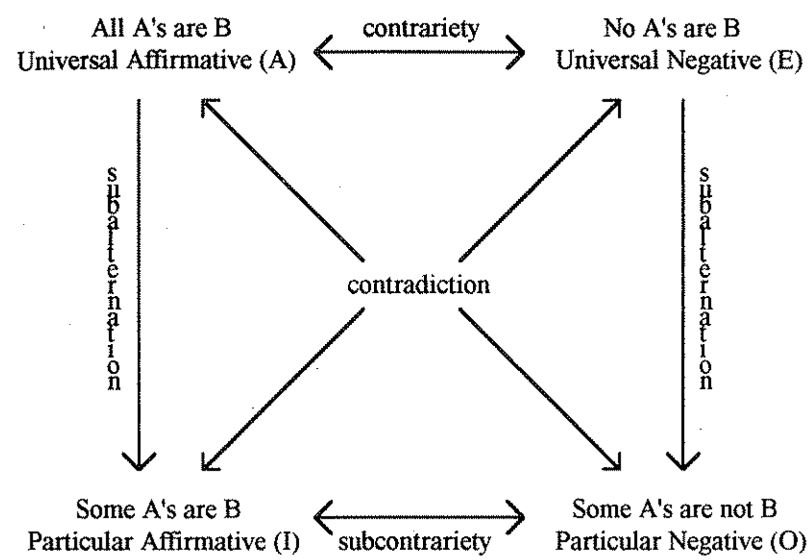
APPENDIX 1

Syllogistic Theory

Syllogistic theory is concerned with the systematic elucidation of the logical relations between the four types of proposition – of *quantity* either universal or particular and of *quality* either affirmative or negative – contained in the traditional square of opposition (as first described by Aristotle in *De Interpretatione*, 7, 10):



Following Smiley (1973) and Lear (1980), I formalize 'All *A's* are *B*' as '*Aab*', 'No *A's* are *B*' as '*Eab*', 'Some *A's* are *B*' as '*Iab*' and 'Some *A's* are not *B*' as '*Oab*'. (The most common alternative formalization in the literature is '*AaB*', '*AeB*', '*AiB*' and '*AoB*', respectively, the first four vowels of the alphabet traditionally being used to distinguish the four forms.) The relationships embodied in the square of opposition may be defined as follows:

(CD) Two propositions stand in the relation of *contradiction* if they can neither both be true nor both be false. The following are thus implicit rules of syllogistic theory: (a) 'From *Aab* infer not *Oab*'; (b) 'From not *Aab* infer *Oab*'; (c) 'From *Eab* infer not *Iab*'; (d) 'From not *Eab* infer *Iab*'.

(CR) Two propositions stand in the relation of *contrariety* if they cannot both be true but may both be false. This results in the following implicit rule: 'From *Aab* infer not *Eab*'.

(SC) Two propositions stand in the relation of *subcontrariety* if they cannot both be false but may both be true. The implicit rule derived here is thus: 'From not *Iab* infer *Oab*'.

(SA) Two propositions stand in the relation of *subalternation* if one proposition cannot be true without the other being true, but the latter may be true without the former being true. The latter is termed the *subaltern* of the former. In this case we arrive at the two traditional rules of subalternation: (a) 'From *Aab* infer *Iab*'; (b) 'From *Eab* infer *Oab*'.

In addition to these two rules of subalternation, and the rules of conversion discussed in §1.4, the following rules of *obversion* – relating propositions of the same *quantity* – were also recognized:

(OB) Two propositions stand in the relation of *obversion* if they have the same subject and quantity, but differ in quality, and possess predicates that are the negations of one another. Formalizing the predicate 'not-*b*' as '*b**', we thus have the following four rules: (a) 'From *Aab* infer *Eab**'; (b) 'From *Eab* infer *Aab**'; (c) 'From *Iab* infer *Oab**'; (d) 'From *Oab* infer *Iab**'.

A *syllogism* may be defined as an argument with two premisses and a conclusion, each proposition taking one of the above four forms. In addition, as noted in §1.1, for the conclusion to establish that a predicate term ('*P*') applies to a subject term ('*S*'), the two premisses must be linked by a 'middle term' ('*M*'), which can occupy either the subject or the predicate position in each premiss. Using the traditional formalization, then, with the conclusion expressed as '*SxP*' (where '*x*' stands for either '*a*', '*e*', '*i*' or '*o*'), the first premiss will take the form of either '*MxP*' or '*PxM*' and the second premiss the form of either '*SxM*' or '*MxS*'. This means that syllogisms fall into one of four *figures*, representing the four possible arrangements of the premisses:

Figures

I	II	III	IV
MxP	*PxM*	*MxP*	*PxM*
SxM	*SxM*	*MxS*	*MxS*
SxP	*SxP*	*SxP*	*SxP*

Aristotle himself, however, recognized only *three* figures, and this has generated much controversy. The most convincing explanation has been provided by Patzig (1968: §25). If one appreciates that Aristotle's main concern in distinguishing the figures is with the *distribution* of the middle term, then it does seem plausible to suppose that there are only three possibilities: either the middle term in the two premisses occupies both subject positions, or both predicate positions, or one of each (cf. *Prior Analytics*, I 4-6). Aristotle recognized the validity of the syllogisms of the fourth figure, but treated them as derivatives of the first figure syllogisms, in effect collapsing together the first and fourth figures. However, he failed to realize that this conflicts with the other criteria he provides for first figure syllogisms (as given in I 4), so that Aristotle cannot be totally absolved of confusion on this issue.

Accepting that there are four figures, then, and that each of the three propositions in each figure takes one of four forms, there are 4 x 4^3, or 256, possible arrangements. Of these, only 24 are valid syllogisms, and these have traditionally been classified into *moods* and ingeniously given names that reflect their structure and method of proof. 19 of these moods constitute the heart of the syllogistic system, the other 5 being the 'weakened' or 'subalternated' derivatives of one of the main moods (in accordance with one of the two rules of subalternation). The moods are arranged in their figures, and the five subaltern moods italicized, as follows:

Moods

(I)	**Barbara**	*Barbari*	**Celarent**	*Celaront*	**Darii**	**Ferio**
MxP	Abc	Abc	Ebc	Ebc	Abc	Ebc
SxM	Aab	Aab	Aab	Aab	Iab	Iab
SxP	Aac	Iac	Eac	Oac	Iac	Oac

(II)	**Cesare**	*Cesaro*	**Camestres**	*Camestrop*	**Festino**	**Baroco**
PxM	Ecb	Ecb	Acb	Acb	Ecb	Acb
SxM	Aab	Aab	Eab	Eab	Iab	Oab
SxP	Eac	Oac	Eac	Oac	Oac	Oac

(III)	**Darapti**	**Felapton**	**Disamis**	**Datisi**	**Bocardo**	**Ferison**
MxP	Abc	Ebc	Ibc	Abc	Obc	Ebc
MxS	Aba	Aba	Aba	Iba	Aba	Iba
SxP	Iac	Oac	Iac	Iac	Oac	Oac

(IV)	**Bamalip**	**Calemes**	*Calemop*	**Dimatis**	**Fesapo**	**Fresison**
PxM	Acb	Acb	Acb	Icb	Ecb	Ecb
MxS	Aba	Eba	Eba	Aba	Aba	Iba
SxP	Iac	Eac	Oac	Iac	Oac	Oac

Traditionally, the 19 main moods were learnt with the help of a mnemonic verse, the following being one of the more familiar:

Barbara, *Celarent* primae, *Darii Ferio*que;
Cesare, *Camestres*, *Festino*, *Baroco* secundae;
tertia grande sonans recitat *Darapti*, *Felapton*,
Disamis, *Datisi*, *Bocardo*, *Ferison*; quartae
sunt *Bamalip*, *Calemes*, *Dimatis*, *Fesapo*, *Fresison*.

(The most common alternative has *Bramantip*, *Camenes* and *Dimaris* for *Bamalip*, *Calemes* and *Dimatis*, but the names are otherwise the same; cf. Keynes, 1906: §258). In addition, the names themselves cleverly revealed the formulation and proof of each syllogism. The three vowels represented the basic form of each of the two premisses and conclusion (i.e. whether it was an A, E, I or O proposition; ***Ferio*** standing for ***Ebc***, ***Iab*** ⊢ ***Oac***, for example); and the consonants indicated how the syllogism, in either the second, third or fourth figures, was to be 'reduced' to a first figure syllogism. The first letter showed to what first figure syllogism it was to be reduced (***Cesare*** being reduced to ***Celarent***, for example), and the letters immediately following the vowels suggested the precise method of proof, in accordance with the following instructions:

c (*conversio*): suppose that the conclusion does *not* hold (i.e. take its negation) and proceed by *reductio ad absurdum*, showing how it contradicts the premiss denoted by the preceding vowel on the basis of the syllogism reached with the other premiss ('changing round' the syllogism).

m (*muta*): transpose the premisses.

p (*per accidens*): 'convert' the premiss denoted by the preceding vowel into its subaltern ('conversion by weakening').

s (*simpliciter*): 'convert' the premiss denoted by the preceding vowel in accordance with one of the two rules of conversion 'From *Eab* infer *Eba*' and 'From *Iab* infer *Iba*' ('simple conversion').

Three examples may help clarify these instructions:

(BC) Proof of *Bocardo* (*Obc*, *Aba* ⊢ *Oac*), illustrating *conversio*: Suppose not *Oac*, then *Aac* (its negation), in which case, with *Aba*, *Abc* can be reached (by *Barbara*); but not *Abc*, since *Obc* (its contradictory), so the initial supposition is false; hence *Oac*.

(CM) Proof of *Camestres* (*Acb*, *Eab* ⊢ *Eac*), illustrating transposition and conversion *simpliciter*: If *Acb* and *Eab* imply *Eac*, then *Eab* and *Acb* imply *Eac* (by transposition). But if *Eab*, then *Eba*, and if *Eac*, then *Eca*, so *Eba* and *Acb* imply *Eca*, which is just the form of *Celarent*.

(FL) Proof of *Felapton* (*Ebc*, *Aba* ⊢ *Oac*), illustrating conversion *per accidens*: If *Aba*, then *Iab* (its subaltern); so with *Ebc*, *Oac* (by *Ferio*).

How coherent is the syllogistic system that emerges from this account? As noted in §1.4, one of the main objections concerns the *existential import* of syllogistic propositions. There appears to be *no* plausible assignment of existential import that preserves *all* the relationships embodied in the traditional square of opposition – whether explicit, as formulated in (CD) to (SA) above, or implicit, as captured in (OB) above. By 'plausible assignment' I mean one which meets the following intuitive constraint (which effectively underlies all treatments of the issue):

(PA) If any syllogistic proposition *possesses* existential import, then (at the very least) it is the I form; and if any syllogistic proposition *lacks* existential import, then (at the very least) it is the E form.

If this constraint is met (which rules out ten of the sixteen possible sets of assignments), and we ignore all cases where only one of the four forms of syllogistic proposition either possesses or lacks existential import (which rules out another two possible sets – there are eight such cases altogether, all of which, at the minimum, invalidate one of the two relations of contradiction), then there are just four sets of assignments of existential import to syllogistic propositions that require consideration:

(A) *All four types of proposition – A, E, I and O – carry existential import.* This runs into the problem that whenever the subject term fails to refer, all four propositions are *false*, invalidating the rules of *contradiction* and *subcontrariety* – see (CD) and (SC) above.

(B) *No syllogistic proposition carries existential import.* This results in the breakdown of the rules of *contradiction* and *contrariety* – see (CD) and (CR) above – since, whenever the subject term fails to refer, all four propositions are *true*.

(C) *The I and O forms, but not A and E forms, carry existential import.* In this case, whenever the subject term fails to refer, '*Aab*' and '*Eab*' are *true*, and '*Iab*' and '*Oab*' *false*, invalidating the rules of *contrariety*, *subcontrariety* and *subalternation*. (Surprisingly, on the *reverse* assignment – the A and E forms, but not I and O forms, possessing existential import – *all* the relationships are preserved, but this fails to meet the intuitive constraint expressed in (PA).)

(D) *The A and I forms, but not E and O forms, carry existential import.* This assignment preserves all four relations *explicitly* captured in the traditional square of opposition, but at the cost of destroying the *implicit* relation that intuitively holds between propositions of the same quantity, as reflected in the rules of *obversion* – see (OB) above. In particular, it is implausible that 'Some *A's* are *B*' should carry existential import, but not 'Some *A's* are not *B*'. (The reverse assignment also preserves all relations except obversion, but fails to meet the intuitive constraint.)

What possibilities are there for restoring coherence to syllogistic theory? The two most promising strategies (conserving at least the *explicit* relations) consist in modifying (A) and (D):

(A*) *Syllogistic theory as a whole presupposes that all terms have reference.* The strategy here is to assign existential import not to each syllogistic proposition individually, as in (A), but to the system as a whole. In a sense, this constitutes a synthesis of (A) and (B), the point being that the existential import is *presupposed* rather than *entailed* by each proposition. If reference-failure is ruled out from the start, then *all* the traditional rules are valid. The only drawback is that syllogistic theory now appears to be even more limited, applying only to non-empty terms.

(D*) *Existential import is assigned only to the A and I forms, but the O form is construed as 'Not all A's are B' rather than 'Some A's are not B'.* This strategy still invalidates the rules of *obversion*, but it removes their intuitive appeal, and preserves the relations explicitly captured in the square of opposition.

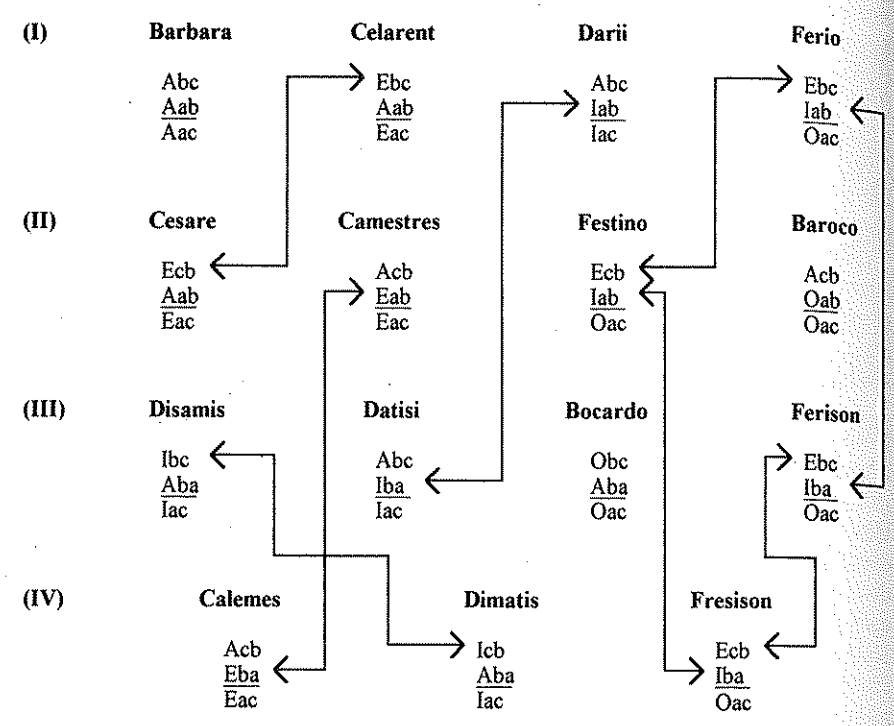
(A*) accords better with Aristotle's *scientific* intentions (see §1.3 above), though the issue as to which strategy Aristotle himself favoured will remain controversial, since he seems not to have appreciated the problem and the texts themselves appear inconsistent (for the alternative view, that (D*) best captures Aristotle's position, see Thompson, 1953, and Wedin, 1978). However, (A*) represents the most familiar contemporary strategy (see e.g. Strawson, 1952: ch. 6, §7; Lukasiewicz, 1957: p. 4; Kneale, 1962: pp. 58-60; Smiley, 1962: §4; Lemmon, 1965: pp. 175-7; Patzig, 1968: §3; Corcoran, 1974a: §3; Ackrill, 1981: pp. 92-3). It was Strawson who first presented this strategy as based on the notion of *presupposition* (though it was anticipated by certain previous writers – see e.g. Johnson, 1892: pp. 24, 28; 1921: p. 139; Nelson, 1946), but Strawson himself only initially proposed that all *subject* terms had to refer. Thinking through the rules of conversion, however, should quickly convince us of the need for *all* terms to refer. It is this strategy that is pursued in §2.4, with the additional suggestion that syllogistic theory *can* be successfully applied in fictional contexts – providing the 'domain of discourse' is specified.

(D*) also represents a coherent strategy, and was familiar in medieval times, but has suffered relative neglect in recent years. (For details of the history of the question of existential import, see Church (1965) and Wu (1969). The strategy is noted by Prior (1962: p. 169; 1976: p. 123), who reports finding it in Keynes' extensive examination of the issue (1906: Part II, ch. 8). But Keynes, in fact, only mentions the strategy very briefly in a footnote (p. 226, n. 1), where it is immediately ruled out because of its invalidation of obversion. More recently, however, the strategy has been adopted by Clark (1980), who raises objections to (A*) (see esp. chs. 5-6), though they are not, I think, insuperable. Thom (1981: §§18 and 30) also discusses both strategies, but bases his full system on (A*).) There is nothing *incoherent* about construing O-propositions as 'Not all *A's* are *B*', nor in thereby dispensing with the rules of *obversion* (since their intuitive appeal does only depend on a specific interpretation of negative propositions), but if the strategy is seen as *more* plausible than (A*), because it does not make *any* presuppositions, then this is mistaken. This strategy too *presupposes* that we understand the various syllogistic propositions in a certain way; and if this is allowed, then (A*) might still seem preferable, for the reasons stated (preserving not only the explicit relations, but also obversion; and lending itself, in any case, to application in fictional contexts).

How does syllogistic theory fare when 'translated' into Fregean logic? Although all four strategies can be formalized, it is (C) that represents the construal that Frege himself implicitly adopted. This results in the destruction of all but the relation of contradiction within the traditional square of opposition. (It is worth noting that when Frege reproduced the square at the end of Part I of the *Begriffsschrift* (which has been excised from the translation in *TPW*), revealing his own formalizations of the four types of syllogistic proposition, he misleadingly *retained* the invalidated relations of contrariety, subcontrariety and subalternation. This suggests that at the time he had failed to think through the semantic implications of his new logical system, providing further support for the view that Frege developed that system for other reasons than any perceived difficulties in syllogistic theory, and only *later* sought to justify it – cf. ch. 2, n. 14.) However, as suggested in §2.3, this was not the only effect of the Fregean formalizations. The failure of subalternation invalidates all five subaltern moods as well as *Darapti*, *Felapton*, *Bamalip* and *Fesapo*, leaving a residue of fifteen moods. Given the equivalences between *Ebc* and *Ecb* (reflecting what we may call the *E rule of conversion*), and between *Iab* and *Iba* (reflecting the *I rule of conversion*), however, the number can be reduced further, since several of the moods then collapse into each other. This results in just eight 'moods' – *Barbara*, *Celarent / Cesare*, *Darii / Datisi*, *Ferio / Festino / Ferison / Fresison*, *Camestres / Calemes*, *Baroco*, *Disamis / Dimatis*, and *Bocardo* – as is shown by the diagram on the following page (arrows indicating equivalences in accordance with the E and I rules of conversion).

But even these eight 'moods' can be reduced further, by using the simple rules of *substitution* and *transposition of premisses* (both were taken for granted by Aristotle, and the latter was codified in medieval times). For example, '*Icb*, *Aba* ⊢ *Iac*' (*Dimatis*) is equivalent to '*Aba*, *Icb* ⊢ *Iac*' (by tranposition), which is of the same logical form as '*Abc*, *Iab* ⊢ *Ica*' (substituting '*c*' for '*a*' and '*a*' for '*c*'), which is equivalent to '*Abc*, *Iab* ⊢ *Iac*' (by the I rule of conversion), which is *Darii*. With these additional rules, we arrive at just six 'moods': *Barbara*, *Celarent / Cesare / Camestres / Calemes*, *Darii / Datisi / Disamis / Dimatis*, *Ferio / Festino / Ferison / Fresison*, *Baroco* and *Bocardo*. This result is interesting, because it shows what is meant by

Residual Moods



the 'reduction' of the *C-moods* to *Celarent*, the *D-moods* to *Darii*, and the *F-moods* to *Ferio* – they all end up as logically equivalent, where this can be proved using only the E and I rules of conversion, and the rules of substitution and transposition. *Baroco* and *Bocardo*, however, cannot be so reduced to *Barbara* – they require a *reductio ad absurdum* argument (proceeding by *conversio*; see (BR) of §1.2, where *Baroco* is proved, and (BC) above, where *Bocardo* is proved). However, if we also admit the rules of *obversion* (see (OB) above), equally legitimate within Fregean logic, then these two moods too can be 'reduced away'. *Baroco* (*Acb*, *Oab* ⊢ *Oac*), for example, is equivalent to '*Ecb**, *Iab** ⊢ *Oac*' (by two of the rules of obversion – (a) and (d) of (OB) above), which is of the form '*Ecb*, *Iab* ⊢ *Oac*' (substituting '*b*' for '*b**'), which is equivalent to '*Ebc*, *Iab* ⊢ *Oac*' (by the E rule of conversion), which is *Ferio*. Likewise, *Bocardo* can be 'reduced' to *Darii* (by transposition, obversion of the O-propositions, substitution, and conversion). So does this then mean that the two moods were wrongly named? It remains the case that they cannot be 'directly reduced' (by merely conversion, substitution and transposition), but they do turn out

to be logically equivalent to *Ferio* and *Darii*, rather than *Barbara*, which does indicate a certain confusion.

So have we then shown that there are just four basic 'moods', as Aristotle himself suggested – the four moods of the first figure? Unfortunately, however, if we allow ourselves the use of obversion, further reductions can be made. For *Ferio* can be 'reduced' to *Darii* (by obversion of the E- and O-propositions, and substitution), and *Celarent* can be 'reduced' to *Barbara* (by obversion of the E-propositions, and substitution). In the end, then, there are just *two* logically distinguishable 'moods' – *Barbara* (with the *C-moods*) and *Darii* (with the rest). But from a Fregean perspective, this need hardly surprise us. For what these correspond to are the only two patterns of inference that are valid within the 'translated' syllogistic system:

(AA) $(\forall x)(Bx \rightarrow Cx), (\forall x)(Ax \rightarrow Bx) \vdash (\forall x)(Ax \rightarrow Cx)$.
(AE) $(\forall x)(Bx \rightarrow Cx), (\exists x)(Ax \,\&\, Bx) \vdash (\exists x)(Ax \,\&\, Cx)$.

(AA) involves only universal propositions, and (AE) contains one premiss that is universal, and one premiss and a conclusion that are particular. Of course, such a basic division is already present in syllogistic theory, and with obversion, all the 'reductions' are provable; but it took Fregean logic, with the emphasis on logical rather than grammatical form, to highlight the simplicity of the structure. The appreciation of this might well have dismantled the (grammatically sound but logically shaky) traditional edifice of moods and figures, but Aristotle himself would have been impressed with the reduction.