

CHAPTER VII

CONCERNING GAMES, WAGERS & LOTTERIES

Games and Wagers have been lawful, provided that they would be moderate, & they would be made from honorable matters, & the one side would be equal to the other, i.e. the risk of losing and the hope of gaining on both sides would have a proportion to the event, about which there is contention; e.g. in a game which on the dexterity of the body or nature is fixed, if the dexterity of one will have exceeded the dexterity of the other by twice as much, there is equality, when here also he <the other> would put down the double premium; or if there should be ten, of whom one at a time they would set down a gold coin, and also they would compete with this agreement, so that he who will have thrown the most, he would carry off all <of the ten>, indeed it may seem unfair, when he who experienced a risk of only one gold coin, may gain nine, but at the same time it must be considered, here the risk of losing is nine-fold greater, than the hope of winning. Pufendorf. *de I. N. & G. l. 5. c. 9. § 7.*

48. Hence it is evident how in this matter the Art of Conjecture would be necessary, for when the conditions of those gaming or competing would be able to vary completely & to be subject to innumerable cases, on that account no one without the assistance of this Art will have been able to discover easily the number of cases, which would favor or oppose this or that fellow gambler or guarantor of the wager, and thereupon to determine, whether each of the two equally must be determined in the condition of winning & losing, or not. Because in order that more be revealed, we shall mention an example in wagers, for in what manner in a game the lots or the expectations of fellow-gamblers would be able to be defined, copiously Huygens in the unique pamphlet *On a system for playing dice*, & my famous Uncle in the entire third part of his *Tractate on the Art of Conjecture* have revealed.

Quite famous today are those Wagers of the Genoese, which are established publicly on the occasion of the elections, which happen in the individual years in Genoa, when five out of one hundred Senators are selected by lot, who in this year discharge the more principal offices; therefore then the wealthy merchants are accustomed at this certain time, before the decision by a lottery would occur, to enter upon a contest with others in this law, in order that whoever will have been willing to compete, he may set out a token for as much as he wishes, & it would name five of these one hundred <Senators>, & if afterwards the lottery will have produced, that one of the named will have been elected, he would be about to receive a certain sum of money, upon which it has been agreed; if two will have been elected, a greater <sum>; if three, an even greater <sum>; if four, again a greater <sum>; if five, again a greater <sum>; if indeed none of the nominated will have been elected, he would make an expense of the losing token; it is sought, for how much the prize in each chance ought to be established, in order that an equality would be strived for in the lots?

Let the token or price to be expended =  $a$ , the prize has been established or rather must be established, if one of the nominated is selected =  $t$ , if two =  $u$ , if three =  $x$ , if four =  $y$ , if five =  $z$ ; now for the sake of brevity the number of cases or of lots would be set, in which five are being selected =  $b$ , in which four =  $c$ , in which three =  $d$ , in which two =  $e$ , in which one =  $f$ , in which none =  $g$ . Therefore there are  $b$  cases with the result that someone would acquire  $z$ ,  $c$  cases that  $y$ ,  $d$  cases that  $x$ ,  $e$  cases that  $u$ ,  $f$  cases that  $t$ , &  $g$  that none, and for that reason the expectation will be worth

$$\frac{b \cdot z + c \cdot y + d \cdot x + e \cdot u + f \cdot t + g \cdot 0}{b + c + d + e + f + g}$$

which ought to be equal to the price paid  $a$ , whence the equation will be held as  $bz + cy + dx + eu + ft = ba + ca + da + ea + fa + ga$ , therefore with the problem it would be indeterminate, indeed four are able to be chosen at will, certainly because there is a combination in the nature of the thing, as the prizes are reciprocally proportional to the number of chances, i.e. for which reason as the chances are fewer, those prizes would be presented as more prized, hence in the place of  $z$ ,  $y$ , &  $c$ . we place a single unknown  $z$  only, with the remaining existing having been made proportionate to that one  $bz/c, bz/d, bz/e$  &  $bz/f$ , whence the equation would be of such kind

49.

$$5bz = ba + ca + da + ea + fa + ga \quad \text{or} \quad z = \frac{ba + ca + da + ea + fa + ga}{5b}$$

or (if for the number of all the cases  $b + c + d + e + f + g$  let  $h$  be put)

$$z = \frac{ha}{5b}, \quad \text{whence} \quad y = \frac{bz}{c} = \frac{ha}{5c}, \quad x = \frac{bz}{d} = \frac{ha}{5d}, \quad u = \frac{bz}{e} = \frac{ha}{5e}$$

& finally

$$t = \frac{bz}{f} = \frac{ha}{5f}.$$

If now there is a token which ought to be set down or  $a$  would be one gold coin, & for the letters  $b, c, d,$  &c. the values or themselves would be substituted, these which are found by means of the noted combination rules, indeed

$$\begin{aligned} b &= 1, & c &= \frac{5 \cdot 4 \cdot 3 \cdot 2}{1 \cdot 2 \cdot 3 \cdot 4} \times \frac{95}{1} = 475, & d &= \frac{5 \cdot 4 \cdot 3}{1 \cdot 2 \cdot 3} \times \frac{95 \cdot 94}{1 \cdot 2} = 44650, \\ e &= \frac{5 \cdot 4}{1 \cdot 2} \times \frac{95 \cdot 94 \cdot 93}{1 \cdot 2 \cdot 3} = 1384150, & f &= \frac{5}{1} \times \frac{95 \cdot 94 \cdot 93 \cdot 92}{1 \cdot 2 \cdot 3 \cdot 4} = 15917725, \\ g &= \frac{95 \cdot 94 \cdot 93 \cdot 92 \cdot 91}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5} = 57940519, & h &= \text{the sum of the preceding numbers} \\ &= \frac{100 \cdot 99 \cdot 98 \cdot 97 \cdot 96}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5} = 75287520. \text{ the prizes are found } z = 15057504 \text{ gold.} \\ y &= 31700 \frac{4}{475} \text{ gold, } x = 337 \frac{6227}{22325} \text{ gold, } u = 10 \frac{608002}{692075} \text{ gold, } t = \frac{15057504}{15917725} \text{ of one gold coin.} \end{aligned}$$

Hence it is evident how much the Genoese merchants would engage in fraud, while for one gold coin regularly they promise only 10,000 gold coins, if five, 1500. if four, 300 if three, 10 if two, & one if one of the named will have been elected, for granted that in this last case those, who contend with the merchants in the way named, would have something of a profit, nevertheless by much more there is a loss, which they suffer by the remaining four chances, which clearly is evident, if we should seek the expectation ourselves, but that is

$$\begin{aligned} &= \frac{1 \cdot 10000 + 475 \cdot 1500 + 44650 \cdot 300 + 1384150 \cdot 10 + 15917725 \cdot 1 + 57940519 \cdot 0}{75287520} \\ &= \frac{43876725}{75287520} = \frac{2925115}{5019168} \end{aligned}$$

of the one gold coin, but they ought to expect so much, as much as they have set down, i.e. one gold coin, therefore the Genoese dealers defraud the individual gamblers the  $2094053/5019168$  parts of one gold coin, from which it is appropriate, not least a Ministry is able to permit such gambling, seeing that <it is> completely unjust, by public authority, & the merchants are held to the restitution of it, which they have received as more than just.

50. Indeed unjust are these Genoese wagers, which Juan Caramuel<sup>1</sup> calls by the proper name *Concertationes Cosmopolitanas*, also he asserts in *Mathesi Nova, syntagm. 7* but nevertheless the prizes, which he assigned to the single cases, still in truth fall short, moreover his errors & paralogisms, which here and there he commits in the expectations to be defined of the players, it would be excessively obliging to show in this.

We would add rather something about the Jars of Fortune of Lotteries, thus in the words of the Belgian voice *Loten*, Latin *sortiri*, of which the use today is most frequent. But the jar of fortune is set up in this way: With a certain number of tickets having been thrown down into an urn, of inscribed and of empty <tickets>, a chance is returned for the price of removing the same, thus so that this one extracting would receive, what the inscription of those exhibits on themselves; it is demanded for the justice of it, that the value of all the tickets chosen successively would not greatly exceed the value of the things having been set out there: for because also the costs must be made, & such jars are generally used towards the collection of money to be expended upon public works, or also the alleviation of others in need, on that account from exact equality a portion is held back, to such an extent so that such a difference, by which commonly the value of all the tickets together exceeds the prizes having been put forth, would have a reason of a certain

<sup>1</sup> Juan Caramuel 1606-1682. was a Cistercian and author of some seventy works. He is most famous for the work *Mathesis biceps: Vetus et nova* [Two-headed mathematics: old and new]. Bernoulli, in a letter to Montmort (p. 387), mentions Caramuel. "A Jesuit named Caramuel, who I mentioned in my Thesis, wished to push these matters, and even criticize M. Huygens in the Treatise that he names KYBEIA [Dicing], and what he has inferred in his great Works of Mathematics; but as all this that he gives is only a heap of faulty reasonings, I reckon it as nothing."

voluntary tax, or of the alms having been enticed in a merry way, as well the esteemed Pufendorf speaks *loc. supra cit.*

51. Such a jar of fortune, being composed out of pure annuities, not long ago in Belgium has been defined under the fairest conditions, of which we have a description in the New Laws of Bern, in which on the date Amsterdam 15 March 1709 these words are held as having been written in the French language:

Here is the plan of the Lottery of the Life Annuity Pensions, that one proposes to make in this Country, & which has been sent to the Cities, in order to have their approbation. There are 8000 gold Tickets at 250 Florins each, that which amounts to 2000000. Of these 8000 Tickets there are 1300 Black or Prize & 6700 White. These latter will carry 6% interest during life. This interest, just as the prize, will be exempt from the 100th & 200th tax & all other Charges. The Prize will be shared in the following manner. Two Lots at 3000 Florins of Pension each, making 6000. Four Lots at 2000 fl. of P. making 8000. Four L. at 1000 fl. of P. making 4000. Eight L. at 500 fl. of P. making 4000. Fourteen L. at 250 fl. of P. making 3500. Thirty L. at 150 fl. of P. making 4500. Thirty L. at 100 fl. of P making 3000. Twelve hundred eight L. at 30 fl. of P. making 36240. This which makes 1300 Lots & 69240 fl. of Life Annuity Pensions. The 6700 White Tickets at 6% produce 100500 florins of Life Annuity Pensions. The first and last Ticket each of 150 florins of Life Annuity Pensions. That which returns at 8000 Tickets of 250 florins each, making 2000000 of Florins in fund or Principal, producing per year 170040 Florins of Life Annuity Pensions. Those, who will wish to convert their Life Annuity Pensions into obligations at 4% on the State, will be able to make in all or in part, of the kind that the Prize of 3000 Florins of Life Annuity Pensions can be changed into the sum of 35250 fl. & the other prizes in proportion. One relates to draw this Lottery on 1 May next, & 6 weeks after one will be obliged to give the names of those, who one wishes to place in the letters of the Life Annuity Pensions. One will be able to put on many Heads, but no less than 100 florins of pension on each.

- From these words it is clear, that the value of all the tickets taken together is 2,000,000 florins, according to which in the individual years 170,040 florins are paid in the annuities, but because someone, if he wishes, is able to change annuities into redeemables, with the result that for the return of 3,000 florins he would receive a lot of 35,250 florins, & from this sum in the individual years I put 4% interest, i.e. 1410 florins, and also to such an extent the annuity of 3000 florins would be estimated to be worth just as much as 35250 florins, therefore 170,040 florins yearly will be worth just as much as 1,997,970 florins however much therefore the value of all the tickets <will be worth> the value of all the payments taken together, that is 1,997,970 florins, it would exceed by 2030 florins, & the annuity of one florin yearly would be put to be worth  $11\frac{3}{4}$  florins (for it is  $3000 \cdot 35250 : : 1 \cdot 11\frac{3}{4}$ ) when the value of it, as we found above in *Chapter 4*, at most would be in the ratio of one to  $10\frac{600}{1000}$  or  $10\frac{3}{5}$ , abundantly however such a difference is balanced with this interest, evidently seeing that such annuities would be exempt from the tax of 1% or 2% coin, and from other taxes; add that, if in *d. cap. 4*. where for the interest we have set down 5%, the annuities would have been computed on the standard of interest of 3 or 4% (as this ought to happen) a richer premium would have been produced than  $10\frac{3}{5}$  for 1 whence it is clear, how much more popular this lottery would be, than the others which commonly are accustomed to be established, where one who brings forth a white or blank ticket, entirely because he exposed silver, one certainly <who brings forth> a black <ticket>, regularly loses a tenth part of his ticket, since in this lottery that man, who draws an empty ticket, would
52. have in single years 6%, this is, for 250 Dutch florins or 100 Imperial thalers, six Imperials, which if they should be multiplied by  $11\frac{3}{4}$  are discovered to be worth  $70\frac{1}{2}$  Imperials, accordingly this man, who extracts a white ticket, out of 100 Imperials, for which he has purchased the ticket, loses nonetheless  $29\frac{1}{2}$  Imperials.

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