

DE WITT'S

Treatise on Life Annuities

IN A SERIES OF

LETTERS TO THE STATES-GENERAL.

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“NOBLE AND MIGHTY LORDS:

“IN so extensive an administration as that of the united country of Holland and West Friesland, it is better, as I have several times stated to your Lordships, for several reasons perfectly well known to you, to negotiate funds by life annuities, which from their nature are infallibly terminable, than to obtain them at interest, which is perpetual, or by redeemable annuities; and that it is likewise more useful for private families, who understand economy well, and know how to make a good employment of their surplus in augmenting their capital, to improve their money by life annuities, than to invest it in redeemable annuities or, at interest at the rate of 4 per cent. per annum; because the above mentioned life-annuities, which are sold even at the present time at 14 years' purchase, pay, in fact, much more in proportion than redeemable annuities at 25 years' purchase. I have consequently respectfully to submit to your Lordships the unchallengeable proof of my assertion, and at the same time to respond to the wish manifested by the members of this body to have such a proof in writing. That proof, founded on a solid basis, is proposed to your High Mightinesses in the following manner:—

“Value of Life Annuities in Proportion to Redeemable Annuities.

“I lay down the following presupposition, in order to determine the proportion of a life annuity to a redeemable annuity. For example, in presupposing that the redeemable annuity is and will be current at 25 years' purchase, or at the rate of 4 per cent. per annum, we must find at how many years' purchase the life annuity should be sold, to be in proportion to the aforesaid redeemable annuity, in such manner that the life annuity may, if not with mathematical precision, at least in its discovered value, be more advantageous to the purchaser than an annuity redeemable with the same capital.

“FIRST PRESUPPOSITION

“I presuppose that the real value of certain expectations or chances of objects, of different values, must be estimated by that which we can obtain from equal expectations or chances, dependent on one or several equal contracts. Let us take, for example, a small matter, and under circumstances intelligible at first sight:— A person has 2 different expectations or chances which may easily lead, the one to nothing, the other to 20 stuyvers. If, by one or several equal contracts, he can obtain for 10 stuyvers 2 like expectations, we must estimate

¹The title of the Treatise in the original (now in the State Archives at the Hague) is “*Waardye van lyf-renten naer proportie van Losrenten*,” which Mr. Hendriks has translated into English R. G. B.

that the 2 aforesaid chances are worth to him exactly 10 stuyvers, because he can really obtain for 10 stuyvers these 2 expectations or chances, by making an agreement with another person that each of them should stake 10 stuyvers, and then gamble or draw lots, by odd or even, head or tail, blank or prize, or in some such way, to determine which of the two should have the 20 stuyvers; thus by the said contract, equal in every regard, he evidently finds himself in the position of having in reality the 2 expectations or chances, the one of nothing, the other of 20 stuyvers.

“SECOND PRESUPPOSITION

“That in taking at pleasure some years of a man’s life, limited to the time when he is in full vigour, and neither too young, nor too advanced in age; (this space of years shall be here 50 years, namely from the third or fourth year of his age, up to the fifty-third or fifty-fourth year;) it is not more likely that this man should die in the first half-year of a given year, than in the second year; similarly, it is not more likely that he should die in the second half-year of the aforesaid year than in the first half. But although it depends entirely on chance whether this man, after having lived to the given year, and dying in the course of that year, should demise in its first or second half, one finds nevertheless in this regard an equality of likelihood or chance similar to the case of a tossed penny, where there is an absolute equality of likelihood or chance that it will fall head or tail, although it depends entirely upon chances as to the side on which it shall turn, and this to so high a degree that the penny may fall head 10, 20, or more times following, without once calling tail; and vice versa.

“THIRD PRESUPPOSITION

“That a man having passed the aforesaid vigorous time of his life, namely the fifty-third or fifty-fourth year of his age, it begins to be more likely that he should die in a given year or half-year of the second period that has previously been the case; or that it is not more likely with respect to another man of like constitution or state of body, that the latter should die in less than a year or half-year of the said vigorous time of his life; whilst this likelihood or chance of dying in a given year or half-year of the first 10 following years, namely from 53 to 63 years of his age, taken inclusively, does not exceed more than in the proportion of 3 to 2 the likelihood or chance of dying in a given year or half-year during the aforesaid vigorous period of life: so that, taking for example two persons of equal constitution, one aged 40 years, and the other aged 58 years, if these two persons made such a contract, that in case the person of 58 years should happen to die in less than six months, the one aged 40 were to inherit a sum of 2,000 florins from the property of the defunct; but that if, on the other hand, the person aged 40 years should die in less than six months, the one aged 58 years here to have 3,000 florins from the property of the deceased; such a contract cannot be considered disadvantageous for the person who would have the 3,000 florins, if the event were favourable to him, and who, in the contrary event, would only lose 2,000 florins.

“I then presume that the greatest likelihood of dying in a given year or half-year of the second series of the ten following years (that is, from 63 years to 73, taken one with the other, rather than in a given year or half-year of the period of the vigour of life) cannot be estimated at more than double, or as 2 is to 1; and as the triple, or as 3 is to 1, during the 7 following years, that is, from 73 years to 80.

“Finally, in supposing that life necessarily ends at the twenty-seventh year after the expiration of 50 years of age above presumed, this time is neither assumed too high, not too low a standard, as experience manifestly teaches us that the life of some men exceeds by a considerable period the age of 80 years, the age of 81 years, and even more.

“These three at articles being presupposed, we have, by a demonstrative calculation, mathematically discovered and proved that the redeemable annuity being fixed at 25 years’ purchase, as above, the life annuity should be sold at 16 years’ purchase, and even higher, to be in equality, one with the other; so that in the purchase of 1 florin of life annuity, on a young and vigorous nominee, more than 16 florins should be paid, as is proved by the following demonstration:—

“FIRST PROPOSITION

“The value of several equal expectations or chances, a certain sum of money or other objects of value pertaining to chance, is found to be exactly determined by adding the money or other objects of value represented by the chances, and by then dividing the sum of this addition by the number of chances: the quotient or result indicates with precision the value of all these chances.

“To give greater clearness to the demonstration, let a person named John have, for example, 3 equal expectations or chances—one of a certain pearl, or of 2,000 florins; the second of a certain ruby, or of 3,000 florins; and the third of a certain diamond, or of 4,000 florins; as beneath.

Chances.				
1	pearl,	or	2000	florins.
1	ruby,	or	3000	“
$\frac{1}{3}$	diamond,	or	4000	“
		3)	9000	“
			3000	florins.

I say that the 3 above-mentioned expectations or chances are together worth to him precisely the third of the above-mentioned objects or sums of money, first added up, and then divided by 3, which is the number of chances.

“DEMONSTRATION

“In the first place, let John have purchased, in community with two other persons, namely, Peter and Paul, and let each of the two have paid one-third of the value of the 3 jewels before mentioned; or rather that John with Peter and Paul have made common purse, by each contributing 3,000 florins, which has evidently been an equal contract.

“In the second place, let John have agreed to cease his communityship with his two partners, or for other reasons to draw lots by three tickets, namely, two tickets blank and one ticket prize, for the 3 above-named jewels, or the aforesaid common purse of 9,000 florins of capital, so that each of them may draw one of the aforesaid tickets, and that fortune may thus point out to which of them she assigns the above-named jewels or the whole purse; which is again evidently an equal contract.

“In the third place, let John have agreed with Peter in particular, that if fortune favors one of them, in drawing the 3 aforesaid jewels or the whole purse, the winner should give the loser the pearl, or 2,000 florins out of the purse; which is likewise evidently an equal contract.

“In the fourth place, let John have agreed with Paul in particular, that if the jewels or purse should fall by lot to one of the two, the winner should in compensation give the loser the ruby, or the 3,000 florins out of the purse; which is indisputably an equal contract.

“The four conventions or contracts being thus entered upon, the matter as concerns John is reduced to this, that he has 3 easy and equal expectations or chances - that is to say, one chance of the pearl, or of 2,000 florins, if fortune favors Peter, who, in compliance with and in virtue of the third above-named contract, made with him in particular, must give up to John the pearl, or 2,000 florins; one chance of the ruby, or of 3,000 florins, if fortune favor Paul, who, from the tenor of the fourth contract made with him in particular, must give John the ruby, or 3,000 florins; and, lastly, one chance of the diamond, or of 4,000 florins, if fortune favor himself, (John) since, by virtue of the two aforesaid, particular contracts, John having to hand over to Peter the pearl, or 2,000 florins, and to Paul the ruby, or 3,000 florins, yet retains for himself the diamond, or 4,000 florins; which chances all proceed from the aforesaid jewels, or from the purse of 9,000 florins, drawn by lot; so that, because John can obtain the proposed expectations by a third share of the 3 jewels, or by a capital of 3,000 florins, such third of the 3 aforesaid jewels, or the capital of 3,000 florins, is the real value of the expectation or chances proposed in the first presupposition. We will in the same manner demonstrate the proposition when there are 2, 4, 5, 6, equal expectations or chances, and even more, of objects of different value, provided that we assume in greater or lesser value, provided that we assume in greater or lesser proportion as many contractors with or partners of John, as also in greater or lesser proportions as many particular contracts made with each of his partners; therefore, the proposition is generally demonstrated.

“COROLLARY

“From that which precedes, we may easily conclude that the before-described rule is not the less decisive, although some of the expectations or chances be of zero or nothing; because in such case the demonstration requires no further change than to suppose one associate or partner more than the number of objects of value, relatively to the expectations; and further, that no contract like the above is made with the partners or associates.

“If, for example, John has the following expectations or chances, namely,—

Chances.

1	of zero or nothing	“	“	“	“	0	florins.
1	of a certain pearl	“	“	“	“	2,000	“
1	of a certain ruby	“	“	“	“	3,000	“
1	of a certain diamond	“	“	“	“	7,000	“
4						4) 12,000	“
						3,000	florins.

I say that the four above mentioned chances are together worth to him precisely the quarter of the three above named jewels, or the sum of 3,000 florins; for supposing that John, having bought with Peter, Paul and Nicholas, the three aforesaid jewels, or that each of them having furnished 3,000 florins, they have made a common purse of 12,000 florins, and then that he makes a general agreement with them, and with Peter and Paul, each separately, but not with Nicholas, a special contract, (similar to that made by him above,) the matter as concerns John is reduced to this, that he has four equal expectations or chances, namely, one chance of zero or nothing, if fortune favor Nicholas, with whom he has not entered into a special agreement relative to any reciprocal reimbursement;— one chance of the pearl, or of 2,000 florins, if fortune favor Peter, who, in such case, and by virtue of the special contract made between the two, has to give up the pearl to him, or make good 2,000 florins;— one chance of the ruby, or of 3,000 florins, if fortune favor Paul, who, in such case, and according to the special contract, must hand over to John the ruby, or 3,000 florins;— and, lastly, one chance of the diamond, or of 7,000 florins, if fortune favor himself, (John,) because, by virtue of the above mentioned special contracts made with Peter and Paul, to whom respectively he has to hand the pearl, or 3,000 florins, and the ruby, or 3,000 florins, he yet retains for himself the diamond, or 7,000 florins;— which chances all proceed from the aforesaid jewels, or from the purse of 12,000 florins, drawn by lot.

“And it is to be observed, that I have here expressly made use of an example or case of three objects of value, without expression of any sum, as in speaking of a pearl, a ruby, or a diamond, so as to cause the demonstration to be applicable to all sorts of numbers, to fractions as well as integer numbers, to irrational as well as to rational numbers, since all imaginable numbers may be applied to the value of these jewels.

“SECOND PROPOSITION

“If any one has different equal expectations or chances, of which some will cause him to obtain each a certain sum of money or other object of value, and the others will produce him nothing at all; if, besides, he possesses several other chances, each of a certain sum of money or object of value; and further, if he has some other chances, each of a certain sum of money or object of value, and so on;— we find the actual value of the aforesaid chances, by multiplying each item or sum of money, relative to each expectation in particular, by the quantity or number of existing chances, then adding the products of the resulting multiplications of these partial operations: we finally divide the sum, or mass of

partial products, by the collective number of chances, and the quotient indicates exactly the value of all these chances.

“Suppose, for example, that a person has the following chances of the objects or value annexed:—

Chances.	Each Chance of								0		
6	“	“	“	“	“	0	“	“	“	“	0
6	“	“	“	“	“	1,200	“	“	“	“	7,200
4	“	“	“	“	“	2,100	“	“	“	“	8,400
3	“	“	“	“	“	3,600	“	“	“	“	10,800
2	“	“	“	“	“	4,200	“	“	“	“	8,400
<hr style="width: 100%; border: 0.5px solid black;"/>											<hr style="width: 100%; border: 0.5px solid black;"/>
21											34,800

21)34,800(1,657 1-7 2

“DEMONSTRATION

I say that all the above mentioned chances are together worth to this person exactly $1,657\frac{1}{7}$; a value which we find, as is mentioned in the proposition, by multiplying each item, namely, 1,200 by 6; 2,100 by 4; 3,600 by 3; and 4,200 by 2; then adding the products of these multiplications, that is to say, 7,200, 8,400, 10,800, and 8,400, and dividing the sum total, or 34,800, by 21, which is the collective number of chances.

“Because we can represent the above chances reduced to their unities, as well as their values, in the following manner:—

Chances									of
6	{	1	0
		1	0
		1	0
		1	0
		1	0
		1	0
6	{	1	1,200
		1	1,200
		1	1,200
		1	1,200
		1	1,200
		1	1,200
4	{	1	2,100
		1	2,100
		1	2,100
		1	2,100
3	{	1	3,600
		1	3,600
		1	3,600

				II.				
Chances				of				
1	“	“	“	0	“	“	“	0
1	“	“	“	1,200	“	“	“	1,200
$\frac{2}{3}$	“	“	“	2,100	“	“	“	1,400
$\frac{1}{2}$	“	“	“	3,600	“	“	“	1,800
$\frac{1}{3}$	“	“	“	4,200	“	“	“	1,400
<hr style="width: 100%; border: 0.5px solid black;"/>								<hr style="width: 100%; border: 0.5px solid black;"/>
$3\frac{1}{2}$,	divisor							5,800
$5,800 \div 3\frac{1}{2} = 1,657\frac{1}{7}$								

				III.				
Chances				of				
18	“	“	“	0	“	“	“	0
18	“	“	“	1,200	“	“	“	21,600
12	“	“	“	2,100	“	“	“	25,200
9	“	“	“	3,600	“	“	“	32,400
6	“	“	“	4,200	“	“	“	25,200
<hr style="width: 100%; border: 0.5px solid black;"/>								<hr style="width: 100%; border: 0.5px solid black;"/>
63,	divisor							104,400
$104,400 \div 63 = 1,657\frac{1}{7}$								

“From the reasons before mentioned, we obtain in the above three examples, by means of the operation of the rule, one and the same quotient to determine the total value of all the chances, namely $1,657\frac{1}{7}$ (It would be the same in every similar case.)

THIRD PROPOSITION

“Each half-year of life is equally destructive or mortal to a person aged 3 or 4 years, to 53 or 54 years; in such a period he is neither too young, nor too aged, to be wanting in the vigour needful for the prolongation of his days: so that there is not greater hazard nor likelihood that the day of his death should arrive in the first than in the second half-year of this vigorous period, and vice-versa; nor that the day of his decease should occur rather in these two aforesaid half-years, considered each in its individuality, than in the third half-year, and vice-versa. And thus with the other half-years during the aforesaid space of time.

“DEMONSTRATION

“Any year of the vigorous period of life of the aforesaid person, being taken at pleasure, the first half of that year, or the first six months, is as destructive or mortal to him as the second six months. (According to the second proposition.)

“And taking a second or other year of this period of the vigour of his life, in setting out from the second half-year of the first year taken, which ends consequently just six months after the expiration of that first year, the first half of the second year, which thus becomes the second half-year of the first year, is quite as destructive or mortal to him as the second half of the second year,

which is thus the third half-year, reckoning as before. But as the first half-year, as well as the third, is as destructive, or mortal as the second, the first half-year and the third, compared with each other, are so likewise, since each of them in particular is as destructive or mortal as the second half-year; therefore, the aforesaid half-years, namely, the first, the second, and the third, each separately considered, are equally mortal.

“We might also demonstrate in the same manner that the second half-year and the fourth, when the one is compared with the other, are equally mortal; and again, that consequently the first half-year, the second, the third, and the fourth, each considered by itself, has the same chance of destructiveness: it is the same thing for all the preceding or subsequent half-years, comprised in the above time of the vigour of life;— which was to be demonstrated.

“COROLLARY

“It results from what precedes, and from the third presupposition, that as life annuities are paid in all the offices of Holland and West Friesland by half-yearly instalments, or from six months to six months, that the annuitant loses all his capital, and receives no return whatever from it, if the life upon which the annuity is sunk happen to die in the first half-year after the purchase, or do not live six whole months. The annuity sunk is supposed to be 1,000,000 of florins, or 20,000,000 stuyvers, per annum, in order that an exact calculation may be made without fractions: therefore, if the above-mentioned life survive a complete half-year and do not die in the course of the second half-year, the annuitant has then drawn 10,000,000 stuyvers, from which a deduction is made of 4 per cent. per annum for a half-year, it would have been worth to him in ready cash (that is to say, on the day of purchase of the said annuity,) 9,805,807 stuyvers, which he would have had to pay, if taken at the true value. If the above life survive so long as two complete half-years, and die in the third half-year, the annuitant has then drawn 10,000,000 stuyvers after the expiration of the first half-year, and after that of the second half-year likewise 10,000,000 stuyvers; which sums, deduction being made at 4 per cent. per annum, one for a half year or six months, and the other for a complete year, would have been worth to him in ready cash, or upon the day of purchase of the said annuity, 19,421,192 stuyvers, and so on, according as the day of decrease were to occur in the fourth, fifth, sixth, or further number of half-years, which would have been worth to him each time as many terms or half-yearly sums of 10,000,000 stuyvers as complete half-years had elapsed from the purchase of the annuity, deduction being made as above of the respective discounts. The computed amounts are specially given in the following table:—

<i>If the Nominee survive</i>				
<i>the following Term of Life.</i>				
<i>Half-years.</i>				<i>Stuyvers</i>
0	.	.	.	0
1	.	.	.	9,805,807

2	19,421,192
3	28,849,853
4	38,095,415
5	47,161,435
6	56,051,398
.
98	431,055,833
99	432,490,825
100	433,897,951
101	435,277,751
.
118	455,030,042
119	455,999,472
120	456,950,076
121	457,882,220
.
138	471,226,168
139	471,881,080
140	472,523,275
141	473,152,998
.
152	479,322,884
153	479,820,563
.
199	494,754,836
200	494,952,836

[³The above table having been calculated very accurately by us the undersigned Bookkeepers to My Lords the States-General, each separately, and having been collated by us, we find that a perfect agreement exists, without there being any error in the figures.

(Signed)

‘T. BELLECHIERE - JACOB LENSE.’³

³The above table, computed to such a nicety by De Witt's directions, is composed of the progressive summations of the present values of 1 Million Florins or 20 Million Stuyvers per annum, receivable in 100 half-yearly instalments for 50 years. The second and every even term will be found correct, on the supposition of discount at 4 per cent. per annum; but the first and every odd term erroneous, in the same way that the remark is applicable to Smart's and Tetens' (or Von Drateln's) Tables, at intermediate half-years, by reason of the interest being reckoned by a geometric instead of by an arithmetic mean. In the original a complete table is given from 1 to 200 half-years, which, however, it is useless to repeat in full, as the even terms may be obtained by an easy process from the data in other works, and the odd terms are inapplicable to modern purposes.

Struyck, in his *Uitrekening van de Lyfrenten* has some remarks on the “prodigious labor” of the two bookkeepers who calculated the Table, although when we compare it with similar ordinary computations of more modern times it is relatively not worthy of such an appellation. At the present date, the tendency is certainly to underestimate such labors; a reaction to the *juste milieu* may, however, take place after a surfeit of Statistics.

“Thus, then, since an annuitant, having purchased and sunk a life annuity upon a young nominee, has in possession, or in his favor, as many different expectations or chances as there are half-years in which the death of the nominee may occur;— since the first 100 different expectations or chances (comprising the term of 50 years, reckoning from the day of the constitution or purchase of the annuity,) may result with the same facility, and relatively to their probability are equal;— since during this term each second half-year of the aforesaid nominee’s life is equally destructive or mortal; (which is demonstrated in the third proposition;); since the following 20 chances or expectations (comprising the first 10 years after the expiration of the 50 years above cited), considered one with the other, each in proportion to each of the first 100 chances, are not in a lower ratio than 2 to 3; (according to the third proposition;);—since the 20 expectations or chances of the 10 years after the expectation of the first 50 years), also considered one with the other, each in proportion to each of the first 100 expectations or chances, are not in a lower ratio than 1 to 2; (according to the third presupposition;);—since the 14 following expectations or chances (comprising the 7 years after the expiration of the two preceding decennial terms, the epoch at which we here suppose the man to terminate his life), taken one with the other, each in proportion to each of the first 100 expectations or chances, are not in a lower ratio than 1 to 3;— it follows that the aforesaid annuitant has in possession, or in his favor, more chances or expectations than there are in the following table:—

Chances.		of Stuyvers		The Life to survive Half-years.
1	.	0	.	0
1	.	9,805,807	.	1
1	.	19,421,192	.	2
1	.	28,849,853	.	3
1	.	38,095,415	.	4
1	.	47,161,435	.	5
1	.	56,051,398	.	6
				7 to 97 } given in original }
1	.	431,055,832	.	98
1	.	432,490,825	.	99
	Sum	<u>28,051,475,578</u>	Once =	28,051,475,578 ⁴
				100
				101
				102 to 117 } in original }
				118
				119

	Sum	8,911,946,713		Two-thirds =	5,941,297,809
				Carried forward =	33,992,773,387
Chances.	of Stuyvers			The Life to	survive Half-years.
$\frac{1}{2}$.	456,950,076	.		120
$\frac{2}{3}$.	457,882,220	.		121
					122 to 137
					in original } 138
$\frac{1}{2}$.	471,226,168	.		139
$\frac{2}{3}$.	471,881,080	.		
	Sum	9,297,075,282		One-half =	4,648,537,641
$\frac{1}{3}$.	472,523,275	.		140
$\frac{2}{3}$.	473,152,998	.		141
					142 to 151
					in original } 152
$\frac{1}{3}$.	479,322,884	.		153
$\frac{2}{3}$.	479,820,563	.		
	Sum	6,668,408,125		One-third =	2,222,802,708
			Total		40,864,113,736

⁵ 40,964,113,736 divided by 128 gives 320,032,130 8 9-16, which divided by 20 gives 16,001,606 18-9.

“Whence it follows that we can immediately determine, by a mathematical calculation, according to the principle of the second proposition above enunciated, the worth to the aforesaid annuitant of all the above-mentioned chances, taken together, always presupposing that such value is payable in ready money on the day of purchase of the annuity; and the method is as follows:—

“Since the first 100 items, each taken once, or each multiplied by the number 1, form the sum of 28,151,475,578 stuyvers; since the 20 following items, two-thirds of each being taken, or each multiplied by $\frac{2}{3}$ (or, which is the same thing, two-thirds of the sum of the aforesaid 20 items,) produce a sum of 5,941,297,809 stuyvers; since then the half of the 20 following items gives a sum of 4,648,537,641 stuyvers, and the third of the 14 following and last items that of 2,222,802,708 stuyvers; these sums being combined, amount together to the sum of 40,964,113,736 stuyvers; which being divided by 128 (that is to say, the real and exact value of all the collective chances,) the sum of 320,032,132 stuyvers, or 16,001,607 florins: so that 1,000,000 per annum of life annuities, sunk or purchased on a young life should consequently be sold for more than 16,001,607 florins,⁶ preserving the right proportion above mentioned, i.e., that

⁵40,964,113,736 is here correctly given by De Witt.

⁶De Witt's calculation may be simplified and explained as follows: *Firstly*. Out of 128 lives, aged say 3 years, 1 is supposed to die in every half-year of the first hundred half-years, or 2 per annum for 50 years, leaving 28 alive, aged 53 years, at the end of the term; out of

proportionately each florin per annum of life annuity is worth more at 16 florins than the interest of a redeemable annuity at 4 per cent. per annum,- and consequently the person who for 16 florins has purchased a young, vigorous, and healthy life, has made a remarkably advantageous contract; I assert it to be remarkably advantageous for the following reasons:-

“Because, in the first place, we have not been able to rate at a certain price, by perfect calculation or correct estimation, the power which the annuitant possesses (power which is of very great value to him) of choosing a life, or person in full health, and with a manifest likelihood of prolonged existence, upon whom to constitute or purchase his annuity, and there is much less risk or danger of a select, vigorous, and healthy life, dying in the first half-year than in some of the following half-years at the beginning of which the aforesaid life might perhaps prove to be in a weak state of health or even in a fatal illness; and such greater likelihood of prolongation of life in the the purchase of an annuity upon a select, healthy, and robust life, may further extend itself to the the second, third, and some following terms or half-years.

“In the second place, the advantage resulting from the aforesaid selection is so much more considerable, as one half year of life, at the commencement of and

whom 1 dies in every 9 months, being 0.66 per half-year during the next 20 half years, or 1.33 per annum for 10 years, leaving 16.66 alive aged 63 years at the end of the second term; of whom 1 dies in every year for 10 years, being 0.5 per half-year during the next 20 half-years, leaving 5.66 alive aged 73 at the end of the third term; of whom 1 dies in every year-and-a-half for 7 years, being 0.33 per half-year during the next fourteen years, leaving 1 alive aged 80 at the end of the fourth term; which survivor does not live over another half-year. *Secondly.* Out of the 128 lives, those who die in the respective half-years between the ages of 3 and 80, will receive an annuity certain in half-yearly instalments. for a term equal to the number of *completed* half-years elapsed between age 3 and the date of their death; therefore, the sum of the present values of half-yearly annuities certain, for the corresponding terms multiplied into the numbers *dying* within such respective terms, gives the present worth of all the annuities which will be enjoyed by the 128 lives, one-hundredth and twenty-eighth of which represents the present value of the single-life annuity at age of, say, 3 years. The system of valuation is therefore identical with the fifth method described by *Tetens*, whose formula I have had the pleasure to refer to on a previous occasion. (See the *Assurance Magazine*, No. 1, pp. 9 and 18; and No. 11, p. 18).

If arranged in the modern form of a life table, the following abstract would represent the course of the results of De Witt's suppositions as to mortality.

Half-year Number.	Age.	Number of living.	Decrements.
1	3	128	1
2	3½	127	1
.	.	.	.
99	52½	29	1
100	53	28	0.66
101	53½	27.33	.
.	.	26.66	0.66
120	63	15.66	0.50
121	63½	15.16	0.50
.	.	.	.
140	73	5.66	0.33
.	.	.	.
154	80	1.00	1.00

shortly after the purchase of the life annuity, is of greater value to the annuitant, with respect to the price of the purchase, than eighteen half-years during which the person upon whom the annuity is purchased might live after the said purchase, from the age, for example, of 70 to 79 years,—a circumstance which, although at first sight it might appear strange and paradoxical, is nevertheless real and susceptible of demonstration.

“In the third place, although each of the first 100 half years expiring after the purchase be considered as equally destructive or mortal, according to the principle of the before-established calculation, by reason of the scarcely appreciable difference existing between the first and second half of each year, it is, however, certain, when we examine the matter very scrupulously, that the likelihood of decrease of the nominees upon whom life annuities are usually purchased is less purchase than in the subsequent years, seeing that the said life annuities are oftenest purchased and sunk upon the lives of young and healthy children of 3, 4, 5, 6, 7, 8, 9, 10 years, or thereabouts. During that time, and for some years ensuing, these young lives, having become more robust, are less subject to mortality than about 50 years afterwards, and than for some years anterior to these 50 years; and so much the more, as during the first aforesaid years they either are not, or are but little, exposed to external accidents and extraordinary causes of death, such as those from war, dangerous voyages, debauch, or excess of drink, of the sex, and other dangers;— for females, there are also confinements and other like causes;— so that the first years after the purchase or foundation of the annuity are the least dangerous, which is a considerable advantage for the annuitant, particularly if we reflect, as I have above stated, that one of the said first years may, as regards the original price of purchase, balance a great number of subsequent years.

“Finally, and in the fourth place, it might also evidently occur, that the life upon which the annuity has been sunk were to live more than 77 years after the purchase, being the time supposed in the above calculation as the term of human life, although such considerations cannot be of much importance; for, notwithstanding that by presupposing the aforesaid nominee living still longer than the expiration of the said term, and preserving life up to the hundredth year inclusive, so that the annuitant or his heirs were to receive 46 more entire half years of annuity, after the expiration of the aforesaid 77 years, this could not, however, increase the price of the life annuity (calculated, as precedes, at about 16 years’ purchase, i.e. at more than 16 florins of capital for 1 florin of annuity per annum,) by more than 14 stuyvers of the same capital; and even if the annuitant were, after the expiration of the above 100 years, to enjoy the life annuity from half year to half year, and that perpetually, the value of the capital at the time of first purchase would not thereby be increased by 10 stuyvers.

“Whence likewise, although it may be considered that the latter years are not established as sufficiently destructive and mortal in the aforesaid presuppositions and in the calculations upon which I have based them, when compared with the anterior years and the time of life’s vigor, we easily conclude that it could not cause an appreciable rise in the price of the purchase found by the above calculation, which in fact is true, even on the presupposition of each half

year of the 10 years after the sixtieth year of purchase being, instead of twice, three times more destructive and mortal than each half year of the first 50 years, and of each half year of the 7 subsequent years being, instead of three times, five times more destructive and mortal than each of the aforesaid first years; and even on the presupposition again, as above, that the said nominee would not survive beyond 77 years after the first purchase. All these presuppositions (which, however, manifestly represent the life as subject to too high mortality) could scarcely reduce by 6 stuyvers the aforesaid 16 florins or value of the before-described annuity. In consequence of all these reasons, we may assume it as established and demonstrated, that the value of a life annuity, in proportion to the redeemable annuity at 25 years' purchase, is really not below, but certainly above 16 years' purchase; so that a person, wishing to purchase a life annuity in such proportion and according to its real value, ought to pay more than 16 florins for 1 florin of annuity per annum.

“Besides the consideration that this calculation has been made on the principle of a deduction of 4 per cent. per annum, at compound interest, and this with such benefit to the purchaser of the life annuity that he would realize not only the interest per annum, but also, without any intermission, interest upon interest at 4 per cent. per annum, as though he could always thus advantageously make use of his money in purchase of annuity; it is constant that one could not always find such opportunity of investing it, and that one is sometimes obliged to let it lie fallow for some time, and often to lend it at a materially smaller interest, to provide against a greater loss.

“Even besides this, as the capital of life annuities is not subject to taxation, not to a reduction to a lower amount of annuity or interest, it follows, that if the blessing of the Almighty continue to be vouchsafed to this country, we may consider the life annuity as much more advantageous to the annuitant than the redeemable annuity, as may manifestly be judged by the example of foregoing times,— by reflecting, in fact that My Lords the States of Holland and West Friesland have in the course of a few years not only increased the charge for life annuities from 11 years' purchase to 12 years' purchase, and from 12 years' purchase to 14 years' purchase, but that these annuities have been sold, even in the present century, first at 6 years' purchase, then at 7 and at 8, and that the majority of all life annuities now current and at the country's expense were obtained at 9 years' purchase; which annuities, by reason of the successive reductions of the rate of interest from 6 to 5 per cent., and then from 5 per cent. to 4 per cent., produce to the annuitants an actual profit of nearly one-half of each half-year's payment, and of more than one-half in the case of those annuities which were obtained at 8 years' purchase or under.

“JOHN DE WITT.”