1. The Logical Background

O insensata cura de' mortali,
quanto son difettivi silogismi
quei che ti fanno in basso batter l'ali.
(Dante, *Paradiso*, XI 1-3.)

[O senseless care of mortals,
how defective are the syllogisms
which make you beat your wings down.][1]

Whilst Frege is the father of modern logic, it is Aristotle who invented logic itself. Indeed, not only did he construct the first formal system, but the system he constructed, except for a period of rivalry with Stoic logic, dominated the intellectual world for twenty-two centuries, right up until Frege's time. Any account of the revolution effected by modern logic, then, and the development of Frege's philosophy as part of this, must begin with Aristotle's theory of the syllogism. It is not just that this introduces the issues concerning the nature, role and scope of logic, which are crucial to an understanding of Frege's work; analysis of Aristotelian logic also highlights key features of Fregean logic that might not otherwise be appreciated. In particular, my aim in the next chapter is to show how Frege's early notion of 'conceptual content' was formulated through an awareness of differences between the two systems and a perceived need to *justify* the new one. Since it is 'conceptual content' that later bifurcated into 'Sinn' and 'Bedeutung', any account of these notions must be set against this logical background.

After a sketch of the emergence and nature of the syllogism (§1.1), I elucidate the operation of proof in syllogistic theory (§1.2), and then explore in turn certain epistemological and semantic aspects of Aristotelian logic (§§ 1.3 and 1.4). My purpose is to demonstrate that Aristotle developed a coherent system of logic *without* a conception of 'sense' (as that might now be understood), something that has implications for an appreciation of Frege's own work. In the final section, I leap through the intervening centuries to bring us to Frege's *Begriffsschrift*.

1.1 The Syllogism

In the *Sophistici elenchi*, Aristotle remarks that before him, logic as a discipline did not exist, and he had 'to work things out over a long time by trial and error' (183b34-184b3).[2] The core of his logical theory is contained in the *Prior Analytics*, and although scholars disagree over its dating, few deny that in its final form it is one of the mature works of his *Organon*. Certainly, the care taken in formalization and presentation suggest a long gestation. Details of this period are unknown, but it is possible to sketch some of the preceding developments that facilitated the emergence of syllogistic theory.[3]

Since logic involves reflection upon forms of reasoning, it presupposes the establishment of those forms. Over two centuries of argumentation were available to Aristotle, from Thales, whom Aristotle himself called the founder of natural philosophy (*Metaphysics*, 983b20), to Plato, under whom Aristotle studied; and by the time of Socrates, self-consciousness about reasoning had also evolved. Plato occasionally states principles in his dialogues (e.g. a principle of opposites in the *Republic*, 436b), and his characterization of the sophists as purveyors of fallacious arguments in the *Euthydemus* reveals concern with correctness of reasoning. These fallacious arguments or 'sophisms' were later analysed by Aristotle in the *Sophistici elenchi*, and he is clearly drawing on a tradition of public disputation.

Socratic enquiry and the search for definitions were also important, and these were refined by Plato into the notion of 'dialectic' explained in the later dialogues.[4] This notion, applied to the method of collection and division (*collecting* things generically, and then *dividing* them by a succession of dichotomies into species), influenced Aristotle in his invention of the syllogism by suggesting a chain of terms related by class-inclusion.[5] Aristotle saw his own procedure as an improvement on Plato's, which he criticizes in the *Prior Analytics* (I 31; 46a31-b40), calling the division by genera a 'weak syllogism, since it begs the point which it is required to prove' (46a33-4). His idea was that the division that is effected is already determined by the definition desired, and hence cannot be considered to *prove* anything. As we shall see, Aristotle regarded the ability to function as a *proof* as an essential feature of the syllogism.

Developing a logical system, however, involves more than a critique of existing forms of reasoning: it also requires formalization and axiomatization within a given area. Aristotle's *De Interpretatione* shows that he was interested in propositions which either affirm or deny that a predicate applies to a subject, and more specifically, with the relationships between universal and particular propositions ('All *A's* are *B*' and 'Some *A's* are *B*'), as captured in the traditional square of opposition. It was in the systematic clarification of these relationships – between what I shall call *syllogistic propositions* – that Aristotelian logic arose. (For details, see Appendix 1.)

Although Aristotle indicates at the beginning of the *Prior Analytics* (24a11) that his concern is with proof or demonstration (*apodeixis*), he in fact offers his theory of the syllogism as a characterization of valid argument in general, proof being a special kind of valid argument or syllogism where the premisses are both true and grounded (cf. 24b9-10, 25b28-31), an issue taken up in the *Posterior Analytics* (see §1.3 below). Initially, Aristotle defines a syllogism as 'a form of words [*logos*] in which certain things are assumed and something other than the things assumed follows necessarily from their being so' (24b18-20; my tr.). By 'from their being so', as Aristotle himself explains, 'I mean that it is because of them that the conclusion follows; and by this I mean that there is no need of any further term to render the conclusion necessary' (24b20-2).

If we ignore limiting cases where the conclusion repeats one of the premisses, what we have here is a reasonable, if informal, characterization of a valid argument with two or more premisses. However, in syllogistic theory, a syllogism is more than just a valid argument: it also involves a certain relationship between its parts. It is tempting to interpret Aristotle as having in mind a further *epistemological* criterion – that in a proper syllogism the premisses can be known without the conclusion being known, and hence that a syllogism may *teach* us something. This is how the Peripatetics interpreted Aristotle, as their disagreement with the Stoic logicians shows (see §1.3 below). But Aristotle himself provides a 'general principle' that is purely *logical*: 'we shall never have any syllogism proving that one term is predicated of another unless some middle term is assumed which is related in some way by predication to each of the other two' (41a2-5). This can be illustrated by considering the syllogism traditionally called *Barbara*:

(BB) All *A's* are *B*, All *B's* are *C*; therefore All *A's* are *C*.

'*B*' represents the 'middle term', which, in mediating between '*A*' and '*C*', links the premisses and enables the conclusion to be derived.

According to Aristotle, syllogistic propositions are essentially of subject-predicate form. But since expressions of the form '*A* is *B*' ('*A estin B*') are ambiguous, in his formalizations Aristotle generally uses '(The) *B* applies to *A*' ('*to B tō A huparchei*') instead, to make explicit which is the subject and which the predicate.[6] Strictly speaking, then, (BB) should be rephrased thus:

(BB′) *B* applies to all *A*, *C* applies to all *B*; therefore *C* applies to all *A*.

However, as we shall shortly see, it is essential to Aristotle's project that the valid syllogisms of the first figure, especially *Barbara* and *Celarent*, exhibit their validity as transparently as possible, and this entails *transposing* the premisses:

(BB″) *C* applies to all *B*, *B* applies to all *A*; therefore *C* applies to all *A*.

Since the transitivity of the relation *applies to* is now quite obvious, the validity of the syllogism can be immediately intuited.[7] Using the notation explained in Appendix 1, then, the correct (Aristotelian) representation of *Barbara* is as follows:

(BB†) *Abc*, *Aab*; therefore *Aac*.

Leaving aside qualifications concerning the ordering of the premisses, though, the syllogism can be regarded as a combination of three propositions – two premisses and a conclusion – arranged in the form of a *proof*, and obeying Aristotle's 'general principle'; and this has been the traditional conception. However, some commentators, notably Lukasiewicz (1957: ch. 1) and Patzig (1968: §2 & App.), have argued that this gives the misleading impression that the premisses are *asserted*, whereas a *valid* argument merely requires that *if* the premisses are true, then the conclusion must also be true. Instead, they propose, the syllogism should be characterized as a *single* proposition in the form of a conditional, since embedding the premisses in an *if*-clause removes any suggestion that they are asserted:

(BB#) If *Abc* and *Aab*, then *Aac*.

Now Aristotle certainly allowed that there could be legitimate syllogisms with one or both premisses false (see e.g. 54b17f.); and we have already noted that he distinguished between a *syllogism* (or *valid* argument) and a *proof* (or *sound* argument, i.e. a valid argument with true premisses). But a syllogism nevertheless has the *form* of a proof – it is still an *argument* rather than a single proposition – and construing it as a conditional obscures its *deductive structure*. An argument comprises premisses and a conclusion, and a single proposition (whatever internal complexity it may have) can hardly be regarded as *valid*.[8]

But how, then, should a syllogism be characterized, if the premisses are not to be seen as asserted? The solution is to use the modern device of the *syntactic turnstile* '⊢':

(BB*) *Abc*, *Aab* ⊢ *Aac*.

This is read as saying that *Aac* can be derived from *Abc* and *Aab* by the rules of the relevant logical system.[9] This preserves deductive structure, without implying that the premisses are true; it merely states that a certain conclusion can be inferred from them. Understood like this, then, there is nothing wrong with the traditional construal of the syllogism.

1.2 Proof in Syllogistic Theory

In the paragraph following his initial definition of a syllogism, Aristotle goes on: 'I call a syllogism perfect if it requires nothing, apart from what is comprised in it, to make the necessary conclusion apparent; imperfect if it requires one or more propositions which, although they necessarily follow from the terms which have been laid down, are not comprised in the premisses' (24b23-6). This distinction between perfect and imperfect syllogisms is crucial to Aristotle's procedure for *proving* syllogisms. Perfect syllogisms are those valid syllogisms of the first figure, whose validity is transparent (in the manner indicated in the last section). These, then, are the *axioms* of the system; and all other valid syllogisms can be shown to be valid by 'reducing' them to perfect syllogisms (cf. 29b1-3; 40b17ff.).[10]

Aristotle has two basic methods of 'reduction': *conversion*, constituting a direct (or ostensive) proof, and *reductio ad impossibile*, constituting an indirect proof.[11] A proof of *Cesare* (*Ecb*, *Aab* ⊢ *Eac*), for example, proceeds directly:

(CS) *Ecb*, so by conversion, *Ebc*; so with *Aab Celarent* can be reached: *Ebc*, *Aab*; therefore *Eac*. Hence we can conclude *Eac*. (Cf. 27a5-9.)

A proof of *Baroco* (*Acb*, *Oab* ⊢ *Oac*), by contrast, proceeds indirectly:

(BR) Suppose *Aac*, then since *Acb* it follows by *Barbara* that *Aab*; but that is impossible since *Oab*; therefore *Oac* (the contradictory of *Aac*). (Cf. 27a37-b4.)

As recent commentators agree, all valid imperfect syllogisms can be proved using one or both of these methods.[12] But what has been disputed is whether the rules that these methods embody are part of syllogistic theory. Lukasiewicz (1957: §§15-16) and Patzig (1968: §27) believe that certain theorems of *propositional* logic are presupposed by Aristotle in his methods of 'reduction' and that this contradicts his view that all proofs are syllogisms (as stated e.g. in 25b30-1). (BR), for example, would be seen as presupposing the rule of contraposition:

(CPS) $(P \rightarrow Q) \rightarrow (\neg Q \rightarrow \neg P)$.

(CPS) is not itself syllogistic in form, and according to Lukasiewicz and Patzig, in a proper axiomatization, Aristotle should have formulated *all* the rules that he was using.

However, such a criticism depends on the misguided identification of syllogisms with single propositions in conditional form. In order to *prove* one proposition from another, of course some rule of inference must be recognized; but for Aristotle, imperfect syllogisms *already* have the deductive structure of proofs. All that 'reduction' does is to fill out this deductive structure: it shows *that* a syllogism *is* a syllogism (i.e. is a valid argument).[13] The premisses (to be distinguished from suppositions) and conclusion of both (CS) and (BR), for example, are precisely the premisses and conclusion of *Cesare* and *Baroco* respectively. The deductive structure has simply been elucidated. Aristotle himself seems clear enough about this, as is suggested by both his characterization of an imperfect syllogism (24b24-6; see above), and his later remarks about the 'same conditions' governing both direct and indirect proofs (I 29; cf. 42a32-5). He writes, for example, that 'we must have regard to the same terms whether it is required to prove a conclusion ostensively or to employ reduction *ad impossibile*' (45b14-15; cf. II 14). Proofs by 'reduction', then, are indeed syllogisms themselves.

It may be helpful here to draw two distinctions. Let us first distinguish between *natural syllogisms*, which are simply valid arguments satisfying Aristotle's initial definition, and *technical syllogisms*, which constitute that subset of valid arguments that fulfil Aristotle's additional logical criterion, that is, those valid arguments that possess the form of one (or a series) of the standard syllogistic moods (as set out in Appendix 1). Such technical syllogisms already have the deductive structure of a proof; so let us call the kind of proof referred to here a *syllogistic proof* (a technical syllogism constitutes such a proof when its premisses are true). This must in turn be distinguished from a *theoretical proof* – a demonstration *that* a syllogistic proof *is* a syllogistic proof. When we talk of the proof-procedure within syllogistic theory, then, we mean the mechanism of theoretical proof *that* technical syllogisms *are* technical syllogisms.

We can now clear up the confusion in Lukasiewicz's and Patzig's criticism. Providing it is appreciated what a syllogistic proof is (i.e. that it has a deductive structure), there can be no dispute that syllogistic proofs are syllogisms (both natural and technical). Nor can it be denied that theoretical proofs are *natural* syllogisms (i.e. valid arguments). The issue hinges on whether they are also *technical* syllogisms. But if by 'technical syllogism' we mean whatever has the premisses and conclusion(s) formalizable as one (or a series) of the syllogistic moods, then, as we have seen, theoretical proofs are indeed such syllogisms. The proof expressed in (CS) may not look like the traditional formulation of *Cesare*, nor the proof in (BR) like *Baroco*; but the difference simply consists in the degree of revelation of deductive structure.

Nevertheless, is it not the case that the theoretical proof in (BR) *presupposes* (CPS), and that it is therefore a *defect* in syllogistic theory that the rule involved is not explicitly formulated? The theoretical proof certainly makes implicit use of such a rule, but why should this amount to a *fault*? It is not that there is a missing *premiss*: the argument is valid as it stands; it is just that the relevant deductive structure could be yet further elucidated. This is hardly to charge Aristotle with any *logical error*; it is simply to accuse him, unfairly, of not having invented propositional logic as well.

Lukasiewicz's and Patzig's criticism, then, involves a failure to recognize the deductive structure of syllogistic proofs and hence their connection with the corresponding theoretical proofs. But are they still not right in accusing Aristotle of not properly axiomatizing his system? He may not have produced the neat system that some of his successors have reconstructed;[14] but that does not mean that he did not recognize what

'reduction' involved. The rules of conversion, for example, are clearly stated (see §1.4 below); and the operation of *reductio ad impossibile* is also explained (I 23; 41a23-41). Admittedly, he did not *formalize* the propositional rule involved in indirect proof; but that does not mean that the rule is not part of syllogistic theory. The rules that his methods of 'reduction' involve are essential to the workings of his logical system, and that system is coherent and admirably axiomatized as it is.[15]

Aristotle does, in fact, provide more than just an informal characterization of *reductio ad impossibile*; he also comments on its nature and status within syllogistic theory. He talks, for example, of the 'unanalysability' of arguments which are established *per impossibile*: 'The reduction *ad impossibile* can be analysed, because it is proved by a syllogism; but the rest of the argument cannot, because the conclusion is drawn from a hypothesis' (50a29-32). *Per impossibile* arguments (proving a proposition by deriving a contradiction on the supposition of its falsity), whilst counting as *natural* syllogisms (Aristotle himself refers to them as 'syllogisms', e.g. in 50b3), do not qualify as *technical* syllogisms, and this is really what he means by calling them 'unanalysable'. Of course, they can be 'analysed' within propositional logic, but then, as we have noted, it would be unfair to construe this as a criticism of Aristotle. However, he does recognize the importance of arguments *per impossibile*, remarking that they require 'further study and clear explanation' (50a39-40); and he even promises such elucidation later (50b1-2), though this is not, unfortunately, a promise he appears to have kept.[16]

As a formal system, then, Aristotle's syllogistic theory cannot fundamentally be faulted. But this is not to say that the theory has an extensive application. Certainly, from our contemporary perspective, its limitations are obvious, since *technical* syllogisms form only a small subset of *natural* syllogisms. Propositional logic has already been mentioned, although this required, and indeed received, independent development; and hence could coexist alongside syllogistic theory. More fatally for Aristotelian logic, statements of *multiple* generality (such as 'Every *A* loves some *B*') and *relational* propositions (such as '*A* is larger than *B*') also proved resistant to 'analysis'. As we shall see, it was only with the arrival of Fregean logic that satisfying treatments of these were finally provided, and this had the effect of simply swallowing up syllogistic theory.

1.3 Aristotelian Logic and Epistemology

In late antiquity, there was a well-known dispute between the Stoics, who held that logic was *part* of philosophy, the Peripatetics, who viewed logic as an *instrument* of philosophy, and the Platonists, who maintained both.[17] Now whether or not one regards the dispute itself as in the end merely verbal, the Peripatetics do appear to be following in their master's own footsteps. Aristotle's interests were *epistemological* rather than semantic, and he developed his logic more for scientific use than for the purpose of understanding the nature of reasoning.[18] It is the aim of the next section to illustrate the negative thesis – Aristotle's relative lack of concern with semantic issues, and the task of the present section to expand on the positive thesis.

The root of the disagreement between the Stoics and the Peripatetics lay in the denial by each side that the other's syllogisms really were syllogisms. The Peripatetics held that a proper syllogism should enable us to *learn* something, and argued that in the standard cases of Stoic syllogisms, such as inferences of the form 'If *P* then *Q*; but *P*; therefore *Q*', no one could legitimately claim to know the premisses without knowing the conclusion. This may well be true, but it does not imply that Aristotelian syllogisms are themselves genuinely informative. The Stoics, on the other hand, in dropping the epistemological criterion, defined a syllogism as a valid argument which could be proved within their own system – of essentially propositional logic, which they were the first to develop and explore. Since Aristotelian syllogisms could not be so proved, they in turn denied that these were proper syllogisms. Neither School, then, had a fully developed conception of a valid argument; had they done so, they would have realized that Aristotelian and Stoic logic were not rivals, but complemented one other.[19]

Was Aristotle himself clear about the nature of valid argument? *Prima facie* it would seem not, since if he had been, one would not have expected the dispute to have arisen. Yet, as we have seen, he distinguishes between validity and proof, and recognizes that not every syllogism is a *technical* syllogism. So how full an appreciation of validity did he have? There are certainly passages that support the Peripatetic interpretation. Aristotle writes, for example:

> There is no reason why a man who knows both that *A* applies to the whole of *B* and again that *B* applies to *C* should not think that *A* does not apply to *C*: e.g., if he knows that every mule is sterile, and that *X* is a mule, he may think that *X* is in foal; because he does not comprehend that *A* applies to *C*, unless he considers both premisses in conjunction. (67a33-8; Loeb tr.)

This conception may well be problematic (does the person *really* know both premisses?), but its implication is clear. Since the obvious way of correcting the man's faulty judgement is to get him to consider both premisses 'in conjunction' by presenting him with the relevant syllogism, an epistemological role for the syllogism is indeed indicated. So did Aristotle think that it was a mark of a genuine syllogism that it could be informative?

Epistemological concerns were undoubtedly a major inspiration behind Aristotle's development of syllogistic theory. His guiding vision was of the sciences as neatly axiomatized deductive systems – arranged as chains of (technical) syllogisms. The aim of the *Prior Analytics* was to set out the

necessary logical structure, whilst its companion, the *Posterior Analytics*, sought to provide the philosophical rationale for the vision itself. Divided into two parts, Book I of the *Posterior Analytics* explains Aristotle's conception of *demonstration* (the method by which the scientific system is ordered) and Book II discusses the nature of the axioms. By 'proof' or 'demonstration' (*apodeixis*), Aristotle writes, he means a 'scientific syllogism' (*syllogismos epistēmonikos*), that is, 'one in virtue of which, by having it, we understand something' (71b17-19). This understanding is effected, according to Aristotle, if the premisses of the syllogism are 'true and primitive and immediate and more familiar than and prior to and explanatory of the conclusion' (71b21-2). These fundamental premisses or principles must themselves be *non*-demonstrable, Aristotle argues, since otherwise an infinite regress threatens: if we could only understand something through demonstration, and this rested on premisses that themselves needed to be understood, then we could never understand anything (72b5-73a20). Given Aristotle's *epistemological* conception of proof, the non-demonstrability of principles clearly follows; but it should be noted that on a (modern) *semantic* conception of proof, all that is required is that the premisses be true – their epistemological status is irrelevant – in which case it would be possible to 'prove' some basic proposition from premisses that are *less primitive* (epistemologically speaking).[20] There may be nothing *incoherent* about Aristotle's epistemological conception of proof, but it does need to be recognized as such, a conception that gains its legitimacy within the context of his scientific vision.

However, it would still be misleading to attribute to Aristotle the extreme Peripatetic view that the whole purpose of demonstration is to enable us to learn things. In the very first sentence of the *Posterior Analytics* he writes: 'All teaching and all intellectual learning come about from already existing knowledge' (71a1-2). But, he goes on, 'nothing, I think, prevents one from in a sense understanding and in a sense being ignorant of what one is learning; for what is absurd is not that you should know in some sense what you are learning, but that <you should know it> in this sense, i.e. in the way and sense in which you are learning it' (71b6-9). What you learn in a syllogism, then, is not the propositions themselves, which you may already know (the verb used here is *eidenai*), but the logical relations between them. As we shall see in §5.1, it is the appreciation of the connections between propositions, in an axiomatized system, that constitutes 'scientific' knowledge (*epistēmē*).

The point deserves emphasis. The Peripatetic misunderstanding of logic is a persistent confusion in the history of philosophy. It lay behind Descartes' rejection of Aristotelian logic in the 17th century. As he put it in his *Rules for the Direction of the Mind*, the syllogistic art of reasoning 'contributes nothing whatever to knowledge of the truth' (Rule Ten; p. 36); and he advocated its abandonment in favour of his own method (see §1.5

below). Locke was even more forthright. In a famous passage of the *Essay*, he wrote: 'If Syllogisms must be taken for the only proper instrument of reason and means of Knowledge, it will follow, that before *Aristotle* there was not one Man that did or could know anything by Reason; and that since the invention of Syllogisms, there is not one of Ten Thousand that doth.' But, he went on, 'God has not been so sparing to Men to make them barely two-legged Creatures, and left it to *Aristotle* to make them Rational' (IV xvii 4). However, whilst Descartes and Locke may be right in refusing to grant syllogisms privileged status, they are wrong in their consequent rejection of logic itself. As Aristotle himself recognized, elucidating the deductive structure of a certain class of inferences has, at the very least, value in the organization and articulation of a scientific system.[21]

Nevertheless, as far as the dispute with the Stoics is concerned, it does seem that Aristotle would have sided (with some qualifications) with the Peripatetics. Since Stoic syllogisms could never constitute 'proofs' as Aristotle understood them (i.e. as potential constituents in an axiomatized deductive science), then they were not genuine (technical) syllogisms. And this is presumably why Aristotle overlooked such syllogisms, and was not led into inventing propositional logic as well. His epistemological interests also explain why Aristotle failed to appreciate certain *semantic* features of his logical theory; and I consider these in the next section.

1.4 The Semantics of Conversion

As we have seen, Aristotle's two basic methods of proof are *conversion* and *reductio ad impossibile*; and his understanding of the latter was considered in §1.2. The rules of conversion are stated in chapter 2 of Book I of the *Prior Analytics*, and can be formulated as follows:

(EC) From *Eab* infer *Eba*.
(AC) From *Aab* infer *Iba*.
(IC) From *Iab* infer *Iba*.

Although these are *logical inferences*, it should be noted that by 'conversion' (*antistrophē*) Aristotle just meant the *transition* from one syllogistic proposition to another, the term 'necessary conversion' being used for logical inference. Aristotle does not state that *Oab* is *non*-convertible, for example, merely that it is 'not necessarily convertible' (since e.g. 'Some animals are not men' does not imply 'Some men are not animals'; cf. I 2); and he writes elsewhere of propositions being 'converted' into their *negations* (e.g. in II 8). However, even suggesting that Aristotle understood 'necessary conversion' as logical inference is being charitable, since he talks of propositions themselves being 'necessarily convertible' rather than the inferences in which they are involved possessing the necessity.[22] So what exactly did Aristotle understand of the semantics of conversion?

There are three further 'rules of conversion', which, for completeness, should also be noted:

(AS) From *Aab* infer *Iab*.
(ES) From *Eab* infer *Oab*.
(ECS) From *Eab* infer *Oba*.

(AS) and (ES) correspond to the two traditional 'rules of subalternation', and (ECS) simply follows from (EC) and (ES). In the *Topics* Aristotle appears to appreciate the two rules of subalternation (cf. 109a3-6), but he does not discuss them when developing his theory of the syllogism in the *Prior Analytics*, and hence fails to recognize the five additional *subaltern* moods that result from their use (see Appendix 1). Aristotle's failure to formulate these further rules does not imply an incoherence in syllogistic theory itself (it merely shows that his own work was slightly incomplete), but it does suggest that he had only a partial understanding of conversion. Yet is there more to it then a mere (uncharacteristic) lapse in his systematizing efforts?

There is an important difference between (EC) and (IC) on the one hand, and (AC) – and the three further rules – on the other. The modern logician would express this by saying that in the former but not the latter case, the corresponding conditionals can be strengthened to biconditionals ('*Eab* → *Eba*' to '*Eab* ↔ *Eba*', and '*Iab* → *Iba*' to '*Iab* ↔ *Iba*'). Aristotle seems to be aware that there is a difference, but it is not obvious that he has appreciated exactly what it is:

> In universal statement the negative premiss is necessarily convertible in its terms: e.g., if no pleasure is good, neither will anything good be pleasure; but the affirmative, though necessarily convertible, is so not as a universal but as a particular statement: e.g., if every pleasure is good, some good must also be pleasure. In particular statements the affirmative premiss must be convertible as particular, for if some pleasure is good, some good will also be pleasure; ... (25a5-11; Loeb tr.)

Unlike the universal negative, which is 'necessarily convertible in its terms', the universal affirmative is 'necessarily convertible ... not as a universal but as a particular statement' ('not however universally, but in part', as the Oxford tr. puts it). The Kneales have interpreted this as showing that Aristotle *did* recognize the difference between the two cases – that the universal affirmative is only 'partially convertible' (1962: p. 58). This certainly suggests that the logical relation between *Aab* and *Iba* is *weaker* than that between *Eab* and *Eba*, as captured by the modern logician in the conditional rather than biconditional. However, what Aristotle goes on to say about the *third* rule undermines this interpretation. (IC), like (EC), is also a case where the stronger *bi*conditional can be used, so one would have thought that Aristotle would also talk of it being 'necessarily convertible in its terms'. Yet he treats it like the *second* rule:

it too is only 'convertible as particular' ('in part', as the Oxford tr. again puts it). Aristotle's own understanding of a difference is characterized in terms of the *status* of the consequent, that is, whether it is universal or particular, *not* in terms of the strength of the relation between the antecedent and consequent. The division is thus between (EC) on the one hand and (AC) and (IC) *together* on the other, *not* the distinction we wanted. (The Oxford tr. may well be responsible for the Kneales' undue charity, since the phrase 'in part' is certainly misleading.) The conclusion can only be, then, that Aristotle did *not* understand the real logical difference here.

The modern logician would explain the difference *semantically*. The Kneales, for example, in using the phrase 'strictly equivalent' to characterize the relationship between *Eab* and *Eba*, and *Iab* and *Iba*, state that 'particular affirmative and universal negative statements are *convertible* without alteration of sense' (1962: p. 57). Can Aristotle be attributed the notion of 'sense' suggested here? Aristotle could certainly have provided a purely *logical* account of the difference between the rules here (in terms of *Eab* necessarily converting to *Eba*, *Iab* necessarily converting to *Iba*, *and vice versa*; *Aab* necessarily converting to *Iba*, *but not vice versa*), but given that he did not even do this, *a fortiori* he cannot have had a *semantic* conception of the difference. If the notion of 'sense' here is introduced to characterize (or 'explain') logical equivalence, then since Aristotle failed to appreciate logical equivalence – as opposed to mere logical transition or inference – he cannot have been in a position to start thinking about 'sense'.

Admittedly, though, Aristotle's text can easily suggest otherwise – if read too quickly in the light of modern logic. In a parenthetical remark in chapter 5 of Book II, Aristotle states that 'the premiss "*B* applies to no *A*" [*Eab*] is the same as "*A* applies to no *B*" [*Eba*]' (*hē ... autē protasis to B mēdeni tō A kai to A mēdeni tō B huparchein*; 58a27-9). Does this imply an understanding of logical equivalence? Once again, any positive answer is soon undermined. In the next but one chapter, he specifically *denies* that *Iab* and *Iba* are the same, even though, as in the case of *Eab* and *Eba*, the one necessarily follows from the other: 'it is necessary, if *C* belongs to some *B*, that *B* should belong to some *C*. But it is not the same that this should belong to that, and that to this: but we must assume besides that if this belongs to some of that, that belongs to some of this' (59a10-13; Oxford tr.). I shall return to what is 'assumed' here shortly, but the message is clear. Since Aristotle again exhibits his *different* treatment of the two pairs *Eab* / *Eba* and *Iab* / *Iba*, it remains the case that he cannot be attributed a proper conception of logical equivalence.

So what *did* Aristotle understand by the 'identity' of *Eab* and *Eba*? The most plausible suggestion that I can offer (and any view must be regarded as underdetermined by the text) makes use of the possibly revealing point that in the passage where Aristotle defines his three rules of conversion (25a14-23) he in effect *proves* the second and third rules by appeal to the

first.[23] Given his epistemological conception of proof, then, one might conjecture that Aristotle regards the first rule as more 'primitive and immediate' than the others, a rule, that is, that cannot itself be demonstrated. Perhaps in calling *Eab* and *Eba identical*, Aristotle is voicing this view. Two propositions are identical, according to Aristotle, if and only if the transition from one to the other is *immediate*.[24] The thought would be this: if the transition from *P* to *Q* requires some mediating assumption, then *Q* cannot be the same as *P*, since *Q* partly depends on the additional input. This is just what Aristotle thinks happens in the case of the third rule. Though also a 'necessary conversion', this rule, unlike the first rule, requires proof: some extra assumption is needed in the transition from *Iab* to *Iba*.

What *is* this assumption? Aristotle writes that 'we must assume besides that if this belongs to some of that, that belongs to some of this' (59a12-13). But this, of course, is precisely the rule that we are seeking to prove. Surely Aristotle is 'begging the question' in just the way he condemns – 'proving by means of itself that which is not self-evident' (65a24-5)? If the interpretation being offered is correct, though, what Aristotle means is that the rule must be *proved* from the more primitive first rule: the conversion is not itself an 'immediate' inference.[25] However, this merely displaces the objection that Aristotle is 'begging the question' to a different level, since it is precisely the status of the rule that is at issue. Aristotle's thesis that basic principles (in this case identities) must be *non*-demonstrable, whilst not in itself incoherent, does seem – in this particular case – to be leading him astray. That the third rule *can* be proved is no reason to deny that it encapsulates an equivalence. It is Aristotle's *epistemological* assumption, then, that appears to have blocked a *semantic* understanding.

Even Aristotle's (correct) view that *Eab* and *Eba* are identical, however, might be thought to raise a difficulty for syllogistic theory. Consider, for example, the pair of syllogisms *Celarent* (*Ebc*, *Aab* $\vdash$ *Eac*) and *Cesare* (*Ecb*, *Aab* $\vdash$ *Eac*). If Aristotle had regarded *Ebc* and *Ecb* as identical, then he would presumably not have distinguished the syllogisms, yet the former is a first figure and the latter a second figure syllogism. Michael Frede has suggested that Aristotle 'should not really want to say' that *Eab* and *Eba* are identical, noting that several of the moods would then be collapsed together (1987a: p. 114; cf. Appendix 1). However, this problem disappears once we recognize again his conception of proof. *Eab* and *Eba* are identical, according to Aristotle, because the transition from one to the other is 'immediate', but the syllogisms *Celarent* and *Cesare* are not themselves identical, since the transition is 'mediated' by the first rule of conversion. (The situation is thus the same as in the case of the third rule.) Whilst *Celarent*, as a first figure syllogism, is *non*-demonstrable, *Cesare* requires proof – the 'reduction' to *Celarent* proceeding in one step by simple conversion.[26] The difficulty for syllogistic theory is removed, then, once we appreciate that Aristotle's conception of identity was *epistemological* rather than semantic; and if we wish to talk of a correlative notion of 'sense' (two propositions having the same 'sense' if they are 'identical'), then that too must be understood as epistemological. Aristotle cannot be attributed a *semantic* conception of sense.

Aristotle's (relative) semantic naivety can be further illustrated by considering one of the standard objections to his logical system. According to (AS) above (itself derivable from (AC) and (IC), which Aristotle states), *Aab* ('All *A's* are *B*') entails *Iab* ('Some *A's* are *B*'). *Aab* must therefore carry *existential import* (it must imply that there is at least one *A*), in which case it cannot be the *contradictory* of *Oab* ('Some *A's* are not *B*'), as assumed in the traditional square of opposition. Two propositions are *contradictories* if they can neither both be true nor both be false, but are merely *contraries* if they cannot both be true but may both be false (cf. *De Interpretatione*, 17b16ff.). Since *Aab* and *Oab* can both be false in the case where there are no *A's*, they are *contraries*, not contradictories. Now although this objection can be defused once it is accepted that the system as a whole *presupposes* that there are *A's* and *B's* (see §2.4 and Appendix 1), Aristotle's failure to recognize the problem does suggest a lack of semantic sensitivity.

It seems fair to conclude, then, that Aristotle did not appreciate the semantics of his logical system, even though the system itself is coherent. This is not to say that Aristotle had no semantic intuitions at all, since he talks about the meanings of terms on a number of occasions. He does, for example, grasp the significance of the *scope* of the negation term.[27] But he did exhibit only a partial understanding of the semantic relations between syllogistic propositions. The failure to recognize the difference between his three rules of conversion suggests that he had no genuine conception of logical equivalence, and this implies that he lacked a semantic conception of sense. As we shall see in the next chapter, this conception had to await the work of Frege.

1.5 From Aristotle to Frege

I have considered certain aspects of syllogistic theory in some detail in this chapter, for reasons that will soon become clear. I have also mentioned some of the developments in logic, both formal and philosophical, from the time of its birth in the *Prior Analytics*, and further elements in the story will emerge in the chapters that follow; but it may be helpful here to provide a synopsis of the main events up to the time of Frege.[28]

Although syllogistic theory was the dominant force in logic until the middle of the last century, this is not to say that there were no advances at all in the twenty-two centuries that passed between Aristotle's death in 322 BC and the publication of Frege's *Begriffsschrift* in 1879. We have already mentioned the other important development in antiquity, the emergence of propositional logic in the work of the Stoics. As we saw in §1.2, Aristotle himself presupposed rules of propositional logic in the (theoretical) proofs he offered of imperfect syllogisms, and whilst he should not be condemned for failing to formulate these, this does suggest that there is a form of reasoning that is more basic than syllogistic argumentation – more basic in the sense that its codification does not itself presuppose any of the syllogistic forms of inference.[29]

Chrysippus (*c.* 280-207 BC) is generally regarded as the father of Stoic logic, though his work was shaped by the interest in dialectic transmitted by the Megarians from Zeno of Elea. Dialectical argument took such forms as 'If *P* then *Q*; but not *Q*; therefore not *P*'; and this stimulated debate about the nature of conditionals. Chrysippus formalized a number of valid inferences in propositional logic, though our knowledge of what these were can only be reconstructed from later commentators such as Alexander of Aphrodisias and Sextus Empiricus (who both flourished in the 2nd century AD). The Stoics were also more sensitive than the Peripatetics to the semantic features of language, which we might use their own term in calling *semainomena*. They distinguished, for example, between a sentence and what that sentence means or signifies – the *lekton* – such that two sentences, verbally different, may express the same *lekton*. Such a conception can be seen as a distant ancestor of Frege's later notion of a *thought* – what is expressed by a sentence on a given occasion of use – though a crucial difference is that *thoughts*, unlike *lekta*, also involve abstraction from the context of utterance.[30]

Aristotelian and Stoic logic were considered rivals in the centuries that immediately followed their emergence, for the reasons outlined in §1.3; but a process of fusion gradually took place, culminating in the work of Boethius (*c.* 480-524). Boethius provided Latin translations of Aristotle's logical works, and wrote several commentaries on them, and these were the main source for the revival of Aristotelianism after the Dark Ages. In the Middle Ages, logic established itself as a fundamental discipline, with syllogistic theory providing its framework. Abelard (1079-1142) is the major figure in early scholasticism, and Duns Scotus (*c.* 1266-1308) and William of Ockham (*c.* 1285-1349) are two of the more well-known logicians of the later period. Abelard seems to have been the first to address the issue of existential import and the coherence of the traditional square of opposition (see Appendix 1). This developed into the elaborate theory of *suppositio*, concerning the various kinds of 'suppositions' involved in using singular and general terms. Medieval logicians were hampered by failing to distinguish between the semantic roles of names and predicates, and it was only with Frege's introduction of function-argument analysis that the distinction was finally clarified.[31] Abelard was also responsible for stimulating discussion in the other main debate of scholastic logic – the theory of *consequentiae*. Abelard used the term 'consequentiae' to refer to conditional propositions, but it came to apply to any piece of valid reasoning.

Once again, great ingenuity was exhibited in drawing subtle distinctions between various kinds of *consequentiae*, but the theory was generally seen as supplementing syllogistic theory rather than embedding it in a broader framework, so that no comprehensive theory emerged in the work of any one logician. Furthermore, many thinkers only succeeded in demonstrating the difficulty involved in trying to analyse statements of generality more complex than those with which Aristotle had dealt without an adequate notation for quantification.

The frustration that this must have caused, and the elaborate distinctions that were drawn, were partly responsible for the disillusionment with logic that was increasingly felt as the movements of the Renaissance gained ground. The humanists of the late 15th and 16th centuries tended to reject logic as arid and sterile, and turned instead to classical literature – to Cicero and Plutarch rather than Aristotle – for inspiration. Those humanists who did write on logic, most notably, Ramus (1515-72), valued it purely pedagogically, as providing the framework for the classification and presentation of the knowledge that was then being recovered from antiquity. But this soon gave way to outright hostility, as the scientific revolution of the 17th century gathered momentum. Two particular features of this deserve mention. Firstly, from Francis Bacon (1561-1626) onwards, there was a growing emphasis on the role of empirical observation in the attainment of knowledge, and on inductive rather than deductive methods. Secondly, the emergence of mathematical physics in the work of Galileo (1564-1642) suggested a more powerful intellectual tool for the understanding of the world than syllogistic theory.

It was this suggestion that Descartes (1596-1650) incorporated so influentially in the new outlook that heralded the birth of modern philosophy. What may be noted here is the revised view of the roles of analysis and synthesis that this embodied. As traditionally conceived, *analysis* involved the working back to epistemologically primitive truths, and *synthesis* involved the presenting of truths in a chain of deductions from the primitive ones. Synthesis was regarded, in the Aristotelian tradition, as the more important – exemplified not only in syllogistic theory but also, paradigmatically, in Euclid's geometry. According to Descartes, however, synthesis merely reveals what 'is contained in what has gone before', and 'does not show how the thing in question was discovered'.[32] It is *analysis* that enables us to discover things; so that syllogistic theory, associated purely with synthesis, was seen as of little scientific value.

Rejection of syllogistic theory on epistemological grounds became a familiar feature of 17th- and 18th-century philosophy, the passage quoted from Locke's *Essay* in §1.3 capturing the prevailing view. It is only Leibniz (1646-1716) who stands out in this period as someone who appreciated the importance of logical theory, as well as some of the technical and philosophical problems it involved. According to Leibniz, analysis and synthesis simply reflected the two directions of movement along the same deductive chain, so that there was no genuine conflict between a 'logic of discovery' and a 'logic of proof'. Although, like the medieval logicians, he too failed to recognize that the scope of logic required *expansion* rather than mere supplementation, he did write extensively on logic and the philosophy of logic, and his vision of a *characteristica universalis*, or logical language, inspired many later thinkers, not least of all Frege himself.[33] I shall say something about this Leibnizian influence in the next chapter, and more about the issue of analysis in chapter 5.

Although his own understanding of formal logic was unsophisticated, in distinguishing between *analytic* and *synthetic* propositions, and *a priori* and *a posteriori* propositions, Kant (1724-1804) shaped philosophical debate about logic and mathematics irreversibly.[34] Within this Kantian framework, Leibniz might be characterized as holding the view that both logic and arithmetic are systems of *analytic a priori* truths.[35] Kant himself, however, whilst retaining the conception of logic, rejected the Leibnizian view of arithmetic, largely for the epistemological reason that it seemed to allow no room for the informativeness of arithmetic, and he regarded it instead as a system of *synthetic a priori* truths. The possibility of such truths, although fundamental to Kant's critical philosophy, has proved highly contentious, and in the 19th century, John Stuart Mill (1806-73), working within the British empiricist tradition, rejected both the Leibnizian and Kantian conceptions, and argued for a third position – that arithmetic consists of empirical generalizations that are *synthetic a posteriori*. It was within this Kantian framework that Frege's own project was conceived, his aim being, as we shall see in chapter 3, to substantiate the original Leibnizian view.

Despite Leibniz's pioneering attempts to link logic and mathematics, it was only in the 1840s that the 'algebra of logic' finally germinated. Boole (1815-64) is often regarded as the father of mathematical logic, with the publication of *The Mathematical Analysis of Logic* in 1847, but, as we shall see in the next chapter, this must not be allowed to obscure the far more substantial advance that Frege made in 1879, when the two parts of Boole's calculus were incorporated into one powerful and comprehensive theory, justifying Frege's claim to be the real founder of modern logic. De Morgan (1806-71), Peirce (1839-1914) and Schröder (1841-1902), amongst others, all contributed to the algebra of logic, and Frege was not only aware of these developments but also wrote several papers comparing his own system with the achievements of his contemporaries.[36]

As well as refinements of Boolean algebra, the latter third of the 19th century also witnessed a revival of Kantianism in reaction to the various forms of 'scientific philosophy' that had become dominant in the middle of the century – empiricism, materialism, naturalism, psychologism – themselves responses to the excesses of German idealism regarded as culminating in Hegel's work.[37] The Kantian dichotomies of form and content, philosophy and science, the *a priori* and the *a posteriori*, were once again emphasized. Within this scheme, the Leibnizian vision of a *characteristica universalis* became the vision of a formal logical language adequate for representing the objective contents of scientific thought. Mathematics may have been seen from the 17th century as providing the necessary framework for empirical science, but with logic (in its widest sense) now being elevated into this role, construing mathematics as part of logic seemed a natural step to take. Logicism – the thesis that arithmetic is reducible to logic – was endorsed by Lotze (1817-81) in his *Logik* of 1874, which had at least some influence on Frege's own work.[38] Frege's motivation as well as his technical ability to actually pursue a demonstration of the logicist thesis were also dependent on the developments in mathematics in the 19th century, as we shall see in chapter 3. But even though the climate was conducive, Frege's transformation of logic, which made such a demonstration feasible, was still a remarkable achievement. I discuss some of the details of this transformation in the next chapter.

2. Frege's 'Begriffsschrift'

Originality. – Not that a man sees something new as the first one to do so, but that he sees something old, familiar, seen but overlooked by everyone, *as though it were new*, is what distinguishes true originality. ...

Error of philosophers. – The philosopher believes that the value of his philosophy lies in the whole, in the building: posterity discovers it in the bricks with which he built and which are then often used again for better building: in the fact, that is to say, that that building can be destroyed and *nonetheless* possess value as material.

(Nietzsche, 'Assorted Opinions and Maxims' (1879), §§200-1, in *Human, All Too Human*, p. 261.)

Despite the developments sketched in the last section, it is really only Frege who truly ushered in the age of modern logic, with the publication of his *Begriffsschrift* in 1879, which refuted once and for all the view that logic had essentially emerged fully formed in the *Prior Analytics*. The classic statement of that view had occurred in Kant's *Critique of Pure Reason*, where it had been claimed that 'since Aristotle ... logic has not been able to advance a single step, and is thus to all appearance a closed and completed body of doctrine' (B viii). Whilst there is indeed a sense in which syllogistic theory is both coherent and self-contained, as we have seen, it is plainly absurd to suggest that it provides 'an exhaustive exposition and a strict proof of the formal rules of *all* thought' (ibid., B ix; my emphasis). Frege showed definitively how wrong this was; in his work, logic took not just a step forward but a huge leap.

As the initial reviews indicate, however, the importance of the *Begriffsschrift* was not immediately appreciated.[1] The main criticism, voiced by Schröder (1880) in particular, was that Frege had simply reproduced, rather poorly, the Boolean system. This induced Frege to write a lengthy paper called 'Boole's logical Calculus and the Concept-script' (*BLC*), where he compared in some detail his system with Boole's, pointing out the advantages of his own. However, no one wished to publish this, and even a much shorter version (*BLF*) was also rejected. But a far less technical paper, 'On the Scientific Justification of a Conceptual Notation' (*SJCN*) was eventually published, and this was followed by 'On the Aim of the "Conceptual Notation"' (*ACN*), originally given as a lecture.[2] In the next two sections, I shall consider these papers, together with the *Begriffsschrift* itself, in an attempt to clarify both the Leibnizian aim of his