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Uncertainty in Healthcare Policy Decisions: An Epidemiological Real Options Approach to COVID-19 Lockdown Exits

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Uncertainty significantly influences healthcare policy decisions, especially regarding lockdown exits amid pandemics. This paper examines different uncertainties affecting stay-at-home and essential workers concerning COVID-19 exposure. Using an epidemiological real options model, we assess how uncertainty impacts the decision to end lockdown without causing disease resurgence. Results indicate higher policy-change value lengthens the lockdown delay. Greater shock correlation between stay-at-home and mask-only populations reduces this value, leading to earlier lockdown exits. The findings bridge discrepancies between public health recommendations and policymakers' actions. Integrating options theory enables more informed decision-making by considering uncertain risks and outcomes.

Keywords: Public Health Strategy; Uncertainty; Real Options; Mitigation Measures; Lockdown

JEL classification: C61, H12, I18

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1. Introduction

The global health and economic challenges instigated by the COVID-19 pandemic have been unparalleled. Lockdowns, demanding people to stay home and avoid outdoor activities, became a key strategy in many countries to contain the virus spread (Giordano, et al., 2020; Greenstone & Nigam, 2020). However, these lockdowns entail significant economic and social costs (Fink, et al., 2022; McCoy, et al., 2021; Singh, et al., 2020; Thunström, et al., 2020; Meyerowitz-Katz, et al., 2021; Kugler, et al., 2023; Adams-Prassl, et al., 2020), needing a delicate equilibrium between virus control and maintaining socio-economic stability (Rowthorn & Maciejowski, 2020; Baqaee, et al., 2020; Broughel & Kotrous, 2021; Acemoglu, et al., 2020; Thunström, et al., 2020). Government decisions regarding lockdown exit are complicated by future uncertainties about the virus spread and the irreversible nature of some politically-motivated decisions (Pindyck, 1988; Dixit, 1989a; Dixit, 1989b; Dixit & Pindyck, 1994). As such, it could be beneficial to delay lockdown exit, allowing time to gather more information about the disease progression and the efficacy of potential mitigation strategies (Oraby, et al., 2021; Misra, et al., 2022; D'angelo, et al., 2021).

Real options theory, often applied in evaluating healthcare decisions, suggests that under uncertain conditions, delaying the decision until more information becomes available has an economic value (Palmer & Smith, 2011; Claxton, 1999; Meyer & Rees, 2012; de Mello-Sampayo, 2014; de Mello-Sampayo, 2015; de Mello-Sampayo, 2022; Zivin & Neideill, 2009; Smith, 2007). Viewing lockdown exit as an option with potential to reduce ongoing and future pandemic damages, the real options framework can guide the optimal timing for lockdown exit lockdown. Davies and Grimes (2022) demonstrate the importance of considering option values in policy decisions. Locking down early created the option to eliminate COVID-19, which may not have been available without an early decision. Delaying lockdown would have preserved the option to enter later, but this option was less valuable due to the (potentially large) health losses accrued in the interim and the government's aversion to such losses. This paper contributes to the literature by applying the real option theory to an epidemiological model of disease spread, demonstrating that the higher the uncertainty in the future progress of infection and the larger the share of the population under lockdown, the longer the optimal lockdown exit should take.

Epidemiological models have proven extremely useful in guiding decision-making during the COVID-19 pandemic. These models posit that infection rates depend on interactions between susceptible and infectious populations. This paper uses a variation of the Susceptible, Infected, and Recovered (SIR) epidemiological model (Avery, et al., 2020; Blackwood & Childs, 2018), the SEIR model (Hethcote, 2000; Li & Muldowney, 1995), which includes an additional compartment for infected but non-contagious individuals (E). The SEIR model has been instrumental in decision-making related to COVID-19 (He, et al., 2020; Wood, et al., 2021; Al-Raeei, et al., 2021; Mwalili, et al., s.d.; Reno, et al., 2020; Ng & Gui, 2020), estimating the effectiveness of control strategies like quarantine and hospitalization (He, et al., 2020), and simulating and predicting virus spread in different countries (Al-Raeei, et al., 2021).

In 2020, when the COVID-19 pandemic was first declared, there was a lack of specific knowledge and literature about the virus. However, the SEIR model has been used in many epidemics before, and the lack of specific knowledge about COVID-19 at the time did not render the model useless. Rather, researchers had to make assumptions and estimations based on the available data to construct the model and interpret its results. In this way, the SEIR model and similar models provided a valuable tool for decision-makers in the early stages of the pandemic, allowing them to evaluate different intervention strategies and predict the potential impact of the virus on populations (Wood, et al., 2021).

However, there can be a wide range of uncertainty in the estimates produced by epidemiologic models due to factors such as model assumptions, data quality, and stochasticity in the disease transmission process (Gugole, et al., 2021; Duarte, et al., 2023). Sensitivity analysis and uncertainty quantification techniques can be used to evaluate model uncertainty and identify the most sensitive parameters (Gugole, et al., 2021). Risk-based cost-benefit analysis and quantitative risk assessment can further support decision-making by assessing the potential risks and benefits of different interventions and providing transparent and objective information (Fischhoff, 2015; Wang, et al., 2021; Rocha-Melogno, et al., 2023; Yasutaka, et al., 2022; Robinson, et al., 2021; Fu, et al., 2023; Kanga, et al., 2021). Effective risk communication is crucial during a pandemic to provide timely and accurate information to the public and ensure the success of public health interventions (Varghese, et al., 2021; Abrams & Greenhawt, 2020). Overall, integrating epidemiological models, uncertainty analysis, real option theory, risk assessment, and risk communication can enhance decision-making in managing the COVID-19 pandemic and exiting lockdown.

This manuscript began to be written in 2021 when there were not many stochastic models for COVID-19 available in the literature. The objective was to introduce stochasticity into these models, thereby presenting a more authentic portrayal of the variations inherent in the disease transmission process (Romiti & Talerico, 2021). The growing importance of uncertainties within the context of the pandemic has progressively surfaced in research related to the COVID-19 outbreak. Previous works addressing the impact of uncertainty on the spread of disease have explored individual behaviors and the externalities of individual decision-making regarding treatments, vaccinations, and social distancing (Avery, et al., 2020; Stock, 2020; Jia & Chen, 2021). Olivares and Staffetti (2021) consider mitigation measures entails a degree of uncertainty and quantify the effects of the uncertainty about the application of social distance actions and testing of susceptible individuals on the disease transmission.

Although the impact of uncertainty in disease spread on the optimal timing of mitigation measures has been previously studied, this paper fills a gap in the literature where an artificial separation between traditional epidemiological models and those used within the real options framework has typically existed. This paper derives a real option model from a SEIR framework to analyze the probability and timing of lockdown exit. In the context of the pandemic, the decision to exit lockdown is uncertain because it depends on the progression of the virus and the effectiveness of control measures. We motivate our analysis by assuming that lockdown is required because there is no perfect vaccine, and the scale of infection is too large for an effective testing and tracing (Cleevely, et al., 2020). During lockdown, the government mandates the use of masks in closed spaces, crowded outdoor areas, and public transportation, especially for workers unable to stay at home due to the nature of their jobs, such as healthcare workers and refuse collectors. Uncertainty in disease spread is incorporated by assuming there is variability in the exposed population, driven by external forces. For example, fluctuations in temperature and climate have been shown to modify the infection rate (Chen, et al., 2021). The uncertainty affecting the exposed population to COVID-19 is characterized differently for those under stay-at-home orders versus those who are not. Our approach is general, but for illustration we focus on the optimal timing of exiting lockdown using data for Portugal from March 3, 2020, to July 12, 2021. As such, this study helps to clarify the often-contradictory research in the field of public health mitigation measures.

The vaccination in Portugal began to be administered free of charge in January 2021 in health centers, homes, and occupational health services, and the vaccine is taken in two dosages. The plan defined priority groups and the respective stages of vaccination. In mid-2021 only the elderly population (>65 years old) took the first dose, and the rest of population began taking the dosage (Portuguese Government, 2020). Though, there were a considerable part of the population not vaccinated in July 2021, there are limitations for not taking into consideration the COVID-19 vaccination. The extent of protective immunity is uncertain, which limits our accuracy and ability to provide reliable predictions about the pandemic (Holmdahl & Buckee, 2020). Marinov and Marinova (2022) show that considering vaccination is central in making accurate predictions. A SIRV model which considers individuals as Susceptible, Infected, Removed, and Vaccinated was employed to study the dynamics of the COVID-19 pandemic and the effect of vaccination in controlling the spread of the disease.

A review of several exit strategies, including phase-wise exit, hard exit, and constant cyclic patterns of lockdown, concluded that phase-wise exit is the optimal exit strategy (Misra, et al., 2022). Studies suggest that adopting a multi-pronged strategy consisting of different approaches may be effective in exiting lockdowns (Oraby, et al., 2021; Misra, et al., 2022; D'angelo, et al., 2021; Freiberger, et al., 2022). Well-timed lockdowns can split the peak of hospitalizations into two smaller distant peaks while extending the overall pandemic duration (Oraby, et al., 2021). In this paper, the end of lockdown means that commercial and social life is permitted, schools are re-opened, the work-at-home ends, while using masks continue to be required. The principal objective of this paper is to know the timing to exit lockdown so as to avoid another wave of the disease. The decision when to exit the lockdown is a dynamic optimization problem because it involves finding the best solution over time. In the context of pandemic, the aim is to minimize harm, such as a resurgence of the cases, while maximizing the beneficial effects of lifting the lockdown, such as societal normalcy. This paper's main value-added resides in the concrete application of the dynamic stochastic model to a problem in public health decision-making, i.e. to governments waiting to exit the lockdown without causing a resurgence of cases.

Our findings indicate that uncertainty concerning potential infected population tends to postpone the decisions to end lockdowns. Additionally, when a substantial proportion of individuals comply with stay-at-home directives (thereby reducing the number of people using solely masks), and when mitigation measures are effectively curbing disease transmission, less decisions to end lockdowns are observed. This can be attributed to the inherent challenge in reversing lockdown lifting decisions. Furthermore, the transition from lockdown to a situation where the public is allowed to live with fewer restrictions, limited to mask-wearing in enclosed spaces and public transportation, is expected to occur sooner and become more probable as the population staying at home and just using masks behave similarly, consequently reducing the associated uncertainty regarding the decisions to end lockdowns.

The rest of the paper is organized as follows. In *Methods* we introduce the deterministic SEIR model and derive the dynamic stochastic model to examine when decision-makers should exit lockdown under uncertainty. In *Results* the simulations of the timing and probability of lockdown exit are analyzed. Then we *discuss* the results and present the *Conclusion*.

2. Methods

In this section we develop an epidemiological-based real option model to illustrate the role that uncertainty can play in determining the decision regarding when to exit the lockdown without provoking a resurgence of the disease.

2.1 Epidemiological Model

An appropriate model for a COVID-19 pandemic where there is a considerable post-infection incubation period in which the exposed person is not yet contagious, is the Susceptible-Exposed-Infected-Removed (SEIR) model (Hethcote, 2000; Li & Muldowney, 1995; Avery, et al., 2020; Avery, et al., 2020; Blackwood & Childs, 2018). Ignoring births and deaths from non-COVID-19 causes, the population at the start of the epidemic is normalized to 1, and at each time t, the population is compartmentalized based on their infection status: susceptible (S) to infection; exposed (E), i.e., likely to be infected when exposed to the virus, but not yet contagious; infected (I) and contagious; and removed (R), who have recovered or died from the disease (See Fig. 1). The interaction between the susceptible and infected is permitted through the productive contact SI. Defining β as the rate of susceptible to become infected, the increase in the number of infected individuals is given by the product of the per capita rate at which a susceptible contracts infection times the number of susceptible individuals, i.e. βIS. The progression rate from exposed (latent) to infected is given by δ and the removal rate is γ. Variables and parameters used in the deterministic SEIR model are shown in Table I.

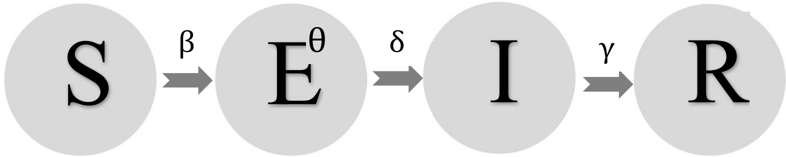

Figure 1. Standard SEIR model of disease propagation. Susceptible (S); Exposed (E); Infected (I); and Removed (R). β transmission coefficient; 1/δ is the mean incubation period; γ is the removal rate; θ is mitigation measure.

Initially, under certain common assumptions (Jones, 2007), i.e. individuals who are infected remain infectious until they recover or die, infected individuals who recover acquire complete immunity, and ignoring uncertainty, the SEIR model translates in a system of four differential equations to relate the rates at which the population moves from one stage to another, where the time unit is one day:

$$\frac{dS}{dt} = -\beta IS, \tag{1}$$

$$\frac{dE}{dt} = \beta IS - \delta E^{\theta}, \tag{2}$$

$$\frac{dI}{dt} = \delta E^{\theta} - \gamma I, \tag{3}$$

$$\frac{dR}{dt} = \gamma I, \tag{4}$$

where β gives the transmission coefficient of the infected cases to the susceptible; 1/δ is the mean incubation period; γ is the rate at which infected individuals cease to be infectious or die; and θ is the degree of the social distancing measure taken by the government.

The exponential surge of the pandemic raises awareness of individuals who have greater potential for disease transmission, i.e. "COVID-19 super-spreaders", who infect several individuals during community gatherings or indoor sports events. The gathering creates the opportunity for increased pathogen spreading. With the rapid spread of the disease COVID-19, epidemiologists have applied various levels of social distancing to "flatten the curve" of the infected population. The notion of flattening the curve by social distancing is no more than reducing opportunities for a pathogen to spread. Social distancing or mitigation measures (θ) range from careful handwashing, use of masks, and in the extreme, the government implements the lockdown on economic and social life.

At the optimum, the government uses social distancing or mitigation measures to stop the infected population from increasing, i.e. $\frac{dI}{dt} = 0$, Equation (3) is now given by:

$$I_t = \frac{\delta E_t^{\theta}}{\gamma}, \tag{5}$$

where the parameter that captures the flattening effect of the curve due to the social distancing measures is assumed to be θ, $0 < \theta \leq 1$. If θ is one, there are no mitigation measures, whereas if it is close to zero, the population has to stay at home.

Table I. Deterministic SEIR Model - Variables and Parameters

Notation	**Definition**
S_t	Share of population susceptible to infection at time *t.*
E_t	Share of population exposed, likely to be infected when exposed to the virus, but not yet contagious at time *t.*
I_t	Share of population infected and contagious at time *t.*
R_t	The share of population recovered and the share of population who have died from the disease at time *t.*
β	The transmission coefficient of the infected cases to the susceptible.
δ	The rate at which latent individuals become symptomatic.
γ	The removal rate at which infected individuals cease to be infectious or die.
θ	The degree of the social distancing measure taken by the government, when θ is one, there are no mitigation measures, whereas if it is close to zero, the population has to stay at home.

2.2 Uncertainty in the Spread of the Disease

There is uncertainty in the future levels of infection due to environmental and demographic noise associated with the transmission process for infection. The decision-maker is faced with the following choice: should mitigation measures be stopped immediately or should the decision-maker wait to learn more about the progression of the epidemic? Waiting allows the decision-maker to determine whether the level of infection gets worse or better over time.

Traditional net-present-value (NPV) analysis would advocate stopping mitigation measures providing the benefit is greater than the cost. However, due to uncertainty in disease dynamics combined with the irreversibility of the decisions, there is value in delaying treatment so as to learn more about the progress of the disease (Dixit & Pindyck, 1994). That is, there is a value associated with the option to remove mitigation measures.

To include uncertainty into the decision-making approach, we assume that the level of exposed, E, can be described by a stochastic process[1]. Traditionally, the geometric Brownian motion which assumes that the mean level of infection grows exponentially has been used to characterized stochastic processes (Dangerfield, et al., 2018). While such an assumption is a good approximation in the early stages of the epidemic, it does not capture the slowdown in the rate of infection as the level of infection becomes large due to the limited number of susceptible individuals. Thus, we assume the exposed population follows a Driftless stochastic process, i.e. our best guess of the exposed population for tomorrow is what we have at the present.

Suppose that the government applies the lockdown (L) as the social distancing measure (θ), i.e. a mandate for citizens to stay-at-home (h), and for citizens who because of the nature of their work (e.g. nurses, doctors, refuse collectors) cannot stay at home, the government mandates the use of masks (m) in closed spaces, outside crowded spaces, and in public transportation. Since the factors that affect the exposed population behave differently

[1] Stochastic process, also known as a random process accounts for certain levels of unpredictability or randomness.

between stay-at-home to not-stay-at-home citizens, we must characterize exposed citizens who stay at home and those using masks differently. As our best guess of the exposed population at time t+1 (E_{t+1}) is what we have at time t (E_t), we assume that E follows a Driftless stochastic process (also called Martingale[2]) by which:

$$dE_h = \sigma_h E_h dz_h, \tag{6}$$

$$dE_m = \sigma_m E_m dz_m, \tag{7}$$

where the subscripts h and m denote stay-at-home and use-of-masks social distancing measures, respectively. The increment of the Wiener process[3] is $dz = \varepsilon_t \sqrt{dt}$ and $\varepsilon_t \rightarrow N(0,1)$, $E(\varepsilon_t,\varepsilon_s) = 0$ *for* $s \neq t$.

Equations (6) and (7) imply that the current value of the random shock[4] is known, but the future values are log-normally distributed with a variance growing linearly with the time horizon. Exposed population's variability, σ, can be interpreted as the uncertainty affecting the Exposed population, with $\sigma_h, \sigma_m \geq 0$, $E(dz_h, dz_m) = \rho dt$ and ρ is the correlation coefficient between the random shocks affecting Exposed citizens subject to stay-at-home measure and use-of-masks.

The exposed population under lockdown becomes an average of both exposed at home and using masks weighted by the share of population assigned to each group:

$$E_L = E_h^{\phi} E_m^{\psi}, \tag{8}$$

where E_L denotes the exposed population under lockdown, ϕ and ψ are the shares of population of stay-at-home and not-stay-at-home, respectively, with $\phi + \psi = 1$.

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[2] Martingale process: the fluctuations in exposed population are a sequence of random variables for which, at a particular time, the conditional expectation of the next value in the sequence is equal to the present value, regardless of all prior values.

[3] Wiener process is a stochastic process. The initial value of all Wiener processes is 0; past values of these processes do not influence any future changes in their value (this is what makes the processes stochastic).

[4] The random shock exposes individuals to a random change.

Now, assume that the government is searching for a way to exit the lockdown without provoking a resurgence of the disease. For that we analyze the timing and probability of exiting the lockdown under uncertainty. Variables and parameters used in the model derived from the epidemiological model accounting for the uncertainty in the progression of the virus are shown in Table II and Fig. A1 in Appendix A.II.

Table II. Epidemiological-Based Real Option Model - Variables and Parameters

Notation	Definition
E_t	Share of population exposed, likely to be infected when exposed to the virus, but not yet contagious at time *t*.
I_t	Share of population infected and contagious at time *t*.
δ	The rate at which latent individuals become symptomatic.
γ	The rate at which infected individuals cease to be infectious or die.
θ	The degree of the social distancing measure taken by the government.
m	The social distancing measure is using masks in closed spaces, outside crowded spaces, and in public transportation.
h	The social distancing measure is to stay at home.
L	The social distancing measure is the lockdown, i.e. stay at home and if not possible, use masks.
E_h	The Exposed population to Covid-19 that stays at home or in lockdown. E_h follows a Driftless stochastic process.
E_m	The Exposed population that uses facial masks in closed spaces, outside crowded spaces, and in public transportation. E_m follows a Driftless stochastic process.
E_L	The Exposed population under lockdown, i.e. average of both exposed at home and using masks weighted by the share of population assigned to each social distancing measure. E_L follows a geometric Brownian motion.
I_L	The Infected population to Covid-19 that is in lockdown.
I_m	The Infected population that uses facial masks in closed spaces, outside crowded spaces, and in public transportation.
$i = \frac{I_L}{I_m}$	The relative infected population, i.e. population infected that is in lockdown relative to infected population that just uses masks.

i^*	Threshold at which the value from exiting lockdown immediately is maximal, that is, the value at which one should end lockdown immediately.
$\frac{\tilde{E}_h}{\tilde{E}_m}$	Critical value of the ratio of the exposed population defines the line that divides the $(E_{h,}E_m)$ space into two regions: one in which it is optimal to exercise the change of social distancing measures option and the other in which it is not.
ϕ	Share of population that stays at home.
ψ	Share of population that do not stay at home and just uses masks.
σ_h	Uncertainty affecting exposed population who to stay-at-home.
σ_m	Uncertainty affecting exposed population who use-of-masks.
ρ	The correlation between the random shocks affecting Exposed citizens who stay-at-home and use-of-masks.
μ	Risk free discount rate. Rate to adjust future outcomes of healthcare interventions to present value, i.e. rate at which time is discounted.

2.3 The Lockdown Exit Decision

Under uncertainty the decision to adopt different mitigation measures is based on expected present discounted value of the infected population so that, when comparing two strategies to mitigate the spread of the disease, the optimal strategy is simply the one with the lowest expected present discounted value. See Appendix A.I.1 for the derivation of the expected infected population under each policy. The optimal choice regarding the social distancing measures depends exclusively on the relative value of the infection attained before and after the change of social distancing measures has been undertaken, that is, on the ratio $i = I_L/I_m$, where I_L is the infected population to COVID-19 that is in lockdown, I_m is the infected population that uses facial masks in closed spaces, outside crowded spaces, and in public transportation. Using standard methods from dynamic programming, the value of the option to exit lockdown, $f(i)$, gives the threshold at which the value from exiting lockdown immediately is optimum, i^*, that is, the value at which one should end lockdown immediately (see Appendix A.I.2). It represents the boundary between the *continuation region*, in which it is optimal not to exercise the option, and the region in which the lockdown ends.

Proposition 1: *The government will end the lockdown when the expected infected population just using masks is lower than that attained when in lockdown.*

Proof: See Appendix A.I.3 .

Equation A11 in Appendix I, states that $i^* > 1$, meaning that the government will engage in relaxing the social distancing measures if $I_m < I_L$, i.e. the expected infected population when just using masks is lower than that attained when in lockdown. For values lower than i^* it is optimal not to end the lockdown. Conversely, the government should stop the lockdown. It follows i^* defines one region in which it is optimal to exercise the decision stop lockdown and the other in which it is not. This threshold value can easily be converted to a critical value in terms of exposed population.

Proposition 2: *The government will end the lockdown when the expected Exposed population staying at home is higher than that attained when just using masks.*

Proof: See Appendix A.I.4 .

The decision-maker will choose to relax the social distancing measures only if the Exposed population associated with staying at home exceeds that of a situation of only using masks, i.e.

$$\frac{E_h}{E_m} \leq \frac{\tilde{E}_h}{\tilde{E}_m} = \left\{\frac{\mu - \frac{\theta}{2}\left(\phi(\theta\phi-1)\sigma_h^2 + \psi(\theta\psi-1)\sigma_m^2 + 2\theta\rho\phi\psi\sigma_h\sigma_m\right)}{\mu - \frac{\theta(\theta-1)}{2}\sigma_h^2} \times i^*\right\}^{\frac{1}{\theta\psi}}, \quad (9)$$

where μ is the discount rate to adjust future outcomes of healthcare interventions to present value[5]; E_h denotes the exposed population that must stay at home; and E_m denotes the Exposed when subject to the use-of-masks measure. For values of the ratio $\frac{E_h}{E_m}$ lower than $\frac{\tilde{E}_h}{\tilde{E}_m}$ it is optimal not to relax the social distancing measures. Conversely, if the value of the ratio is greater than the critical value, the government should stop the lockdown. It follows that Equation (9) defines the line that divides the $(E_{h,}E_m)$ space into two regions: one in which it is optimal to exercise the change of social distancing measures option and the other in which it is not.

From the comparative statics shown in Appendix A.I.5 of Equation (9) if the uncertainty (σ^2) affecting exposed population is high, the decision-maker tends to prefer the lockdown, i.e. stay-at-home and use-of-masks. The more correlated (ρ) are the shocks affecting the exposed at home and using masks, the less the change of the policy option is worth. The reason is that the more correlated the shocks are, the more closely both Exposed processes move and so the lower the uncertainty that results from the switch from a situation in which both measures are applied to the population that only has to use masks. With regard to the discount rate (μ), a higher time preference increases the decision-maker's opportunity cost of not immediately stopping the stay-at-home measure. When more population is staying at home (high ϕ), the change of policy option is worth more. The lower the value of the mitigation policy (θ), i.e. the closer we are to full lockdown, the more the option is worth. This is because a change of policy always has some degree of irreversibility, if population

[5] Discounting seeks to consider the impact of time on how outcomes are valued. Typically, individuals prefer to consume a product or service now rather than delay that same consumption until sometime in the future. This reflects a positive rate of time preference, or discount rate.

starts using just masks (not staying at home) raises the society's overall risk of exposure. The reduced form of Equation (9) can be written as follows:

$$\text{Exit Lockdown} = f\begin{pmatrix} \sigma^2 & \rho & \mu & \phi & \theta \\ - & + & + & - & - \end{pmatrix}.$$

Thus, the model presented gives clear indications regarding the exit of lockdown decision under uncertainty. It predicts that the higher the uncertainty (σ^2) affecting the exposed population, the later the exit of lockdown is made, the higher the correlation (ρ) between the exposed population staying at home and using masks, the less valuable the option of exiting lockdown will be, and so the more exits of lockdown will be observed. With regard to the discount rate (μ), a higher time preference increases the decision-maker's opportunity cost of not immediately stopping the stay-at-home measure. Conversely, the higher the share of population staying at home (the lower the share of population just using masks) and the higher the effect of the mitigation measure (low θ), the less exits of lockdown one would expect to observe. This is because change of policy always includes some degree of irreversibility such that an increase in the share of population using just masks (not staying at home) raises the society's overall risk of exposure.

2.4 The Probability and Expected Time of Lockdown Exit

It is important for the decision-maker to know the expected time that will elapse until the decision of stopping the stay-at-home measure becomes optimal. Using standard properties of the Brownian motion and the lognormal distribution, (see Dixit (1993) and Øksendal (2003)) closed-form solutions for the probability $Q(\frac{E_h}{E_m})$ and expected time $T(\frac{E_h}{E_m})$ for the process $\frac{E_h}{E_m}$ to hit the barrier $\frac{\tilde{E}_h}{\tilde{E}_m}$ from any point inside the continuation region are given by:

$$Q\left(\frac{E_h}{E_m}\right) = \begin{cases} 1 & \text{if} \quad \sigma_m^2 \geq \sigma_h^2 \\ e^{\left[\frac{(\sigma_m^2-\sigma_h^2)\left[ln\left(\frac{\tilde{E}_h}{\tilde{E}_m}\right)-ln\left(\frac{E_h}{E_m}\right)\right]}{\sigma_h^2+\sigma_m^2-2\rho\sigma_h\sigma_m}\right]} & \text{if} \quad \sigma_m^2 < \sigma_h^2 \end{cases} \tag{10}$$

$$T\left(\frac{E_h}{E_m}\right) = \begin{cases} \infty & \text{if} \quad \sigma_m^2 \leq \sigma_h^2 \\ \frac{ln\left(\frac{\tilde{E}_h}{\tilde{E}_m}\right)-ln\left(\frac{E_h}{E_m}\right)}{(\sigma_m^2-\sigma_h^2)/2} & \text{if} \quad \sigma_m^2 > \sigma_h^2 \end{cases} \tag{11}$$

where $(\sigma_m^2 - \sigma_h^2)/2$ and $\sigma_h^2 + \sigma_m^2 - 2\rho\sigma_h\sigma_m$ are the drift and variance parameters of the process $\frac{E_h}{E_m}$, respectively. Equations (10) and (11) indicate that the probability and expected time until stopping the stay-at-home measure to become optimal depend on the relative variability of exposed population associated with stay-at home and using-masks. The lower is the uncertainty affecting exposed population staying at home relative to the population using masks, the higher is the probability that exiting lockdown will never become optimal.

3. Results

Simulation of the exposed and infected population from deterministic SEIR model described by Equations 2 and 3, for the Portuguese case, used data from different sources. The National Epidemiological Surveillance System (BI SINAVE) gathered retrospective data on 707,795.0 confirmed cases of SARS-CoV-2 / COVID-19 infection data between March 3, 2020 until July 12, 2021 (497 days). The data contain the information on symptoms and signs collected at the time of notification: Asymptomatic or Symptomatic. Regarding the health level of the confirmed cases, the data also indicated if the patient had comorbidities and died. The average age was approximately 51 years old for all patients. A summary of the dataset is shown in Table III. The mortality rate of the population is around 2% of the total. There were around 70% asymptomatic cases and 30% of symptomatic cases. From these data, we use the mortality rate that will be added to the rate that individuals cease to be infectious which is around 0,09 (Teles, 2020; Spinner, et al., 2020), to calculate the removal rate, i.e. $\gamma \sim 0{,}11$. From the literature on COVID-19, we used the rate at which latent individuals become symptomatic (δ) approximately 0,19 (Lauer, et al., 2020). The transmission coefficient of the infected cases to the susceptible (β) used was approximately 0,20 (Ferguson, et al., 2020; Chae, et al., 2020). Fig. 2 shows the effect on social distance measures in exposed and infected populations, Equations (2) and (3), respectively. When there are no social distancing measures ($\theta = 1$), the exposed and infected populations are represented by solid line curves. Assuming that the government imposes the use of facial masks, the exposed and infected populations are represented by dashed line curves. However, if the government increases the social distance measures, e.g. bans large public gatherings, the curves are now represented by the dotted line. In effect, under social distance measures the area under the curves in the model is spread over time (see Fig. 2), i.e flattened.

Table III. Descripted Statistics

Avg. Age	SE	Nr. Infected Cases	Nr. Deaths (%)	Sym/Asym (%)	
50.78	6.83	707,795	15,232 (2.2)	Symptomatic	218,715 (30.9)
				Asymptomatic	489,080 (69.1)

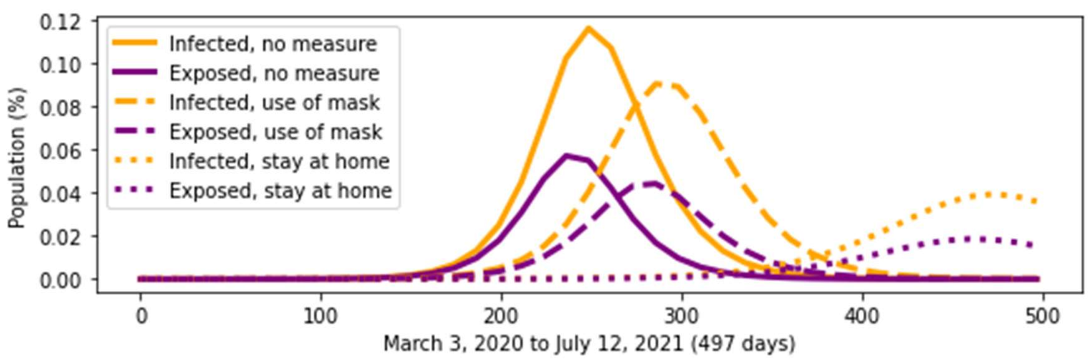

Figure 2. The Flattening Effect of Social Distancing on the Exposed and Infected under the deterministic model. When there are no social distancing measures (θ=1), the exposed and infected populations are represented by solid line curves. When the use of facial masks is imposed, the exposed and infected populations are represented by dashed line curves. If

the government increases the social distance measures, the curves are now represented by the dotted line.

Fig. 2 shows that with no mitigation measures, there would be approximately 12% of the Portuguese population infected by September 2020. If the face masks had been imposed from the beginning of the pandemic, the peak of the infection would be around 9% of the population in October 2020. Finally, the lockdown would lower the peak of the infected population to approximately 4% in March 2021.

Now, introducing uncertainty in the future path of the infected population, the values of the parameters considered in the benchmark case, as well as the ranges used in the simulations of the critical ratio as given by Equation 9, were also drawn from various sources. Table IV presents the range and the base value of each parameter according and its sources.

Table IV. Parameter values used in numerical simulations

Model parameter	Description	Base case (range)	Source
β	The transmission coefficient of the infected cases to the susceptible	0.20 ([0.1, 0.8])	(Ferguson, et al., 2020; Chae, et al., 2020)
δ	The rate at which latent individuals become symptomatic	0.19 ([0.1, 0.8])	(Lauer, et al., 2020)
γ	The removal rate at which infected individuals cease to be infectious or die.	0.11 ([0.05, 0.5])	(Teles, 2020; Spinner, et al., 2020) a)
θ	The degree of the social distancing measure taken by the government, when θ is one, there are no mitigation measures, whereas if it is close to zero, the population has to stay at home.	0.3 ([0.01, 0.9])	b)
ψ	Share of population that do not stay at home and just uses masks. During the	0.25 ([0.1, 0.9])	(Bureau of Labor

	coronavirus pandemic, about 1 in 4 employed people did not telework.		Statistics, 2020)
ϕ	Share of population that stays at home, $\phi = 1 - \psi$.	0.75 ([0.1, 0.9])	(Bureau of Labor Statistics, 2020)
σ_h	Uncertainty affecting exposed population who to stay-at-home on a scale of 0–1, with 0 representing no uncertainty and 1 representing high uncertainty.	0.3 ([0.1, 0.8])	c)
σ_m	Uncertainty affecting exposed population who use-of-masks on a scale of 0–1, with 0 representing no uncertainty and 1 representing high uncertainty.	0.5 ([0.1, 0.8])	c)
ρ	The correlation between the random shocks affecting exposed population who stay-at-home and use-of-masks.	0.5 ([-1, 1])	d)
μ	Risk free discount rate. Rate at which time is discounted.	0.03 ([0.02, 0.05])	(O'Mahony & Paulden, 2014)

a) includes the mortality rate derived from our dataset; b) Data on the degree of the social distancing measure taken by the government are not available, the benchmark value and the range of variation for both volatilities were picked arbitrarily. c) Since the data on the volatility of the exposed population who stay at home or not are not available, the benchmark value and the range of variation for both volatilities were picked arbitrarily. d) Since the data on the correlation are not available, the benchmark value and the range of variation for both volatilities were picked arbitrarily.

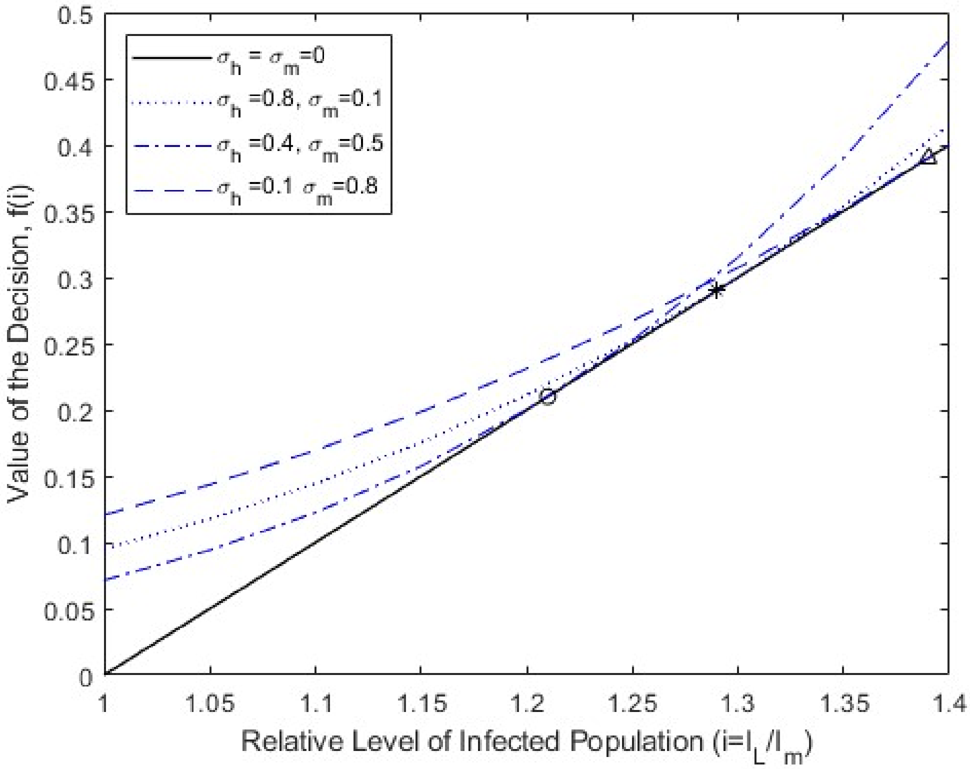

Figure 3. Value of the option to exit lockdown as a function of the relative infected population. The standard NPV, i.e. there is no uncertainty, is shown as a black line. Dash line is for the case when $\sigma_m = 0.8$ and $\sigma_h = 0.1$, and the threshold $* = 1.39$. Dotted line is the case when $\sigma_m = 0.1$ and $\sigma_h = 0.8$, and the threshold $\Delta = 1.29$. Dash-dotted line is for the case when $\sigma_m = 0.5$ and $\sigma_h = 0.4$, and the threshold $\circ = 1.21$. The other parameter values are given in Table IV.

Fig. 3 shows the value of the option, $f(i)$, to exit lockdown as a function of the relative infected population, $i = I_L / I_m$, for different levels of uncertainty (see Table IV for the variables and parameters used in the model). Providing the value of the option is greater than the NPV of immediate exit, there is value in retaining the option to exit, and so it is beneficial to wait. When the value of the option and the NPV is the same, there is no additional gain in waiting and so exit should be applied immediately. The value of exit at which $f(i)$ first equals the NPV, the threshold value of treatment, i^*, is the boundary between the waiting region and the immediate exit region and is also shown for each uncertainty level in Fig. 3. The highest thresholds are obtained when the uncertainty affecting infected under lockdown is very different than that affecting infected population just using masks. The lowest threshold appears when the uncertainty affecting infected in lockdown is closer to that affecting infected population just using masks.

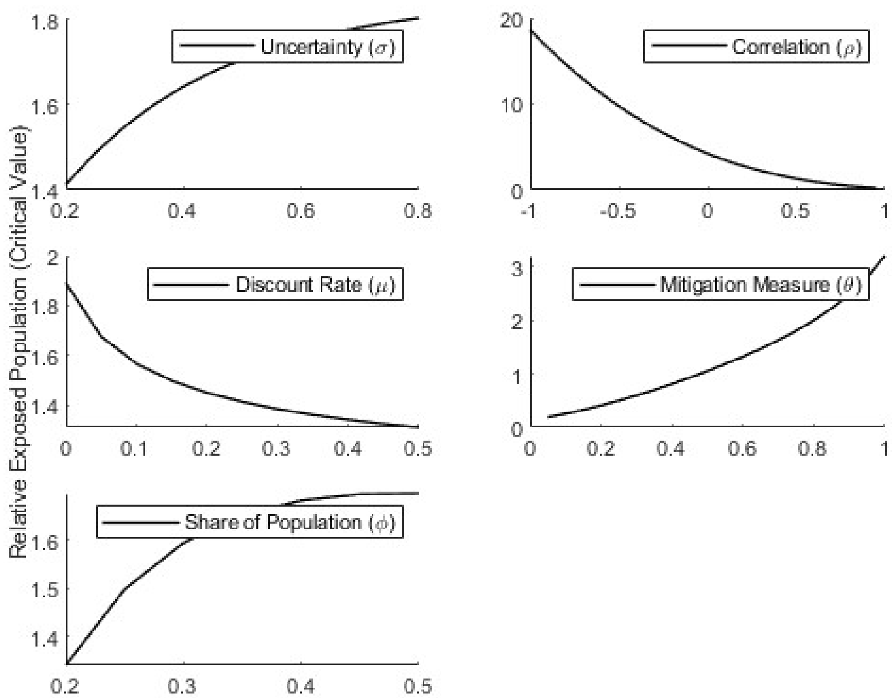

Figure 4. The critical value in terms of relative exposed population $\frac{\tilde{E}_h}{\tilde{E}_m}$. Plots showing the region in which exit should be applied immediately (above the line) and where exit should be delayed (below the line) for each of the variable in the model. Other parameter values are given in Table IV.

The implications for decision makers of the model results can be seen in plots of the critical value in terms of relative exposed population. Fig. 4 shows simulations of critical value, $\frac{\tilde{E}_h}{\tilde{E}_m}$, with respect to the model's variables (see Table IV for the variables and parameters used in the model) and it corroborates the comparative statics presented in Appendix A.I.5. The plots suggest that the higher the uncertainty (σ) affecting the exposed population, the later the exit of lockdown is made, therefore, when uncertainty is large, the higher the attainable threshold at which to exit lockdown. The higher the correlation (ρ) between the exposed population staying at home and using masks, the less valuable the option of exiting lockdown will be, and so the more exits of lockdown will be observed. The higher the discount rate (μ), the more the decision to stop the stay-at-home measure is worth. Conversely, the higher the share of population staying at home (ϕ) and the higher the effect of the mitigation measure (low θ), the more exits of lockdown one would expect to observe.

Fig. (5) and Fig. (6) illustrate respectively the impact of σ_m and $\frac{E_h}{E_m}$ on the expected probability and on the expected time of changing the mitigation measures, as given by Equations (10) and (11). The parameters of the model are calibrated with the values described in Table IV.

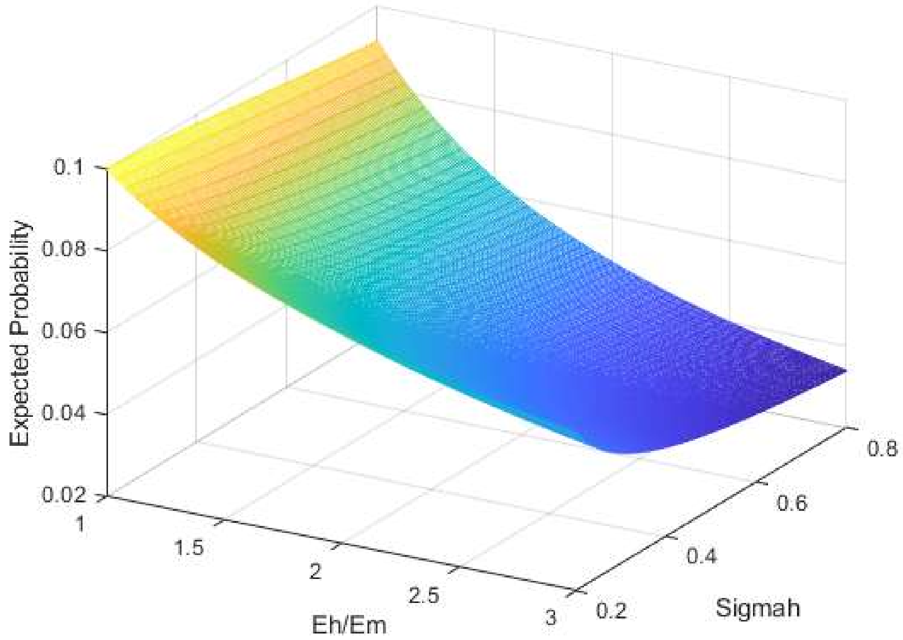

Figure 5. Expected Probability of Exiting Lockdown for Values of Uncertainty, σ_h, and Critical Values $\frac{E_h}{E_m}$. Other parameter values are given in Table IV.

Fig. 5 illustrates the impact of σ_h and $\frac{E_h}{E_m}$ on the probability of optimal policy change. When $\sigma_m^2 < \sigma_h^2$, the probability that $\frac{\tilde{E}_h}{\tilde{E}_m}$ will be hit in the future is decreasing in both σ_h and $\frac{E_h}{E_m}$. For relative low levels of $\frac{E_h}{E_m}$, i.e. $E_h \sim E_m$., the two processes behave similarly, the probability of optimal policy change rises as σ_h^2 converges to σ_m^2, that is, as the drift of $\frac{E_h}{E_m}$ converges to zero[6].

[6] For the limiting case where $\sigma_m^2 = \sigma_h^2$, even though the probability that the government will change the social distancing measures in the future is 1, the expected time for it to occur is infinite. The intuition behind this result is that if the drift of E_h / E_m is zero, long diversions away from the barrier $\tilde{E}_h / \tilde{E}_m$ might occur. Then, the probabilities for successfully longer hitting times not falling sufficiently quickly, and the expectation, which is the average of the possible hitting times weighted by their respective probabilities, diverges (Dixit, 1993).

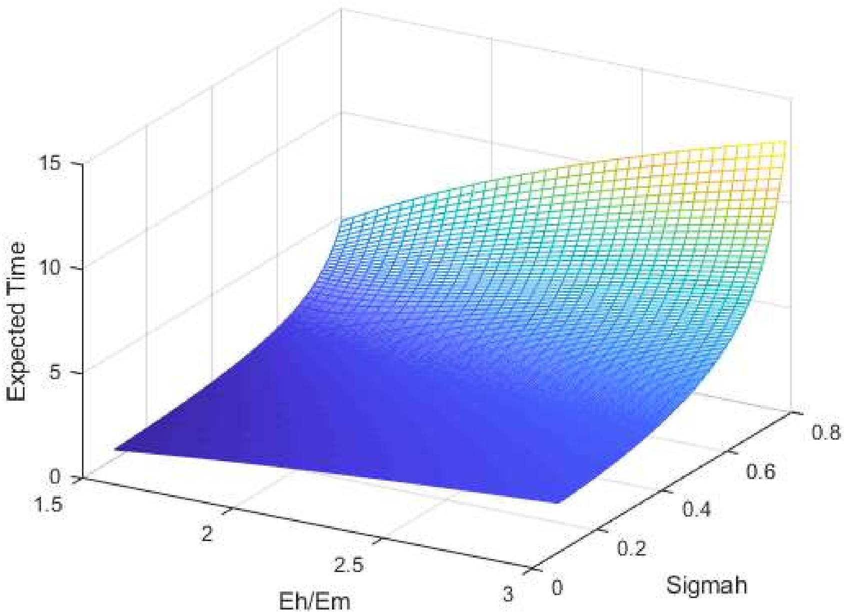

Figure 6. Expected Time of Exiting Lockdown for Values of Uncertainty, σ_h, and Critical Values $\frac{E_h}{E_m}$. Other parameter values are given in Table IV.

Fig. 6 simulates the effect of changes in σ_h and $\frac{E_h}{E_m}$ on the expected time for optimal policy change. It shows that the lower is σ_h and the lower is $\frac{E_h}{E_m}$, the sooner the policy change is expected to occur.

In summary, the lower the uncertainty affecting the exposed population being at home, the more likely policy change is to become optimal and the sooner it is expected to occur. Moreover, exiting lockdown becomes likelier and is expected sooner, the closer the two processes are, i.e. $E_h \sim E_m$, and so the lower the uncertainty that results from the switch from a situation in which social distancing measures of staying at home and using masks to one in which the population can live freely and just use masks in closed spaces and public transportation.

4. Discussion

Simulations presented in this paper and their underlying theory may clarify the timing and probability of governments' decisions to exit lockdown under uncertainty. When using the deterministic epidemiologic model, Fig. 2 shows lockdown is implemented in order to smooth the spread of the disease. Governments began planning to exit the lockdown but were always afraid of a new surge of infection, continuing to implement it after the flattening effect was accomplished (Rowthorn & Maciejowski, 2020). This paper extends the existing literature by applying the real option theory, a framework for decision-making under uncertainty, to an epidemiological model of disease spread. By incorporating uncertainty into the analysis, the study aims to determine the optimal timing for exiting lockdown. The real option approach allows for the evaluation of the economic value of deferring the decision to exit lockdown until more information becomes available (Fig. 3).

The implications for decision makers of the epidemiological-based real option model can be seen in policy plots (Fig. 4). The plots suggest that the higher the uncertainty (σ) affecting the exposed population, the later the exit of lockdown is made, therefore, when uncertainty is large, the higher the attainable threshold at which to exit lockdown. The more correlated (ρ) are the shocks affecting the exposed at home and using masks, the less the change of the policy option is worth. The reason is that the more correlated the shocks are, the more closely both exposed processes move and so the lower the uncertainty that results from the switch from a situation in which both measures are applied to the population that only has to use masks. With regard to the discount rate (μ), a higher time preference increases the decision-maker's opportunity cost of not immediately stopping the stay-at-home measure. When more population is staying at home (high ϕ), the change of policy option is worth more. The lower the value of the mitigation policy (θ), i.e. the closer we are to full lockdown, the more the option is worth. This is because a change of policy always has some degree of irreversibility, if population starts using just masks (not staying at home) raises the society's overall risk of exposure.

The uncertainty affecting exposed population discourages exiting the lockdown, as shown by Fig. 5. The stochastic model presented in this paper also illustrates that the more the change of policy option is worth, the greater is the impact of uncertainty on the delay to exit the lockdown (Fig. 6). Several key insights emerge. The existence of an option value means that a seemingly unpopular policy, such as the lockdown, may be the better choice when considering that decisions taken by governments may be irreversible for political reasons and in this case aggravated by the possibility of a resurgence of the virus. Optimal decision making requires a careful comparison of an unpopular social distancing measure today with a popular mitigation measure in the future. The intuition for these results is deepened when we recognize that one of the principal features driving our results is that the population exposed to the COVID-19 virus under different social distancing measures, such as the lockdown and just using face masks, have particular characteristics that make the change of policy uncertain. Exiting lockdown becomes likelier and is expected sooner, the more both populations (exposed in lockdown and exposed just using masks) behave similarly (Fig. 5), and so the lower is the uncertainty that results from the switch from a situation in which social distancing measures of staying at home and using masks to a situation in which the population can live freely and just use masks in closed spaces and public transportation.

A corollary of our results is that low-risk exposed population staying at home may exit lockdown as soon as possible, which corroborates previous studies that suggest targeted lockdowns in different groups. Acemoglu et al. (2020) examine targeted lockdowns in different groups: the young, the middle-aged, and the old and concluded that targeted policies can minimize both economic losses and deaths. As the costs of reversing the change of policy falls, it becomes more feasible to exit lockdown of the high-risk population.

It is important to acknowledge the limitations of the model proposed here. One limitation concerns overlooking endogeneity of parameters and heterogeneity in the SEIR model. First, our epidemiological model assumes that social distancing is determined by government. However, as the disease spreads, people voluntary start social distancing (Farboodi, et al., 2021; Toxvaerd, 2020; Campos-Mercade, et al., 2021). Reluga (2010) develops a SIR model in which agents take a social distancing action that reduces their probability of infection. Goolsbee and Syverson (2021) study empirically the endogenous social distancing. Their results suggest that ignoring endogeneity could lead researchers to misinterpret the effects on disease dynamics of government policies. Our results may overestimate the effects of mitigation measures imposed by government when compared with a policy of no social distancing measures.

Another limitation assuming a homogeneous population with uniform mixing, which may not be realistic in some contexts. The severity of symptoms, health outcomes, degree of infectiousness, and development of immunity vary among patients. These factors are subject to individual, spatial, and temporal heterogeneity. One of the striking features of COVID-19 is that mortality rates vary with age (Ferguson, et al., 2006). Therefore, the social and economic impact of policies vary with age. Several studies suggest that age-dependent policies can provide substantial gains relative to uniform policies (Acemoglu, et al., 2020; Rampini, 2020; Favero, et al., 2020). Ellison (2020) take a broader view of heterogeneity and suggest that those who use public transportation or frequent bars will have many more contacts than others in their age group. In age and spatially structured models the spread of diseases is less than in a homogeneous SIR-type model (Hébert-Dufresne, et al., 2020) and may lead to persistence of the disease (Britton, et al., 2020).

Finally, it is assumed that everyone is susceptible to infection, but there could be some people already vaccinated during the period under analysis since a vaccine became available. Furthermore, the virus mutated between March 2020 and July 2021, making the vaccines less effective. Analysis of the SARS-CoV-2 genome show different variants of the virus during the pandemic. In the beginning of 2020, the results of the study of Korber et al. (2020) suggested that the COVID-19 variant most common was more infectious than the strain that was dominant in Wuhan. Another limitation of our results is that our data consider all COVID-19 variants, but not the connections among them. Data from contact tracing has information about the source of infection and the resulting infections. As more data become available, contact tracing will become difficult at the peak of the epidemic, and the quality of the resulting data will diminish. Additional data might lead to poorer estimates (Ferretti, et al., 2020). These limitations should be considered when interpreting the model results and using them to inform decision-making.

Our model was developed in 2021, a pivotal year in the global fight against COVID-19. The key strength of the model is its ability to generate robust insights from relatively sparse data. During 2021, there was a lack of consolidated information and comprehensive literature on COVID-19. Despite these challenges, our model is able to extrapolate useful patterns, guiding the public health response during a critical period. Moreover, the potential utility of this model extends beyond the immediate retrospective analysis of the COVID-19 pandemic. While it has been specifically tailored to the unique circumstances of COVID-19, the model's core framework is sufficiently flexible to adapt to other viral outbreaks. It is not merely a retrospective tool but a prospective one as well, designed to provide critical insights during the early and uncertain stages of future pandemics.

Future work could examine the interaction of a real option-epidemiologically-based model with more complex models of costs. Economic uncertainty, which is potentially correlated with infectious risk could be included into the decision problem. Finally, the model presented here could be extended to incorporate individual and space heterogeneity, in order to better inform decision-makers.

5. Conclusion

Viewed from the perspective of real option theory, this paper sheds new light on some debates about the mitigation measures to control COVID-19 dissemination. The theoretical model corroborates previous studies arguing that in the presence of uncertainty the possibility of deferring the decision until some later time when better information may become available has an economic value (Fornaro, et al., 2021). Information about how the disease spreads, clear public health guidance, and vaccines, decreases the uncertainty regarding exposure and transmission of COVID-19 (Chalkiadakis, et al., 2021; Varghese, et al., 2021; Abrams & Greenhawt, 2020).

Our results indicate that uncertainty concerning potential infected population tends to postpone the decisions to end lockdowns. Furthermore, the exit from lockdown is expected to occur sooner and become more probable as the population staying at home and just using masks behave similarly, consequently reducing the associated uncertainty regarding the decisions to end lockdowns. Additionally, when a substantial proportion of individuals comply with stay-at-home directives, and when mitigation measures are effective, less decisions to end lockdowns are observed. This can be attributed to the inherent challenge in reversing decisions. As the costs of reversing the change of policy falls, it becomes more feasible to exit lockdown. These results are critical given that they may help clarify current inconsistencies between recommendations and practical behaviors of policy and public health experts and the expressed set of preferences and expectations of these same decision-makers.

This work provides a valuable basis for the development of dynamic optimization model derived from epidemiological models and opens up avenues for future research. This work also presents the opportunity for interdisciplinary collaboration, bringing together experts in epidemiology, economics, and behavioral sciences.

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Appendix I

A.I.1 Infection Before and After the Decision

We characterize exposed population who stay at home (E_h) and those using masks (E_m) differently, and the exposed population who is under lockdown (E_L) is an average of both exposed weighted by the share of population assigned to each group (see Equations 6 **to** 8 in the main text). The infected population under lockdown is given by:

$$I_{Lt}^{*} = \frac{\delta(E_L)^{\theta}}{\gamma} = \frac{\delta\left(E_h^{\phi} E_m^{\psi}\right)^{\theta}}{\gamma}. \tag{A1}$$

Using Ito's lemma, it can be shown that the stochastic process followed by exposed under lockdown, E_L is a geometric Brownian motion, as follows:

$$dE_L(t) = \phi\psi\left(\rho\sigma_h\sigma_m - \frac{\sigma_h^2+\sigma_m^2}{2}\right) E_L dt + (\phi\sigma_h dz_h + \psi\sigma_m dz_m)E_L. \tag{A2}$$

Assume the decision-maker will apply two social distancing measures for the rest of its life. As such, the decision-maker will calculate the expected present discount value (PDV) of each mitigation measure and select the one with the highest return, i.e. the lower infected population. Since the government can adjust the distancing measures according to the present realization and future expectations of exposed population (the state variable), it will do so in order to minimize the following intertemporal infected population function:

$$I_{Lt} = \mathrm{E}\left[\int_t^{\infty}\left\{\frac{\delta\left(E_h^{\phi} E_m^{\psi}\right)^{\theta}}{\gamma}\right\} e^{-\mu(\tau-t)} d\tau\right], \tag{A3}$$

where μ is the time-discount rate. The expectation (E) must be taken since E_h and E_m (and therefore also I_{Lt}) are stochastic processes.

Bearing in mind that $E_L(t)$ follows a geometric Brownian motion, the properties of the lognormal distribution (see Aitchison and Brown (Aitchison & Brown, 1957)) can be used to transform Equation (A3) into:

$$I_L = \frac{\delta\left(E_h^{\phi} E_m^{\psi}\right)^{\theta}}{\gamma\left[\mu - \frac{\theta}{2}\left(\phi(\theta\phi-1)\sigma_h^2 + \psi(\theta\psi-1)\sigma_m^2 + 2\theta\rho\phi \quad {}_h\sigma_m\right)\right]}, \tag{A4}$$

provided that $\mu - \frac{\theta}{2}(\phi(\theta\phi - 1)\sigma_h^2 + \psi(\theta\psi - 1)\sigma_m^2 + 2\theta\rho\phi\psi\sigma_h\sigma_m) > 0$, which will be assumed here. Intuitively, this condition forces intertemporal infected population under lockdown to be bounded by imposing the time preference (μ) to be higher than the rate at which E_{Lt}^{θ} is expected to increase.

If the decision-maker decides to apply only the social distancing measure of mask-use, Equation (A3) is now given by:

$$I_m = \frac{\delta E_m^{\theta}}{\gamma\left[\mu - \frac{\theta(\theta-1)}{2}\sigma_m^2\right]}, \tag{A5}$$

provided that $\mu - \frac{\theta(\theta-1)}{2}\sigma_m^2 > 0$, which will be assumed here.

A.I.2 The Decision to Stop the Lockdown Problem

The decision of whether or not to relax the social distancing measures, i.e. citizens only have to use the masks, constitutes an optimal stopping problem (Thijssen, 2013) for which the relevant Bellman equation is:

$$F(I_L, I_m, t^*) = Max\{I_L - I_h;\ lim_{dt\to 0}\frac{1}{\mu dt}E_E[dF(I_L, I_m)]\}\ , \tag{A6}$$

where $F(I_m, I_o, t)$ is the value of the option to relax the social distancing measures, $I_L - I_h$ accounts for the expected decision-maker's value gain (i.e. decrease the infected population) that results from opting for not-staying-at-home measure (payoff of exercising the option to use only masks), and the second term in curly brackets yields the time-discounted expected increment in the value of the option that arises from keeping the option unexercised for an additional amount of time, *dt*. Here t^* is the time in the future at which the decision is made. The range of values for which the second term in curly brackets is greater than the first defines the continuation region, in which it is optimal not to exercise the option. In this region the Bellman equation is given by:

$$\mu F(I_m, I_L, t) = \lim_{dt \to 0} \frac{1}{dt} E_E[dF(I_L, I_m)]. \tag{A7}$$

Applying Ito's lemma to the right-hand side (RHS) of Equation (A7) yields the partial differential equation:

$$\left(\frac{\theta^2\sigma_h^2}{2}\right) I_h{}^2 \frac{\partial^2 F}{\partial I_h{}^2} + \left[\frac{\theta^2(\phi^2\sigma_h^2+\psi^2\sigma_m^2+2\rho\phi \quad m)}{2}\right] I_o{}^2 \frac{\partial^2 F}{\partial I_o{}^2} - [\theta^2(\phi\sigma_h^2 + \rho\psi\sigma_h\sigma_l)]I_o I_h \frac{\partial^2 F}{\partial I_m I_o} + \left[\frac{\theta(\theta-1)\sigma_h^2}{2}\right] I_h \frac{\partial F}{\partial I_h} + \left[\frac{\theta(\theta-1)(\sigma_h^2\phi+\sigma_m^2\psi)}{2} - \frac{\theta^2\phi\psi(\sigma_h^2+\sigma_m^2-2\rho\sigma_h\sigma_m)}{2}\right] I_o \frac{\partial F}{\partial I_o} - \mu F = 0. \tag{A8}$$

Since the optimal choice regarding the social distancing measures depends exclusively on the relative value of the infection attained before and after the change of social distancing measures has been undertaken, that is, on the ratio $i = I_L / I_m$, we can impose homogeneity of degree one of $F(I_L, I_m, t)$ in (I_L, I_m), such that: $F(I_L, I_m, t) = I_m f\left(\frac{I_L}{I_m}\right) = I_m f(i)$.

Such transformation allows us to re-write Equation (A8) as a function of i:

$$\left[\frac{\theta^2\psi^2(\sigma_h^2+\sigma_h^2-2\rho\sigma_h\sigma_h)}{2}\right] i^2 \frac{d^2 f}{di^2} + \frac{\theta\psi\left((\theta\sigma_h^2+\sigma_h^2)(\theta\psi-1)+2\theta\rho\phi \quad _h\sigma_m\right)}{2} i \frac{df}{di} + \left[\frac{\theta(\theta-1)\sigma_h^2}{2} - \mu\right] f = 0, \tag{A9}$$

which turns out to be an ordinary differential equation. The corresponding boundary conditions become: $f(i^*) = i^* - 1$, $\frac{df(i^*)}{di} = 1$, and $\frac{df(i^*)}{di} = \frac{1-f(i^*)}{i^*}$. The first condition is called the *value matching condition*, which states that when exit of lockdown is undertaken the option value equals $I_L - I_h$. The other two conditions are the *smooth pasting conditions* which ensures i^* to be optimal, since if f was not continuous at i^* then one could do better by deciding at a different point. Notice that Equation (A9) imposes a supplementary boundary condition: $f(0) = 0$, i.e. if the value of exiting lockdown goes to 0 it remains at 0.

A.I.3 Solution to the Optimal Decision to Stop the Lockdown

To solve the optimal stopping problem given by Equation (A8) and the respective boundary conditions, one must search for a solution and test its validity by substituting it into Equation (A8). Considering $f(i) = Pi^{\pi}$, one finds that it constitutes a solution to Equation (A8) if and only if π is a root of the following quadratic form of equation (A9).

$$Q(\pi) = \left[\frac{\theta^2\psi^2(\sigma_h^2+\sigma_m^2-2\rho\sigma_h\sigma_m)}{2}\right]\pi^2 + \frac{\theta\psi(2\theta\rho\sigma_h\sigma_m-\sigma_m^2-\sigma_h^2)}{2}\pi + \frac{\theta(\theta-1)\sigma_h^2}{2} - \mu = 0. \qquad \text{(A10)}$$

The general solution for Equation (A10) is then, $f(i^*) = P_1 i^{*\pi_1} + P_2 i^{*\pi_2}$, where P_1, P_2 are constants, and π_1, π_2 are the roots of the characteristic equation. Since for $-1 < \rho < 1$ the coefficient of π^2 in Equation (A10) is positive, $Q(\beta)$ is an upward pointing parabola. Moreover, since $Q(1) = \frac{\theta}{2}(\phi(\theta\phi - 1)\sigma_h^2 + \psi(\theta\psi - 1)\sigma_m^2 + 2\theta\rho\phi\psi\sigma_h\sigma_m) - \mu$ and $Q(0) = \frac{\theta(\theta-1)\sigma_h^2}{2} - \mu$ are both negative by previous assumptions, it follows that $\pi_1 > 1$ and $\pi_2 < 0$.

The solution is $\pi_1 = \frac{-b+\sqrt{b^2+2\theta^2\psi^2(\sigma_h^2+\sigma_m^2-2\rho\sigma_h\sigma_m)\left(\mu-\frac{\theta(\theta-1)\sigma_h^2}{2}\right)}}{\theta^2\psi^2(\sigma_h^2+\sigma_m^2-2\rho\sigma_h\sigma_m)}$, where

$b = \frac{\theta}{2}\psi[2\theta\rho\sigma_h\sigma_m - \sigma_m^2 - \sigma_h^2]$, since $P_2 = 0$ in order to satisfy the boundary condition, $f(0) = 0$.

$$\frac{-b + \sqrt{b^2 + 2\theta^2\psi^2(\sigma_h^2 + \sigma_m^2 - 2\rho\sigma_h\sigma_m)\left(\mu - \frac{\theta(\theta - 1)\sigma_h^2}{2}\right)}}{\theta^2\psi^2(\sigma_h^2 + \sigma_m^2 - 2\rho\sigma_h\sigma_m)}$$

Making use of the value-matching and smooth-pasting conditions, the expression for the critical ratio is obtained and likewise for the constant P_1 as:

$$i^* = \frac{\pi_1}{\pi_1 - 1}, \tag{A11}$$

$$P_1 = \frac{(\pi_1 - 1)^{\pi_1 - 1}}{{\pi_1}^{\pi_1}}. \tag{A12}$$

Now, since $\pi_1 > 1$, Equation (AA11) implies that $i^* > 1$, meaning that the government will engage in relaxing the social distancing measures if $I_m < I_L$, i.e. the expected infected population when just using masks is lower than that attained when in lockdown. This solution depends on a restriction on π_1 in relation to θ, such that: $\pi_1 > \theta$, related to superharmonicity of the value function. This condition shows that the value function can only be superharmonic if the PDV does not increase faster than the value of waiting, i.e. if $f(i)$ is more convex than PDV (Thijssen, 2013).

A.I.4 Threshold in Terms of Exposed population

In order to obtain the critical value as a function of the ratio of the model's state variables, i.e. the exposed population subject to be at home and exposed just using masks, Equations (A4) and (A5) are used to obtain $i^* = \frac{I_L}{I_m}$. Then, substituting Equation (AA11) for i^*, the critical value is:

$$\frac{\tilde{E}_h}{\tilde{E}_m} = \left\{ \frac{\mu - \frac{\theta}{2}\left(\phi(\theta\phi - 1)\sigma_h^2 + \psi(\theta\psi - 1)\sigma_m^2 + 2\theta\rho \qquad {}_h\sigma_m\right)}{\mu - \frac{\theta(\theta - 1)}{2}\sigma_h^2} \times \frac{\pi_1}{\pi_1 - 1} \right\}^{\frac{1}{\theta\phi}} \tag{A13}$$

Equation (A13) is the trigger value of exposed population separating the region in $\frac{E_h}{E_m}$ space where the decision-maker's option of relaxing the social distancing measures (i.e. for $\frac{\tilde{E}_h}{\tilde{E}_m} >$

$\frac{E_h}{E_m}$) from the one where immediate exercise of that option is perceived as optimal (i.e. for $\frac{\tilde{E}_h}{\tilde{E}_m} \leq \frac{E_h}{E_m}$).

A.I.5 Comparative Statics

In order to do some comparative statics, the original setup will be simplified and it will be considered that $\sigma = \sigma_h = \sigma_l$. It follows from Equation (A13) that:

$$\frac{E_h^*}{E_m^*} = \left\{ \frac{\phi\psi\theta^2\sigma^2(1-\rho)}{\mu - \frac{\theta(\theta-1)}{2}\sigma^2} \times \frac{\theta\sigma(1-\rho)+\sqrt{(1-\rho)\{4\mu-\theta\sigma^2[\theta(1-\rho)+2(1-\theta)]\}}}{\sigma(1-\rho)\phi+\sqrt{(1-\rho)\{4\mu-\theta\sigma^2[\theta(1-\rho)+2(1-\theta)]\}}} \right\}^{\frac{1}{\theta\phi}} \quad \text{(A14)}$$

The greater the volatility of the exposed (i.e. the higher σ^2), the higher the critical value has to be to make it optimal for the decision-maker to use only masks as the social distancing measure, i.e.:

$$\frac{\partial\left(\frac{E_h^*}{E_m^*}\right)}{\partial\sigma^2} > 0 \text{ and } lim_{\sigma \to \infty}\, i^* = \infty. \quad \text{(A15)}$$

The more correlated are the shocks affecting the exposed both staying at home and just using masks, the less the change of policy option is worth and so the lower is the value of the relative exposed that triggers the use of mask, i.e.

$$\frac{\partial\left(\frac{E_h^*}{E_m^*}\right)}{\partial\rho} < 0 \quad \text{(A16)}$$

The reason is that the more correlated shocks are, the more closely both exposed processes move and so the lower the uncertainty that results from the switch from a situation in which social distancing measures of staying at home and using masks to a situation in which the population can live freely and just use masks in closed spaces and public transportation. This means that just using masks is likelier to succeed in cases when exposed at home are similar to exposed using masks.

With regard to the discount rate, the greater the decision-maker's time discount rate, the less it values the change of social distancing measures option, and thus the lower the value $\frac{E_h}{E_m}$ that triggers optimal change, i.e.:

$$\frac{\partial\left(\frac{E_h^*}{E_m^*}\right)}{\partial\mu} < 0. \tag{A17}$$

This result stems from the fact that a higher time preference increases the government's opportunity cost of not immediately changing the policy. The economic intuition for this is that, if the decision-maker cares more about the future (low μ), he wants to use just masks instead of both social distancing measures; and if he cares more about the present, he wants to lock down instead of using masks. In the extreme case in which the decision-maker cares only about the present moment, so that $\mu \to\infty$, then $\lim_{\mu\to\infty}\left(\frac{\beta_1}{\beta_1-1}\right) = 0$ and $\frac{E_h^*}{E_m^*} = 0$, so that uncertainty is disregarded and the value of the change of policy option collapses to zero.

The higher the share of population staying at home and the higher the threshold for changing the policy, the higher the critical value, i.e.:

$$\frac{\partial\left(\frac{E_h^*}{E_m^*}\right)}{\partial\theta} < 0 \; and \; \frac{\partial\left(\frac{E_h^*}{E_m^*}\right)}{\partial\phi} < 0, \tag{A18}$$

Intuition suggests that it should be the case. This means that when in lockdown (high ϕ and low θ), the more the change of policy option is worth. This is because change of policy always includes some degree of irreversibility such that an increase in the share of population using just masks (not staying at home) raises the society's overall risk of exposure.

Appendix II Flowchart

Mitigation measures to stop the infected population from increasing, i.e.

$$\frac{dI}{dt} = 0 \Leftrightarrow I_t = \frac{\delta E_t^{\theta}}{\gamma}$$

E_t follows a Driftless stochastic process.

Exposed population at home: $dE_h = \sigma_h E_h dz_h$

Exposed population just using masks: $dE_m = \sigma_m E_m dz_m$

Exposed population under lockdown is a weighted average of both exposed at home and using masks

$$E_L = E_h^{\phi} E_m^{\psi}$$

Infected population under Lockdown

$$I_{Lt}^{*} = \frac{\delta \left(E_h^{\phi} E_m^{\psi}\right)^{\theta}}{\gamma}$$

Expected present discount value for infected population under Lockdown

$$I_L = \frac{\delta \left(E_h^{\phi} E_m^{\psi}\right)^{\theta}}{\gamma \left[\mu - \frac{\theta}{2}\left(\phi(\theta\phi - 1)\sigma_h^2 + \psi(\theta\psi - 1)\sigma_m^2 + 2\theta\rho\phi\psi\sigma_h\sigma_m\right)\right]}$$

Expected present discount value for infected population just using masks

$$I_m = \frac{\delta E_m^{\theta}}{\gamma \left[\mu - \frac{\theta(\theta - 1)}{2}\sigma_m^2\right]}$$

The value of the option, $f(i)$, to exit lockdown as a function of the relative infected population, $i = I_L / I_m$, for different levels of uncertainty

$$f(i^*) = P_1 i^{*\pi_1}$$

The threshold value of exiting lockdown, i^*

$$i^* = \frac{\pi_1}{\pi_1 - 1}$$

The threshold value of exiting lockdown in terms of exposed population, $\frac{\tilde{E}_h}{\tilde{E}_m}$

$$\frac{\tilde{E}_h}{\tilde{E}_m} = \left\{\frac{\mu - \frac{\theta}{2}\left(\phi(\theta\phi - 1)\sigma_h^2 + \psi(\theta\psi - 1)\sigma_m^2 + 2\theta\rho\phi\psi\sigma_h\sigma_m\right)}{\mu - \frac{\theta(\theta - 1)}{2}\sigma_h^2} \times \frac{\pi_1}{\pi_1 - 1}\right\}^{\frac{1}{\theta\phi}}$$

Note: $\pi_1 = \frac{-b + \sqrt{b^2 + 2\theta^2\psi^2(\sigma_h^2 + \sigma_m^2 - 2\rho\sigma_h\sigma_m)\left(\mu - \frac{\theta(\theta-1)\sigma_h^2}{2}\right)}}{\theta^2\psi^2(\sigma_h^2 + \sigma_m^2 - 2\rho\sigma_h\sigma_m)}$ and $P_1 = \frac{(\pi_1 - 1)^{\pi_1 - 1}}{\pi_1^{\pi_1}}$

Figure A1. Real option model derived from the SEIR model. The variables and parameters are defined in Table II.