GEOMETRIC SYMMETRY
SLIDES

ORDER FOR FRIEZE GROUPS

- t  L
- tg  LΓ
- tm  V
- mt  C
- t2  N
- t2mg  VA
- t2mm  H

ORDER FOR WALLPAPER GROUPS

- p1  p2  pm  pg  cm
- pmm  pmg  pgg  cmm
- p4  p4m  p4g
- p3  p3m1  p31m
- p6  p6m
GEOMETRIC SYMMETRY—SLIDES

010
(Cypriot Vase with the Name of Thales).

020

030
Hedgehog

040
Palindrome.
THE END - Frederic Brown.

050
Palindrome
BLACK AND WHITE by J.A. Lindon.
Lived as a dog - o no! God, as a devil
Doom lives ever, it’s astir, Eve’s evil mood,
Live, O devil, revel ever, live, do evil!
Do, O God, no evil deed, live on, do good!

060
m

070
m
Mixtec culture: a sacrificial obsidian knife with mosaic handle. A two-handled mosaic serpent (below). These objects are believed to have been among the treasure presented to Cortés by Moctezuma. Hammond Innes, *The Conquistadors*, Collins 1969, repr. by Fontana 1972, p.110.
Flags of Europe I: 1, Denmark; 2, Norway; 3, Iceland; 4, Sweden; 5, Finland; 6, Greece; 7, Czechoslovakia; 8, Poland.
Numbers 1, 2, 3, 4 (and 5 apart from the badge) have symmetry m but number 7 needs inversion of red and yellow to go with the mirror operation.

Fig. 129. M83. This picture of a face-on spiral galaxy was made of Kodak Vericolor II negative film, rather than by the three-colour superposition technique and shows much less intense but similar colours to the picture of NGC 2997 [Fig. 128 in the same book].
David Malin and Paul Murdin, *Colours of the Stars*, Cambridge University Press 1984, Fig. 129.

Fig. 133. Centaurus A. Basically an elliptical galaxy with a population of yellowish stars, Cen A is crossed by a band of dust which reddens the stars still further. Formed from the dusty material, blue stars and pink H II regions can be perceived along the rim of the dust band.
David Malin and Paul Murdin, *Colours of the Stars*, Cambridge University Press 1984, Fig. 133.

Back cover and p.13. The yin-yang symbol [tai-ji] surrounded by the eight trigrams. This arrangement is known as the Sequence of Earlier Heaven, or the Primal Arrangement (Roland Michaud).

Figure 6.8 portrays two ancient symbols belonging to the same group, the runic symbol for death and the well-known Chinese yin-yang symbol of universal duality.
P.S. Stevens, *Handbook of Regular Patterns*, MIT Press 1974, p.43, Figs. 6.8a and 6.8b.
An arrangement of diatoms from the South Pacific, photographed in polarized light × 150.

Fig. 20 Aurelia insulinda. Example of an organism possessing a four-fold symmetry axis (Haeckel).
A.V. Shubnikov and V.A. Koptsik, Symmetry in Science and Art, Plenum 1974, Fig. 20, p.20.

8. A sprig of Codlins and Cream (Epilobium hirsutum)
The whole plant grows to five or six feet
Iolo A. Williams, Flowers of Marsh and Stream, Penguin 1946 (King Penguin No. 27), Plate 8.

Fig. 1. The starfish Pentaceraster manillatus, one of the most conspicuous inhabitants of deeper sand bottoms, is 20 to 25cm across and bright red or orange in colour. This individual was brought up from the bottom at 15m and photographed on the beach. [Jana Island].
Philip W. Basson, John E. Burchard, Jr., John T. Hardy, and Andrew R.G. Price (illustrated and designed by Lisa Bobrowski), Biotopes of the Western Arabian Gulf, Aramco (Dharan, Saudi Arabia) 1977, Fig. 1.

W.A. Bentley and W.J. Humphreys, Snow Crystals, McGraw-Hill 1931 and Dover 1962.

W.A. Bentley and W.J. Humphreys, Snow Crystals, McGraw-Hill 1931 and Dover 1962.

(top left) Antedon bifida (top right) Laida ciliaris (always has 7 flattened rays)
(centre left) Astropecten aranciacus (centre right) Astropecten irregularis
(bottom left) Ceramaster placenta (bottom right) Porania pulvillas

210
8m, 9m, 12m and 5m
(top left) *Crossaster papposus* (8-13 blunt rays) (12 shown)
(top right) *Solaster endeca* (7-13 rays) (9 shown)
(centre left) *Marthasterias glacialis*
(centre right) *Asterias rubens*
(bottom left) *Stichastrella rosea*
(bottom right) *Coscinasterias tenuispina* (6–10 rays often of very different lengths) (8 shown)

A.C. Campbell (illustrated by James Nicholls), *The Seashore and Shallow Seas of Britain and Europe*, Collins 1946, p. 245.

220
12
9. *Sea anemone* (*Halcampa chrysanthellum*).
[It] is very much enlarged and inhabits sandy and muddy places, burying its worm-like body and extending its tentacles at its surface.

230
20m (approx.)
Floating sea slug attacking a jellyfish. Great Barrier Reef.

240
Fig. 14–6 Asante brass weights in geometric shapes, used for measuring gold dust currency. Many of the designs have symbolic meaning; for example, a zigzag line represents the fire of the sun. British Museum.

250
The Seven Friezes

```
L L L L L L L
L Γ L Γ L Γ L
V V V V V V V
C C C C C C C
N N N N N N N
V A V A V A V
H H H H H H H
```

260
Plate XV, Greek No. 1

270
Plate VI, Egyptian No. 3

280
Plate XCIII Leaves from Nature No. 3

285
Granada. Sala de Reposo [del Bao]
[Baths. The Rest Hall or Hall of Repose in the Comares Palace Bath]
(Postcard)

290
p1
Plate X, No. 11. Ornament on the walls, Hall of the Abencerrages.

300
p2
24.9a Peruvian textile design
24.9b Peruvian textile design
24.9c Peruvian textile design

310
pm (diamonds); cm (lozenges)
Plate 31
Plate 32

320
pg
M.C. Escher, Study for the lithograph “Encounter”, pencil and ink 1944. Note that in this case it is impossible to bring, e.g., the two white men to coincidence by a rotation.


Figure 27.4
(a) (top right) American Indian, Nez Percé
(b) (bottom left) Romanesque
(c) (bottom right) mosaic pavement, Florence baptistry

Chinese lattice designs
25.7 (a) (top) (Dye N2b, p.215)
(b) (bottom left) (Dye I3b, p.157)
(c) (top right) (Dye 7, p.307)
This pattern presents a tricky problem. It is formed by an array of starfishes, clams and snail shells. At first sight, one would mark as cell corners the points where four clams and four starfish meet. However, on closer inspection it is seen that the edges of this square are not true translations of the patterns; the snail shells halfway between these pseudocell corners are not repeated in identical orientation by such shifts. . . . On the other hand, the mid-point of the pseudocell, where four starfishes and four snail shells meet, is indeed a true fourfold rotation point. . . .

row and find in the pattern three other fourfold points in the same orientation. These outline a square unit cell which is tilted at 45 to the borders of the picture.


450

$p4gm$

33.6a (top) mother-of-pearl inlay motif, Turkey

33.6b (bottom left) Arabian

33.6c (bottom right) Arabian


460

$p3$

Plate XLVIII No. 58. Mosaic from the portico of the Generalife.


465

$p3$

Postcard with caption “La Alhambra (Granada) Estucado”.

470

$p3m1$

30.3 (b) (top left) Persian

30.3 (c) (bottom right) Chinese (Dye C12b, p.79)


480

$p31m$

31.4 (a) (top left) Chinese

31.4 (b) (top right) Chinese (Dye C5a, p.72)

31.4 (c) (bottom) Russian


490

12m or 4m (top); $p6$ (bottom)

Plate XLII, Moresque No. 4 (Do not confuse with Plate XLII*, Moresque No. 4*)

No. 6 (top) ≡ Albert F. Calvert, *The Alhambra*, John Lane, The Bodley Head 1906, Plate LXXVIII, No. 106, Part of the Portico of the Court of the Fish Pond.

No. 5 (bottom) ≡ Calvert, *op. cit.*, Plate LXXX, No. 108, Ornaments on the walls, House of Sanchez.

36.6 (a) (left) An ancient design from the palace at Nimrud, Mesopotamia
36.6 (b) (right) A Japanese design


44. Some of the myriads of elegant basalt columns that make up the Giant’s Causeway in Antrim, Northern Ireland. Most, but not all, are hexagonal. (Institute of Geological Sciences photograph).


530

Plate 10 B. Polygonal jointing in Tholeiitic Basalt lava. Giant’s Causeway.

540

Plate 52. Workers of the dwarf honey bee gnawing wax off an abandoned comb and putting it into their leg baskets (see p.95).


550

Fig. 137 (above) M.C. Escher, Study of Regular Division of the Plane with Horsemen, Indian Ink and Watercolour 1946 (Magic Mirror, Fig. 225)
Fig. 138 (below), M.C. Escher, Horsemen, Woodcut in three colours 1946 (≈ Magic Mirror, Fig. 224 ≈ Graphic Work, Fig. 6)

Colour plane symmetry
This is a more typical case of colour symmetry.
The pattern is composed of fishes in four different colours and orientations. All
the fishes of one colour have the same orientation and surroundings, so there is
one fish of each colour per cell. . . .
[Discussed also by Arthur L. Loeb, Color and Symmetry, Wiley 1971, Chap. 22,
pp.162–165]
Caroline H. MacGillavry, Symmetry Aspects of M.C. Escher’s Periodic Draw-

570
M.C. Escher, The Graphic Work of M.C. Escher, Oldbourne 1961, Fig. 20
Bruno Ernst, The Magic Mirror of M.C. Escher, Ballantine 1976, Fig. 244
Plate VII and Fig. 239

580
Tilings
(Frontispiece) Johannes Kepler (1580-1630) is well known for his pioneering
work in astronomy. He also made fundamental contributions to the theory of
tilings, and some of his ideas have still not been fully investigated. We reproduce
here tilings from his book Harmonica Mundi (volume 2) published in 1619. The
tiling marked Aa can be extended over the whole plane as shown in Figure 2.0.1
and described in Section 2.5.

590
Tilings
Figure 8.0.1. An attractive and ingenious pattern of squares from Islamic art
(Bourgoin [1879, Plate 13]) perfectly coloured in four colours. The pattern
is of type PP48A and it can be perfectly k-coloured if k has one of the values 3n,
6n, n², 2n², 3n² or 6n² for a positive integer n (see Senechal [1979a]).

600
Regular solids
M.C. Escher, Study for the wood engraving ”Stars”, Woodcut 1948.
J.L. Locher (ed.), The World of M.C. Escher, Harry N. Abrams 1971, Fig. 151.

610
Regular solids
M.C. Escher, Stars, Wood engraving 1948.
M.C. Escher, The Graphic Work of M.C. Escher, Oldbourne 1961, Fig. 51
Bruno Ernst, The Magic Mirror of M.C. Escher, Ballantine 1976, Fig. 209
J.L. Locher (ed.), The World of M.C. Escher, Harry N. Abrams 1971, Fig. 152
Regular solids

Fig. 26. Skeletons of various radiolaria (after Haeckel).

Regular solids
Fig. 46. Here (Fig. 46) is his [Kepler’s] construction, by which he believed he had penetrated deeply into the secrets of the Creator. The six spheres correspond to the six planets, Saturn, Jupiter, Mars, Earth, Venus, Mercurius, separated in this order by cube, tetrahedron, octahedron, dodecahedron, icosahedron.

Table 9. The thirteen semi-regular (Archimedean) polyhedra and their duals
(a) truncated octahedron (b) truncated icosahedron (c) great rhombicuboctahedron or truncated cuboctahedron (d) snub cube (e) truncated tetrahedron (f) cuboctahedron (g) icosidodecahedron (h) small rhombidodecahedron (i) small rhombicosidodecahedron (j) truncated cube (k) truncated dodecahedron (l) truncated dodecahedron (m) snub dodecahedron.

Table 10. Some examples of space filling systems.

Crystal
Fig. 191. The fourteen Bravais lattices. If we assume spherical symmetry for the lattice points, the space groups of the fourteen Bravais lattices will be (1) P1 (2) P2/m (3) C2/m (4) Pmmm (5) Cmmm (6) Immm (7) Fmmm (8) P6/mmm (9) R3m (10) P4/mmm (11) I4/mmmm (12) Pn3m (13) Ia3m (14) Fm3m. The capital letters denote the translation groups: P, R primitive lattices; C lattices centred in the face cutting the edge c; F face centred lattices; I body-centred lattices. (1) is triclinic, (2)–(3) monoclinic, (4)–(7) orthorhombic (8) hexagonal (9) trigonal (10)–(11) tetragonal (12)–(14) cubic.
Crystal

Fig. 73. The 47 simple forms which crystals may take: (1)–(7) Pyramids; (8)–(14) bipyramids; (15)–(21) prisms (22), (23), (25) tetrahedra; (24), (26), (28) trapezohedra; (27) rhombohedron; (34) scalene triangle; (33), (35) scalenohedra; (31) dihedron; (32) pinacoid; (23), (29), (36)–(47) simple forms of the cubic system; (23) tetrahedron; (29) cube; (30) octahedron; (36) trigonal tristetrahedron; (38) pentagonal tristetrahedron; (39) rhombic dodecahedron; (40) pentagonal dodecahedron; (41) tetrahexahedron; (42) hexatetrahedron; (43) dodecahedron; (44) tetragonal trisoctahedron; (45) trigonal trisoctahedron; (46) pentagonal trisoctahedron (47) hexoctahedron.


Figs. 69 & 70. Here are two Laue diagrams (Figs. 69 and 70), both of zinc-blende from Laue’s original paper (1912); the pictures are taken in such directions as to exhibit the symmetry around an axis of order 4 and 3 respectively.


Crystal

14. Fluor spar

15. Fluor spar


Crystal

12. Iron pyrites

13. Fluor spar ‘Bluejohn’


Spiral

NAUTILOIDS, GONIATITITES AND CERATITES [on left] 64 Orthoceras [top left]; 65 Nautilus [top right of left]; 66 Glyphioceras [bottom left]; 67 Ceratites.

LIASSIC AMMONITES [on right] 68 Promicroceras [top right of left]; 69 Phylloceras [top right]; 70 Dactilioceras [bottom right of left]; 71 Harpoceras.

Plate XXXV Shell, Logarithmic Spiral and Gnomonic Growth (Photographs: Kodak Ltd)
Plate XXXV gives two examples of X-rayed shells, showing clearly the direct-
ing spiral of *Nautilus Pompilius* and the gnomonic growth of *Triton Tritonis* (p.97). Gnomonic growth means growth from inside outwards as opposed to agglutination as is common in crystal [paraphrased from p.90].

740
Spiral
Fig. 44. For everybody looking at this picture (Fig. 44) of a giant sunflower, *Hel-
ianthus maximus*, the florets will naturally arrange themselves into logarithmic
spirals of opposite sense of coiling.

750
Helical
(334) No. 23 HIGH PETERGATE, c. 1779 (left) (helical)
(334) No. 62 LOW PETERGATE, c. 1725 (right)
Royal Commission on Historical Monuments, *City of York. Volume 5: The

760
Helical
J.D. Watson, *The Double Helix: A New Critical Edition*, Weidenfeld and Nichol-
son 1981, p.121.

770
Helical

780
Conical spiral
HOLOSTOMATOUS AND SIPHONOSTOMATOUS GASTROPODS
110. Left, *Turitella*, Right *Cerithium*
Bartonian (Upper Eocene), Gisois, Normandy, France × 1 ½.
J.F. Kirkaldy (Photographs by Michael Allman), *Fossils in Colour*, Blandford