Mathematical programming for the optimal allocation of healthcare resources

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BACKGROUND

The standard decision rules of cost-effectiveness analysis either require the decision maker:
• to set a threshold willingness to pay for additional health care or
• to set an overall fixed budget.

In practice, neither are generally taken, but instead an arbitrary decision rule is followed which:
• may not be consistent with the overall budget
• may lead to an allocation of resources which is less than optimal
• is unable to identify the programme which should be displaced at the margin.

AIMS

We aim to show, using a policy-relevant example, how mathematical programming (MP) can be used as a generalisation of the standard decision rules.

This allows us:
• to examine alternative budgetary rules about when expenditure can be incurred,
• to show that indivisibility in a patient population and other equity concerns can be represented as constraints in the programme.

METHODS

The objective is to determine the optimal values of the available healthcare treatments \( x_{ik} \) so as to maximise the gross benefit \( B \) subject to an overall budgetary constraint \( \delta \), and constraints that ensure all members of each independent healthcare programme \( k \) and population group \( i = 1, \ldots, J_k \) receive one and only one treatment \( j = 1, \ldots, J_k \).

\[
\text{max } \sum_i \sum_k \sum_j c_{ijk} x_{ijk} \delta \sum_i \sum_k x_{ijk} = 1 \text{ for } i = 1, \ldots, J_k \text{ and } k = 1, \ldots, K
\]

\[
0 \leq x_{ijk} \leq 1 \text{ for } i = 1, \ldots, J_k, j = 1, \ldots, J_k, k = 1, \ldots, K
\]

\[
\text{Decision variables are the proportion of population } i \text{ and current care for treatments } j \text{ and programme } k.
\]

POLICY EXAMPLE

We demonstrate the method using data taken from the 6th and 7th wave of NICE appraisals. We assume all treatments must be provided from within a fixed budget. We show results for a range of possible budgets.

RESULTS

Figure 1: The shadow price (QALYs per additional £1m) of the overall budget constraint at different values of the overall budget, and corresponding threshold cost per QALY.

The shadow price falls (or the threshold cost per QALY rises) as the budget increases.

Table 1: QALY loss with alternative budget rules

<table>
<thead>
<tr>
<th>Budget rule</th>
<th>Health gain (QALY)</th>
<th>Opportunity Loss (QALY)</th>
<th>Budget spent (£m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No constraint</td>
<td>7317</td>
<td>0</td>
<td>£180</td>
</tr>
<tr>
<td>Equal phasing</td>
<td>3586</td>
<td>3731</td>
<td>£103</td>
</tr>
<tr>
<td>All in 1st 5 years</td>
<td>4879</td>
<td>2438</td>
<td>£75</td>
</tr>
</tbody>
</table>

Table 2: QALY loss with indivisibilities (horizontal equity)

<table>
<thead>
<tr>
<th>Health gain (QALY)</th>
<th>Opportunity Loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>No equity constraint</td>
<td>3586</td>
</tr>
<tr>
<td>Indivisibility in popn. 1</td>
<td>3066</td>
</tr>
<tr>
<td>Indivisibility in popn. 2</td>
<td>3547</td>
</tr>
<tr>
<td>Indivisibility in all patient populations</td>
<td>3066</td>
</tr>
</tbody>
</table>

CONCLUSIONS

We show that MP can be used to allocate resources to treatments within and between patient populations, using a policy-relevant example. The outcome is equivalent to the rules of Johannesson and Weinstein (1993), using a cost-effectiveness threshold, if one is willing to accept the resulting level of expenditure as the budget. MP is able to generalise these rules, to accommodate constraints such alternative budgetary rules about the timing of expenditure, or incorporate indivisibilities (and other equity concerns). We show the efficiency loss from these additional constraints and show that the effect of equity concerns will vary from patient population to population.