Most Honourable Remembrance: The Life and Work of Thomas Bayes

Most beginners no doubt believe that Bayes' Theorem was proved by Thomas Bayes. In fact his most famous paper considered the following problem, "*Given* the number of times in which an unknown event has happened and failed: *Required* the chance that the probability of its happening in a single trial lies somewhere between any two degrees of probability that can be named." He assumes *a priori* that all values of the unknown probability θ are equally likely and concludes (in modern notation) that, conditional on the observation of $X = p$ successes in n trials,

$$\mathsf{P}(b < \theta < f \mid X = x) = \frac{\mathsf{P}(b < \theta < f \cap X = p)}{\mathsf{P}(X = p)} = \frac{\int_b^f \binom{n}{p} \theta^p (1-\theta)^{n-p} \, \mathrm{d}\theta}{\int_0^1 \binom{n}{p} \theta^p (1-\theta)^{n-p} \, \mathrm{d}\theta}.$$

This amounts to a special case of the result now known as Bayes' Theorem, but a recognizable form of the general result allowing for unequal prior probabilities did not appear until Laplace's proof in 1783.

As the statement above shows he not merely proved a special case of the theorem but also used it to conduct what we now call a Bayesian inference. Note that he used an "objective" prior distribution, in which he was followed by, e.g., by Sir Harold Jeffreys, but not by the majority of modern investigators. He gave an ingenious argument for his prior the interpretation of which has been a matter of some controversy. To my mind, the most convincing description of Bayes' argument is to be found in Stigler[1] (pp. 127–129).

The increasing importance of Bayesian methodology has naturally led to an increasing interest in the inventor (even though he might wish to disown some of the things done in his name). Consequently, a man about whom little has so far been written has recently been the subject of the biography under review and substantial biographical essays by Bellhouse[3,2]. In a review of Dale's book Bellhouse has also drawn attention to some earlier work by Bayes on chance.

It happens that at the same time I was reading this book I read Levi's 1988 book, "The Life and Times of William Shakespeare," and was struck by the fact that the paucity of real evidence about Bayes' life is reminiscent of that about Shakespeare's. Nevertheless, Professor Dale has assembled a respectable amount of evidence about Bayes' education at the University of Edinburgh, his ministry in Tunbridge Wells and his burial in Bunhill Fields (very near the present-day headquarters of the Royal Statistical Society).

The book's length is mainly due to the inclusion of reprints of his published tracts *Divine Benevolence* and *An Introduction to the Doctrine of Fluxions* (which is an attempt to rebut Bishop Berkeley's views), his published letter to John Canton on a semi-convergent series and his "Essay on the Doctrine of Chances" and the supplement to it. In addition there are reprints of various unpublished letters and of a notebook partly in shorthand which is in the archives

of the Equitable Life Assurance Society. All of the works are complemented by full discussion and notes, so that while there is still a limited amount known about his personality we can now have no excuse for unfamiliarity with his work.

The book has some delightful light touches, with references to Matilda (who told such dreadful lies), Augustus Carp, Lucky Jim and goodness knows what else, and yet maintains a high standard of scholarship throughout. Altogether it is a monumental achievement which suitably follows Professor Dale's previous work on the history of inverse probability[4].

References

1. S M Stigler, *The History of Statistics: The Measurement of Uncertainly Before 1900*, Cambridge, MA: Belknap Press of Harvard University Press 1986.

2. D R Bellhouse. On some recently discovered manuscripts of Thomas Bayes, *Historia Mathematica* **29** (2002), 383–394.

3. D R Bellhouse, Thomas Bayes, F.R.S.: A biography to celebrate the tercentenary of his birth (with discussion), *Statistical Science*, **19** (2004), 3–43.

4. A I Dale, *A History of Inverse Probability from Thomas Bayes to Karl Pearson*, New York: Springer 1991 and 1999.