INTERPRETING THE EXPECTED VALUE OF PERFECT INFORMATION ABOUT PARAMETERS

Susan Griffin and Karl Claxton, Centre for Health Economics, University of York, UK

CHE Centre For Health Economics

Introduction

Expected value of information (EVI) analysis can quantify the expected gain in net benefit (NB) from obtaining further information to inform a decision. The expected value of perfect information (EVPI) for a decision problem must exceed the cost of research to make additional investigation worthwhile.
Expected value of partial perfect information (EVPPI) can be used to identify:
(i) parameters that contribute most to the EVPI for the whole decision problem;
(ii) parameters with no EVPPI that may be disregarded as potential targets for further research.
Recently it has been noted that parameters with zero EVPPI for a one-off research design may be associated with high EVPPI when considered as part of a sequential research design.

A simple two parameter decision problem is used to illustrate the calculation of EVPI, EVPPI for single parameters, joint EVPPI for groups of parameters, conditional EVPPI and sequential EVPPI.

- Conditional EVPPI is the expected value of perfect information about parameter two, conditional on having obtained perfect information about parameter one.
- Sequential EVPPI is the expected value of perfect information for a sequential research design to investigate first parameter one, then parameter two.

Value with current information

The (NB of a decision can be calculated as a function of a set of input parameters ($\theta=\theta_1, \theta_2$) which are known with uncertainty. With current information a decision maker should select the intervention (j) that maximises expected net benefit. The value of such a decision based on current information can be expressed as:

(1) $$\max_j E_{\theta_1,\theta_2} NB(j,\theta_1,\theta_2)$$

Value of perfect information

With perfect information the decision-maker could select the intervention that maximises the net benefit for a particular value of θ ($\max_j NB(j,\theta_1,\theta_2)$). As the true values of θ are unknown, the expected value of a decision taken with perfect information is found by averaging the maximum net benefit over the joint distribution of θ:

(2) $$E_{\theta_1,\theta_2} \max_j NB(j,\theta_1,\theta_2)$$

Value of partial perfect information

With perfect information about one parameter, θ_1, the decision-maker could select the intervention that maximises expected net benefit over the remaining uncertain parameters, θ_2, conditional on the particular value of θ_1 ($\max_j E_{\theta_2|\theta_1} NB(j,\theta)$). Again, the true values of θ_1 are unknown. So for a two parameter decision the expected value of a decision with perfect information about θ_1 and θ_2 respectively are:

(3) $$E_{\theta_1} \max_j E_{\theta_2|\theta_1} NB(j,\theta_1,\theta_2);$$

(4) $$E_{\theta_2} \max_j E_{\theta_1|\theta_2} NB(j,\theta_1,\theta_2)$$

Benefits and costs

The value of a decision made with additional information must be compared to the value of that decision based on current information.

- Additional information only has value when it would change adoption decision.
- Information is non-rival and can be used to in every future decision
- thus EVPI should be multiplied by population of patients expected to benefit, $pop_{T\theta}$;
- $pop_{T\theta}$ decreases with the duration of research on θ, $T\theta$.
- The population EVPI must be compared to the costs of further research, C_θ
- in order to establish whether there may be a positive payoff, $\Pi_\theta>0$. The costs of research include not only investigative and organisational costs, but also any opportunity cost to patients included in the research.

The research decision space

i) Do not conduct further research?

ii) Conduct research to inform a single parameter in isolation?

iii) Conduct research to inform a group of parameters simultaneously?

iv) Conduct research to inform all parameters (whole decision problem) simultaneously?

(v) Conduct research to inform a set of parameters sequentially?

One-off research designs

Existing EVI analyses have focussed on questions about one-off research designs. These address the questions of whether an additional study is required, and if it is which or how many parameters should be included.

i) Do not conduct research?

For there to be any value to further research the maximum achievable value must at least exceed the minimum expected cost of research. The population EVPI represents the upper bound for the value that could be gained from further research.

$$EVPI = E_{\theta_1,\theta_2} \max_j NB(j,\theta_1,\theta_2) - \max_j E_{\theta_1,\theta_2} NB(j,\theta_1,\theta_2)$$

$$\Pi = EVPI*pop_{T\theta_1,\theta_2} - \min(C_{\theta_1},C_{\theta_2})$$

With this necessary condition met we may calculate the value of obtaining further information for a range of more specific research questions.

ii) Conduct research to inform a single parameter in isolation?

$$EVPPI_{\theta_1} = E_{\theta_1} \max_j E_{\theta_2|\theta_1} NB(j,\theta_1,\theta_2) - \max_j E_{\theta_1,\theta_2} NB(j,\theta_1,\theta_2)$$

$$\Pi_{\theta_1} = EVPPI_{\theta_1}*pop_{T\theta_1} - C_{\theta_1}$$

$$\Pi_{\theta_2} = EVPPI_{\theta_2}*pop_{T\theta_2} - C_{\theta_2}$$

iii) Conduct research to inform groups of parameters simultaneously?

$$EVPPI_{\theta_1,\theta_2} = EVPI; \quad \text{where } \theta = \theta_1, \theta_2$$

$$\Pi_{\theta_1,\theta_2} = EVPPI_{\theta_1,\theta_2}*pop_{T\theta_1,\theta_2} - C_{\theta_1,\theta_2}$$

Note that in a two-parameter model the joint EVPPI (perfect information about both θ_1 and θ_2) is equal to the EVPI for the whole decision problem.

iv) Conduct research to inform the whole decision problem?

This is answered by Π_{θ_1,θ_2}, but more generally:

$$\Pi_{\theta_1,\ldots,\theta_n} = EVPI*pop_{T\theta_1,\ldots,\theta_n} - C_{\theta_1,\ldots,\theta_n}$$

Sequential research designs

Decisions about one-off research designs cover only a portion of the total research decision space. We may to consider the value of a series of studies, each investigating a different parameter or group of parameters in turn. This gives us a new question not previously addressed in EVI analyses:

(v) Conduct research to inform a set of parameters sequentially?

Conditional EVPPI

We could calculate the value of obtaining additional information on θ_2, conditional on having obtained perfect information about θ_1.

- The value of further research on θ_2 given additional information about θ_1 will not be the same as the value of further research to inform θ_2 alone:
- knowledge about θ_1 changes the maximum expected net benefit with current information;
- additional information about θ_2 then gives us perfect information on θ.

$$EVPPI_{\theta_1|\theta_2} = E_{\theta_1,\theta_2} \max_j NB(j,\theta_1,\theta_2) - E_{\theta_2} \max_j E_{\theta_1|\theta_2} NB(j,\theta_1,\theta_2)$$

$$\Pi_{\theta_1|\theta_2} = EVPPI_{\theta_1|\theta_2}*pop_{T\theta_1} - C_{\theta_1}$$

$$\Pi_{\theta_2|\theta_1} = EVPPI_{\theta_2|\theta_1}*pop_{T\theta_2} - C_{\theta_2}$$

Conditional EVPPI addresses the question of whether further research would be justified after additional information has been obtained on other input parameters. However it does not provide us with the expected value of a sequential research design because it does not incorporate the value or cost of the information obtained on the first parameter in sequence.

Sequential EVPPI

A sequential research design would allow for revision of the research strategy on the basis of additional information gathered on the first parameter(s) in sequence. The maximum net benefit achievable will be a function of the benefit following research to inform only θ_1 where $EVPPI_{\theta_2|\theta_1}=0$ and the benefit with research to inform θ_1 and θ_2 (θ) where $EVPPI_{\theta_2|\theta_1}> 0$.

- The decision maker should select the research strategy (r) that maximises the expected net benefit of the sequential research design.

$$seqEVPPI_{\theta_1,\theta_2} = E_{\theta_1,\theta_2} \max_r\{\max_j NB(j,\theta_1,\theta_2), \max_j E_{\theta_2|\theta_1} NB(j,\theta_1,\theta_2)\} - \max_j E_{\theta_1,\theta_2} NB(j,\theta_1,\theta_2)$$

If research was costless we would obtain any information that reduced the probability of making an incorrect decision:

$$EVPPI_{\theta_1} + EVPPI_{\theta_2|\theta_1} =$$

$$E_{\theta_1} \max_j E_{\theta_2|\theta_1} NB(j,\theta_1,\theta_2) - \max_j E_{\theta_1,\theta_2} NB(j,\theta_1,\theta_2) + E_{\theta_1,\theta_2} \max_j NB(j,\theta_1,\theta_2) - E_{\theta_1} \max_j E_{\theta_2|\theta_1} NB(j,\theta_1,\theta_2)$$

$$= E_{\theta_1,\theta_2} \max_j NB(j,\theta_1,\theta_2) - \max_j E_{\theta_1,\theta_2} NB(j,\theta_1,\theta_2) = EVPI$$

However, research is not costless. In a sequential design the cost of research into the first parameter in the sequence is always incurred but research on the second parameter will only proceed and costs be incurred if the expected payoff is positive ($\Pi>0$).

$$\Pi_{seq\theta_1,\theta_2} = E_{\theta_1,\theta_2} \max_r\{[\max_j NB(j,\theta_1,\theta_2) - \max_j E_{\theta_2|\theta_1} NB(j,\theta_1,\theta_2)]*pop_{T\theta_2+T\theta_1} - C_{\theta_2}, 0\} + \Pi_{\theta_1}$$

$$\Pi_{seq\theta_2,\theta_1} = E_{\theta_1,\theta_2} \max_r\{[\max_j NB(j,\theta_1,\theta_2) - \max_j E_{\theta_1|\theta_2} NB(j,\theta_1,\theta_2)]*pop_{T\theta_1+T\theta_2} - C_{\theta_1}, 0\} + \Pi_{\theta_2}$$

- Sequential EVPPI can be used to identify a parameter or group of parameters that should be initiated first.
- It evaluates the potential impact of any additional information on any parameters to be investigated subsequently.
- Once information has been collected on the first parameter previous EVI calculations are redundant and a new set incorporating the additional information is required.

Comparing alternative research designs

With few parameters it is feasible to enumerate the payoff from all possible research designs. However, as the number of parameters increases, the number of potential research designs also increases.

Comparing benefit

- The value of a sequential design will be at least as great as the value of information about the first parameter in the sequence.
- The value cannot exceed the value of information for both parameters in combination: $EVPPI_{\theta_1} \le seqEVPPI_{\theta_1,\theta_2} \le EVPI_{\theta_1,\theta_2}$
- Where the payoff from investigating the second parameter in sequence is always negative, $seqEVPPI_{\theta_1,\theta_2} = EVPPI_{\theta_1}$
- Where the payoff from investigating the second parameter is always positive, $seqEVPPI_{\theta_1,\theta_2} = EVPI_{\theta_1,\theta_2}$

Comparing costs

- The cost of investigating two parameters sequentially will exceed the cost of investigating them simultaneously:
- due to economies realised by running a single trial
- due to increased opportunity costs because the length of time taken to gain the additional information on both parameters will be greater:
- $C_{\theta_1} + C_{\theta_2} > C_{\theta_1,\theta_2}$
- Where the payoff from research on the second parameter in sequence is always positive a sequential design will be dominated by a one-off research design that investigates both parameters simultaneously.

Comparing populations

Comparing the duration of research for parameters simultaneously, individually or sequentially: $T\theta_1,\theta_2 = \max(T\theta_1, T\theta_2) < T\theta_1 + T\theta_2$

- The population benefiting from a sequential design is $pop_{T\theta_1}$ for θ_1 and $pop_{T\theta_1+T\theta_2}$ for θ_2
- For a simultaneous design it is $pop_{T\theta_1,T\theta_2}$ which is $<pop_{T\theta_1}$ and $>pop_{T\theta_1+T\theta_2}$
- If $T\theta_1$ is short and $T\theta_2$ long then $seqEVPPI_{\theta_1,\theta_2}$ could exceed EVPI

The value of the sequential approach derives from the ability to avoid the cost of research on the second parameter. The benefits may be highest when the research costs for the second parameter are significant and the payoff for the second parameter varies widely between positive and negative depending on the value of the first parameter in sequence.

Discussion

Previous EVI analyses have focussed on questions about one-off research designs. It may have been tempting to prioritise further studies in order of the size of the individual payoffs (EVPPI).

We have shown an alternative by calculating the expected value from sequential research designs. This:

- accounts for learning from the information gathered on each parameter in sequence in deciding to proceed with research on the next parameter;
- may imply a different order of research to that indicated by naively ordering the parameters in terms of the individual EVPPIs.

Of course the value of perfect information represents the maximum value of further research, and in practice we will obtain only imperfect or sample information. Even so, estimating the maximum payoff from further research is computationally feasible and can provide a useful indication of which research designs are likely to represent a worthwhile use of available resources. Currently, optimising over the total research decision space on the basis of the expected value of sample information is generally and sometimes prohibitively computationally expensive. The computational expense increases if sequential research designs are considered. Therefore this type of EVPI analysis can be used to prioritise the research decision space, allowing expensive EVSI calculations to focus on those research designs which are likely to be most valuable.

- Sequential EVPPI calculations in addition to EVI analyses for one-off research designs can aid decision-makers in determining the most appropriate use of research funding.

Acknowledgements: We would like to thank colleagues who commented on an early version at the 71st Health Economists' Study Group meeting in Brunel, UK

Contact info: Email scg3@york.ac.uk

THE UNIVERSITY *of York*