

Frege

Michael Beaney

Gottlob Frege (1848-1925) was primarily a mathematician, logician and philosopher of mathematics rather than a philosopher of language as that might be understood today. However, in inventing modern logic and pursuing his main goal – the demonstration that arithmetic can be reduced to logic – he was led to reflect on how language works, and the ideas he introduced in doing so laid the foundations for the development of philosophy of language, especially within the analytic tradition, in the twentieth century.

1 Life and works

Frege was born in Wismar, on the Baltic coast in northern Germany, in 1848, and he studied mathematics, physics, chemistry and philosophy at the Universities of Jena and Göttingen from 1869 to 1873. In 1874 he was appointed to teach mathematics at Jena, where he remained for the rest of his academic career. He retired in 1918 and moved back to the Baltic coast, where he died in 1925.¹

Frege published three books in his lifetime: *Begriffsschrift (Conceptual Notation)* in 1879, *Die Grundlagen der Arithmetik (The Foundations of Arithmetic)* in 1884, and *Grundgesetze der Arithmetik (Basic Laws of Arithmetic)* in 1893 (Volume I) and 1903 (Volume II). Frege's main aim in these books was to demonstrate the logicist thesis that arithmetic is reducible to logic. To do so, Frege realized that he needed to develop a better logical theory than was then available: he gave his first exposition of this in *Begriffsschrift*, 'Begriffsschrift' (literally, 'concept-script' or 'conceptual notation') being the name he gave to his logical system. (In what follows, I shall use 'Begriffsschrift' in italics to refer to the book and without italics to refer to Frege's logical system.) In *Grundlagen* he criticized other views about arithmetic, such as those of Kant and Mill,

¹ For a detailed chronology of Frege's life, see Thiel and Beaney 2005, where further references to biographical accounts can be found.

and offered an informal account of his logicist project. In *Grundgesetze* he refined his logical system and attempted to provide a formal demonstration of his logicist thesis. In 1902, however, as the second volume was in press, he received a letter from Russell informing him of a contradiction in his system – the contradiction we know now as Russell’s paradox. Although Frege hastily wrote an appendix attempting to respond to this, he soon realized that the response failed, and was led to abandon his logicism. He continued to develop and defend his philosophical ideas, however, writing papers and corresponding with other mathematicians and philosophers. It is these philosophical writings, and his four most important papers, in particular, which have established his status as one of the founders of modern philosophy of language. These papers are ‘Function and Concept’ (1891), ‘On Sense and Reference’ (1892a), ‘On Concept and Object’ (1892b), and ‘Thought’ (1918). The main ideas in these four papers form the subject of the present chapter. We start with the idea that lay at the heart of his new logical system.

2 Frege’s use of function–argument analysis

The key to understanding Frege’s work is his use of function–argument analysis, which he extended from mathematics to logic. Indeed, it is no exaggeration to say that all Frege’s main doctrines follow from his thinking through the implications of this use.² We can begin with a simple example from mathematics. In analytic geometry, the equation for a line is $y = ax + b$: this exhibits y as a function of x , with a and b here being constants (a being the gradient of the line, and b the point where the line cuts the y -axis on a graph). Let $a = 2$ and $b = 3$. If $x = 4$, then $y = 11$: we say that 11 is the value of the function $2x + 3$ for argument 4. We call x and y the variables here: as x varies, so too does y , in the systematic way reflected in the function. By taking different numerical values for x , we get different numerical values for y , enabling us to draw the relevant line on a graph. Frege thought of sentences – and what those sentences express or represent – in the same way: they can be analyzed in function–argument terms. Sentences, too, have a ‘value’

² Although written 12 years after the completion of the *Begriffsschrift*, the clearest account of this can be found in Frege 1891.

which can be seen as the result of an appropriate function for an appropriate argument or arguments. If we think of this value as in some sense its ‘meaning’, then we can regard this meaning, too, as a function of the ‘meanings’ of what we might loosely call its ‘parts’, i.e. of what function–argument analysis yields as the constituent elements of the sentence. What are these ‘meanings’ and ‘parts’? This is the question that Frege attempted to answer in his philosophical work, and it is the main aim of the present chapter to explain his answer.

Let us see how Frege applied function–argument analysis in the case of sentences, starting with simple sentences such as ‘Gottlob is human’. In traditional (Aristotelian) logic, such sentences were regarded as having subject–predicate form, represented by ‘ S is P ’, with ‘ S ’ symbolizing the subject (‘Gottlob’) and ‘ P ’ the predicate (‘human’), joined together by the copula (‘is’). According to Frege, however, they should be seen as having function–argument form, represented by ‘ Fa ’, with ‘ a ’ symbolizing the argument (‘Gottlob’) and ‘ Fx ’ the function (‘ x is human’), the variable x here indicating where the argument term goes to complete the sentence. The sentence ‘Gottlob is human’ is thus viewed as the value of the functional expression ‘ x is human’ for the argument term ‘Gottlob’. Besides the terminological change, though, there might seem little to choose between the two analyses, the only difference being the absorption of the copula (‘is’) into the functional expression (‘ x is human’).³

The advantages of function–argument analysis begin to emerge when we consider relational sentences, which are analyzed as functions of two or more arguments. In ‘Gottlob is shorter than Bertrand’, for example, ‘Gottlob’ and ‘Bertrand’ are taken as the argument terms and ‘ x is shorter than y ’ as the relational expression, formalized as ‘ Rxy ’ or ‘ xRy ’. This allows a unified treatment of a wide range of sentences which traditional logic had had difficulties in dealing with in a single theory.

The superior power of function–argument analysis only fully comes out, though, when we turn to sentences involving quantifier terms such as ‘all’ and ‘some’. Take the sentence ‘All logicians are human’. In traditional logic, this was also seen as having subject–predicate form, ‘All logicians’ in this case being the subject, with ‘human’ once

³ On this, see e.g. Frege’s letter to Marty of 1882, repr. in Frege 1997, p. 81.

again the predicate, joined together by the plural copula ‘are’. According to Frege, however, such a sentence has a quite different and more complex (quantificational) form: in modern notation, symbolized as ‘ $(\forall x) (Lx \rightarrow Hx)$ ’,⁴ read as ‘For all x , if x is a logician, then x is human’. Here there is nothing corresponding to the subject; instead, what we have are two functional expressions (‘ x is a logician’ and ‘ x is human’) linked together by means of the propositional connective ‘if ... then’ and bound by a quantifier (‘for all x ’). In developing his *Begriffsschrift*, Frege’s most significant innovation was the introduction of a notation for quantification, allowing him to formalize – and hence represent the logical relations between – sentences not just with one quantifier term but with multiple quantifier terms.

Traditional logic had had great difficulty in formalizing sentences with more than one quantifier term. Such sentences are prevalent in mathematics, so it was important for Frege to be able to deal with them. Consider, for example, the sentence ‘Every natural number has a successor’. This can be formalized as follows, ‘ Nx ’ symbolizing ‘ x is a natural number’ and ‘ Syx ’ symbolizing ‘ y is a successor of x ’:

$$(S) \quad (\forall x) (Nx \rightarrow (\exists y) (Ny \& Syx)).^5$$

This can be read as ‘For all x , if x is a natural number, then there is some natural number which is its successor’. This shows that there is a much more complex (quantificational) structure involved here than the original sentence might suggest. Since what inferences can be drawn from such complex sentences depend on their quantificational structure, it is only when we have an adequate way to represent this structure that we can properly exhibit the relevant logical relations.

For a good illustration of this, consider the sentence ‘Every philosopher respects some logician’, which is ambiguous. It can mean either (1) that whatever philosopher we take, there is some (at least one) logician whom they respect (not necessarily the same one), or (2) that there is indeed some one (at least one) logician whom every philosopher

⁴ Here, and in what follows, I use modern notation rather than Frege’s own two-dimensional *Begriffsschrift*. For an explanation of Frege’s symbolism, see App. 2 of Frege 1997.

⁵ It should be noted that Frege does not himself use a separate symbol for the existential quantifier, relying on the equivalence, as we now write it, of ‘ $(\forall x) Fx$ ’ and ‘ $\neg(\exists x) \neg Fx$ ’ (cf. 1879, § 12), nor for conjunction, which he defines in terms of conditionality and negation (1879, § 7).

respects. Quantificational logic provides us with a succinct way of exhibiting this ambiguity:

$$(1) \quad (\forall x) (Px \rightarrow (\exists y) (Ly \& Rxy)).$$

$$(2) \quad (\exists y) (Ly \& (\forall x) (Px \rightarrow Rxy)).$$

The first can be read as ‘For all x , if x is a philosopher, then there is some y such that y is a logician and x respects y ’; the second can be read as ‘There is some y such that y is a logician and for all x , if x is a philosopher, then x respects y ’. The formalization makes clear that the difference lies in the order of the quantifiers – $\forall\exists$ versus $\exists\forall$. While the second implies the first, the first does not imply the second: mistakenly reasoning from the first to the second is known as the quantifier shift fallacy. Quantificational logic allows us to diagnose the error and avoid making it in our own reasoning.

3 Some key distinctions

Quantificational logic has proved itself to be far more powerful than any of the logical systems that existed prior to Frege’s *Begriffsschrift*. As far as the development of philosophy of language is concerned, its significance lies, at the most general level, in the gap it opened up between grammatical form and logical form. The grammatical structure of a sentence is not necessarily a good guide to its logical structure, in other words, to the inferential relations it has with other sentences. Take the example of ‘All logicians are human’ again. This might look as if it has a similar structure to ‘Gottlob is human’, a view reflected in the treatment of both as having subject–predicate form, ‘All logicians’ and ‘Gottlob’ being the subjects. On Frege’s view, however, there are two quite different relations here: subordination and subsumption, respectively. To say that all logicians are human is to assert a relationship between being a logician and being human, namely, that the first implies the second: the concept of being a logician is subordinate to the concept of being human. To say that Gottlob is human, on the other hand, is to say that the object Gottlob falls under the concept of being human: the object is subsumed under the concept. When we say that ‘All logicians are human’, we do not mean that the collection or set of logicians is human (the set is not being subsumed under the concept of being

human); rather, we mean that any member of this collection is human, i.e., that anything that is a logician is also human. This is what is captured in its formalization in quantificational logic, which makes clear that from it, together with any sentence ‘ A is a logician’, we can infer ‘ A is human’. Frege stresses the distinction between subordination (which holds between two concepts) and subsumption (which holds between an object and a concept) throughout his work.⁶ For him, this is a fundamental logical distinction which is obscured by focus merely on the subject–predicate structure of sentences.

Presupposed by this distinction is the most fundamental distinction of all for Frege: that between object and concept. This distinction follows directly from Frege’s use of function–argument analysis. Take ‘Gottlob is human’ again. Analyzing this in function–argument terms yields ‘Gottlob’ as the argument term and ‘ x is human’ as the functional expression. Frege takes ‘Gottlob’ as the name of an object and ‘ x is human’ as the name of a concept. On Frege’s view, in other words, concepts *are* functions. Furthermore, just as functions are different from objects, so concepts are different from objects. Indeed, this distinction is absolute for Frege, which he expresses by saying that while objects are ‘saturated’, concepts (and functions) are ‘unsaturated’. What he means by this is that while names of objects are ‘complete’, concept-words (functional expressions) contain ‘gaps’ to indicate where the argument terms go. This gap is indicated by the use of the variable x in, for example, ‘ x is human’. This seems reasonable at the linguistic level: the expression for a concept must show where the argument term goes to produce a well-formed sentence. It is more problematic at the level of what the words refer to. Frege talks of concepts themselves being ‘unsaturated’, mirroring the unsaturatedness at the linguistic level. He admits that this talk is metaphorical, but the ontological status of concepts has proved controversial.⁷

The final distinction to be explained here is the distinction between first-level and second-level concepts. To illustrate this, let us consider the central idea of Frege’s logicism, the claim that a number statement involves an assertion about a concept.⁸ To say that Jupiter has four moons, for example, is to say that the concept *moon of Jupiter*

⁶ See, for example, Frege 1882/1997, pp. 80-1; 1892b/1997, pp. 189-90.

⁷ See, for example, the papers in Beaney and Reck 2005, IV, Part 12; Oliver 2010; Ricketts 2010.

⁸ See esp. Frege 1884, §§ 53, 57/1997, pp. 103, 106-7. For further discussion, see Beaney 2005, § 4.

has four instances. More precisely, it is to say that the first-level concept *moon of Jupiter* (under which fall the four objects that are the individual moons of Jupiter) itself falls within the second-level concept *has four instances*. So first-level concepts are concepts under which objects fall, and second-level concepts are concepts within which first-level concepts fall.⁹

This construal of number statements is important in Frege's logicist project because the concept *is instantiated* can be defined purely logically, thus setting Frege on the path of providing logical definitions of all numerical concepts. To say that a concept *F* is instantiated is to say that there is some *x* such that *x* is *F*, symbolized logically as:

$$(EQ) \quad (\exists x) Fx.$$

The concept *is instantiated* is thus represented by means of the existential quantifier, which thus turns out to be a second-level concept. The same is true of the universal quantifier, since to say that everything is *F* is to say that the concept *F* is universally instantiated:

$$(UQ) \quad (\forall x) Fx.$$

Frege's construal also has significance outside the context of logicism. Consider negative existential statements such as 'Unicorns do not exist'. We might be tempted to construe this as attributing to unicorns the property of non-existence. But if there are no unicorns, then how is this possible? On Frege's view, however, the statement is to be interpreted as 'The concept *unicorn* is not instantiated', which can be readily formalized as follows, '*Ux*' symbolizing '*x* is a unicorn':

$$(U) \quad \neg(\exists x) Ux.$$

This makes clear that there is no mysterious reference to unicorns, only to the *concept* of a unicorn. What the statement says is that there is nothing that instantiates this concept. We have here another example of how the subject–predicate form of a sentence can be

⁹ Since Frege thinks that the relation between objects and first-level concepts (subsumption) is different from, though analogous to, the relation between first-level and second-level concepts, he talks of objects 'falling under' first-level concepts and first-level concepts 'falling within' second-level concepts; see Frege 1892b/1997, p. 189. Both relations are different from subordination, which is a relation between concepts of the same level.

misleading. Representing it in quantificational theory reveals that its logical form is quite different.

Frege was well aware of the philosophical significance of this construal. As he points out, it offers a diagnosis of what is wrong with the traditional ontological argument for the existence of God.¹⁰ On his account, to say that something exists is not to attribute a property to something, in other words, it is not to say that an object falls under a first-level concept (the concept of existence, so supposed), but to say that a first-level concept falls within a second-level concept, namely, the concept of being instantiated. The concept of existence thus turns out to be a second-level rather than a first-level concept, although, strictly speaking, what we should say is that talk of existence is to be analyzed in terms of the second-level concept of being instantiated. The general strategy here – reformulating a potentially misleading sentence to reveal its ‘real’ logical form – was to become a central idea of analytic philosophy.¹¹ It also illustrates why philosophy of language became so fundamental to analytic philosophy. Careful attention to the meaning and logical structure of sentences opens the way to successful resolution of philosophical problems.

4 Conceptual content

We have seen how Frege interprets ‘All logicians are human’ as ‘For all x , if x is a logician, then x is human’, for example, and ‘Jupiter has four moons’ as ‘The concept *moon of Jupiter* has four instances’. But what justifies these interpretations? A traditional logician might object to them by saying that they *change the subject*: ‘All logicians are human’ is about all logicians, not about the relationship of subordination between two concepts; and ‘Jupiter has four moons’ is about the planet Jupiter, not about the concept *moon of Jupiter*. According to Frege, however, subject–predicate analysis is of no logical significance: all that is relevant is what he calls – in his early work – ‘conceptual content’. It is this that determines the inferential relations between sentences, which is what is logically important.

¹⁰ See e.g. Frege 1882/1997, p. 82; 1884, § 53/1997, p. 103.

¹¹ I have called this ‘interpretive analysis’, which I see as characteristic of the Fregean strand in analytic philosophy. Cf. e.g. Beaney 2007b, 2007c.

The example Frege gives to introduce the idea is the following pair of sentences:¹²

(GP) At Plataea the Greeks defeated the Persians.

(PG) At Plataea the Persians were defeated by the Greeks.

Although there is a grammatical difference here, ‘the Greeks’ being the subject in (GP) and ‘the Persians’ being the subject in (PG), there is clearly something that they have in common, which Frege calls their ‘conceptual content’. Two sentences have the same conceptual content, on Frege’s view, if and only if whatever we can infer from one, together with any additional assumptions, can also be inferred from the other, together with those same assumptions. In effect, this is to say that two sentences have the same conceptual content if and only if they are logically equivalent. If (GP) is true, then (PG) is true, and vice versa.

What Frege is suggesting, then, is that for logical purposes, we do not need to take into account all those aspects of our use and understanding of sentences that might be called their ‘meaning’ in the widest sense of the term: we need only concentrate on their ‘conceptual content’, which we can think of as constituting the logical core of their meaning.¹³ But what exactly *is* this content? In his early work, Frege seems to have thought of it as something like the ‘circumstance’ or ‘state of affairs’ represented.¹⁴ In the case of (GP) and (PG) above, this seems plausible: what they have in common is that they both refer to the same battle: the battle at Plataea between the Greeks and the Persians which the former won and the latter lost. Whether we describe this battle as the Greeks defeating the Persians or the Persians being defeated by the Greeks seems irrelevant: it is one and the same event to which we are referring.

Returning to Frege’s use of function–argument analysis, we can now state what Frege saw – at any rate, in his early work¹⁵ – as the ‘value’ of a sentence when construed

¹² Frege 1879, § 3/1997, pp. 53-4.

¹³ In his later work, Frege calls those aspects of meaning that lie outside this logical core ‘shading’ or ‘colouring’ (see e.g. 1892a, p. 31/1997, p. 155; 1892b, p. 196, fn./1997, p. 184, fn. G; cf. 1918, p. 63/1997, pp. 330-1; and esp. 1897, pp. 150-5/1997, pp. 239-44). These aspects have also been called ‘tone’.

¹⁴ See e.g. Frege 1879, §§ 3, 9. Cf. Beaney 2007d.

¹⁵ By Frege’s ‘early work’, I mean his work up to and including the *Grundlagen* of 1884, when the notion of conceptual content was operative. From 1891 onwards, this notion is split into the dual notions of sense and reference, as we will shortly see.

in function–argument terms: its conceptual content. We have also noted that functional expressions are taken to represent concepts or relations, and names to represent objects. So we have one answer to the question posed at the beginning of § 2 above. The value or logical meaning of a sentence is its conceptual content, regarded as the result of an appropriate function (a concept or relation) for an appropriate argument or arguments (object or objects). Unfortunately, however, the notion of conceptual content was to prove problematic, as Frege realized when he reconsidered his earlier views about identity statements.

5 The problem of identity statements

Consider the example Frege gives at the beginning of his paper ‘On Sense and Reference’ (see Diagram 1 below): “Let a , b , c be the lines connecting the vertices of a triangle [PQR] with the midpoints of the opposite sides. The point of intersection of a and b is then the same as the point of intersection of b and c .”¹⁶

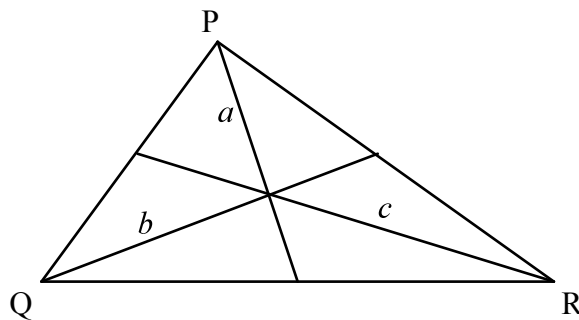


Diagram 1

What we have here is an identity statement:

(AB) The point of intersection of a and b = the point of intersection of b and c .

At the time of *Begriffsschrift*, Frege regarded the ‘content’ of a name as the entity designated. But what is the content of (AB)? If the conceptual content of a sentence is the circumstance or state of affairs represented, then what is this in the case of an identity

¹⁶ Frege 1892a, pp. 25-6/1997, p. 152. Frege does not provide the diagram himself, but it has been added here for illustration (as in *The Frege Reader*).

statement? It would seem to be more than just the entity designated by the two names, which would not distinguish between the sentence and the name. Another possibility is the self-identity of the entity designated, but this does not seem plausible, either. One reason why both of these possible answers are unsatisfactory is that neither would distinguish (AB) from the following identity statement:

(AA) The point of intersection of a and b = the point of intersection of a and b .

This identity statement is trivially true, while (AB) tells us something. (AA) is an instance of a logical truth, namely, $a = a$, while (AB) is not. So there is a logical difference, which means that there must be a difference in conceptual content, according to Frege's criterion for sameness of conceptual content.

In § 8 of the *Begriffsschrift*, where he discusses the matter, Frege states that the content of an identity statement is “the circumstance that two names have the same content”.¹⁷ This allows us to distinguish between (AB) and (AA), for in the first case, the content is the circumstance that the names ‘the point of intersection of a and b ’ and ‘the point of intersection of b and c ’ have the same content, while in the second case, the content is the circumstance that ‘the point of intersection of a and b ’ and ‘the point of intersection of a and b ’ have the same content. However, this still seems not to capture the real difference here, for it is not just the mere fact that two names have the same content that is significant. As Frege puts the objection at the beginning of ‘On Sense and Reference’, where he criticizes his earlier view, names can be arbitrary. I could arbitrarily give something the names ‘ m ’ and ‘ n ’, for example, but then saying ‘ $m = n$ ’ is not to express any “proper knowledge”, as Frege puts it.¹⁸ What is important is what the names tell us about the object designated. What (AB) tells us, for example, is that the point designated in one way, namely, as the point of intersection of a and b , turns out to be the same point designated in another way, namely, as the point of intersection of b and c .

In fact, Frege recognized this at the time of the *Begriffsschrift*, too, when he considers a similar geometrical example. Here he distinguishes between content and mode of determination of content, and writes that “different names for the same content

¹⁷ Frege 1879, p. 14/1997, p. 64.

¹⁸ Frege 1892a, p. 26/1997, p. 152.

are not always merely a trivial matter of formulation, but touch the very heart of the matter if they are connected with different modes of determination”.¹⁹ The name ‘point of intersection of a and b ’ determines the same point as the name ‘point of intersection of b and c ’, for example, but in a different way. Despite recognizing the relevance of modes of determination, however, Frege continues to maintain that the content of an identity statement is the circumstance that two names have the same content, which does no justice to the role that modes of determination play in our use and understanding of identity statements.

As we have noted, (AB) tells us something whereas (AA) does not: both are true, but while (AB) is informative, (AA) is trivial. Since the same point is determined (twice) in both cases, the difference must clearly lie in the fact that (AB) involves two different modes of determination – and this must be reflected in the ‘content’ of (AB), at least in some sense of ‘content’. What Frege comes to see is that *two* notions of content are needed, and from 1891 onwards he distinguishes between ‘sense’ (*Sinn*) and ‘reference’ (*Bedeutung*).²⁰ In the case of names, the reference of a name is the object designated, while the sense of a name is what contains the mode of determination of the object designated – or its ‘mode of presentation’, as Frege also calls it in ‘On Sense and Reference’.²¹

Armed with this distinction, Frege can then give an account of why (AB) can be informatively true while (AA) is only trivially true. The two names ‘point of intersection of a and b ’ and ‘point of intersection of b and c ’ have the same reference, which is why both identity statements are true, but they express different senses, which is why (AB) can tell us something – namely, that determining a point in one way gives the same result as determining a point in another way. So too, in the other – and more famous – example that Frege gives at the beginning of ‘On Sense and Reference’, ‘the Morning Star’ and ‘the Evening Star’ have the same reference, but express different senses, which is why we can learn something in being told that the Morning Star is the Evening Star. In general,

¹⁹ Frege 1879, p. 15/1997, p. 65.

²⁰ The distinction is first drawn in Frege 1891, but receives its fullest explanation in Frege 1892a. The translation of ‘*Bedeutung*’ has generated enormous controversy. In *The Frege Reader* (Frege 1997) I left it untranslated, for the reasons I give in § 4 of my introduction. But I also say there that my own preference is for ‘reference’, and this is what I use in the present chapter.

²¹ Frege 1892a, pp. 26-7/1997, p. 152.

identity statements of the form ' $a = b$ ' are true if and only if ' a ' and ' b ' have the same reference, and are informative only if ' a ' and ' b ' express different senses.

6 The sense and reference of sentences

Frege's distinction between sense and reference seems well motivated in the case of names. (We will turn to some problems in §8 below.) But Frege also applied the distinction to sentences. On Frege's early view, as suggested above, the content of a sentence is the 'circumstance' represented; on his later view, with content split into sense and reference, the sense of a sentence is the thought expressed, and the reference of a sentence is its truth-value (if any). It seems plausible to identify the sense of a sentence with the thought it expresses – at any rate, if we consider grammatically well-formed sentences in the indicative mood, which are used correctly in an appropriate context.²² In uttering (AB), for example, we express the thought that the point of intersection of a and b is the same as the point of intersection of b and c . In general, in uttering an indicative sentence ' p ' (correctly, in an appropriate context), we express the thought that p .

Frege's claim that the reference of a sentence is its truth-value seems far less plausible, especially when we take into account Frege's view that the truth-values are *objects*, which he calls 'the True' and 'the False'. All true sentences, according to Frege, refer to the True, while all false sentences refer to the False. Once again, though, the key to understanding Frege's claim is his use of function–argument analysis, and identity statements remain at the heart of his concern. Consider the functional expression ' $x =$ the point of intersection of b and c ', which yields the following identity statements when ' x ' is replaced by the respective names (see Diagram 1 above):

(AB) The point of intersection of a and $b =$ the point of intersection of b and c .

(BB) The point of intersection of b and $c =$ the point of intersection of b and c .

(CB) The point of intersection of c and $a =$ the point of intersection of b and c .

²² It is common to see the indicative (declarative) mood as basic and other moods, such as the imperative and interrogative, as derivative. If so, then the sense of a non-indicative sentence can be taken as the sense of the corresponding indicative sentence. For recent discussion of the issues raised here, see e.g. Boisvert and Ludwig 2006.

(PB) Vertex P = the point of intersection of b and c .

(QB) Vertex Q = the point of intersection of b and c .

(RB) Vertex R = the point of intersection of b and c .

Of these, (AB), (BB) and (CB) are true, while (PB), (QB) and (RB) are false. It thus seems natural to take the ‘value’ of (AB), (BB) and (CB), which results from applying the function to the respective arguments, as their truth, and the ‘value’ of (PB), (QB) and (RB) as their falsity.²³ Indeed, this makes talk of *truth-value* seem entirely appropriate.

Why does Frege think that the truth-values are objects? Here the answer hinges on his distinction between concept and object: concepts are represented by ‘unsaturated’ expressions, objects by ‘saturated’ expressions. Sentences, like names, are saturated expressions, so they must represent objects. On Frege’s view, then, sentences refer to either the True or the False just as names such as ‘the point of intersection of b and c ’ refer to the objects they stand for.²⁴

What we have in these considerations, though, is not so much an argument as merely a motivation for identifying the reference of a sentence (understood as the value that results from applying a function to an argument or arguments) with its truth-value. Frege does not consider other possibilities. Why should we not take the reference of true identity statements as the self-identity of the relevant object, for example, and the reference of false identity statements as the non-identity of the relevant objects? (This still allows us to regard their sense as the thought expressed.) There may be objections to such a suggestion, but Frege does not present them.²⁵ With his new view in place, however, Frege was able to streamline his ontology and offer a simple account of concepts. In Frege’s ontology, there are just objects and functions, of which one

²³ Cf. Frege 1891, p. 13/1997, p. 137. Frege gives the example of the function $x^2 = I$ and considers the four identity statements (equations) that result from taking -1 , 0 , 1 and 2 as arguments: -1 and 1 yield true identity statements, while 0 and 2 yield false identity statements.

²⁴ Cf. *ibid.* I discuss Frege’s introduction of truth-values as objects in more detail in Beaney 2007d.

²⁵ A further possibility is to develop Frege’s earlier idea of the ‘circumstance’ or ‘state of affairs’ represented. It may be harder to think what this might be in the case of identity statements (which may be one reason why Frege offered a metalinguistic construal of their ‘content’ in his early work), but it looks attractive in the case of simple sentences such as ‘Gottlob is human’, and is arguably what Frege needs to fill the gap that opens up between sense and reference. I discuss some of the issues raised here in Beaney 1996, esp. § 8.1.

important type are concepts. Concepts are functions whose value is always a truth-value.²⁶ Concepts, in other words, map objects (the references of names) onto one of two particular objects, the True and the False (the references of sentences).

7 The sense and reference of concept-words

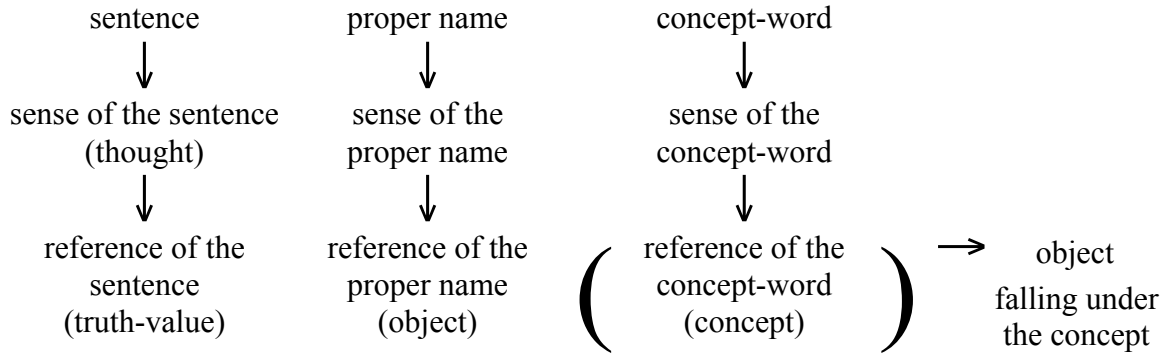
According to Frege, the distinction between sense and reference applies to both names and sentences (which turn out to be a type of name). In none of his three published papers of the early 1890s, however, does Frege discuss its application to concept-words – or to functional expressions in general. This led to much dispute in the early secondary literature, which was only resolved when other writings from the period were published in 1969 and 1976.²⁷ It then became clear that Frege did indeed regard concept-words as having both a sense and a reference, at least when properly used in logic and science: the reference of a concept-word is the concept itself and the sense contains the mode of determination of the concept.

Traditional logicians had distinguished two features of concepts: their extension and their intension. The extension of a concept is the set of objects that fall under the concept, while the intension (roughly) is what we understand in grasping the concept: the condition that must be met for something to fall under the concept (which might be exhibited in a set of properties). It would be tempting to interpret Frege as lining up the sense/reference distinction with the intension/extension distinction. But while, as a first approximation, sense corresponds to intension, the reference of a concept-word is not its extension, on Frege's view, but the concept itself, understood as a function – an unsaturated entity. Frege does indeed see concepts as having extensions, but since these are sets, they count as objects, not concepts, and must be distinguished from the concepts themselves.

²⁶ Cf. Frege 1891, p. 15/1997, p. 139.

²⁷ See Frege 1969, and in particular his '[Comments on *Sinn* and *Bedeutung*]', tr. in Frege 1997, pp. 172-80; and Frege 1976, and in particular his letter to Husserl of 24 May 1891, partly tr. in Frege 1997, pp. 149-50.

Frege's view of the sense and reference of the three types of expression – sentences, names and concept-words – is schematically summarized in a letter he wrote to Edmund Husserl in 1891:²⁸



Frege never changed his views on sense and reference after he had formulated them in 1891. The diagram above thus represents his final and considered position.

8 Two problems about names

Frege's views on sense and reference set much of the agenda for the development of philosophy of language in the twentieth century. In the remaining two sections of this chapter, we briefly look at some of the problems raised by these views, the debates about which have fuelled this development. In this section we consider two problems about the sense and reference of names, the first concerning whether names can have a sense without a reference, and the second concerning whether names can have a reference without a sense.

According to Frege, the sense of a name contains the 'mode of determination' – or as he also sometimes puts it, the 'mode of presentation' – of its reference. But if there is no reference, then how can there be a mode of presentation of it? Does sense, therefore, not require reference? Yet Frege himself admits that there can be names that have sense but lack reference. Fictional names such as 'Odysseus' provide one set of examples, and

²⁸ See Frege 1976, pp. 96-7; tr. in Frege 1997, p. 149.

there are other examples such as ‘the greatest number’ which have a sense that *entails* a lack of reference.²⁹ Frege’s views might look inconsistent if we think of senses solely as ‘modes of presentation’, but Frege rarely uses this term himself; he talks far more often of ‘modes of determination’, and this is the better term to use because it carries no implication of reference. If we were to think, say, of a mode of determination as a specification of a condition that something must meet (in order to be the reference of a name whose sense contains that mode of determination), then it may turn out that nothing meets that condition, even though the condition itself is perfectly coherent. On Frege’s view, for example, ‘the present King of France’ has a sense even though it lacks a reference.

A qualification needs to be made here, however. For Frege nevertheless insisted that in an ideal language, that is, in a logical language suitable for scientific purposes (where truth is the goal), all names must have a reference (since otherwise sentences that contain them would lack a truth-value). What we can do here, though, Frege suggested, is *stipulate* a reference for terms that might otherwise be thought to lack a reference – for example, we can stipulate that they refer to the number 0 (seen as itself a kind of object).³⁰ On Frege’s view, then, names can indeed have a sense without a reference, although this is to be avoided in an ideal language.

Can there be names that have reference but not sense? By ‘name’ Frege understood any expression referring – or purporting to refer – to an object. It is customary now to distinguish between proper names, such as ‘Aristotle’, and definite descriptions, such as ‘the present King of France’. The conception of sense just suggested – the sense of a (Fregean) name being specified as a condition that something must meet in order to be the reference of the name – clearly applies to definite descriptions: this is what enables them to have sense but lack reference. But does it apply equally well to proper names? What is the sense of ‘Aristotle’, for example? In a notorious footnote in ‘On Sense and Reference’, Frege writes that opinions may differ here, depending on what we take as the relevant condition. One person may take ‘Aristotle’ to have the same sense as ‘the pupil

²⁹ Cf. e.g. Frege 1892a, pp. 28, 32-3/1997, pp. 153, 157.

³⁰ See e.g. Frege 1892a, p. 41/1997, p. 163.

of Plato and teacher of Alexander the Great’, for example, while another may take it to have the same sense as ‘the teacher of Alexander the Great who was born in Stagira’.³¹

There are objections, however, to such a ‘description theory’ of proper names. One problem is the threat of infinite regress or circularity, since any definite description contains names whose sense must in turn be specified. A second problem, which Kripke was to highlight, is that such a view fails to do justice to certain modal considerations.³² One response is simply to deny that proper names have a sense at all. This was Russell’s view: genuine (or logically) proper names merely refer.³³ According to Frege, however, to deny that proper names have sense is to deny that sentences that contain those names have sense. (Russell needed his theory of descriptions to avoid this implication.) Frege’s own response was to stress again the differences between our use of language in everyday life and the demands of an ideal logical language. In the latter we must indeed specify a unique sense for each name, definition or demonstration being required wherever two names have the same reference. On Frege’s own view, then, all names – whether proper names or definite descriptions – have both a sense and a reference, or at least should have in an ideal language; in other words, they do so wherever sentences containing them both express thoughts and have a truth-value.

9 Two problems about contexts

Fundamental to Frege’s application of function–argument analysis to language is the idea that the ‘value’ of a sentence is determined by the ‘value’ of its (logically significant) parts. In his early work, Frege called this value ‘content’. With content later split into sense and reference, however, this yields two principles of determination (sometimes also called principles of compositionality):

(PDS) The sense of a sentence is determined by the sense of its (logically significant) parts.

³¹ Frege 1892a, p. 27, fn./1997, p. 153, fn. B.

³² See e.g. Kripke 1980, esp. Lecture 1. See the chapter on Kripke in this volume.

³³ See the chapter on Russell in this volume.

(PDR) The reference of a sentence is determined by the reference of its (logically significant) parts.

These principles have motivated a great deal of work in subsequent philosophy of language, but they are not unproblematic. In this final section we briefly consider two problems.³⁴

The first concerns their application in so-called intensional contexts, that is, in contexts where there are attributions of propositional attitudes, such as saying of someone that they believe that *p*. Consider, for example, the following two sentences:

(GAA) Gottlob believes that Aristotle is Aristotle.

(GAP) Gottlob believes that Aristotle is the author of the *Prior Analytics*.

‘Aristotle’ and ‘the author of the *Prior Analytics*’ have the same reference, so by the principle (PDR), (GAA) should have the same reference, i.e. truth-value, as (GAP). But (GAA) can clearly be true without (GAP) being true. Frege recognized the problem here and argued that in these cases, the reference of the embedded sentence, i.e. what follows the ‘that’, is not its customary or direct reference but its indirect reference, where this is taken as its customary sense.³⁵ In other words, in intensional contexts, the reference of the embedded sentence is the sense, i.e. thought, that the sentence (normally) expresses. This is plausible in itself, for what determines whether (GAP), say, is true is the *thought* that Gottlob takes to be true (not the truth-value of the embedded sentence). The principles of determination can thus be preserved, albeit at the cost of complicating the account of sense and reference. There are further complications when we consider cases of multiply embedded sentences (such as ‘Bertrand hopes that Gottlob believes that Aristotle is Aristotle’); and the issue continues to be debated in philosophy of language today.³⁶

There is a general message suggested here: that what counts as the sense and reference of an expression depends on the context. This message is reinforced when we consider the problem of indexicality. Indexicals are expressions such as ‘I’, ‘you’, ‘here’, ‘there’, ‘now’ and ‘then’, whose reference depends systematically on the context; and

³⁴ For a fuller account of these problems, see my introduction to Frege 1997, pp. 29-35, on which I draw here.

³⁵ See Frege 1892a, p. 37/1997, p. 160.

³⁶ For a brief recent account of the issue, see Textor 2011, pp. 182-91.

they pose a serious problem for Frege's conception of sense. For if the reference changes, then so too must the sense. Consider the following:

(TS) Today is sunny.

Uttered today, this expresses a different thought from that that would be expressed if uttered tomorrow; so the sense of 'today' – as partly determining the thought, in accord with (PDS) – must differ on the two occasions. But then what is it?

Here is one suggestion as to how (TS) might be understood:

(DS) Monday 8 November 2010 is sunny.

But it is clearly possible to hold (TS) as true but (DS) as false, and vice versa, in which case they cannot be taken to express the same thought; and the same will apply to any attempt to 'cash out' the indexical in terms of a definite description.³⁷ The only response might seem to be to take the sense of any indexical as used on a given occasion as primitive and irreducible. It is clear, however, that Frege allowed that the thought expressed by (TS) uttered today could be expressed on other occasions.³⁸ To express tomorrow what I said today, I would have to utter:

(Y₁S) Yesterday was sunny.

But how far can this go on? Next Monday, I might say:

(Y₇S) The day before the day before the day before the day before the day before the day before yesterday was sunny.

Yet there is obviously a better sentence:

(LMS) Last Monday was sunny.

This introduces the concept *day of the week*, so arguably goes beyond what is expressed in uttering (TS). And if we consider what I would say in a few years' time, then it seems that I am going to end up using (DS) to express the thought, and we are back with the

³⁷ This has been called 'the problem of the essential indexical' (Perry 1979). It is similar to the problem that arises if we identify the sense of a proper name with the sense of some corresponding definite description, as discussed in the previous section.

³⁸ Cf. Frege 1918, p. 64/1997, p. 332.

dilemma. In trying to capture the same thought through time, we seem to have been led to an arguably *different* thought.

Frege never provided a satisfactory solution to the problem of indexicality. As far as he was concerned, this was a problem that affected our use of ordinary language – and once again, was to be avoided in an ideal logical language (in which all indexicality is to be cashed out, as we might put it). Nevertheless, indexicality – and contextuality generally – is a pervasive feature of ordinary language, and the problems cannot be brushed aside if we want to apply Frege’s ideas.³⁹ They do suggest that the principles of determination and the account of sense and reference need, at the very least, to be qualified in important ways. This task was one of the many things that Frege bequeathed to his successors.

Further reading

The Frege Reader (1997) contains the four key papers discussed in this chapter, as well as substantial selections from Frege’s three books and other works. For elaboration of the account offered in this chapter, see Beaney’s *Frege: Making Sense* (1996). The pioneering work on Frege was Dummett’s *Frege: Philosophy of Language* (1973, 2nd ed. 1981). Many of its claims are now seen as controversial, but the book did much to secure Frege’s place in modern philosophy of language. The four-volume collection of papers edited by Beaney and Reck (2005) contains the most influential papers, on all aspects of Frege’s philosophy, published between 1986 and 2005. Volume 4 is on Frege’s philosophy of thought and language. The recent (but long heralded) *Cambridge Companion to Frege* (2010), edited by Potter and Ricketts, contains further papers. Textor’s *Frege on Sense and Reference* (2011) offers a useful book-length guide to many of the ideas covered in this chapter.

³⁹ For an introduction to these problems, see Beaney 1996, § 7.4; Textor 2011, pp. 154-70.

Bibliography

- Beaney, Michael, 1996, *Frege: Making Sense*, London: Duckworth
- _____, 1997, 'Introduction' to Frege 1997, pp. 1-46
- _____, 2005, 'Frege, Russell and Logicism', in Beaney and Reck 2005, Vol. I, pp. 213-40
- _____, 2007a, ed., *The Analytic Turn: Analysis in Early Analytic Philosophy and Phenomenology*, London: Routledge
- _____, 2007b, 'Conceptions of Analysis in the Early Analytic and Phenomenological Traditions: Some Comparisons and Relationships', in Beaney 2007a, pp. 196-216
- _____, 2007c, 'Analysis', in *The Stanford Encyclopedia of Philosophy*, online at: plato.stanford.edu/entries/analysis
- _____, 2007d, 'Frege's Use of Function-Argument Analysis and his Introduction of Truth-Values as Objects', in Dirk Greimann, ed., *Essays on Frege's Conception of Truth*, *Grazer Philosophische Studien*, 75, pp. 93-123
- Beaney, Michael and Reck, Erich H., 2005, eds., *Gottlob Frege: Critical Assessments*, 4 vols., London: Routledge
- Boisvert, Daniel and Ludwig, Kirk, 2006, 'Semantics for Nondeclaratives', in Ernest Lepore and Barry C. Smith, eds., *The Oxford Handbook of Philosophy of Language*, Oxford: Oxford University Press, pp. 864-92
- Dummett, Michael, 1973, *Frege: Philosophy of Language*, London: Duckworth; 2nd ed. 1981
- Frege, Gottlob, 1879, *Begriffsschrift*, Halle: L. Nebert; Preface and most of Part I (§§ 1-12) tr. in Frege 1997, pp. 47-78
- _____, 1882, 'Letter to Marty, 29.8.1882', tr. in Frege 1997, pp. 79-83
- _____, 1884, *Die Grundlagen der Arithmetik*, Breslau: W. Koebner; selections tr. in Frege 1997, pp. 84-129
- _____, 1891, 'Funktion und Begriff', Jena: H. Pohle; tr. as 'Function and Concept' in Frege 1997, pp. 130-48
- _____, 1892a, 'Über Sinn und Bedeutung', *Zeitschrift für Philosophie und philosophische Kritik*, 100, pp. 25-50; tr. as 'On Sinn and Bedeutung' in Frege 1997, pp. 151-71
- _____, 1892b, 'Über Begriff und Gegenstand', *Vierteljahrsschrift für wissenschaftliche Philosophie*, 16, pp. 192-205; tr. as 'On Concept and Object' in Frege 1997, pp. 181-93
- _____, 1893/1903, *Grundgesetze der Arithmetik*, Jena: H. Pohle, Vol. I 1893, Vol. II 1903; selections tr. in Frege 1997, pp. 194-223, 258-89
- _____, 1918, 'Der Gedanke', *Beiträge zur Philosophie des deutschen Idealismus*, 1, pp. 58-77; tr. as 'Thought' in Frege 1997, pp. 325-45
- _____, 1969, *Nachgelassene Schriften*, ed. H. Hermes, F. Kambartel & F. Kaulbach, Hamburg: Felix Meiner; tr. as Frege 1979
- _____, 1976, *Wissenschaftlicher Briefwechsel*, ed. G. Gabriel, H. Hermes, F. Kambartel, C. Thiel and A. Veraart, Hamburg: Felix Meiner; abr. and tr. as Frege 1980

- _____, 1979, *Posthumous Writings*, tr. of Frege 1969 by P. Long and R. White, Oxford: Blackwell
- _____, 1980, *Philosophical and Mathematical Correspondence*, tr. of Frege 1976, ed. B. McGuinness, tr. by H. Kaal, Oxford: Blackwell
- _____, 1997, *The Frege Reader*, ed. with an introd. by M. Beaney, Oxford: Blackwell
- Kripke, Saul, 1980, *Naming and Necessity*, Oxford: Blackwell
- Oliver, Alex, 2010, 'What is a predicate?', in Potter and Ricketts 2010, pp. 118-48
- Perry, John, 1979, 'The Problem of the Essential Indexical', *Noûs*, 13, pp. 3-21
- Potter, Michael and Ricketts, Tom, 2010, eds., *The Cambridge Companion to Frege*, Cambridge: Cambridge University Press
- Ricketts, Thomas, 2010, 'Concepts, objects and the Context Principle', in Potter and Ricketts 2010, pp. 149-219
- Textor, Mark, 2011, *Frege on Sense and Reference*, London: Routledge
- Thiel, Christian and Beaney, Michael, 2005, 'Frege's life and work: chronology and bibliography', in Beaney and Reck 2005, Vol. I, pp. 23-39

Final draft 26 March 2011