{r2}]$, therefore we need to analyze what happens for $p_r \in [p_{r1}, p_{r4}]$ and $p_r \in [p_{r4}, p_{r2}]$. For $p_r \in [p_{r1}, p_{r4}]$ we have already seen that $k < k_c$ and $k < k_d$ Therefore we can state that $p_i^c < p_i^{rp}$ and $q_i^c > q_i^{rp}$. For $p_r \in [p_{r1}, p_{r4}]$ as $p_r > p_{r4}$ then $k_c > k_d$. We will then have three possible cases: $\{k > k_c, k > k_d\}$ and $\{k < k_c, k > k_d\}$ and $\{k < k_c, k < k_d\}$. Nevertheless, since $k_c > k_{2p}$ then for $[k_{6c}, k_{2p}]$ it must be the case that $k < k_c$. So we can rule out the case $\{k > k_c, k > k_d\}$. Moreover,as $k_d > k_{2p}$ we can also rule out the case $\{k < k_c, k > k_d\}$. Consequently we are left with $\{k < k_c, k < k_d\}$ what implies that $p_i^c > p_i^{rp}$ and $q_i^c > q_i^{rp}$. On what concerns market coverage we have that $M^c(=1) > M^{p_r}(<1)$. For $k \in [k_{2p}, k_{3p}]$

under both reference pricing and co-payment the market is fully covered and the equilibria are characterized by (101) and (100) respectively. In this area we have that,

$$\begin{aligned} q_i^c - q_i^{rp} &= p_r > 0 \\ p_i^c - p_i^{rp} &= \frac{\alpha}{3(1-\alpha)} > 0 \\ M^c - M^{p_r} &= 0 \end{aligned}$$

Finally for $k > k_{3p}$ there exists no equilibrium under reference pricing while under co-payment the equilibrium is characterized (101) and the market is (endogenously) fully covered. For $p_r \in [p_{r2}, p_{r3}]$ we have that for $k < k_{ii2p}$ no equilibrium exists under reference pricing and the equilibria under co-payment is characterized by partial market coverage and given by (102). For $k \in [k_{ii2p}, k_{2p}]$ under co-payment and reference pricing the market is partly covered and the SPNE are characterized by, respectively, (102) and (99). In this case we find that, for $p_r \in [p_{r2}, p_{r7}]$

$$\begin{aligned} \text{if } k &< k_e \begin{cases} q_i^c > q_i^{rp} \\ p_i^c < p_i^{rp} \end{cases} \\ \text{if } k &> k_e \begin{cases} q_i^c < q_i^{rp} \\ p_i^c > p_i^{rp} \end{cases} \end{aligned}$$

Instead for $p_r \in [p_{r7}, p_{r3}]$

$$\begin{aligned} q_i^c > q_i^{rp} \\ p_i^c < p_i^{rp} \end{aligned}$$

The proof is done in a similar way as the previous. Indeed analyzing the differences $q_i^c - q_i^{rp}$ and $p_i^c - p_i^{rp}$ we find that $\{q_i^c > q_i^{rp}, p_i^c < p_i^{rp}\}$ if $k < k_e$ where k_e stands for the reservation price that solves $q_i^c - q_i^{rp} = 0$ and $p_i^c - p_i^{rp} = 0$. Since $k_e > k_{2p}$ for $p_r > p_{r7}$ (where p_{r7} is the p_r that solves $k_e - k_{2p} = 0$ and $p_{r7} = \frac{5\alpha}{6}$ and $p_{r7} \in [p_{r2}, p_{r3}]$) we find that for $p_r \in [p_{r2}, p_{r7}]$ then $k_e > k_{ii2p}$ and consequently as $k \in [k_{ii2p}, k_{2p}]$ we will have two cases the one for which $k < k_e$ implying that $q_i^c > q_i^{rp}$ and $p_i^c < p_i^{rp}$ and the one for which $k > k_e$ implying that $q_i^c < q_i^{rp}$ and $p_i^c > p_i^{rp}$Instead, for $p_r \in [p_{r7}, p_{r3}]$ then $k_e > k_{2p}$ and consequently as $k \in [k_{ii2p}, k_{2p}]$ we must have $k < k_e$ implying that $q_i^c > q_i^{rp}$ and $p_i^c < p_i^{rp}$. For $k \in [k_{2p}, k_{6c}]$ under co-payment the market is partly covered and the SPNE is characterized by (102) while under reference pricing the market is (endogenously) fully covered and the equilibrium is characterized (100). Proceeding in the same way as in the previous cases comparing the prices and qualities we find that for $p_r \in [p_{r7}, p_{r3}]$

$$\begin{aligned} \text{for } k &\in [k_{2p}, k_g] \begin{cases} q_i^c > q_i^{rp} \\ p_i^c < p_i^{rp} \end{cases} \\ \text{for } k &\in [k_g, k_h] \begin{cases} q_i^c < q_i^{rp} \\ p_i^c < p_i^{rp} \end{cases} \\ \text{for } k &\in [k_h, k_{6c}] \begin{cases} q_i^c < q_i^{rp} \\ p_i^c < p_i^{rp} \end{cases} \end{aligned}$$

Where k_g and k_h are, respectively, the thresholds that solve $q_i^c - q_i^{rp} = 0$ and $p_i^c - p_i^{rp} = 0$ and p_{r7} is the p_r that solves $k_g - k_h = 0$ (and is the same that solves $k_e - k_{2p} = 0$). For $k \in [k_{6c}, k_{3p}]$ under both reference pricing and co-payment the market is (endogenously) fully covered and the equilibria is characterized by (100) and (101) respectively. Comparing quality, prices and market coverage between the two policies we find that,

$$\begin{aligned} q_i^c - q_i^{rp} &= p_r > 0 \\ p_i^c - p_i^{rp} &= \frac{\alpha}{3(1-\alpha)} > 0 \\ M^c - M^{rp} &= 0 \end{aligned}$$

Finally, for $k > k_{3p}$ there exists no equilibrium under reference pricing while under co-payment the equilibrium is characterized by (101) and the market is (endogenously) fully covered. Finally for $p_r > p_{r3}$ [21]for $k < k_{ii2p}$ no equilibrium exists under reference pricing and the equilibria under co-payment is characterized by partial market coverage and given by (102). For $k \in [k_{ii2p}, k_{2p}]$ under co-payment and reference pricing the market is partly covered and the SPNE are characterized by, respectively, (102) and (99). In this case we find that,

$$\begin{aligned} q_i^c &> q_i^{rp} \\ p_i^c &< p_i^{rp} \end{aligned}$$

The proof is analogous to the previous proofs. On what concerns market coverage calculating the difference between the market coverage under co-payment and under reference pricing we find that,

$$M^c - M^{rp} > 0 \;\text{ if } p_r < p_{r13} (= \alpha q_i^{*c})$$

Where p_{r13} is the reference price p_r that solves $M^c - M^{rp} = 0$. As for $k \in [k_{ii2p}, k_{2p}]$ $p_{r13} < p_{r3}$. Therefore, as $p_r > p_{r3}$ then it must be the case that $p_r > p_{r13}$ and, thus, it follows that $M^c < M^{rp}$. For $k \in [k_{2p}, k_{3p}]$ under co-payment the market is partly covered and the SPNE is characterized by (102) while under reference pricing the market is (endogenously) fully covered and the equilibrium is characterized (100). Comparing the prices and qualities we find that, for $p_r \in [p_{r3}, p_{r9}]$

$$\begin{aligned} k &\in [k_{2p}, k_g] \begin{cases} q_i^c > q_i^{rp} \\ p_i^c < p_i^{rp} \end{cases} \\ k &\in [k_{2p}, k_g] \begin{cases} q_i^c < q_i^{rp} \\ p_i^c < p_i^{rp} \end{cases} \end{aligned}$$

Instead, for $p_r > p_{r9}$,

$$\begin{aligned} q_i^c &> q_i^{rp} \\ p_i^c &< p_i^{rp} \end{aligned}$$

[21]the proofs are analogous and follow from the previous. Therefore, we will omitt them

Where p_{r9} stands for the p_r that solves $k_g - k_{3p} = 0$. Finally, for $k > k_{3p}$ there exists no equilibrium under reference pricing while under co-payment the equilibrium is characterized by partial market coverage and given by (102) for $k \in [k_{3p}, k_{6c}]$ and by full market coverage and given by (101) for $k \in [k_{6c}, k_{3c}]$.

Combining the results found above and knowing that for $p_r > p_{r7}$ we have that $k_e > \{k_h, k_g\}$ the comparisons are such that for $p_r < p_{r2}$

$$\begin{aligned} q_i^c &> q_i^{rp} \\ p_i^c &> p_i^{rp} \end{aligned}$$

For $p_r \in [p_{r2}, p_{r7}]$

$$\begin{aligned}
k &\in [k_{ii2p}, k_e] \begin{cases} q_i^c > q_i^{rp} \\ p_i^c < p_i^{rp} \end{cases} \\
k &\in [k_e, k_{2p}] \begin{cases} q_i^c < q_i^{rp} \\ p_i^c > p_i^{rp} \end{cases} \\
k &\in [k_{2p}, k_g] \begin{cases} q_i^c > q_i^{rp} \\ p_i^c > p_i^{rp} \end{cases} \\
k &\in [k_g, k_{6c}] \begin{cases} q_i^c < q_i^{rp} \\ p_i^c > p_i^{rp} \end{cases} \\
k &\in [k_{6c}, k_{3p}] \begin{cases} q_i^c > q_i^{rp} \\ p_i^c > p_i^{rp} \end{cases}
\end{aligned}$$

For $p_r \in [p_{r7}, p_{r3}]$

$$\begin{aligned}
k &\in [k_{ii2p}, k_{2p}] \begin{cases} q_i^c > q_i^{rp} \\ p_i^c < p_i^{rp} \end{cases} \\
k &\in [k_{2p}, k_g] \begin{cases} q_i^c > q_i^{rp} \\ p_i^c < p_i^{rp} \end{cases} \\
k &\in [k_g, k_h] \begin{cases} q_i^c < q_i^{rp} \\ p_i^c < p_i^{rp} \end{cases} \\
k &\in [k_h, k_{6c}] \begin{cases} q_i^c < q_i^{rp} \\ p_i^c > p_i^{rp} \end{cases} \\
k &\in [k_{6c}, k_{3p}] \begin{cases} q_i^c > q_i^{rp} \\ p_i^c > p_i^{rp} \end{cases}
\end{aligned}$$

Finally for $p_r > p_{r3}$

$$\begin{aligned} q_i^c &> q_i^{rp} \\ p_i^c &< p_i^{rp} \end{aligned}$$

■

D.2 Local Monopolies

In the same line as in the competitive scenario also local monopolies show a multiplicity of results. Comparing prices, qualities and market coverage of the two reimbursement systems, results are summarized in the proposition that follows.

Proposition 19 *When firm one is closer to the left end of the market, i.e.* $x_1 < \frac{1}{4}$*,*

- *For low treatment instant utilities the two systems deliver the same quality and price differences between the two systems depend on the co-payment rate.*
  - *Namely, for a co-payment rate higher than* 0.5*, prices are higher under co-payment*
  - *While for lower co-payment rates, i.e.* $\alpha < 0.5$*, the reverse holds*

  Co-payment system leads to lower market coverage than a reference pricing system.

- *For medium treatment instant utilities a co-payment system delivers higher quality than the reference pricing system but, at maximum, achieves the same market coverage than reference pricing policies. On what concerns prices, for intermediate treatment instant utilities co-payment leads to lower prices than a reference pricing system, while for high instant utility* k *levels prices, are higher under co-payment.*

Proof. For $x_1 < \frac{1}{4}$ for $k < 2x_1 - p_r - \overline{Q}$ the equilibrium under reference pricing is characterized by (76) while under co-payment by (20). Therefore we have that,

$$\begin{aligned} q_i^c - q_i^{rp} &= 0 \\ p_i^c - p_i^{rp} &< 0 \text{ for } \alpha < 0.5 \\ p_i^c - p_i^{rp} &> 0 \text{ for } \alpha > 0.5 \\ M^c - M^{rp} &= -p_r < 0 \end{aligned}$$

for $i = 1, 2$. For $k \in \left[2x_1 - p_r - \overline{Q}, 2x_1 - \overline{Q}\right]$ the equilibrium under reference pricing is characterized by (78) while under co-payment by (20). The market is fully covered under the reference pricing system and partly covered under co-payment. Therefore we have that,

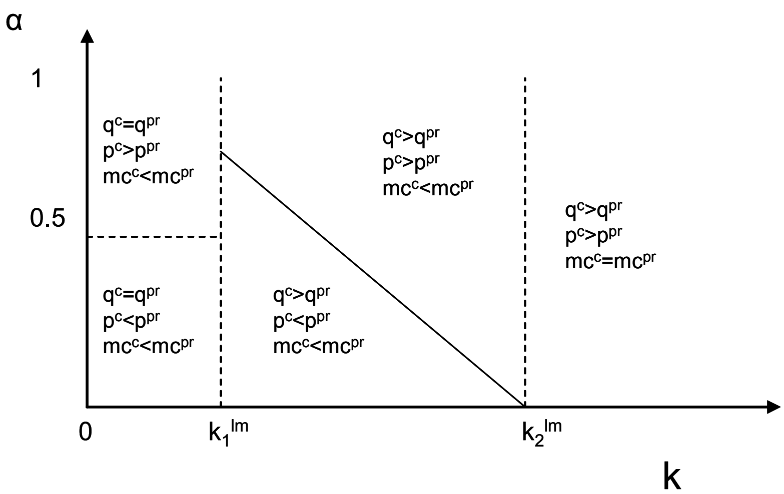
$$\begin{aligned} q_i^c - q_i^{rp} &> 0 \\ p_i^c - p_i^{rp} &> 0 \text{ for } k > 2x_1(1-\alpha) - \overline{Q} \\ p_i^c - p_i^{rp} &< 0 \text{ for } k < 2x_1(1-\alpha) - \overline{Q} \\ M^c - M^{rp} &< 0 \end{aligned}$$

for $i = 1, 2$. Finally for $k > 2x_1 - \overline{Q}$ the equilibrium under reference pricing is characterized by (78) while under co-payment by (22) and the market is fully covered under both systems.. Therefore we have that,

$$\begin{aligned} q_i^c - q_i^{rp} &> 0 \\ p_i^c - p_i^{rp} &> 0 \\ M^c - M^{rp} &= 0 \end{aligned}$$

for $i = 1, 2$. ■

The following graph illustrates the results described in the propositions above,



Reference Pricing versus co-payment equilibrium qualities and prices for $x_1 > \frac{1}{4}$