without holding (MBL) as true, for example, have I not *expressed* the thought without having *grasped* it? But if I can express thoughts with really grasping them, then have we not further distanced ourselves from Frege's original conception of sense? *If context* is allowed as a determinant of sense, then do principles of compositionality not need to be modified? I take up these questions in §8.2.

One obvious way out of the problem is to deny that I do express a complete thought when I utter (IHN), say, without being able to fully cash out its indexicality. But if all sentences involve some kind of indexicality, and there is no such thing as a genuinely 'timeless' linguistic expression of a thought, then the result would seem to be that *no* thoughts can ever be fully expressed or grasped. In fact, however, Frege's own later conception of 'grasping' thoughts seems to point in the other direction. According to Frege, anyone can in principle grasp the thought that someone else expresses, but will do so in a different way, since 'grasping' involves ideas, and ideas are essentially private. But the obvious objection to this is hardly less devastating. For, even if someone else does grasp the thought I express, I can never *know* that they do so, since I do not myself have access to their ideas.[58] In the end, then, this seems a high price to pay for the objectivity of thoughts. Thoughts have become so ethereal that either grasp of them is no longer possible at all or else I can never know whether anyone else grasps them. Like locking up one's money in a chest, burying it on some remote island, and then throwing away the key, to prevent anyone from stealing it, what we aimed to protect has been rendered valueless. This is the ultimate paradox of Frege's semainomenalism: the logical realm has become so sublimated that it is hard to see how ordinary linguistic life can go on at all.

8. The Crystallization of Sense

> The more narrowly we examine actual language, the sharper becomes the conflict between it and our requirement. (For the crystalline purity of logic was, of course, not a *result of investigation:* it was a requirement.) The conflict becomes intolerable; the requirement is now in danger of becoming empty. – We have got on to slippery ice where there is no friction and so in a certain sense the conditions are ideal, but also, just because of that, we are unable to walk. We want to walk: so we need *friction.* Back to the rough ground! (Wittgenstein, *Philosophical Investigations,* §107.)

In the preceding chapters, we have traced the development of Frege's philosophy, focussing on his conception of 'sense', from the emergence of his early notion of 'conceptual content', through the bifurcation of that notion into 'Sinn' and 'Bedeutung' in response to the problem of the status of his logicist definitions, to his late account of 'timeless thoughts'. In this final chapter, I want to draw together some of the threads of the discussion, and consider a number of outstanding problems. At various points we have suggested that there is a certain tension in Frege's conception of sense, and in the first section I elucidate this further by considering the criteria for sameness of sense that Frege himself offers and that might be formulated on his behalf. In §8.2 I explore the relationship between Frege's context principle and his principles of compositionality, all of which we have seen playing key roles in his thought; and in the following two sections, I consider what conclusions can be drawn concerning the coherence of Frege's conception of sense. In the final section, I return to the central theme of this book, highlighting the way that 'sense' is *made* or *crystallized* in our attempts to deepen our understanding in a given area through the process of analysis.

8.1 Criteria for Sameness of Sense

In §2.5 we formulated the following criterion for Frege's early notion of 'conceptual content':

(CC) Two propositions have the same *conceptual content* iff they are logically equivalent.

Intuitively, the idea seems simple. If *P* implies *Q*, and *Q* implies *P*, then *P* and *Q* have the same 'conceptual content', however else they may differ. But as we also noted, the 'logical equivalence' here must be interpreted as

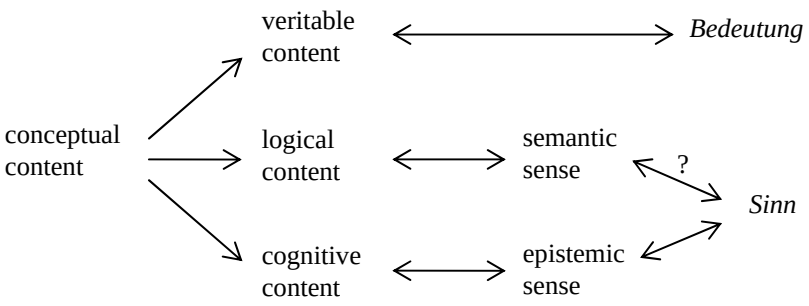
more than mere 'material equivalence'; and even 'provable material equivalence' fails to do justice to all of our intuitions about sameness of 'content'. Frege's logicism shows the need for an additional, finer-grained conception: it is just because the propositions of arithmetic have different 'cognitive contents' that their sameness of 'logical content' requires demonstration. This led us to distinguish the following three versions of (CC):

(CC#) Two propositions have the same *veritable content* iff they are materially equivalent.
(CC*) Two propositions have the same *logical content* iff they are logically equivalent (provably materially equivalent).[1]
(CC†) Two propositions have the same *cognitive content* iff they are cognitively equivalent.

(CC#) provides us with the criterion for Frege's later notion of *Bedeutung*, whilst (CC†) was intended as a place-holder for the elucidation of *Sinn* that it has been our subsequent task to provide. We also noted that (CC*) is offered in one place as the criterion for *Sinn*, but that this cannot be considered as Frege's 'official' view.

Nevertheless, it remains the case that there is a certain tension in Frege's conception of *Sinn*, and this was highlighted in chapter 5. The paradox of analysis shows that the notion of 'content' requires disambiguation. In analysing anything, the *analysans* and the *analysandun* must, at the very least, be logically equivalent; yet if the analysis is to be 'fruitful', it would seem that they must also have different 'cognitive contents'. Frege's distinction between 'Bedeutung' and 'Sinn' was offered as a response to the paradox, but as we argued, it is inadequate as it stands. For sameness of *Bedeutung* is too weak a relation to act as the only constraint on the adequacy of an analysis; and whilst the notion of *Sinn* does indeed accommodate informativeness, since the *analysans* and *analysandum* are regarded as having *different* senses, there is no longer any way of making out the notion of 'logical content' to act as the required constraint. As we suggested in §5.4, Frege's conception of *Sinn* appears to be *epistemic* rather than semantic, yet the paradox of analysis demands a *semantic* conception of sense as well. Certainly, Frege's 'official' conception, as presented in 'Über Sinn und Bedeutung', is epistemic; although, as we also hinted, a more semantic conception does seem to surface in the *Grundgesetze.* The position is summarized in the diagram on the page opposite.

Although Frege operated with an undifferentiated notion of 'content' in his early work, he did distinguish between 'content' and 'way of splitting up content', which anticipated to some extent his later distinction between 'Bedeutung' and 'Sinn'. As we saw in §5.2, Frege held that the same 'content' can be 'split up' in different ways, a conception that was grounded in the idea of alternative function-argument analyses. This early distinction appeared to offer an attractive resolution of the paradox of analysis, 'content' being understood as 'logical content' and 'way of splitting up



content' as 'cognitive content'. But as we showed in §5.3, the legitimacy of certain kinds of argument (where 'cross-identifications' are involved) seems to depend on 'cognitive content' and not just 'logical content'; and Frege was by no means clear about this at the time. In any case, it is implausible to maintain that the relationship between the two members of what we termed a 'Fregean pair' (Fregean pairs being fundamental to Frege's logicist project, as we saw in chapter 4) is simply one of alternative function-argument analyses: introducing terms for abstract objects, or engaging in conceptual ascent, seems to generate an arguably *different* (albeit logically equivalent) proposition. Recall, for example, (Da), (Db) and (Dc):

(Da) Line *a* is parallel to line *b*.
(Db) The direction of line *a* is identical with the direction of line *b*.
(Dc) The concept 'line parallel to line *a*' is coextensive with the concept 'line parallel to line *b*'.

Compare these with one of Frege's paradigm examples of alternative function-argument analyses of the same 'conceptual content' (see §5.2 above), with parentheses highlighting the argument place:

(HLC) (Hydrogen) is lighter than carbon dioxide.
(CHH) (Carbon dioxide) is heavier than hydrogen.

Although the concept *lighter than carbon dioxide* is different from the concept *heavier than hydrogen*, it does seem that it is the same relation between the same two objects with which (HLC) and (CHH) are concerned, so that the same 'content' is involved.[2] But it is hard to see how (Da), (Db) and (Dc) fit the same model: despite their logical equivalence, it is not just that different concepts are used, but that different relations are being stated to obtain between things of different kinds (parallelism of lines, identity of directions, coextensiveness of concepts).

One way of expressing the difference between these two cases is to say

that whilst it is impossible to hold that (HLC) is true without holding that (CHH) is true, and vice versa, it *is* possible to hold that (Da) is true without holding that (Db) is true, and vice versa. To use the distinction discussed in §6.5, whilst (Da) and (Db) are *extensionally equivalent*, they are not *intensionally equivalent:* substituting one for the other in intensional contexts does not necessarily preserve truth-value. If this is right, and Frege's own account of intensionality in 'Über Sinn und Bedeutung' gives an essential clue to his conception of sense, then the implication must be that (Da) and (Db) possess *different senses* – they express *different thoughts*. This suggests the following criterion:

(SIE) Two propositions have the same *sense* iff they are *intensionally equivalent*, i.e. iff they can be intersubstituted *salva veritate in* all intensional contexts.[3]

But did Frege really want to hold that members of Fregean pairs possess different senses? As we have seen, Frege's primary concern in drawing the distinction between *Sinn* and *Bedeutung* was with the status of the 'analytic definitions' of the *Grundlagen* and Axiom V of the *Grundgeseize;* but this suggests that he wants at least the two members of that fundamental Fregean pair, (Va) and (Vb), to possess the *same sense.* It was just this that was indicated by his remark in 'Function and Concept' that each side of Axiom V 'expresses the same sense, but in a different way' (*FC*, p. 27; see §5.4 above). Frege's logicism thus clearly demands a notion of sense that is finer-grained than the notion of *Bedeutung*, yet coarser-grained than his 'official' notion of epistemic sense.

It was no doubt the tension between the requirements of his logicism and his desire to nevertheless uphold the informativeness of identity statements that contributed to Frege's uncharacteristic reluctance to specify exactly what he meant by 'Sinn'. There are just two passages, both written around the same time, 1906, where he offers what seem to be his most careful formulations; but the two criteria he suggests appear to be in marked conflict. In a letter written to Husserl, after stating that 'an objective criterion is necessary for recognizing a thought again as the same, for without it logical analysis is impossible', Frege provides the following criterion:

(SLE) Two propositions *A* and *B* possess the same *sense* (express the same *thought)* iff '*both* the assumption that the content of *A* is false and that of *B* true *and* the assumption that the content of *A* is true and that of *B* false lead to a logical contradiction, and ... this can be established without knowing whether the content of *A* or *B* is true or false, and without requiring other than purely logical laws for this purpose'. (*PMC*, p. 70.)

In a posthumously published piece (*BSLD*), however, Frege offers a rather different criterion:

(SEE) Two propositions *A* and *B* possess the same *sense* (express the same *thought*) iff 'anyone who recognizes the content of *A* as true must straight away [*ohne weiteres*] also recognize that of *B* as true, and conversely, anyone who recognizes the content of *B* must immediately [*unmittelbar*] also recognize that of *A* (*equipollence*)'. (*NS*, p. 213.)

Whilst (SLE) bases sameness of sense on *logical equivalence* (provable material equivalence), (SEE) grounds it on what can be called *epistemic equipollence.* Once more, the conflict seems to be between a fairly coarse-grained notion of sense, echoing the central strand in Frege's earlier notion of 'conceptual content', and a much finer-grained one.[4]

In other places, Frege's notion of sense is left to be gleaned from the examples he gives, or else, where he does offer some characterization, he simply talks loosely of two propositions having different senses if one can be taken as true but not the other.[5] So how are we to offer a criterion that best captures Frege's intentions, and how are we to explain the conflict between the two criteria just stated? The difference between the two criteria can be seen by considering their application to Fregean pairs. According to (SLE), the two members of a Fregean pair, such as (Da) and (Db), or (Va) and (Vb), would possess the *same* sense, since they are certainly logically equivalent, whereas according to (SEE), they would presumably possess *different* senses, since it does seem possible to recognize one as true without immediately recognizing the other as true.

The conflict becomes even sharper if we apply the criteria to arithmetical propositions. If Frege's logicism is right, then, by (SLE), all arithmetical truths turn out to possess the same sense, since they are all provably materially equivalent; yet in the vast majority of cases, someone can take one as true without immediately recognizing another as true (some kind of reasoning will typically be needed), so that, by (SEE), they would possess different senses. However, in his formulation of the first criterion, Frege makes a qualification that arguably rules out its application in just these cases: 'I assume that neither of the two propositions contains a logically self-evident component part in its sense' (*PMC*, p. 70). Despite his talk of 'a logically self-evident component part', Frege's intention here was presumably to exclude the application of the criterion to any pair of propositions whose negations in themselves lead to a logical contradiction, to avoid what would certainly be for Frege an absurdity – that all logical truths (taken as including arithmetical truths) possess the same sense.[6]

A similar qualification is made in his formulation of the second criterion. Frege writes: 'I assume there is nothing in the content of either of the two equipollent sentences *A* and *B* that would have to be immediately accepted as true by anyone who had grasped it properly' (*BSLD*, p. 197). *Self-evident* propositions are excluded for an equally obvious reason. If it

is taken as a mark of a self-evident truth that it is not possible to grasp its content without immediately recognizing it as true, then (SEE) – interpreted literally – would imply that all self-evident truths have the same sense, and this too cannot have been Frege's intention.[7]

Of course, in the case of self-evident logical truths, both criteria yield the result that they all possess the same sense, so that here at least the conflict is removed. But it is clear not only that the conflict would remain in the case of non-self-evident logical truths, but also that such a result is inconsistent with the many illustrations that Frege gives of his conception of sense. '2 + 2 = 4' and '$2^2 = 4$', for example, would by both criteria posses the same sense (assuming that they are indeed self-evident truths, to anyone with a reasonable knowledge of arithmetic); yet Frege himself explicitly states that they possess *different* senses (see e.g. *GG*, I, §2). Since it is hardly likely that Frege failed to recognize the obvious implications of the use of the two criteria in just that area with which he was most concerned, namely, arithmetic, one can only conclude that the qualifications he made were precisely intended to rule out in advance the inconsistencies we have just considered.[8] If this is appreciated, then perhaps the seriousness of the conflict between the two criteria can be played down.

However, neither member of a Fregean pair need in itself be a self-evident logical truth. In the case of (Va) and (Vb), it is the proposition that asserts their equivalence that is meant to be a self-evident logical truth. Within Frege's system, of course, (SLE) yields the sameness of sense of (Va) and (Vb) very trivially, since Axiom V is precisely one of the logical laws. But what of the application of (SEE)? To claim that Axiom V is a self-evident logical truth would indeed be to claim that anyone who recognizes (Va) as true must immediately recognize (Vb) as true, and vice versa; but it is just this that is problematic. As we saw in §7.2, Frege admitted in the preface to the *Grundgesetze* that a dispute might arise as to the status of Axiom V, although he held it to be purely logical. But in the Appendix to Volume II, where he attempted to face up to Russell's paradox, he remarked that 'I have never disguised from myself its lack of the self-evidence that belongs to the other axioms and that must properly be demanded of a logical law' (*FRP*, p. 214). This suggests that whilst Frege *wanted* Axiom V to be self-evident, he had doubts that it really was, doubts only confirmed by the appearance of Russell's paradox.[9] If Frege had these doubts all along, then this would have been reflected in doubts about whether (Va) and (Vb) possessed the same sense, which would certainly explain his vacillation over what criterion to offer. To have insisted that they do possess the same sense would have meant rejecting (SEE) and offering something much more like (SLE), which is what he does do in his letter to Husserl.

It remains the case, then, that (SLE) and (SEE) yield different results, a tension that was reflected in, if not generated by, Frege's uncertainty about the status of Axiom V. According to the first criterion, two (contingent) propositions have the same sense if one cannot be true without the other being true, whilst according to the second criterion, two (non-self-evident) propositions have the same sense if one cannot be recognized to be true without the other being recognized to be true. The first encapsulates a *semantic* conception of sense, and the second an *epistemic* conception.[10] Of the two, however, (SLE) is the anomaly. Most of the illustrations that Frege gives of his conception of sense are more consistent with adherence to (SEE). But as we have suggested, this implies that (Va) and (Vb) – and presumably other members of Fregean pairs – possess *different* senses, which is not what Frege wanted. Indeed, as we argued in §5.4, if Frege's notion of 'Bedeutung' is the only other notion available to us then the epistemic conception of sense encapsulated in (SEE) would render us unable to satisfactorily respond to the paradox of analysis. For whilst a fine-grained conception of sense allows us to account for the informativeness of analyses, sameness of *Bedeutung* provides an insufficient constraint on their correctness.

So can a criterion be found that would provide the right constraint on the correctness of analyses, encapsulating a coarser-grained conception of sense than (SEE) but a finer-grained conception of sense than (SLE)? The best that can be offered, I think, is the following, which can be taken as filling out the required notion of 'cognitive equivalence' mentioned above:

> (SCE) Two propositions *A* and *B* possess the same *sense* (express the same *thought)* iff anyone who understands both propositions at a given time can immediately recognize that *A* is true (or false) if they recognize *B* as true (or false), and vice versa.[11]

At first glance, this might seem no different to (SEE), and it is, indeed, as close as one can get to (SEE) whilst leaving room to assert the equivalence in sense of members of Fregean pairs. The crucial difference is that (SCE) contains the qualification that both propositions must be understood at a given time before the test of immediate recognition can be applied.

To appreciate the difference between (SEE) and (SCE), let us return to Frege's original contextual definition of *direction* in terms of the parallelism of lines. Applying (SEE) implies that (Da) and (Db) *differ* in sense: it is perfectly possible to recognize that (Da) is true without immediately recognizing that (Db) is true – where, for example, someone simply lacks the concept of direction. However, since this is just what we are attempting to specify through contextual definition, to appeal to this possibility in repudiating the view that they possess the same sense would appear to rule out such definitions from the start. Frege's own objection to contextual definitions was only that they were *insufficient* for fully determining the relevant abstract *objects*, not that they cannot legitimately yield new *concepts.*[12] We certainly want (Da) and (Db) to possess the same sense ('content', to use Frege's original term) under some appropriate conception

of sense; and it is just such a conception of sense that (SCE) encapsulates. For if we *do* understand (Db), then we *will* possess the concept of direction and we ought then to immediately recognize that (Db) is true if we recognize that (Da) is true. If someone cannot do this, then we will say that they cannot *really* have understood (Db) after all. So the extra condition that (SCE) embodies does seem to yield the desired result that members of Fregean pairs possess the same sense.

To see that such a criterion still leaves *differences* of sense where Frege typically locates them, let us consider Frege's paradigm example:

(MSB) The Morning Star is a body illuminated by the Sun.

(ESB) The Evening Star is a body illuminated by the Sun. (Cf. *SR*, p. 62.)

It is clearly possible to *understand* both propositions without immediately inferring that (ESB) is true in recognizing that (MSB) is true. I may know what is meant by 'the Evening Star' – it refers to the bright heavenly body that I see in the evening, but I may not realize that the Evening Star *is* the Morning Star. In holding (MSB) as true, for example, I may know that the Morning Star is actually one of the planets in our solar system (and hence is a body illuminated by the Sun), but in not thereby recognizing that (ESB) is true, I may well lack the belief that the Evening Star is a planet. By (SCE), therefore, (MSB) and (ESB) have different senses.

The use of (SCE) also yields the results that Frege wants in at least the vast majority of arithmetical cases. Consider the following two propositions:

(28PN) 28 is an even perfect number [where a perfect number is a number whose proper divisors add up to the number itself].

(28EN) 28 is a Euclid number [where a Euclid number is a number that can be represented in the form $2^{k-1}(2^k - 1)$, where $2^k - 1$ is prime].

It has been proved that all and only even perfect numbers are Euclid numbers (and it is conjectured that all perfect numbers are even, in which case all and only perfect numbers are Euclid numbers), so that by (SLE), (28PN) and (28EN) possess the same sense, which is certainly not what Frege wished. (SEE), on the other hand, yields the result that they possess different senses, since someone can clearly recognize that (28PN) is true without recognizing that (28EN) is true. But (SCE) also yields the desired result, since even if someone does grasp both the concept of an even perfect number and the concept of a Euclid number, they need not immediately recognize (28EN) as true if they recognize (28PN) as true: unless they know the proof, they will typically need to *work out* that (28EN) is true. Only in the case of very elementary propositions such as '2 + 2 = 4' and '2 + 3 = 5' might it be possible to use (SCE) to argue that both propositions possess the same sense; and here we can simply introduce the qualification that Frege made in formulating (SEE) – ruling out its application to such self-evident propositions.[13]

(SCE) does, then, provide a criterion for sameness of sense that best captures Frege's intentions – having, in particular, the desired result that members of Fregean pairs can have the same sense. However, as formulated, (SCE) is open to an obvious objection. For what exactly is involved in 'understanding' the relevant propositions, which is the precondition of applying the test of immediate recognition? If we return to Frege's own specification of the criterion given by (SEE), we might suggest that to 'understand' a proposition is to grasp its 'content'; but this threatens to undermine any criterion incorporating a requirement of 'understanding'. For if the 'content' of a proposition is precisely the 'thought' expressed (and we are indeed trying to capture that notion of 'content' that Frege used in the *Grundlagen* in discussing contextual definitions), then, if two propositions do express the same thought, to grasp the 'content' of one is *ipso facto* to grasp the 'content' of the other, and if it is the 'content' that we recognize as true or false, then we automatically recognize the 'content' of one as true if we recognize the 'content' of the other as true.[14] Not only does this make the criterion useless, but it also violates the constraints on an adequate criterion: it is unacceptable to presuppose on the right hand side of the biconditional precisely that notion that we are attempting to specify. However, it is absurd to suggest that in understanding one proposition – by grasping its 'content' – I thereby 'understand' (in having grasped that 'content') any other proposition that has the same 'content' (I can grasp a thought in English, for example, without understanding its expression in any other language). Clearly, in making (SCE) a workable criterion, there must be a way of 'understanding' a proposition that does not simply consist in grasping its 'content'.

The obvious suggestion, which we have, in fact, been assuming in discussing (SCE), is that we 'understand' a proposition if we understand the concepts involved in its articulation. What is needed here is something along the lines of Frege's earlier distinction between 'content' and 'way of splitting up content', where it is the latter notion that exploits the idea of conceptual articulation. The difficulty for Frege, as we have seen, is that the distinction between 'Bedeutung' and 'Sinn' which replaced this distinction did not inherit the virtues of the earlier one. In restricting 'Bedeutung' purely to truth-value, the notion of 'Sinn' was given too much work to do: it had to capture both our 'understanding' of a proposition as well as the 'content' of that understanding; so that it is not surprising that a definitive criterion could not be laid down.

The problem was only exacerbated by his later adoption of principles of compositionality, which might at first sight appear to fill out the necessary notion of conceptual articulation. According to Frege, the thought expressed by a proposition is determined by the senses of its parts; and it seems natural to then suggest that to understand a proposition is to appreciate the way in which the senses of its parts do determine the sense of the whole. But such a suggestion only generates two further difficulties.

Firstly, it still leaves an ambiguity concerning the individuation of thoughts. Does every 'determination' yield a different thought, or is it possible for the same thought to be 'determined' in different ways? If the former, then thoughts are individuated as finely as 'conceptual articulations', but this seems incompatible with Frege's insistence that the same thought can be expressed in different linguistic forms. If the latter, then how exactly is this to be explained? On the face of it, this might seem no more problematic than Frege's earlier conception of the 'content' of a proposition being 'split up' in different ways. But that earlier conception had been embedded in a contextualism that had granted sentences logical priority over their parts, whereas his later compositionalism reverses the priority. I shall take up the issue of the tension between his earlier contextualism and his compositionalism in the next section.

Secondly, our discussion of indexicality and Frege's later conception of thoughts in §§7.4 and 7.5 suggested that there is a *gap* between our understanding of sentences and our grasp of the thoughts expressed by sentences. The thought I express by using a sentence involving an indexical seems to be determined not just by what I understand by the words I use but also by the *context.* So understanding a sentence seems to be neither a necessary condition for grasping the thought expressed, since that thought can be expressed in a different linguistic form, nor a sufficient condition, since an appreciation of the context also plays a role. I shall return to this in §8.3.

8.2 Contextualism and Compositionality

As we saw in §§2.2 and 5.2, in distinguishing his 'Begriffsschrift' from the systems of previous logicians, Frege emphasized that he proceeds not from concepts but from judgements: 'instead of putting a judgement together out of an individual as subject and an already previously formed concept as predicate, we do the opposite and arrive at a concept by splitting up the content of possible judgement' (*BLC*, p. 17). It was Frege's use of function-argument analysis that was crucial here. The removal of a proper name from a sentence that represents a 'content' (that can be used to make a judgement) yields an incomplete expression that represents a concept; and with a complex sentence there will be more than one way of doing this. Since different concepts thus result, definitions and analyses can be fruitful in revealing new concepts from a given content.

This process was accorded fundamental significance in the central argument of the *Grundlagen,* which formed the basis of Frege's logicist programme (see §§4.2 and 5.3 above). Through contextual definition the concepts of *direction* and *number* can be yielded from contents initially specified in terms of equivalence relations – parallelism and equinumerosity – holding between lines and concepts, respectively. It is therefore clear that judgements or 'conceptual contents' were seen as having logical priority over concepts; and we might capture this in the following principle:

(PDC) The concepts that can be yielded from the content of a judgement are determined by (function-argument) analysis of that judgement, each concept corresponding to a distinct analysis.

Such a principle, however, seems in direct conflict with the two principles that were formulated in §6.2 as underlying Frege's arguments concerning the *Sinn* and *Bedeutung* of expressions:

(PDB) The *Bedeutung* of a complex expression is determined by the *Bedeutung* of its parts.

(PDS) The sense of a complex expression is determined by the sense of its parts.

(PDC) seems to accord *sentences* logical priority, whilst (PDB) and (PDS) seem to accord the elementary *parts* of a sentence logical priority.

The apparent conflict here is only sharpened if we bring in Frege's context principle. The context principle is first used in the *Grundlagen* in combatting psychologism about numbers. That we can form no *idea* of the content of a number word, Frege argues, is no reason for denying all meaning to the word. As long as propositions involving number terms have a sense as a whole, then their parts also must have a content. 'Only in a proposition do the words really have a meaning [*Bedeutung*]' (*GL*, §60; see §4.1 above.) But the principle is formulated again, almost immediately, in §62, where the constructive part of Frege's project gets under way. Its role is clear. We can define the meaning of number terms by defining the sense of sentences in which such terms appear, in particular, identity statements of the form of (Nb). (Nb) is defined contextually by means of (Na), which in turn is definable purely logically. (See §§4.2 and 4.3 above.) The context principle can thus be seen as a generalization of (PDC), according sentences logical priority not only over concept expressions but also over names; and we might formulate it as follows:

(CP) The contents of parts of a sentence are determined by the content of the whole.[15]

Once again, what we have here seems in direct conflict with (PDB) and (PDS).

Now the obvious response is simply to suggest that Frege changed his mind – that his earlier contextualism gave way to compositionalism. Two points might be made in support of this. Firstly, Frege himself came to reject contextual definitions in the *Grundlagen,* and opted instead for explicit definitions. Secondly, despite its emphatic endorsement as one of the three fundamental principles of the *Grundlagen* (p. X), the context principle was never again even mentioned.[16] However, with regard to the first point, we have already replied that contextual definitions were only

seen as *insufficient*, not as incorrect, and that in any case, a contextual definition of *value-ranges* was in fact embodied in Axiom V of the *Grundgesetze* (see §5.3 above). With regard to the second point, we can reply that the context principle was not explicitly repudiated either, and that there was a perfectly good reason why it could not have been formulated again, namely, that it depended on just that undifferentiated notion of 'content' that was later replaced by the notions of 'Sinn' and 'Bedeutung'.

Of course, it would have been simple to have formulated two corresponding versions of (CP), one in relation to *Bedeutung* and the other in relation to sense:

(CPB) The *Bedeutungen* of parts of a sentence are determined by the *Bedeutung* of the whole.

(CPS) The senses of parts of a sentence are determined by the sense of the whole.

However, on Frege's conception of *Bedeutung*, (CPB) would be implausible: the referent of a name hardly seems dependent on the truth-value of the sentence in which the name appears. Here it does seem that it is the *Bedeutung* of the whole that is determined by the *Bedeutungen* of its parts; and as we saw in §6.2, Frege uses precisely (PDB) in motivating his conception of the *Bedeutung* of a sentence.[17] (CPS), on the other hand, might be felt to remain valid, at least to the extent that Frege continues to allow alternative function-argument analyses in yielding different concepts.

That Frege did insist on the logical priority of judgements over concepts throughout his life is made clear in notes that Frege wrote in 1919 for the historian of science Ludwig Darmstaedter:

> I do not begin with concepts and put them together to form a thought or judgement; I come by the parts of a thought by analysing the thought. This marks off my concept-script from the similar inventions of Leibniz and his successors, despite what the name suggests; perhaps it was not a very happy choice on my part. (*NLD*, p. 253.)

This is a firm endorsement of his earlier position. Indeed, in suggesting that the name of his logical notation, 'Begriffsschrift', was misleading, the priority thesis is only underlined.[18] Yet the semantic theory of the *Grundgesetze* seems to be based on the idea that the *Sinn* and *Bedeutung* of complex expressions are determined by the *Sinn* and *Bedeutung* of their parts (see esp. *GG*, I, §§28-32); and his last paper 'Compound Thoughts', published in 1923, opens with a ringing statement of the compositionalist view:

> It is astonishing what language can do. With a few syllables it can express an incalculable number of thoughts, so that even if a thought has been grasped by an inhabitant of the Earth for the very first time, a form of words can be found in which it will be understood by someone else to whom it is entirely new. This would not be possible, if we could not distinguish parts in the thought corresponding to the parts of a sentence, so that the structure of the sentence can serve as a picture of the structure of the thought...
>
> If, then, we look upon thoughts as composed of simple parts, and take these, in turn, to correspond to the simple parts of sentences, we can understand how a few parts of sentences can go to make up a great multitude of sentences, to which, in turn, there correspond a great multitude of thoughts. (*CT*, p. 390.)[19]

This is a frequently quoted passage. An adequate explanation of what is now called *linguistic creativity* – the ability to use and understand new linguistic constructions – requires just that compositionalism that Frege is here articulating; and indeed, it was Frege's attempt to develop such a compositional theory in the *Grundgesetze* that has entitled him to be regarded as the founder of modern semantic theory.[20] But the question remains: how is this compatible with his earlier contextualism?

At least with respect to concepts, an answer can be readily given; and indeed, was provided by Frege himself in one of his earliest works. Immediately after the remark quoted at the beginning of this section, concerning the formation of concepts by 'splitting up' contents, Frege goes on:

> Of course, if the expression of the content of possible judgement is to be analysable in this way, it must already be itself articulated. We may infer from this that at least the properties and relations which are not further analysable must have their own simple designations. But it doesn't follow from this that the ideas of these properties and relations are formed apart from objects: on the contrary they arise simultaneously with the first judgement in which they are ascribed to things. Hence in the concept-script their designations never occur on their own, but always in combinations which express contents of possible judgement. I could compare this with the behaviour of the atom: we suppose an atom never to be found on its own, but only combined with others, moving out of one combination only in order to enter immediately into another. (*BLC*, p. 17.)

The possibility of alternative function-argument analyses *presupposes* that the judgement is already articulated, and this requires that the judgement has elementary parts. What Frege nevertheless maintains is that even these elementary parts are incapable of existing independently.[21]

The idea can be illustrated by taking Frege's *Begriffsschrift* example once again:

(HLC) Hydrogen is lighter than carbon dioxide.

This can be analysed either as saying of hydrogen that it has the property of being lighter than carbon dioxide or as saying of carbon dioxide that it has the property of being heavier than hydrogen. Of course, if we wished

to express these by respecting subject/predicate position (regarded by Frege as of no *logical* significance), then the latter might more accurately be stated as follows:

(CHH) Carbon dioxide is heavier than hydrogen.

But, as we have seen, there is a clear sense in which (HLC) and (CHH) possess the same 'content', since it does seem that it is the same relation between the same two objects that is involved. But this suggests that there is a fuller analysis that would bring this out – that would elucidate the essential structure of the 'content'. Whilst (HLC) and (CHH) can be represented in the form '*Fa*', then, the 'real' logical form that makes the alternative function-argument analyses possible must be represented as '*Rab*' (or '*aRb*') – saying of object *a* that it stands in relation *R* to object *b*. But since the relation involved here cannot be conceived except as relating objects in the relevant way, even the compositionalism that the idea of a complete analysis legitimizes is not incompatible with continued adherence to the principle (PDC). Indeed, the logical priority of judgements over concepts is simply a presupposition of Frege's doctrine of the unsaturatedness of functions: expressions for concepts or relations are formed by removing names from sentences; and this doctrine Frege continued to hold to the end of his life.[22]

Michael Dummett has suggested that what is required in reconciling Frege's views here is a distinction between *decomposition* and *analysis*, taken with a corresponding distinction between *component* and *constituent* concepts.[23] Decomposition is what is involved in yielding *component* concepts – extracting the concept *lighter than carbon dioxide* from the 'content' of (HLC), for example. It is with the process of decomposition that the contextualist principle (PDC) is concerned; and judgements do indeed have logical priority over *component* concepts. Analysis, on the other hand, is what is involved in uncovering the 'real' logical form of a judgement, revealing its *constituent* concepts or relations. To grasp a thought we must understand those elementary parts that *constitute* it, and it is with this process, which analysis elucidates, that Frege's principles of compositionality are concerned. To understand (HLC), for example, I must grasp the relation *lighter than* and know which objects hydrogen and carbon dioxide are. Having done so, I can then form the concept *lighter than carbon dioxide*, but such a decomposition presupposes that I have already grasped the thought in the way revealed by the correct analysis. So contextualism and compositionalism, at least with regard to concepts, are not in conflict: properly understood, contextualism presupposes compositionalism; and it was just this compositionalism that came to the fore in Frege's philosophy as he thought things through more fully.

However, whilst the distinction between decomposition and analysis is undoubtedly useful, the issue is more complex than the account just given might imply. Dummett maintains that whilst there can be alternative

function-argument *decompositions* of a given thought, there is only one correct *analysis* of it. But if we look more carefully at Frege's *Begriffsschrift* example, we can see that this cannot be accepted without qualification. (HLC) and (CHH), we suggested, are both concerned with the same relation between the same two objects – it is this that underpins their sameness of 'content'. But are there not two *different* relations involved here? For if we represent the relation *lighter than* by '*R*', then the relation *heavier than* is its *inverse*, *R'*; so that if *a* is hydrogen and *b* is carbon dioxide, then the judgement can be represented either as '*Rab*' ('*aRb*') or as '*R'ba*' ('*bR'a*'). So even at the deepest level, there seems to be room for alternative analyses.

An obvious response is to make use of Frege's distinction between sense and reference and to say that whilst at the level of *reference*, it is the same relation that is involved, there are two different *modes of presentation* of that relation: we can conceive of it either as the referent of the expression 'ξ is lighter than ζ' or as the referent of the expression 'ζ is heavier than ξ'.[24] However, in admitting that these two expressions have different senses, we would be admitting – if (PDS) were still to be endorsed – that (HLC) and (CHH) themselves have different senses, i.e. express different thoughts; and this would certainly not have been Frege's view. Now it would be possible to deny that we do have two different senses here: once the order of the place-holders for the relata is appreciated, the two expressions can be seen to have the same sense. But at least without the place-holders, what we understand by the expression 'is lighter than' is different from what we understand by the expression 'is heavier than', and if place-holders *are* to enter into the expression for a relation, then we would now be working *back* from the sense of the sentence as a whole to the sense of the part that designates the relation. Indeed, this was just what was involved in Frege's conception of the *unsaturatedness* of functions. Either way, then, (PDS) cannot be endorsed as it stands. If the two relational expressions in (HLC) and (CHH) have different senses, then there can be alternative analyses of one and the same thought; and if the two expressions have the same sense, then this can only be apprehended by working back from the sense of the whole.[25]

However, we can agree with Dummett that there is only one correct analysis of the thought expressed by (HLC) *as expressed by* (HLC). For I do need to understand the expression 'is lighter than' to understand (HLC) itself. But it is not necessary to understand this expression to grasp the thought itself, since if that thought can also be expressed by (CHH), then it can also be grasped by understanding the expression 'is heavier than' – together, of course, with an understanding of the names 'hydrogen' and 'carbon dioxide' and an appreciation of their order in the sentence.

The distinction just drawn here between the thought expressed by a sentence and the thought as expressed by the sentence may seem unnecessarily subtle, but it is essential to respect it if we are to preserve Frege's

fundamental conception that the same thought can be expressed in different linguistic forms. Indeed, it really amounts to no more than Frege's original distinction between 'content' and 'way of splitting up content'. Dummett himself was led to underestimate the importance of retaining such a distinction by the emphasis he places on the following thesis:

(TIS) A thought is isomorphic with the sentence whose sense it is.[26]

Now it is true that this thesis is a presupposition of the compositional principle (PDS), and it is also explicitly endorsed in the statement of the compositionalist view that we quoted above. But what was omitted (at the place indicated) from the passage quoted is the following admission:

> To be sure, we really talk figuratively when we transfer the relation of whole and part to thoughts; yet the analogy is so ready to hand and so generally appropriate that we are hardly even bothered by the hitches which occur from time to time. (*CT*, p. 390.)

Although Frege suggests that 'hitches' only occur occasionally, he is clearly aware that (TIS) is only *generally* applicable; and a similar qualification is implicit in his notes for Ludwig Darmstaedter: 'We can regard a sentence as a mapping of a thought: corresponding to the whole-part relation of a thought and its parts we have, *by and large*, the same relation for the sentence and its parts' (*NLD*, p. 255; my emphasis). So we cannot treat Frege as having ruled out entirely the possibility of different sentences of quite different linguistic forms expressing the same thought.

As far as the example from the *Begriffsschrift* is concerned, however, (TIS) is not in any case incompatible with the possibility of alternative analyses of the relevant thought, for (HLC) and (CHH) are presumably isomorphic with each other, and hence if one is isomorphic with the thought it expresses, then so too is the other. Whether or not we say that the two relational expressions possess the same sense, in other words, the two sentences have the structural similarity that justifies regarding them as isomorphic: they both involve a relational expression and two proper names. So whilst (PDS) may imply (TIS), (TIS) does not on its own imply (PDS).

However, what does threaten (TIS) are examples of *Fregean pairs* – not only *horizontal* ones such as (Da) and (Db) but also *vertical* ones such as (Da) and (Dc):

(Da) Line *a* is parallel to line *b*.
(Db) The direction of line *a* is identical with the direction of line *b*.
(Dc) The concept 'line parallel to line a' is coextensive with the concept 'line parallel to line *b*'.

Members of Fregean pairs are not isomorphic with one another, since, as we remarked in the last section, they are concerned with different relations between objects of different kinds. But before we conclude that Fregean pairs cannot, after all, embody sameness of sense, let us note that such a conclusion would wreck Frege's logicist programme of analysis. For if we recall our discussions in §§4.1 and 5.3, we can see that the relationships between members of Fregean pairs are essentially no different from the relationships between propositions involving the adjectival use of number terms, propositions involving the substantival use of number terms, and propositions that more explicitly attribute properties to concepts. Compare (Da), (Db) and (Dc), for example, with the following:

(Oa) There are 0 *F's*.
(Ob) The number of *F's* is 0.
(Oc) The concept *F* is not instantiated.

As we saw, Frege clearly held that a 'content' can be 'split up' in such a way that the resulting propositions have different subjects (the subject/predicate distinction being regarded as of no logical significance); and even with the bifurcation of 'content' into sense and reference, the basic idea – at the level of sense – was still endorsed. In 'On Concept and Object', Frege is explicit that the following two sentences express the same thought:

($\sqrt{4}$ a) There is at least one square root of 4.
($\sqrt{4}$ c) The concept *square root of 4* is realized.

Prescinding from Frege's worries about referring to concepts by use of phrases of the form 'the concept *F*',[27] we can take his point to be that, as he puts it, 'a thought can be split up in many ways, so that now one thing, now another, appears as subject or predicate' *(CO*, p. 49). He writes:

> The thought itself does not yet determine what is to be regarded as the subject. If we say 'the subject of this judgment', we do not designate anything definite unless at the same time we indicate a definite kind of analysis; as a rule, we do this in connexion with a definite wording. But we must never forget that different sentences may express the same thought. For example, the thought we are considering could also be taken as saying something about the number 4:
>
> [($\sqrt{4}$ b)] 'The number 4 has the property that there is something of which it is the square.'
>
> Language has means of presenting now one, now another, part of the thought as the subject; one of the most familiar is the distinction of active and passive forms. It is thus not impossible that one way of analysing a given thought should make it appear as a singular judgment; another, as a particular judgment; and a third, as a universal judgment. It need not then surprise us that the same sentence may be conceived as saying something about a concept and also as saying something about an object; only we must observe that what is being said is different. (*CO*, p. 49.)

All three propositions here, ($\sqrt{4}$ a), ($\sqrt{4}$ b) and ($\sqrt{4}$ c), are not isomorphic with

one another, yet Frege is insistent that they all express the same thought. So Frege cannot be taken to have held (TIS) in its unqualified form.

However, with regard to vertical Fregean pairs, such as (Da) and (Dc) (Oa) and (Oc), or ($\sqrt{4}$ a) and ($\sqrt{4}$ c), what we arguably have here are cases of *degenerate analyses*, or more accurately, *degenerate decompositions*.[28] There is an obvious sense in which the first of the two propositions is the more fundamental, and the second is dependent upon it. In these cases we have what was called in §5.3 *conceptual ascent*, involving a shift from talking about things of a certain kind to talking about the concepts used in so talking; and there is in principle no limit to how high such conceptual ascent can go. From (Dc), for example, we can in turn generate the following proposition:

(Df) The concept 'concept coextensive with the concept "line parallel to line *a*"' is coextensive with the concept 'concept coextensive with the concept "line parallel to line *b*"'.

A whole hierarchy of such propositions can be constructed, each spun out of the previous one, and each expressing, in an ever more complicated form, the same thought.[29]

The possibility of such conceptual ascent clearly suggests that only the first member of the series generated should be taken as fundamental, and this is obviously compatible with there being just one correct *analysis* of a thought. Only of the first member would it be reasonable to demand that it reflect the essential structure of the thought; all other sentences represent *decompositions* of the thought, becoming increasingly degenerate the further up the hierarchy they go. A simple modification of (TIS) thus readily suggests itself:

(TIS') A thought is isomorphic with the sentence that expresses that thought in its most fundamental form.

But this now reduces the thesis of isomorphism to a tautology, for if a sentence is not isomorphic with the thought it expresses, then it would not be regarded as expressing it in its most fundamental form; and nothing has been said here about what does count as a thought's most fundamental form.

Whilst it is easy to determine which of the two members of a vertical Fregean pair is the more fundamental, there may be no obvious answer as to which member of a horizontal Fregean pair is fundamental. In the case of (Da) and (Db), Frege thought that (Da) was fundamental, since 'everything geometrical must surely originate in intuition', and lines are intuited before directions (cf. *GL*, §64). But in his central numerical case, the issue is more complex:

(Na) The concept *F* is equinumerous to the concept *G*.
(Nb) The number of *F's* is identical with the number of *G's*.

Frege's use of the context principle suggests that here too the first of the two propositions is regarded as more fundamental: (Na), which can itself be characterized purely logically, is to be used to define the 'content' of the number terms in (Nb). But of course Frege comes to reject such contextual definitions, and the reason might be put by saying that the number terms are already taken as referring to objects in the domain, i.e. they are already assumed to have an independent 'meaning'. As we saw in §4.1, this assumption first surfaces in §57 of the *Grundlagen*, where Frege shows that he treats *substantival* constructions such as (Ob) as more fundamental than *adjectival* constructions such as (Oa). And it is this underlying assumption that drives the Caesar problem, Frege's response to which we outlined in §4.2 and discussed in more detail in §§5.3 and 6.4.

Frege's use of the context principle in the *Grundlagen* is thus disingenuous. Although he writes in §60 that 'Only in a proposition do the words really have a meaning', using the word 'Bedeutung' for what is here translated as 'meaning', and talks of the proposition as a whole determining the 'content' ('Inhalt') of its parts, he does not really mean that the *referents* of the number terms, i.e. the numbers themselves, are dependent on the 'content' of the proposition, for he has already taken himself to have shown that numbers are independent objects (*GL*, §57; see §4.1 above). His use of the context principle is *epistemological* rather than ontological. Numbers are not abstract objects parasitic upon something else – our use of sortal concepts in classifying the ordinary objects of the world – but objects that are already there in the universal domain in their own right. But since we cannot apprehend such objects by either our senses (which would make our knowledge of arithmetic synthetic *a posteriori)* or our intuition (which would yield synthetic *a priori* knowledge), we must be taken to apprehend them by pure reason, by understanding the senses of propositions – definable purely logically – in which names for the objects appear.

The obvious strategy for dealing with horizontal Fregean pairs – treating (Nb) as contextually dependent on (Na), for example – was not therefore one that Frege himself felt able to adopt, at least in his central case. So should (Na) be treated instead as dependent on (Nb)? This would preserve both Frege's realism about numbers and also the idea that there is only one ultimate analysis of a thought – once its fundamental form is determined. This would also enable us to cut down on our ontological commitments: referents need only be required for the constituents of a sentence that expresses a thought in its most fundamental form. Given that there is in any case a problem with (Na), on Frege's view, since phrases of the form 'the concept *F*' do not in fact succeed in referring to what they are intended to refer, this is clearly attractive. It is also consistent with the account just given of vertical Fregean pairs, since in conceptual ascent too we make use of such misfiring phrases. But as we saw in §7.1, where the eliminative strategy was suggested as the obvious

way of dealing with the paradox of the concept *horse*, it does not appear to have even crossed Frege's mind. This is significant, for it shows just how little ontological work – in the sense of reducing our ontological commitments – the context principle was regarded as doing. Even when contextual definitions cry out to be used as a way of eliminating the need to treat concepts as any kind of object – and the absoluteness of the distinction between concept and object was as deeply held as anything in Frege's philosophy – Frege fails to make the obvious move. Even phrases of the form 'the concept *F*' are seen as referring; they are just taken to refer not to concepts but to 'concept-correlates' – special objects that go proxy for concepts. It is worth noting that the passage quoted above concerning the different possible analyses of a thought comes from the very paper – 'On Concept and Object' – where Frege confronts most directly the problems raised by the paradox of the concept *horse*.

What could possibly have prevented Frege from making the obvious move? It is true that eliminativism as a strategy in philosophical logic was only first pursued by Russell in his theory of descriptions,[30] but what prevented Frege from pursuing this himself was his *compositionalism*, *implicit* in his thought even at the time when the context principle appeared on the surface to be dominant. For according to Frege, if a proposition is true, then all the names within it must refer. As we have seen, Frege regarded logic as a system of *truths*, and in his very early work had asserted that 'The rules of logic always presuppose that the words we use are not empty' (*DPE*, p. 60; quoted in §2.4 above). Since (Nb) is true if (Na) is true (they are, at least, *logically* equivalent), then regardless of the epistemological question as to which is prior in the order of explanation, the number terms contained in (Nb) must refer to pre-existing objects, as a condition of the truth of the proposition. Even in his early thought, then, some form of compositionalism was operative.

What we are therefore left with is the view that (Na) and (Nb) are *equally* fundamental, in the sense that they both involve reference to things that are viewed as ontologically independent ('*selbstandig*'); and this means that there can after all be alternative *analyses* (and not just decompositions) of one and the same thought. For the thesis that there is just one correct analysis of a thought can only be maintained if there is some fundamental form that each thought takes. But definitional strategy aside, Frege saw no difference in ontological status between the members of a Fregean pair.[31] So even in its modified form, (TIS'), the thesis of isomorphism cannot be attributed to Frege as his considered view. But since, as we have already remarked, (PDS) implies (TIS), does this then not mean that (PDS) too cannot be attributed to Frege, and does this not blatantly contradict the claim that compositionalism was fundamental to Frege's philosophy?

However, just before we introduced (TIS), we drew a distinction that we can now use to suggest a modification of (PDS):

(PDS') The sense *as* expressed by a sentence is determined by the sense of its parts.

This arguably preserves the essence of Frege's compositionalism, whilst leaving room for alternative analyses of thoughts, the possibility of which is intimately connected with Frege's central thesis that the same thought can be expressed in different linguistic forms.[32]

What, then, can we conclude about the relationship between Frege's contextualism and his compositionalism? At the beginning of this section we suggested that the context principle (CP) could be seen as a generalization of (PDC), the principle concerning the logical priority of judgements over concepts. Frege's doctrine of the unsaturatedness of concepts, and indeed the fundamental role that function-argument analysis plays in his logic, implies that this latter principle was central throughout his work.[33] But as we have seen, it had never been Frege's belief that the *referents* of proper names, i.e. the objects themselves, depended on anything else. The use of the context principle was *epistemological* rather than ontological: it was introduced – though no doubt inspired by (PDC) – to explain how anlytic *a priori* knowledge of numbers is possible.

It is not surprising, then, that once the distinction between sense and reference had been drawn, which was a response to the problems that §§62-9 of the *Grundlagen* raised, the context principle could no longer be endorsed. For at the ontological level – the level *of Bedeutung* – there is certainly no dependence of the referents of terms on the truth-values of sentences.[34] Rather, the truth-value of a proposition is dependent on the reference of its parts. But at the epistemological level – the level of *Sinn* – the issue is more complex. Thoughts may be logically prior to concepts, in so far as they permit alternative analyses, but for any given expression of a thought, there is indeed only one correct analysis that explains how that particular expression is understood. It was the subtle complexity of Frege's position here, requiring further distinctions to be drawn of which Frege himself was not fully aware, that resulted in the context principle being neither explicitly endorsed (because contextualism at the level of *Bedeutung* was rejected) nor explicitly repudiated (because contextualism at the level of *Sinn* was in one respect retained).

8.3 The Varieties of Sense

A distinction was drawn in the last section between the thought expressed by a sentence and the thought *as* expressed by a sentence. The idea here is simple: what grasping the thought *as* expressed by a sentence involves is *understanding* the sentence; and it is entirely plausible to suppose that there is a unique analysis of what constitutes our *understanding* of a given sentence, which we might capture in the following principle:

(PDU) The understanding of a complex expression is determined by the understanding of its parts.

But the question that now needs to be addressed is whether this principle is the same as (PDS'), the modification offered of the compositional principle concerning sense, and whether (PDS') is in fact trite. Our discussion of indexicality in §§7.4 and 7.5 suggests otherwise. For there appears to be a *gap* between our understanding of sentences and our grasp of the thoughts even *as* expressed by sentences, since *context* too is a determinant of sense.

I understand the *linguistic role* of an indexical if I understand the *rule* that takes me from the context of utterance to the object denoted. But this is not enough to grasp the *sense* of the indexical, as used on a particular occasion, since its linguistic role is the same from context to context, whilst its sense and reference varies. What is needed is some way of 'cashing out' the indexical which 'presents' the referent in an appropriate way. But on the assumption that the sense of the indexical, as used on a particular occasion, is to be identified with the sense of some definite description, we arrive at an inconsistency in the Fregean position. For as we argued, I can hold that a sentence involving the indexical is true without holding that the corresponding sentence involving the definite description is true, and vice versa – even where I 'understand' all the relevant words. What is operative here is just that criterion of identity for thoughts that we discussed in §8.1, together with the compositional principle concerning sense. Since the two sentences express different thoughts, the sense of the indexical must be different from the sense of the corresponding definite description.

Rather than concluding that the sense of an indexical, as used on a particular occasion, is primitive and irreducible, however, we suggested that we grasp that sense simply if we know *which object* is referred to, something that can be *shown* in any number of different ways, and that typically involves the ability to *track* the relevant object. The suggestion here drew on the analogy with simple names, the sense of which, we argued in §6.4, should also not be identified with the sense of any corresponding definite description. But as we remarked, this represents something of a departure from Frege's own ideas; and we can appreciate this if we return to the example that Frege discusses in 'Der Gedanke', which we considered in §7.5 in relation to the question of *I-thoughts.* This is the passage that we only summarized at the time:

> Consider the following case. Dr. Gustav Lauben says, 'I was wounded', Leo Peter hears this and remarks some days later, 'Dr. Gustav Lauben was wounded'. Does this sentence express the same thought as the one Dr. Lauben uttered himself? Suppose that Rudolph Lingens was present when Dr. Lauben spoke and now hears what is related by Leo Peter. If the same thought was uttered by Dr. Lauben and Leo Peter, then Rudolph Lingens, who is fully master of the language and remembers what Dr. Lauben said in his presence, must now know at once from Leo Peter's report that he is speaking of the same thing. But knowledge of the language is a special thing when proper names are involved. It may well be the case that only a few people associate a definite thought with the sentence 'Dr. Lauben was wounded'. For complete understanding one needs in this case to know the expression 'Dr. Gustav Lauben'. Now if both Leo Peter and Rudolph Lingens mean by 'Dr. Gustav Lauben', the doctor who is the only doctor living in a house known to both of them, then they both understand the sentence 'Dr. Gustav Lauben was wounded' in the same way; they associate the same thought with it. But it is also possible that Rudolph Lingens does not know Dr. Lauben personally and does not know that it was Dr. Lauben who recently said 'I was wounded'. In this case Rudolph Lingens cannot know that the same affair is in question. I say, therefore, in this case: the thought which Leo Peter expresses is not the same as that which Dr. Lauben uttered. (*T*, pp. 358-9.)

The main point here is clear: even when Leo Peter and Rudolph Lingens both associate the same mode of presentation with the proper name 'Dr. Gustav Lauben', Dr. Lauben saying 'I was wounded' and Leo Peter later saying 'Dr. Gustav Lauben was wounded' do not express the same thought since Rudolph Lingens can hear and understand both utterances but not realize that it is the same incident involved. The key assumption of Frege's argument here is that a 'definite thought' is only expressed by a sentence if a unique mode of presentation is associated with every constituent singular term (indexical or proper name). Presumably the thought that Rudolph Lingens grasps when he hears Dr. Lauben saying 'I was wounded' would be expressed by Rudolph Lingens himself saying 'The man now speaking was wounded',[35] and the thought that he grasps when he hears Leo Peter saying 'Dr. Gustav Lauben was wounded' would be expressed by The only doctor living in house *H* [the house known to both Leo Peter and Rudolph Lingens] was wounded'. Clearly, these two thoughts *are* different. But are Leo Peter's corresponding thoughts different? If when he hears Dr. Lauben saying 'I was wounded', Leo Peter already knows that the man speaking *is* Dr. Gustav Lauben, then surely the thought that he hlimself grasps is better expressed by 'Dr. Gustav Lauben was wounded', in which case Leo Peter's two thoughts are the same. But for Frege, this would still raise the question as to what is actually understood by 'Dr. Gustav Lauben'; and it would indeed seem implausible to suggest that Leo Peter has the thought expressed by 'The only doctor living in house *H* was wounded' when he hears Dr. Lauben saying 'I was wounded'.

Of course, if thoughts are determined by their constituent modes of presentation, then there are many other possibilities as to what thoughts are involved here. Perhaps in saying 'I was wounded', Dr. Lauben thinks of *himself* under just that mode of presentation that Leo Peter and Rudolph Lingens associate with the name 'Dr. Gustav Lauben', in which case we might again have the same thought expressed both times. (They

might all think of him as the man named ‘Dr. Gustav Lauben’, for example.) And why should Leo Peter not have a *compound thought* made up of *several* ‘definite thoughts’ when he hears Dr. Lauben saying ‘I was wounded’ – ‘The man now speaking, who is called ‘Dr. Lauben’ and who is the only doctor living in house *H*, was wounded’? All that would then be needed to secure communication would be one particular component ‘definite thought’ that is common to all thinkers. Nevertheless, it is quite clear that the assumption isolated above plays a critical role, and this is only confirmed in the two paragraphs that immediately follow:

> Suppose further that Herbert Garner knows that Dr. Gustav Lauben was horn on 13 September, 1875 in N.N. and this is not true of anyone else; suppose, however, that he does not know where Dr. Lauben now lives nor indeed anything else about him. On the other hand, suppose Leo Peter does not know that Dr. Lauben was born on 13 September 1875, in N.N. Then as far as the proper name ‘Dr. Gustav Lauben’ is concerned, Herbert Garner and Leo Peter do not speak the same language, although they do in fact refer to the same man with this name; for they do not know that they are doing so. Therefore Herbert Garner does not associate the same thought with the sentence ‘Dr. Gustav Lauben was wounded’ as Leo Peter wants to express with it. To avoid the awkwardness that Herbert Garner and Leo Peter are not speaking the same language, I shall suppose that Leo Peter uses the proper name ‘Dr. Lauben’ and Herbert Garner uses the proper name ‘Gustav Lauben’. Then it is possible that Herbert Garner takes the sense of the sentence ‘Dr. Lauben was wounded’ to be true but is misled by false information into taking the sense of the sentence ‘Gustav Lauben was wounded’ to be false. So given our assumptions these thoughts are different.
>
> Accordingly, with a proper name, it is a matter of the way that the object so designated is presented. This may happen in different ways, and to every such way there corresponds a special sense of a sentence containing the proper name. The different thoughts thus obtained from the same sentences correspond in truth-value, of course; that is to say, if one is true then all are true, and if one is false then all are false. Nevertheless the difference must be recognized. So we must really stipulate that for every proper name there shall be just one associated manner of presentation of the object so designated. It is often unimportant that this stipulation should be fulfilled, but not always. (*T*, p. 359.)

As we remarked in §6.4, in relation to Frege’s notorious ‘Aristotle’ footnote in ‘Über Sinn und Bedeutung’, Frege regarded the multiplicity of senses typically associated with a proper name as a defect of ordinary language. Only in a formal language such as that developed by Frege for arithmetic could the problem of variability be avoided. As we outlined in §4.3, we can start by defining the number 0, for example, as the extension of the concept ‘equinumerous to the concept *not identical with itself*’, and then proceed to define each subsequent number in terms of its predecessors. What we have in each case is a ‘constructive definition’, where the *definiendum* is stipulated to have both the same sense and the same reference as the *definiens* (cf. §5.5). Each numeral is indeed then associated with a unique mode of presentation – or what might be better called here a *mode of construction* – which ‘determines’ the relevant number. This clearly provides the model for the revision of ordinary language that Frege proposes in the above passage.

However, what the passage also suggests is that, strictly speaking, there may be nothing wrong with any *individual* person’s language, as long as they consistently associate one particular mode of presentation with each singular term; the problem lies rather in individuals not being able to *communicate* with one another properly, since they may well associate *different* senses with the same name. This seems to be the point of Frege’s remark that Herbert Garner and Leo Peter do not speak the same language in the case imagined: they may use the same words, but their understanding of those words is quite different. The defect of ordinary language that Frege is highlighting, then, is located at the *epistemological* rather than semantic level. It is thus unlike what is wrong, according to Frege, with our use of terms for *vague* concepts. As we saw in §7.3, sentences involving these terms were regarded as *lacking* a truth-value (a *Bedeutung*) and hence as not expressing a genuine thought at all. By his compositional principles, therefore, the terms themselves lack a reference and a sense. An ordinary proper name, as used by someone on a particular occasion, on the other hand, may well have both; it is just that someone else may not realize exactly what they are. The typical problem is not that such a name *lacks* a sense, which would be a semantic deficiency, as that there are *too many* senses with which it might be associated, which raises the epistemological question as to which sense is intended.

This only reinforces the point that we have emphasized throughout this book – that our understanding of the various elements involved in the use of an expression on a particular occasion requires an appreciation of the *context*. Indeed, if Frege is right here, then context-dependence is even more pervasive at the level of sense than it is at the level of reference. For whilst an awareness of the *immediate* context may be all that is needed to recognize the *referent* of a singular term as used by someone on a particular occasion, to grasp its *sense* I may well need to understand something of that person’s *cognitive history* – to ascertain the mode of presentation which they associate with the term.

Carried to their logical conclusion, Frege’s views seem absurd. For when I write ‘Gottlob Frege was the most important influence on Wittgenstein’, for example, the exact way in which I think of Frege (e.g. as the philosopher I am currently writing about) may well be irrelevant to the thought that I intend to convey. As long as you know *to whom* I am referring, and understand the property that is being ascribed, then you can be said to have grasped the thought I express. This can be compared with my saying ‘The author of the *Grundgesetze* was the most important

influence on Wittgenstein', where a grasp of one particular mode of presentation *is* necessary to grasp the thought I express (I might be implying, for example, that it was the *Grundgesetze* that most influenced Wittgenstein). What this points to is the need to draw just that distinction between simple names and definite descriptions for which we argued in §6.4. Only in the case of a definite description is it reasonable to identify its sense with one particular mode of presentation. In the case of simple singular terms (including simple indexicals as well as proper names), a different account is required.

Although the texts show that Frege was unaware of the need to draw this distinction, the suggestion that to grasp the sense of a simple singular term is to know *which object* is referred to is a *refinement* rather than a repudiation of Frege's views, since modes of presentation are still taken as playing a crucial role. Frege's central thesis that the referent of an expression can only be apprehended via some mode of presentation is retained. As we have seen, what is needed here is an elementary scope distinction. I can be said to grasp the sense of a simple singular term if I have *some* means of identifying its referent; it is not necessary that there be one particular mode of presentation with which everyone who grasps its sense associates the term.

The scope distinction revealed here is important, for a similar distinction can be utilized to elucidate Frege's argument in 'Der Gedanke'. The main argument in the first passage quoted above initially seems straightforward. Since Rudolph Lingens can understand Dr. Lauben saying 'I was wounded' and Leo Peter later saying 'Dr. Gustav Lauben was wounded', but not realize that it is the same incident involved, the two utterances do not express the same thought. According to Frege, I understand an expression by associating a mode of presentation with it; but this is ambiguous between the following two readings:

(UMP) Anyone who understands an expression associates some particular mode of presentation of its referent with it.

(MPU) There is some particular mode of presentation of the referent of an expression that is associated with that expression by anyone who understands it.

On the first reading, Rudolph Lingens can certainly be taken to understand Dr. Lauben saying 'I was wounded' – he apprehends Dr. Lauben under *some* mode of presentation (e.g. as the man who is speaking). Yet as we have just seen, Frege's conception of proper names suggests that it is the second reading that he has in mind. But if Dr. Lauben may associate quite a different mode of presentation with the indexical 'I', and if this is the mode of presentation that someone else must grasp in order to understand – on the second reading – what Dr. Lauben himself has said, then Rudolph Lingens does not in fact grasp the thought that Dr. Lauben expresses. So nothing can then be concluded about whether Dr. Lauben and Leo Peter do express the same thought. But if Rudolph Lingens *is* assumed to grasp the thought that Dr. Lauben expresses, then how would he himself express it? Obviously, if he said 'Dr. Gustav Lauben was wounded', then we are back with the original question; but presumably a similar question would arise with regard to any attempt to express Dr. Lauben's thought. If the problem here is not, perhaps, immediately recognized, then it may be due to our unconsciously having in mind that weaker conception of 'understanding' embodied in (UMP).

We can now see why it was so essential for Frege to allow that there *is* a way of understanding someone else's I-thoughts – or else we could never even judge that someone's I-thought was *different* from the thought expressed in some other form. As Frege goes on to suggest, Dr. Lauben saying 'I was wounded' might be expressed by Rudolph Lingens as 'He who is speaking to me at this moment was wounded' (cf. *T*, pp. 359-60; quoted in §7.5). But of course, if we then ask what justifies treating these two sentences as expressing the same thought, and appeal to how some third party might understand them, then we are faced with an infinite regress.

The issue is important in view of our attempt to find a useful role for the criterion for sameness of sense, (SCE), discussed in §8.1. The criterion is implicit in the first passage quoted (and more explicitly operative in the paragraph that follows, concerning Herbert Garner's thoughts). Since Rudolph Lingens can 'understand' both utterances, yet take up different cognitive attitudes towards them, then they cannot express the same thought. Now at the end of §8.1, an objection was raised to (SCE), namely, that on the assumption that 'understanding' a sentence involves grasping the thought expressed, if two sentences do express the same thought, then 'understanding' one sentence is *ipso facto* to grasp the thought expressed by the other. In the light of our discussion in the previous section, however, this is easily answered. One should say, more precisely, that I 'understand' a sentence if I grasp the thought *as* expressed by that sentence, which clearly does not entail that I grasp the thought *as* expressed by any other sentence, even if that sentence does in fact express the same thought. But as the case of I-thoughts reveals, there remains a problem. For even if 'understanding' a sentence involves grasping the thought *as* expressed by the sentence, we are still entitled to ask how we are to tell whether someone *has* grasped the thought as expressed by the sentence; and we cannot then appeal to the criterion itself, on pain of regress.

The problem is only exacerbated if we set Frege's denial that Dr. Lauben saying 'I was wounded' and Leo Peter saying 'Dr. Gustav Lauben was wounded' express the same thought against the conception of the 'timelessness' of thoughts that it was one of the main tasks of 'Der Gedanke' to establish – the 'timelessness' of thoughts being captured by 'complete' sentences in which all indexicality is cashed out. Since the first shot at decontextualizing Dr. Lauben's original utterance is to say 'Dr. Gustav Lauben was wounded', the conflict is obvious. (Fully decontextual-

ized, of course, the implicit spatial and temporal indexicality would also need to be cashed out.) Now as we suggested at the end of §7.5, one response is to distinguish between expressing and grasping a thought. Dr. Lauben saying 'I was wounded' *expresses* the thought cashed out more fully as 'Dr. Gustav Lauben was wounded', even if he himself does not grasp that thought in its 'completeness' (he may not remember, for example, exactly when it was that he was wounded, or how he came to be wounded). The way is then open to hold that Rudolph Lingens also expresses that thought by saying, at the time of Dr. Lauben's original utterance, 'The man speaking to me at this moment was wounded', even if he too cannot fully grasp that thought by cashing out all indexicality. But equally we must allow that Leo Peter later expresses that same thought by saying 'Dr. Gustav Lauben was wounded', exhibiting in doing so a fuller grasp of that thought.

On this conception, the explanation of how Rudolph Lingens can take up different cognitive attitudes towards the thought as expressed by Dr. Lauben and the thought as expressed by Leo Peter is that he has not fully grasped at least the former, and hence does not know that they do in fact express the same thought. If Rudolph Lingens *were* to properly grasp both, then he would indeed immediately recognize that it is the same thought expressed. But this really does threaten to make the criterion (SCE) useless. For if 'understanding' a sentence as used on a given occasion involves fully grasping the thought as expressed by that sentence, then since, as we remarked at the end of §7.4, no one can break out of the realm of the indexical altogether, the criterion can never actually be applied. The criterion may not be invalidated, but its utility is certainly undermined. I shall, however, consider what conclusions might be drawn from this in the next section.

But before doing so, let us return to the question raised at the beginning of this section, since the answer should now be clear. For Frege, there is *no* gap between (PDU) and (PDS'), properly construed, since, as the passages from 'Der Gedanke' reveal, 'understanding' a sentence as used on a particular occasion precisely involves grasping the sense as expressed by that sentence, and a 'full understanding' involves grasping that sense *on the basis of* an appreciation of the context. Where the gap comes out is between *incomplete* and *complete* 'understanding', with corresponding distinctions to be drawn between incomplete and complete senses and incomplete and complete sentences. Any incomplete sense is determined by the sense of the parts of the corresponding incomplete sentence, and any complete sense is determined by the sense of the parts of the corresponding complete sentence. But what is false is the following thesis:

(PDS#) The *complete* sense expressed by the use of an *incomplete* sentence in a particular context is determined by the sense of the parts of that sentence.

A complete sense can still be *expressed* by the use of an incomplete sentence in a particular context, but the incompleteness of the sentence used does not show that the user of the sentence has fully *grasped* that sense.

Nevertheless, it remains the case that on Frege's conception of 'timeless' thoughts, Dr. Lauben saying 'I was wounded' and Leo Peter saying 'Dr. Gustav Lauben was wounded' express the same thought, however incomplete the sentences themselves may be, and however incompletely the thought may be grasped. But in view of the problems with that conception discussed in §7.5, where are we left? It seems clear that Frege did want some notion of thought such that someone else could at least express, if not grasp, just that thought that Dr. Lauben expresses in saying 'I was wounded'. What is needed, in fact, is to resurrect Frege's earlier notion of the 'content' of a proposition, where this is understood as something like the *state of affairs* (possible as well as actual) involved. Extending the refinement offered of Frege's conception of the sense of simple singular terms, it might be proposed that what it is to grasp the sense expressed by a sentence all of whose constituent singular terms are simple is to know *which state of affairs* is being referred to. This construal would not have been open to Frege, of course, because of his conception of the reference of a sentence as its truth-value; but as we suggested in §6.2, states of affairs are in many ways more plausible candidates.

However, such a proposal would have as a consequence that someone could grasp the thought as expressed by a sentence and grasp that same thought as expressed by another sentence but not realize that they were the same thought; and this does seem counterintuitive. But we could still argue that the 'content' of that person's two thoughts is the same, making use of the distinction between *thoughts* and *objects of thought* that Frege himself failed to appreciate (cf. §7.5 above), and here taking 'content' as the *object of thought*.[36] To return to Frege's example, we could maintain that the *content* of Dr. Lauben's, Leo Peter's and Rudolph Lingen's thoughts is the same; they just express or 'split up' that content differently. Thoughts would indeed then be construed epistemically, reflecting what would earlier have been regarded as ways of 'splitting up' content, rather than 'contents' themselves.

The obvious move would have been to take the *Bedeutung* of a sentence as its 'content' and the sense it expresses as the way of thinking of that content. But since Frege later conceived of the *Bedeutung* of a sentence as its truth-value, there remained a need for a notion of 'content', a need revealed not only in Frege's insistence that the same thought can be analysed in different ways (it would have been less controversial to have said that the same 'content' can be articulated in different ways) but also in his response to the problem of indexicality. Frege's conception of timeless thoughts can be seen as an attempt to restore something of his earlier notion of 'content', though with a less attractive metaphysical gloss. States

of affairs may not be ontologically innocuous, but they are not so obviously bizarre as timeless thoughts.

The dilemma for Frege, though, is clear: either thoughts are discriminated so finely that no two different sentences can be regarded as expressing the same thought, or else they are distinguished so coarsely that someone can express the same thought in two different forms without knowing it. The formulation of the criterion (SCE) was an attempt to find a stable intermediate position. But in view of the tension in Frege's account, and the problems with this criterion, can it genuinely be said that Frege's conception of sense is ultimately coherent? I take up this question in the next section.

8.4 Sense and Substitutability

The criterion (SCE) was offered in §8.1 as the best attempt at capturing a notion of sense that is adequate for Frege's logicist purposes – allowing, in particular, the two members of a Fregean pair to possess the same sense. The idea was simple. Someone cannot *really* 'understand' (grasp the thought as expressed by) both (Da) and (Db), or (Na) and (Nb), for example without immediately recognizing that one is true if they recognize that the other is true. But this seems in conflict with the criterion that emerged from Frege's discussion of intensionality in 'Über Sinn und Bedeutung':

(SIE) Two propositions have the same sense iff they are intensionally equivalent, i.e. iff they can be intersubstituted *salva veritate* in all intensional contexts. (See §8.1 above.)

Take the following two propositions:

(LDa) Ludwig believes that line *a* is parallel to line *b*.
(LDb) Ludwig believes that the direction of line *a* is identical with the direction of line *b*.

Since (LDa) can be true without (LDb) being true, it might be argued, the two embedded sentences – (Da) and (Db) – cannot possess the same sense.

But the issue is not as straightforward as this might imply. For it is not absurd to maintain that if (LDa) is true, then (LDb) is true too, even if Ludwig himself lacks the concept of direction. As suggested in §6.5, (LDa) and (LDb) might be glossed as follows:

(LDa') Ludwig holds as true the thought expressed by 'Line *a* is parallel to line *b*'.
(LDb') Ludwig holds as true the thought expressed by 'The direction of line *a* is identical with the direction of line *b*'.

If (Da) and (Db) do express the same thought, then (LDb') will be true iff (LDa') is true. Compare the inference from (LDa') to (LDb') with the inference from (LDa') to the following proposition:

(LDa†) Ludwig holds as true the thought expressed by 'Die Gerade *a* ist parallel der Gerade *b*'.

If (LDa') is true, then (LDa†) is arguably also true, even if Ludwig does not speak German. Certainly, the fact that Ludwig does speak German and only German, say, does not mean that (LDa') could not be used to report one of Ludwig's beliefs.[37]

In view of the distinction drawn in §8.2, however, the obvious response is to point out that (LDa') and (LDb') have missed out the key little word 'as'. (LDa) and (LDb) should be glossed, more precisely, as follows:

(LDa") Ludwig holds as true the thought *as* expressed by 'Line *a is* parallel to line *b*'.
(LDb") Ludwig holds as true the thought *as* expressed by 'The direction of line *a* is identical with the direction of line *b*'.

There would still be room here to dispute whether (LDa") could be regarded as true if Ludwig only spoke German; but the essential point is clear: (LDb") does not necessarily follow from (LDa"). However, even if we reject the inference from (LDa") to (LDb"), this does not mean that the thought expressed by (Da) is *not* the same as the thought expressed by (Db); it just means that the thought *as* expressed by (Da) is not the same as the thought *as* expressed by (Db). It is the *ways* in which the thought is expressed that are different; and this may or may not be relevant in our specification of someone's beliefs.

As it stands, then, (LDa) is ambiguous. It can be glossed either as (LDa') or as (LDa"), depending on whether we want to report just the belief or the belief as Ludwig himself would express it. But as well as (LDa') and (LDb'), there is another way of interpreting (LDa) and (LDb) that enables us to regard them as logically equivalent. To appreciate this, let us consider another pair of propositions:

(L00) Ludwig believes that 0 is 0.
(L0E) Ludwig believes that 0 is the extension of the concept 'equinumerous to the concept *not identical with itself*'.

It might seem indisputable that (LOO) can be true without (L0E) being true; yet this is just what Frege's own views lead us to deny. For what is embedded in (L0E) is precisely that definition of the number 0 that forms the basis of his construction of the natural numbers, and definitions, according to Frege, must embody sameness of sense as well as sameness of reference. So if a belief can only properly be ascribed to someone if they fully grasp the senses of the terms used to express that belief, and if we accept, with Frege, that his logicist definitions do explicate the senses of our number terms, then (L0E) must indeed follow from (L00).

But if we can allow this ploy here, then we can allow it anywhere else. Frege's definition of the number 0, after all, is one of the most unlikely

things that someone could be expected to know. Returning to Frege's paradigm example, what would it be to fully grasp the senses of 'the Morning Star' and 'the Evening Star'? We might think that all that is really required is knowledge of the following definitions:

(MSA) The Morning Star is the bright heavenly body that appears in the morning.
(ESA) The Evening Star is the bright heavenly body that appears in the evening.

It would indeed then be possible for one of the following to be true and the other false:

(LMSB) Ludwig believes that the Morning Star is a body illuminated by the sun.
(LESB) Ludwig believes that the Evening Star is a body illuminated by the sun.

But if we take the Caesar problem seriously, then these definitions are inadequate since they do not enable us to recognize the object defined under any other guise than that involved in the definition. And even if we find Frege's own demands on definitions excessive, we might still feel that within astronomical theory, an adequate definition must say more about what the object actually is. Better definitions might be the following:

(MSV) The Morning Star is Venus as it appears in the morning.
(ESV) The Evening Star is Venus as it appears in the evening.

Clearly, such definitions presuppose knowledge of what Venus is; but this is no different from Frege presupposing what the extension of a concept is in defining the natural numbers. (Venus might be specified, say, as the second planet in our solar system.) If these are the definitions that must be known to be attributed any proper beliefs about the Morning Star or Evening Star, then (LMSB) and (LESB) are logically equivalent. Anyone who understands at least that portion of astronomical theory that concerns our own solar system – and hence fully grasps the senses of the terms – must know that the Morning Star is the Evening Star.

Even in Frege's own paradigm case, then, considerations of substitutability are by no means straightforward, and there is at least room to pull the two criteria (SCE) and (SIE) into line. In much the same way as (SCE) incorporated a requirement of 'understanding' the relevant propositions, what we have done here is to restrict the application of (SIE) to contexts in which the senses of the propositions (those to which an attitude is being ascribed to someone) are fully grasped: we might call them *fully intensional* contexts. Anyone who fully understands (Euclidean) geometry, for example, must believe that line *a* is parallel to line *b* iff they believe that the direction of line *a* is identical with the direction of line *b*.

This might seem a mere fiddle; but there is an important moral here.

For any criterion of the form 'Two propositions have the same sense iff ...' will contain on the right hand side notions that can always be 'adjusted' to deliver the required conception of sense. This obviously threatens to make any specification of a conception of sense circular; but we have already dealt with this problem in discussing, in relation to Frege's earlier notion of 'conceptual content', what was called the 'Fregean circle' in §2.5. It is neither the case that our semantic intuitions underpin our logical inferences, nor that our assessments of logical equivalence underpin our judgements about sameness of 'conceptual content'. What we have instead is a delicate dialectical interplay, each reinforcing the other as our use and understanding of language develops. Exactly the same thing can be said here, since to substitute one word for another (*salva veritate*) just is to make a logical inference, and we have precisely been trying to refine a conception of sense to capture that notion of 'content' that Frege utilized in the central argument of the *Grundlagen.* Our judgements about sense and our practices of substituting words are mutually dependent.

Frege was thus absolutely right in seeing the fundamental connection between sense and substitutability. But what he failed to appreciate was the dialectical interplay between the various notions involved here. The fundamental connection is obvious in the extreme case of what might be called 'perfect synonymy'. Two expressions are *perfectly synonymous* iff they are intersubstitutable in every possible context. But of course, no two words can ever be perfectly synonymous. For they are bound to resonate slightly differently in somebody's ear, making the use of one more appropriate in some particular context. In Fregean terminology, they would thus have different *tones.*[38] But in crystallizing out from the cauldron of linguistic life the notions of *Sinn and Bedeutung,* Frege has to specify the kinds of substitutability that are relevant, specifications that can only be made in 'theory-laden' terms. According to Frege, two expressions have the same *Bedeutung* iff they are intersubstitutable *salva veritate* in all extensional contexts. But as we remarked in §6.2, we are not really given any independent characterization of 'extensional contexts'. All we have is Frege's repeated insistence that he is only concerned with *scientific* contexts, where the goal is *truth,* which begs the question as to whether 'non-scientific' propositions can have a truth-value. So here too what we have is an 'adjustment' of our notion of truth to deliver the requisite notion of *Bedeutung.* And as we saw, Frege's own arguments *underdetermine* what is to be regarded as the *Bedeutung* of an expression. The same problem arises with the criterion for sameness of sense: talk of intensional contexts presupposes an interpretation of statements ascribing a propositional attitude to someone, which already assumes some kind of conception of sense.

This is not in itself to deny the legitimacy of Frege's notions of *Sinn* and *Bedeutung.* It is simply to stress that their rationale cannot be given from any 'neutral' standpoint entirely outside the scope of his logical theory;

and it has, of course, precisely been the aim of this book to provide an account of his logical theory that makes those notions at least explicable. But there are, in fact, as many different notions of 'meaning' – whether called 'content', 'Bedeutung', 'sense', 'role' or whatever – as there are distinguishable types of context in which substitutions of linguistic expressions occur. At the opposite end of the spectrum to the case of perfect synonymy mentioned above is the case of two expressions that are only intersubstitutable on one particular occasion. Here it might be said that on this particular occasion they have the same 'sense'. Between these two extremes there is a whole range of cases. Frege himself only officially recognized two; having isolated a fairly clear-cut notion of 'Bedeutung' (whatever other possibilities we ourselves might contemplate), he assumed that every other notion could be swept up under the heading of 'Sinn', an assumption, as we have seen, that is questionable even for Frege's own philosophy.

Nevertheless, the connection between sense and substitutability remains, and in its general schematic form, the 'sense' of an expression can be contextually defined in the familiar way (cf. §6.5):

(Sa) Expression 'A' is intersubstitutable *salva veritate* in all contexts of type C with expression '*B*'.
(Sb) The sense of '*A*' is identical with the sense of '*B*'.

But this threatens to raise an even more insidious problem. For the legitimacy of such contextual definitions seems to depend on (Sa) and (Sb) having the same sense, but the contextual definition is precisely an attempt to specify what it is to have the same sense. So do we not go round in an even tighter circle? However, the problem here is only serious on the assumption that the abstract objects defined by means of a contextual definition are independently existing objects; and there is every reason to reject this assumption.

The problem essentially concerns the status of Fregean pairs. If the abstract objects referred to in the second member of a Fregean pair were indeed 'independent' objects, then it would certainly be questionable whether the two members of a Fregean pair do possess the same sense. For the second member would appear to involve an ontological commitment not made by the first. On any reasonable conception of belief, someone could surely legitimately believe that two expressions are intersubstitutable in the relevant contexts without taking themselves to be referring to independent objects called 'senses'. Of course, we could argue that (Sa) makes *implicit* reference to senses, and that anyone who *fully* 'understands' (Sa) must know that senses are being referred to as independent objects; but then this really does seem to beg the question, building into our understanding of (Sa) exactly what we wish to extract.

In any case, our discussion of Russell's paradox in §7.2 should already have convinced us of the need to reject the assumption that objects

contextually defined are 'independent'. For Axiom V, asserting the equivalence between (Va) and (Vb), has the form of a contextual definition, legitimizing the introduction of *value-ranges*, of which *extensions of concepts* were a special kind; and it was the assumption that extensions were objects already existing in the domain, and hence objects for which even the concepts of which they were the extensions had to be defined, that generated the contradiction. But as long as the objects contextually defined are seen as *dependent* entities, of a different *type* from that of the objects on which they are dependent, then the status of Axiom V, and contextual definitions in general, is unproblematic. In a genuine contextual definition, the two members of the relevant Fregean pair have the same sense *by stipulation*, since its purpose is not to capture a pre-existing equivalence (which would assume that both members had an independent meaning), but simply to specify how terms for abstract objects are to be understood.

As contextually defined by (Sa) and (Sb), then, senses too must be regarded as dependent entities, dependent on our practices of substituting expressions in certain contexts. That senses are dependent on our use of language might seem obvious. But if we accept Frege's thesis that the sense of a sentence is the thought it expresses, then thoughts too are dependent on our use of language, and this is by no means an uncontroversial claim.[39] It certainly conflicts with Frege's own conception of thoughts as timeless entities inhabiting a 'third realm' of their own, although as we argued in §7.5, this is a view that is fraught with difficulty. But unless senses are treated as dependent entities, then there is no way of responding to the Caesar problem – that the relevant objects have not been adequately distinguished from every other object of the universal domain. And as we shall see in the next section, even Frege did not attempt to provide explicit definitions of senses.

8.5 Analysis and Crystallization

In §6.4 we remarked that if the senses of names are construed as modes of presentation, and these are characterized by means of definite descriptions, then we face an infinite regress, since the sense of any definite description can only be grasped by grasping the senses of its components. So a different account is needed at least of simple names; and we suggested that the sense of such names should be seen as *shown* by saying, in one of any number of appropriate ways, what its referent is. Now although Frege failed to recognize the scope distinction that we drew in explaining the sense of simple names, he was well aware that not everything can be defined, and that there comes a point at which we must rely not on definition but on what he calls *elucidation* (*Erlauterung*), which makes clear the sense of an expression by example.[40] Elucidations, we can say, do

indeed *show* the sense of an expression by *using* the expression in an appropriate way.

According to Frege, however, the crucial feature of elucidations is that they occur at the *pre-theoretical* stage, and at this stage there is always the possibility of misunderstanding. All we can then do is to 'count on a meeting of minds, on others guessing what we have in mind' (*LM*, p. 207) But, Frege emphasizes, 'all this precedes the construction of a system and does not belong within a system. In constructing a system it must be assumed that the words have precise meanings and that we know what they are.' (Ibid.) Once again, the possibility of misunderstanding is pushed aside as revealing the inevitable deficiencies of ordinary language.

To explore this further, however, let us return to Frege's definition of the number 0 that forms the basis of his logicist project:

(E0) The number 0 is the extension of the concept 'equinumerous to the concept *not identical with itself*'. (Cf. §4.3 above.)

This is a definition *within* Frege's theory. But it clearly presupposes knowledge of what the extension of a concept is, the relation of equinumerosity and the concept *not identical with itself*. Frege takes the concept to be purely logical and the relation to be definable in terms of one-one correlation, which can in turn be defined purely logically. At the time of the *Grundlagen*, Frege just assumed that it was known what the extension of a concept was (cf. §5.5 above), but in the *Grundgesetze* extensions were in effect contextually defined in Axiom V. As we have seen, Frege understood extensions as objects, and objects were categorically distinct from concepts; so that even if we allow that all the elements of (E0) can be logically defined, it must still be assumed that the notions of object and concept are understood. Here indeed, as Frege admitted in 'On Concept and Object', we have reached the point at which 'hints' rather than definitions must be appealed to, where Frege has to rely 'upon a reader who would be ready to meet me half-way – who does not begrudge a pinch of salt' (*CO*, p. 54).

But if we put the remarks from 'Logic in Mathematics' and 'On Concept and Object' together, it would be misleading to suggest that this reliance on a meeting of minds occurs purely at the *pre-theoretical* stage, for the concept/object distinction arises from Frege's use of function-argument analysis in developing his logical *theory*. A relatively high degree of sophistication is required to understand this: we need to see how Frege's logical system works before we can grasp the concept/object distinction sufficiently well to even meet him half-way. This is no less so if we consider the other assumptions that Frege makes in offering his definition (E0). Frege clearly assumed that there were no problems in appealing to *logic*, regarded as governing all conceptual thought, and our knowledge of which was taken to be *a priori*. But as we are now more aware, the idea that there is such a thing as *the* logic of conceptual thought is highly problematic, and

is certainly not the case that Frege's own logical theory offers us *the* logic.

None of this *invalidates* Frege's definitions; it just suggests that we need to appreciate the whole working of his logical system to 'guess what has in mind'. If, as in the *Grundgesetze*, (E0) is taken as laying down both the sense and reference of 'the number 0', then there is more to understanding it than simply what can be appealed to prior to the construction of a theory. Indeed, it is not just that we need to understand Frege's logical theory to understand his attempted reduction of arithmetic to logic; we also need to see what he does with his axioms and definitions to appreciate that it is *arithmetic* that he has reduced. Frege recognized as much at the time of the *Grundlagen*. Definitions, he wrote, 'prove themselves by their fruitfulness', where this was understood as enabling us to derive 'the well-known properties of numbers' (*GL*, §70); and this idea was endorsed in his 1914 paper 'Logic in Mathematics': 'a definition must prove itself in the construction of a scientific system' (*LM*, p. 228).

It was this latter idea that ended up taking centre stage in his final answer to the paradox of analysis. As we saw in §5.4, according to his initial response, as revealed, for example, in his reply to Husserl's objections to his *Grundlagen* definitions, the paradox can be solved by distinguishing between *Sinn* and *Bedeutung*. An identity statement is correct if the relevant two terms have the same *Bedeutung*, and informative if the two terms have different senses. Whilst numbers might not seem to us to be extensions of concepts, the difference lies only at the level of sense, not of *Bedeutung*. But as we argued, Frege soon realized that the identity statements that lie at the base of a theory, i.e. the axioms and definitions, require sameness of sense as well as sameness of *Bedeutung* (since otherwise their status would be problematic). But what we ordinarily understand by number terms is clearly not the same as what we understand by the terms as defined by Frege. Frege found himself forced to reject altogether our ordinary understanding. According to his final answer to the problem, as given in 'Logic in Mathematics', the paradox only arises on the assumption that there are clearly grasped senses to begin with, which our analyses are trying to capture. But it is just this assumption that needs to be discharged. The task of the theorist is to associate clearly grasped senses to the relevant terms where there were no such senses associated before. Talk of *reconstruction*, then, seems appropriate, since the aim is to build up again from scratch, and *replace* our ordinary discourse.

Yet this talk of reconstruction from scratch or replacement is highly problematic, for there must surely be *some* constraints on the adequacy of a reconstruction, and these can only have their source in our ordinary discourse. However poor our philosophical understanding of arithmetic might be, it is still *arithmetic* that we are trying to reconstruct. Frege is right that an analysis should not be regarded as simply trying to capture

our pre-existing conceptions (correct analyses would indeed then be uninformative). But it is wrong to conclude that those pre-existing conceptions must be altogether ignored. The aim of analysis is to *extend* or *refine* those conceptions, for particular purposes. What led Frege to his later posit was his *semainomenalism* – his view that senses, in all their determinate fullness, already exist in some 'third realm' waiting to be definitively associated with the appropriate terms. Since this association is not made in ordinary discourse, there is obvious work for the theorist to do. But if senses are contextually dependent on our linguistic practices, and those practices evolve over time, then so too will senses evolve. We can indeed offer reconstructions of our notions, but it is a condition of offering or understanding these that we start *from* the position of our everyday conceptions; and it is wrong to suggest that these can ever be replaced overnight.

Let us illustrate this by taking a simple example from chemistry, an area of science that Frege himself sometimes mentions for comparison. Consider the following two propositions:

(SW) Salt dissolves in water.

(NH) $2H_2O + NaCl \rightarrow H_3O^+ + Cl^- + Na^+ + OH^-$.

(SW) represents a familiar, everyday process, and (NH), we could say, provides its 'chemical analysis'. There is also a sense in which, for the purposes of chemistry, (NH) does 'replace' (SW), in that it is this equation that represents the reaction and that plays its part in more complex analyses of chemical processes. Nevertheless, in no sense does a chemist *discard* its informal characterization, as captured in (SW). Whatever manipulation of chemical formulae a chemist may perform, the informal characterizations always stand in the background, being something that is *presupposed.* In offering (NH) as the analysis of (SW), then, the chemist is *refining* rather than *replacing* ordinary language.

This is not to suggest that there no cases where talk of 'replacement' is appropriate. Remaining with chemistry, the notion of phlogiston – the supposed 'principle' underlying the properties of combustible substances – is an obvious example of a notion that was abandoned once a better understanding of combustion was attained, as a result of the 'chemical revolution' of the late 18th century. However, it should be noted, firstly, that this revolution took place over a period of two decades, and in its initial stages did *not* involve the repudiation of the notion of phlogiston, and secondly, that what was radically transformed was the *theoretical* understanding of combustion. Although new experiments were done, there was a substantial body of empirical data that could be characterized, and hence agreed upon, using 'ordinary' notions (e.g. that sulphur produces sulphuric acid on combustion).[41] So this does not conflict with the claim that theoretical analysis offers a refinement rather than a replacement of our ordinary understanding. The 'paradigm shifts' that occur from

time to time are located at the *theoretical* level, and although theory inevitably filters down and permeates our ordinary discourse, any transformation of our ordinary understanding takes a much longer time, and makes talk of 'paradigm shifts' at the ordinary level inappropriate.[42] So the general point remains: our ordinary understanding is *presupposed* (however vague it may be in certain areas) in the theoretical attempts that are made to *deepen* that understanding.

But does this not return us to Frege's original response to the paradox of analysis – that a correct analysis must preserve *Bedeutung* whilst allowing 'a more richly articulated sense' (see §5.4 above)? But if the *Bedeutung* of a sentence is merely construed as its truth-value, then this is inadequate as the required constraint. What is needed is something more like Frege's original notion of 'content', suitably understood (as referring, say, to the state of affairs involved). Correct and informative analyses 'split up' the same 'content' differently. What makes (NH) a correct analysis of (SW), for example, is that they both refer to the same 'content' – in this case, the same chemical process. However, even this requires qualification. For 'water' as we ordinarily understand it – the 'water' that we drink and wash in – is not just H_2O, but also contains various 'impurities'; and 'salt' too can refer to more than just NaCl. So the 'content' of (NH) is itself an idealized refinement of the 'content' of (SW). But what tends to happen in such cases is adjustment of our ordinary notions to reinforce the analysis. (NH) makes us interpret (SW) as 'Sodium chloride dissolves in "pure" water'; and once the adjustment is made, then we do have sameness of 'content'. As in the case of (Va) and (Vb) discussed at the end of §5.5, the sense that (SW) and (NH) have in common is *crystallized* in the process of offering the analysis. By taking (NH) as the correct analysis, the appropriate sense of (SW) is precipitated out. Senses are themselves like salts dissolved in the fluid of ordinary linguistic life, which are crystallized out with the help of the seeds of our theoretical notions. In answer to the paradox of analysis, then, what is needed is a synthesis of Frege's earlier and later responses. Sameness of 'content' is indeed a constraint on the adequacy of an analysis, but this is not to say that the 'real' content of the sentence of ordinary language is properly grasped prior to the theory in which the analysis is offered. In the end, what makes an analysis a good one is its success, as part of some overall theory, in convincing us that our ordinary discourse is indeed imprecise, and requires refinement for scientific purposes.

But such a strategy presupposes a *realist* view of the relevant "contents'. The paradox of analysis seems easily solved in the case of chemistry, since chemical compounds are indeed taken to have a real essence (water is H_2O, salt is NaCl). But is arithmetic analogous to chemistry? What we are now forced to confront is the problem raised at the end of chapter 4. In setting up his logicist system – in *reconstructing* arithmetic – has Frege revealed for the first time the *real essence* of numbers, or merely introduced

analogues of them? It is clear that, at the time of the *Grundlagen*, Frege saw himself as doing the former. As we noted in §0.3, Frege regarded himself as the culminator of centuries of intellectual effort, as the philosopher who had finally managed to achieve knowledge of the concept of number 'in its purity' (cf. *GL*, p. VII). That the ordinary person might not recognize numbers as extensions of concepts was not seen as an objection. 'To those who might want to declare my definitions unnatural, I would suggest that the question here is not whether they are natural, but whether they go to the heart of the matter and are logically unobjectionable' (*GL*, p. XI).

What motivated Frege's logicist project was the combination of his realism and his Pythagoreanism. For the reasons given in §4.1, Frege was in no doubt that numbers were objects; and as we saw in §3.4, he also regarded enumerability as part of the essence of conceptual thought and hence as purely logical. So the task simply became one of finding a logical characterization of numbers, of showing that numbers were *logical objects*. But in identifying numbers with extensions of concepts, regarded as objects already there in the universal domain, Frege ran into contradiction. And as we noted in §4.5, the subsequent development of mathematical logic has shown that there are alternative set-theoretic characterizations of the natural numbers, so that talk of having discovered *the* logical essence of number now seems misplaced.

The issue should already have struck Frege, even before the emergence of Russell's paradox, since he himself had insisted that the real numbers must be defined quite separately from the natural numbers (see §§4.4 and 4.5). According to Frege, whilst the natural numbers are to be identified with extensions of concepts, the real numbers are to be identified with extensions of relations. But this means that *the natural number 7*, for example, is not identical with *the real number 7*. Even in Frege's own system, then, the members of the corresponding subset of the real numbers, {1, 2, 3,...}, are only *analogues* of the natural numbers. But if this is allowed here, then why should Frege's definitions of the natural numbers not also be regarded as only providing analogues?

But if, at best, Frege's own system could only have specified *analogue*; of the natural numbers, and there is no such thing as *the* logical or set-theoretic reduction of the natural numbers, then how are we now to solve the paradox of analysis? No analysis would seem to be correct. In response, it might be argued that the only 'content' that needs to be preserved in a theoretical account is the *structure* of arithmetic.[43] Exactly what objects are specified is irrelevant, as long as, to use Frege's words, all the 'well-known properties of numbers' can be derived (cf. *GL*, §72). But would this mean that we should abandon talk of numbers as objects at all? However, if preservation of *structure* provides the constraint on adequate analyses, and we allow the legitimacy of contextual definition, then a familiar way of specifying numbers as abstract objects is open to us:

(NSa) Number system S1 is isomorphic to number system S2.
(NSb) The numbers defined by S1 are identical with the numbers defined by S2.

S1 and S2 could either both be ordinary number systems (e.g. the natural number system and the relevant part of the real number system), or both be set theories, or one be an ordinary number system and the other the relevant part of a set theory. Clearly, what is required to judge the relevant isomorphism is a detailed comparison of the two systems, which in turn requires an appreciation of the workings of both. But if such isomorphism could be appealed to, then there would no longer be any essential difference between the cases of arithmetic and of chemistry.

So would this mean that 'the natural number 7' and 'the real number 7' do, after all, have the same referent? Frege's theory of real numbers may suggest that they have different referents, and this may well be compatible with his final response to the paradox of analysis – that new systems are to be set up from scratch. But this leaves him with no way of explaining the relationship between the two. On the traditional account, however, whereby the real number system is generated by successive expansion of the natural number system, they would be regarded as having the same referent (which would be compatible with Frege's original response to the paradox of analysis). But the interesting question here concerns whether they have a different *sense*. However, in the light of what we have just been saying, the question can be seen to hide a confusion. For our understanding of the term 'the natural number 7' itself varies according to which stage in the development of number theory we have reached. If we have only learnt the system of natural numbers, then the question will have no meaning for us. If, on the other hand, we are familiar with the system of real numbers, then as part of our learning of this, we will have come to see that the real number 7 *is* the natural number 7. Once again, what we would have here is a case of the sameness of sense being crystallized as the new theory is developed. The real number system is not set up from scratch; but in moving to it from the system of natural numbers, the latter is *incorporated* into the former. The natural numbers *become* a subset of the real numbers. Our understanding of the natural numbers is *presupposed* in the development of the real number system, and becomes refined in the process.

Contextual definitions themselves provide a good illustration of the subtle transformations involved in developing a theory. Take Frege's central definition of sameness of number in terms of one-one correlation. We might intuitively think that the set of rational numbers has more members than the set of natural numbers, or the set of natural numbers more members than the set of even natural numbers; but once we are persuaded that one-one correlation underlies attributions of sameness of number, as it certainly does in the case of finite sets, then we are prepared

to accept that the infinite sets just mentioned have the same number. Our concept of number has been *refined* in the process, allowing us, in particular, to understand the development of the theory of transfinite numbers.[44]

But if all this is right, then what it suggests is that the paradox of analysis cannot ultimately be 'solved' without taking an historical approach. For if what analysis involves is the crystallization of sense, then since this is a process that occurs over time, there is no ahistorically positioned answer as to whether it is both correct and informative. Before the theory is developed in which the analysis is offered, the analysis, if it is understood at all, will seem incorrect; and after it is developed, with the necessary transformation in our understanding effected, it will be correct but uninformative. To talk of 'correctness' is to make a move *within* a system; yet informativeness is located in the process of developing, learning and using a system. The paradox of analysis arises from trying to account for both features through 'logical justification', and in the end Frege's lack of success in dealing with the paradox results from his denigration of historical considerations. To see how an analysis is both informative, and comes to seem correct, we need to retrace the path by which that analysis was offered.

The point applies as much to logical theory as it does to chemical or mathematical theories. As we argued in chapter 2, understanding Frege's logic is itself to crystallize those semantic intuitions that are taken to lie at the base of the theory. 'All *A's* are *B*' becomes *regimented* as 'If anything is an *A*, then it is a *B*', for example (which is not to say that we cannot represent cases in which there *is* existential import). 'There are 0 *F's*' is interpreted, at the conceptual level, as 'The concept *F* is not instantiated' (cf. §5.3). Does Frege's logic offer a good analysis of our linguistic life? This cannot be straightforwardly answered since that linguistic life is itself refined by learning Frege's logic. To properly answer this question, we must carefully trace the development of that logic historically; and it has been one of the central aims of this book to do just that.

But as we noted in the introduction, such a conception of logic was not Frege's. Frege himself regarded the logical realm as lying entirely outside the domain of ordinary language, a view that found its ultimate expression in the semainomenalism of 'Der Gedanke'. The failure of his logicist project only reinforced him in his belief that there was something inherently deficient about ordinary language, since it tempts us into thinking that proper names such as 'the extension of the concept *F*' denote objects. 'So a great part of the work of a philosopher consists – or at least ought to consist – in a struggle against language.' (*SKM*, p. 270; cf. *DECN*, p. 263; *PWN*, pp. 265-6.) This idea became a guiding theme of analytic philosophy. As Wittgenstein was to put it, 'Philosophy is a battle against the bewitchment of our intelligence by means of language' (*PI*, §109). But although in his earlier work, Wittgenstein had shared Frege's belief in an absolute logic, one of the major motivations of his later work was to repudiate this

conception.[45] As the passage quoted at the beginning of this chapter shows, he realized that whilst the frictionless environment of a perfect logic might seem ideal, we soon find that we are unable to walk. However, as Wittgenstein goes on to remark in the paragraph that follows, this was not to be understood as bargaining the rigour out of logic. 'The *preconceived idea* of crystalline purity can only be removed by turning our whole examination round.' (*PI*, §108.) For Wittgenstein, this meant returning to the 'rough ground' of our ordinary discourse, and 'describing' rather than 'explaining' our linguistic practices. But as I hope I have shown in the present book, adopting an historical approach not only removes the idea that the crystalline purity was there to begin with, but at the same time offers an *explanation* of the process of crystallization.[46]

Such an approach is also needed to explain Frege's conception of sense, which lay at the heart of his philosophy. We are tempted to think – seduced, perhaps, by Frege's semainomenalism – that if that conception is legitimate, then it must have been there all along, fully crystallized, in Frege's thought. But that conception too has a history, which extends indeed into the work of later philosophers, and different demands have been made upon it. I have concentrated in this book on Frege's notion of sense as it developed in his own thought; but this is clearly only one part of the much longer story that reaches to the present. However, in telling just this part of the story, in explaining the role Frege played in *making sense*, ideas have been used that are themselves refinements of Frege's own ideas (e.g. the distinction between semantic and epistemic sense, the deeper apprehension of the roles of context and presupposition, or the historicized synthesis of Frege's earlier and later responses to the paradox of analysis). In making sense of Frege's philosophy, in appreciating how his conceptions were crystallized and can be further crystallized, we too are *making sense.*