

# Upper Critical Values for the Kruskal-Wallis Test

( $k$  samples)

- Notes: 1. In the table below, the critical values give significance levels as close as possible to, but not exceeding the nominal  $\alpha$ . The actual levels of significance are in brackets.
2. When the table below is not applicable and  $k = 3$  with the three group sizes above 5 or  $k > 3$  with all group sizes above 4, use the following approximation:

$$h_{\alpha} = \chi_{\alpha}^2, \quad \text{df} = k - 1.$$

| Group Sizes |   |   | Nominal size $\alpha$ |                |                |                |
|-------------|---|---|-----------------------|----------------|----------------|----------------|
|             |   |   | 0.10                  | 0.05           | 0.025          | 0.01           |
| 2           | 2 | 2 | 4.571 (.06667)        | --             | --             | --             |
| 3           | 2 | 1 | 4.286 (.10000)        | --             | --             | --             |
| 3           | 2 | 2 | 4.500 (.06667)        | 4.714 (.04762) | --             | --             |
| 3           | 3 | 1 | 4.571 (.10000)        | 5.143 (.04286) | --             | --             |
| 3           | 3 | 2 | 4.556 (.10000)        | 5.361 (.03214) | 5.556 (.02500) | --             |
| 3           | 3 | 3 | 4.622 (.10000)        | 5.600 (.05000) | 5.956 (.02500) | 7.200 (.00357) |
| 4           | 2 | 1 | 4.500 (.07619)        | --             | --             | --             |
| 4           | 2 | 2 | 4.458 (.10000)        | 5.333 (.03333) | 5.500 (.02381) | --             |
| 4           | 3 | 1 | 4.056 (.09286)        | 5.208 (.05000) | 5.833 (.02143) | --             |
| 4           | 3 | 2 | 4.511 (.09841)        | 5.444 (.04603) | 6.000 (.02381) | 6.444 (.00794) |
| 4           | 3 | 3 | 4.709 (.09238)        | 5.791 (.04571) | 6.155 (.02476) | 6.745 (.01000) |
| 4           | 4 | 1 | 4.167 (.08254)        | 4.967 (.04762) | 6.167 (.02222) | 6.667 (.00952) |
| 4           | 4 | 2 | 4.555 (.09778)        | 5.455 (.04571) | 6.327 (.02413) | 7.036 (.00571) |
| 4           | 4 | 3 | 4.545 (.09905)        | 5.598 (.04866) | 6.394 (.02476) | 7.144 (.00970) |
| 4           | 4 | 4 | 4.654 (.09662)        | 5.692 (.04866) | 6.615 (.02424) | 7.654 (.00762) |
| 5           | 2 | 1 | 4.200 (.09524)        | 5.000 (.04762) | --             | --             |
| 5           | 2 | 2 | 4.373 (.08995)        | 5.160 (.03439) | 6.000 (.01852) | 6.533 (.00794) |
| 5           | 3 | 1 | 4.018 (.09524)        | 4.960 (.04762) | 6.044 (.01984) | --             |
| 5           | 3 | 2 | 4.651 (.09127)        | 5.251 (.04921) | 6.004 (.02460) | 6.909 (.00873) |
| 5           | 3 | 3 | 4.533 (.09697)        | 5.648 (.04892) | 6.315 (.02121) | 7.079 (.00866) |
| 5           | 4 | 1 | 3.987 (.09841)        | 4.985 (.04444) | 5.858 (.02381) | 6.955 (.00794) |
| 5           | 4 | 2 | 4.541 (.09841)        | 5.273 (.04877) | 6.068 (.02482) | 7.205 (.00895) |
| 5           | 4 | 3 | 4.549 (.09892)        | 5.656 (.04863) | 6.410 (.02496) | 7.445 (.00974) |
| 5           | 4 | 4 | 4.668 (.09817)        | 5.657 (.04906) | 6.673 (.02429) | 7.760 (.00946) |
| 5           | 5 | 1 | 4.109 (.08586)        | 5.127 (.04618) | 6.000 (.02165) | 7.309 (.00938) |
| 5           | 5 | 2 | 4.623 (.09704)        | 5.338 (.04726) | 6.346 (.02489) | 7.338 (.00962) |
| 5           | 5 | 3 | 4.545 (.09965)        | 5.705 (.04612) | 6.549 (.02436) | 7.578 (.00968) |
| 5           | 5 | 4 | 4.523 (.09935)        | 5.666 (.04931) | 6.760 (.02490) | 7.823 (.00978) |
| 5           | 5 | 5 | 4.560 (.09952)        | 5.780 (.04878) | 6.740 (.02475) | 8.000 (.00946) |

| Group<br>Sizes |   |   | Nominal size $\alpha$ |                |                |                |
|----------------|---|---|-----------------------|----------------|----------------|----------------|
|                |   |   | 0.10                  | 0.05           | 0.025          | 0.01           |
| 6              | 1 | 1 | --                    | --             | --             | --             |
| 6              | 2 | 1 | 4.200 (.09524)        | 4.822 (.04762) | 5.600 (.02381) | --             |
| 6              | 2 | 2 | 4.545 (.08889)        | 5.345 (.03810) | 5.745 (.02063) | 6.655 (.00794) |
| 6              | 3 | 1 | 3.909 (.09524)        | 4.855 (.05000) | 5.945 (.02143) | 6.873 (.00714) |
| 6              | 3 | 2 | 4.682 (.08528)        | 5.348 (.04632) | 6.136 (.02294) | 6.970 (.00909) |
| 6              | 3 | 3 | 4.590 (.09773)        | 5.615 (.04968) | 6.436 (.02229) | 7.410 (.00779) |
| 6              | 4 | 1 | 4.038 (.09437)        | 4.947 (.04675) | 5.856 (.02424) | 7.106 (.00866) |
| 6              | 4 | 2 | 4.494 (.09986)        | 5.340 (.04906) | 6.186 (.02453) | 7.340 (.00967) |
| 6              | 4 | 3 | 4.604 (.09997)        | 5.610 (.04862) | 6.538 (.02498) | 7.500 (.00966) |
| 6              | 4 | 4 | 4.595 (.09847)        | 5.681 (.04881) | 6.667 (.02495) | 7.795 (.00990) |
| 6              | 5 | 1 | 4.128 (.09271)        | 4.990 (.04726) | 5.951 (.02453) | 7.182 (.00974) |
| 6              | 5 | 2 | 4.596 (.09807)        | 5.338 (.04729) | 6.196 (.02481) | 7.376 (.00982) |
| 6              | 5 | 3 | 4.535 (.09932)        | 5.602 (.04956) | 6.667 (.02452) | 7.590 (.00999) |
| 6              | 5 | 4 | 4.522 (.09974)        | 5.661 (.04991) | 6.750 (.02473) | 7.936 (.00998) |
| 6              | 5 | 5 | 4.547 (.09835)        | 5.729 (.04973) | 6.788 (.02484) | 8.028 (.00988) |
| 6              | 6 | 1 | 4.000 (.09774)        | 4.945 (.04779) | 5.923 (.02381) | 7.121 (.00932) |
| 6              | 6 | 2 | 4.438 (.09824)        | 5.410 (.04993) | 6.210 (.02443) | 7.467 (.00982) |
| 6              | 6 | 3 | 4.558 (.09948)        | 5.625 (.04999) | 6.725 (.02462) | 7.725 (.00985) |
| 6              | 6 | 4 | 4.548 (.09982)        | 5.724 (.04950) | 6.812 (.02458) | 8.000 (.00998) |
| 6              | 6 | 5 | 4.542 (.09987)        | 5.765 (.04993) | 6.848 (.02489) | 8.124 (.00990) |
| 6              | 6 | 6 | 4.643 (.09874)        | 5.801 (.04905) | 6.889 (.02493) | 8.222 (.00994) |
| 7              | 7 | 7 | 4.594 (.09933)        | 5.819 (.04911) | 6.954 (.02446) | 8.378 (.00992) |
| 8              | 8 | 8 | 4.595 (.09933)        | 5.805 (.04973) | 6.995 (.02485) | 8.465 (.00991) |