Discussion Papers in Economics

No. 15/08

CONVEXITY, QUALITY AND EFFICIENCY IN EDUCATION

David J. Mayston

Department of Economics and Related Studies
University of York
Heslington
York, YO10 5DD

CONVEXITY, QUALITY AND EFFICIENCY IN EDUCATION

David J. Mayston

Department of Economics and Related Studies
University of York

Abstract: While Data Envelopment Analysis (DEA) has many attractions as a technique for analysing the efficiency of educational organisations, such as schools and universities, its efficiency estimates are based upon the assumption that the output possibility set is convex. If this assumption does not hold, DEA may overstate the scope for improvements in technical efficiency through proportional increases in all educational outputs and understate the importance of improvements in allocative efficiency from changing the educational output mix. The paper therefore examines conditions under which such convexity may not hold, particularly when the performance and efficiency evaluation includes measures related to the assessed quality of the educational outputs, and the position of the school or university in national league tables. Under such conditions, there is a need to deploy other educational efficiency assessment tools, including an alternative non-parametric output-orientated technique and a more explicit valuation function for educational outputs.

Keywords: Data envelopment analysis, quality, education, efficiency analysis, allocative efficiency.

JEL classifications: C61, D61, H52, I20, I23.

1. Introduction

One of the most widely used techniques for analysing the efficiency of non-profit organisations in education and elsewhere is that of Data Envelopment Analysis (DEA). DEA has been used, for instance, to assess the scope for efficiency improvements of individual schools by Jesson *et al* (1987) and by Thanassoulis and Dunstan (1994), and of higher education institutions by Athanassopoulos and Shale (1997) and individual academic departments by Johnes and Johnes (1995). It has also been used extensively in the operational research and management science literatures to assess the efficiency and scope for improvement of a wide range of other organisations (see Emrouznejad *et al*, 2008). As an analytical technique for assessing the efficiency of educational decision-making units (EDMUs), such as schools and universities, DEA has many advantages through its ability to incorporate multiple outputs and multiple inputs into its determination of its resultant efficiency scores. As a nonparametric frontier estimation technique, DEA also has the advantage of not requiring the prior specification of a specific functional form for the educational production function between educational inputs and outputs that maps out the frontier of the associated feasible set. However, an important assumption on which the conclusions of the standard models of DEA rest is that that the associated feasible set of achievable outcomes is convex. In this paper we will argue that this assumption may well not be valid for many potential applications of DEA in assessing the efficiency of EDMUs.

In Section 2, we examine how non-fulfilment of the convexity condition may well lead to misleading conclusions on both the technical and the allocative efficiency of educational organisations. In Sections 3 and 4, we examine why the use of educational data in particular may lead to non-convexity. In Section 5, we examine the scope for applying to educational data an alternative output-orientated non-parametric frontier technique which relaxes the convexity assumption. Section 6 contains our conclusions.

2. The Importance of the Convexity Assumption

The role of the convexity assumption in DEA's efficiency assessment can be seen most clearly in the output-orientated form of DEA developed by Banker *et al* (1984), which may be expressed in terms of the linear program:

$$\max \theta_j \ s.t. \ \lambda X \leq X_j, \theta_j Q_j - \lambda Q \leq 0, \lambda e = 1, \lambda \equiv (\lambda_1, ..., \lambda_n) \geq 0, e \equiv (1,...,1)' \qquad (1)$$

where X_j and Q_j are the input and output vectors respectively of the EDMU j, with $X \equiv (X_1, ..., X_n)'$ and $Q \equiv (Q_1, ..., Q_n)'$ from our sample of *n* individual EDMUs. (1) involves seeking the maximum possible proportional expansion θ_j in the existing output vector $Q_j = (Q_{j1},...,Q_{jr})$ of EDMU j's r different outputs from its existing input vector X_j based upon a comparison with a hypothetical EDMU. This is assumed to have an input vector that is a convex combination λ of the input vectors of the actual EDMUs in the sample and an output vector that is the same convex combination λ of the output vectors of the actual EDMUs in the sample. However, unless the feasible production possibility set is itself convex, there is no guarantee that this input-output combination of such a hypothetical EDMU, on which DEA's estimate of θ_j and its associated efficiency assessments are based, will actually be feasible.

Figure 1 illustrates a case in which the actual feasible set is non-convex, with a frontier given by the curve ABEFH between two educational outputs, with the level of inputs held constant in this simple example. The output vector that is achieved by the EDMU j in Figure 1 is given by the point J, which is strictly inside the feasible frontier ABEFH. The output-orientated form of DEA would compare the point J with a convex combination N of the feasible points B and F which are achieved here by other EDMUs in the observed sample, and where N lies on the same ray though the origin as point J. The associated value of the coefficient of technical efficiency, ε_{TD}, of point J under DEA is OJ / ON. However, when the feasible set is non-convex, as in Figure 1, the point N may itself not be a feasible output vector. Instead, the true measure of technical efficiency, defined in terms of the proportional shortfall of its current output vector J compared to the maximum feasible proportional expansion in this output vector J at point E, would be here $\varepsilon_{TT} = OJ / OE$, which is strictly greater here than DEA's coefficient of technical efficiency, ε_{TD}, OJ / ON. The maximum feasible proportional expansion θ_{TT} in the output vector J given by the reciprocal of ε_{TT}, i.e. OE/OJ is smaller than the value of $\theta_{TD} = ON/OJ$ indicated by DEA. An educational organisation that was actually on the efficient frontier at point E in Figure 1 would erroneously be given by DEA a technical efficiency score of less than one, with an implied target for improvement that was not actually feasible.

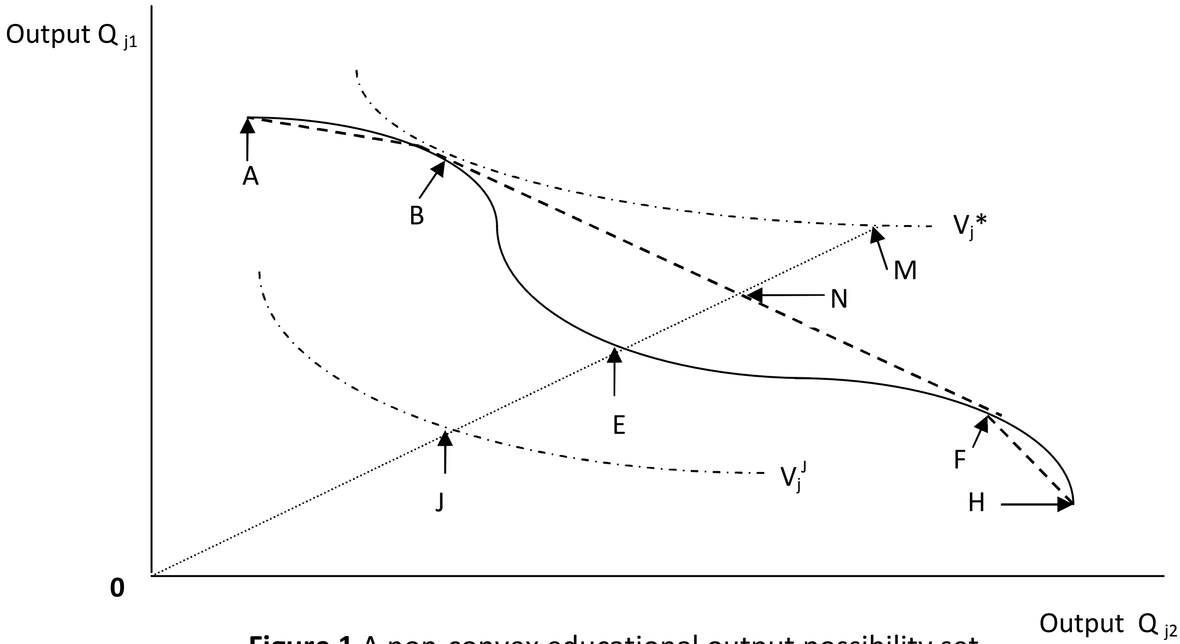

Figure 1 A non-convex educational output possibility set

While published DEA studies in education and elsewhere have concentrated predominantly on the assessment of *technical efficiency*, an important further direction in which DEA may yield biased efficiency assessments in the presence of non-convexity is in its assessment of *allocative efficiency*. Rather than focussing upon simply proportional improvements in the existing output vector, the concept of allocative efficiency seeks to assess which further improvements are feasible by changing the existing proportions in which the educational outputs are produced. One reason that output allocative efficiency has received much less attention in the educational literature than technical efficiency is that there are typically no simple market prices for the different educational outputs which an EDMU might produce. However, progress can be made in the assessment of the important issue of allocative efficiency if a *valuation function* of the form $V(Q_j)$ can be deployed to evaluate the relative value placed upon the different educational outputs. One useful property of such a valuation function in the context of allocative efficiency assessment is that of *homotheticity* (see Henderson and Quandt, 1980, p. 40), which implies that its indifference curves are those generated by a valuation function which is homogeneous of degree one in the elements of the educational output vector Q_j. This in turn means that a relevant true measure of the allocative efficiency of the output vector J in Figure 1 is given by $\varepsilon_{AT} = (OE / OM)$, where M is the point on the ray through the origin on which J lies where it intersects the indifference curve, or *iso-valuation* curve, that is tangential to the feasible set at point B in Figure 1. The inverse of ε_{AT}, i.e. OM / OE, then provides a measure of the further improvements which can be made according to the homogeneous valuation function by changing the *educational output mix*, beyond those which can be achieved by improving its technical efficiency along the ray OE holding its existing output mix constant.

That DEA may overstate the existing allocative efficiency of the educational outputs of an EDMU in the presence of non-convexity is also illustrated in Figure 1, with DEA's measure of allocative efficiency given here by the ratio $\varepsilon_{AD} = (ON / OM)$. We then have:

$$\varepsilon_{AT} = (OE / OM) < (ON / OM) = \varepsilon_{AD} \; with \;\; \theta_{AT} \equiv \varepsilon_{AT}^{-1} > \varepsilon_{AD}^{-1} \equiv \theta_{AD} \qquad (2)$$

with DEA's assessment, θ_{AD}, understating the true scope, θ_{AT}, for improvements in the value of educational outputs from changing the educational output mix from that along the ray OE to that at point B in Figure 1. That questions of output allocative efficiency and improving the educational output mix become more important in the presence of non-convexities than DEA recognises is consistent with the heightened need for educational institutions to make efficient choices regarding the mix of their educational outputs when there may be factors, such as significant increasing gains from specialisation in the production of their individual educational outputs, that can generate such non-convexities, as we note below. It is also notable that the *overall* efficiency measure which DEA generates as the *product* of its technical efficiency measure and its allocative efficiency measure (see Farrell, 1957) is equal here to the true overall efficiency measure even under non-convexity, with:

$$\varepsilon_{OT} = \varepsilon_{TT}\varepsilon_{AT} = (OJ / OM) = \varepsilon_{TD}\varepsilon_{AD} = \varepsilon_{OD} \; with \; \theta_{OT} = \theta_{TT}\theta_{AT} = \varepsilon_{OT}^{-1} = \varepsilon_{OD}^{-1} = \theta_{TD}\theta_{AD} = \theta_{OD} \quad (3)$$

However, DEA's overestimate of the extent to which efficiency improvements can be made by holding constant the existing educational output mix of the EDMU, combined with DEA's underestimate of the scope for improvements in its allocative efficiency, remain important drawbacks to its use for educational efficiency assessment in the presence of non-convexities.

3. Quality and Non-convexity

A notable feature of educational output is the relevance of both the quantity of students which an educational institution may process in any given time period, and the quality of their education during this time. We will denote by m the vector of relevant quantities, such as student numbers at different stages of the educational process in a given institution, and by q the vector of the EDMU's recorded quality achievements. For the sake of simplicity we assume a single real resource input whose total usage is held constant at some level x_0 in defining our output possibility set but whose allocation across different educational activities within the EDMU can be varied to change the educational output mix. The feasible output possibility set associated with a given total input x_0 is given by:

$$\mathrm{S}(x_0) = \{(q,m) : f(q,m,x_0) = x_0 - c(q,m) \geq 0\} \quad (4)$$

where $f(q,m,x_0) = 0$ defines an implicit multiple-output educational production function that maps out the production possibility frontier (PPF) for any given value of x_0, and which we assume can be decomposed into x_0 minus a function $\mathrm{c}(q,m)$ that defines how the total resource input requirement varies with the quality vector q and the quantity vector m. Since the performance of educational institutions, such as schools and universities, is increasingly judged on the basis of the assessed quality of their educational output, of particular interest is the shape of the *quality frontier*, holding m and x_0 constant. The slope of the PPF at any given point on the quality frontier corresponds to its *marginal rate of product transformation* $\Gamma_{kh} = (-dq_k / dq_h)$ between any two relevant quality scores holding constant m, x_0 and any other elements of q. A necessary condition (see Arrow and Enthoven, 1961) for convexity of $\mathrm{S}(x_0)$ is that Γ_{kh} is non-decreasing as q_h is increased along the PPF, and hence that:

$$\chi_{kh} \equiv (d\Gamma_{kh} / dq_h) = (\mathrm{c}_{hh} - 2c_{kh}(\mathrm{c}_h / c_k) + c_{kk}(\mathrm{c}_h^2 / c_k^2)) / c_k \geq 0 \; where \; c_k \equiv \partial c / \partial q_k , c_{kh} \equiv \partial c_k / \partial q_h \; etc \qquad (5)$$

and where we assume that $c_k > 0$ and $c_h > 0$. Condition (5) in turn requires that:

$$(-c_{kh}) \geq 0.5[(-c_{kk}(c_h / c_k) - c_{hh}(\mathrm{c}_k / c_j))] \qquad (6)$$

When we include the quality of educational output within our efficiency analysis, we might expect there to be increasing quality gains from specialisation and a greater focus of resources in particular directions, which would imply here that $c_{kk} < 0$ and $c_{hh} < 0$. The necessary condition (6) for convexity of the feasible set $S(x_0)$ then requires that any such gains from specialisation are offset by sufficiently large gains from the economies of scope associated with the cost complementarities (see Baumol *et al*, 1982, p. 75) that negative c_{kh} terms in (6) reflect. Thus, it may indeed be the case in universities that high quality research does indeed help to inspire high quality teaching, and that additional time spent teaching and preparing for teaching does generate some ideas for improved research activity, so that such cost complementarities may well exist. However, the convexity condition (6) requires not simply that they exist but rather that they are sufficient strong to offset the opportunity costs of the lost gains from specialisation because of a greater spreading of resources more thinly between the two activities.

Whether or not this is the case is essentially an empirical question, rather than one which should automatically be assumed to be true, in the way DEA would do. Empirical evidence for a lack of any positive relationship between research and teaching quality is indeed claimed by Ramsden and Moses (1992). Moreover, the inclusion of quantity data, in the form of student numbers at undergraduate and postgraduate levels, alongside research income as a measure of research quality, in the empirical study by Izadi *et al* (2002) produced estimates of the γ_h parameters in their fitted stochastic frontier estimation of a CES cost function for university j of the form:

$$c^{(j)} = a_o + (\sum_{h=1}^{r} a_h Q_{jh}^{\gamma_h})^{\rho} \tag{7}$$

that were found to be strictly between zero and one. As in Baumol *et al* (1982, p. 461), such parameter values imply a non-convex iso-cost output possibility set between these outputs. Similarly, if the educational production function is of the multiple-output Cobb-Douglas form:

$$\prod_{k=1}^{\eta_1} q_k^{\alpha_k} \prod_{\kappa=1}^{\eta_2} m_\kappa^{\beta_\kappa} = A \prod_{\ell=1}^{\eta_3} x_\ell^{\gamma_\ell} \quad \text{and hence } \Gamma_{kh} = (-dq_k / dq_h) = (\alpha_h q_k / \alpha_k q_h) \tag{8}$$

non-convexity of the output possibility set for any given input vector is again implied for $\eta_1 > 0, \eta_2 > 0$, $\alpha_h > 0$ and $\alpha_k > 0$, with Γ_{kh} decreasing as q_h increases and q_k decreases along the PPF.

A further feature of the available data on the quality of educational output for schools and universities is that they typically result from *assigning grades* within the quality assessment process. For secondary schools in England, GCSE results achieved at grades A*- C have been a primary measure of the quality of their output, with much emphasis placed on the percentage of pupils who achieve 5 or more grades A*-C, including in English and mathematics. For universities in the UK, their research quality has been measured in terms of their submitted research outputs that fall within each of the grades 4*, 3*, 2*, 1* and unclassified (RAE, 2008; REF, 2014). UK university teaching quality is assessed by the percentage of student responses in the annual National Student Survey (NSS) that are awarded grade 5, 4, 3, 2 or 1 according to the strength of their agreement with complimentary statements regarding their university department's teaching and associated provision (NSS, 2014).

The nature of the frontier of the feasible set between the quality scores in different directions facing an EDMU can be illustrated by the case of university research and teaching quality. We will assume that the resources devoted to one direction, such as research, may have spillover effects on the underlying quality achieved in another direction, such as teaching. In particular, we will denote by x_1 the resource expenditure on research per member of staff of any given EDMU and by x_2 its resource expenditure on teaching per student. The *underlying* quality y_{1i} of the ith assessed research output and the underlying quality y_{2i} of the ith assessed teaching episode are assumed to be given by:

$$y_{1i} = \bar{y}_1 + \varepsilon_{1i} \text{ where } \bar{y}_1 = \alpha_{11}x_1 + \alpha_{12}x_2, y_{2i} = \bar{y}_2 + \varepsilon_{2i} \text{ where } \bar{y}_2 = \alpha_{21}x_1 + \alpha_{22}x_2 \qquad (9)$$

and where ε_{1i} and ε_{2i} are terms that reflect additional latent variations between each submission in individual ability and inspiration in research and teaching which impact on the underlying quality of each individual submission around the mean levels, $\bar{y}_1$ and $\bar{y}_2$ of the underlying quality given by the resource expenditures in equation (9). The *assessed* quality of each submission, however, is a result of a *grading process* in which the grade awarded to the ith submission in direction k is given by:

$$\tilde{g}_{ki} = g_k \text{ if } \vartheta_{g_k+1} > y_{ki} \geq \vartheta_{g_k} \text{ for } k = 1,2 \qquad (10)$$

with the grade hurdle $\vartheta_{g_k^{\infty}+1}$ for the highest grade g_k^{oo} assumed to be $+\infty$ and $\vartheta_{g_k^o}$ for the lowest grade g_k^o assumed to be $-\infty$. If ε_{1i} and ε_{2i} have independently normal frequency distributions with zero means and variances s_1^2 and s_2^2 respectively across the multiple individual submissions to the grading process, the *mean value of the assessed quality score* in direction k is given by:

$$q_k = Z_k(\bar{y}_k) \equiv \sum_{g_k} w_k(g_k)\Xi(\bar{y}_k, g_k) \text{ where } \Xi(\bar{y}_k, g_k) = [\Phi((\bar{y}_k - \vartheta_{g_k})/s_k) - \Phi((\bar{y}_k - \vartheta_{g_k+1})/s_k)] \quad (11)$$

and where Φ is the standardised normal cumulative distribution, $w_k(g_k)$ is the relative weight attached to the quality grade g_k in direction k, and $\Xi(\bar{y}_k, g_k)$ is the proportion of submissions in direction k awarded the quality grade g_k for a given value of $\bar{y}_k$. Equation (11) in turn implies that for $\Delta w_k(g_k) \equiv w_k(g_k) - w_k(g_k - 1) \geq 0$ for all g_k and $\Delta w_k(g_k) > 0$ for some g_k :

$$q'_k(\bar{y}_k) \equiv \partial Z_k / \partial \bar{y}_k = \sum_{g_k} \Delta w_k(g_k)\phi(Y_k)/s_k > 0 \quad \text{where } Y_k \equiv (\bar{y}_k - \vartheta_{g_k})/s_k \quad (12)$$

and

$$q''_k \equiv \partial q'_k / \partial \bar{y}_k = \sum_{g_k} \Delta w_k(\mathrm{g}_k)G(Y_k)/s_k^2 \quad \text{where } G(Y_k) \equiv -Y_k\phi(Y_k) \quad (13)$$

where ϕ is the standardised normal density function. $\partial Z_k / \partial \bar{y}_k > 0$ in (12) and (11) imply inverse functions D_k such that $\bar{y}_k = D_k(q_k) \equiv Z_k^{-1}(q_k)$. Using (9), the associated cost function is given by:

$$c(q,m) = m_1x_1 + m_2x_2 = x_0 = z_1(m)D_1(q_1) + z_2(m)D_2(q_2) \quad (14)$$

where m_1 is the number of staff and m_2 the number of students of the EDMU, and where

$$z_1(m) \equiv (m_1\alpha_{22} - m_2\alpha_{21})/z_0, z_2(m) \equiv (m_2\alpha_{11} - m_1\alpha_{12})/z_0, z_0 \equiv (\alpha_{11}\alpha_{22} - \alpha_{12}\alpha_{21}) \quad (15)$$

with $z_1 > 0$ and $z_2 > 0$ under the condition that $\partial c / \partial y_k > 0$ for $k = 1,2$ in (14). From (5), (12) – (15):

$$\chi_{12} = -(\mathrm{z}_2/\mathrm{z}_1)[q''_2 + q''_1(\mathrm{z}_2/\mathrm{z}_1)](q'_1/(q'_2)^2) \quad (16)$$

The sign of χ_{12}, and hence whether or not the convexity condition (5) is broken, depends in (16) upon the behaviour, for both $k = 1$ and $k = 2$, of q''_k, and hence of the function $G(Y_k)$ at each point in the grading process for which $\Delta w_k(g_k) > 0$ in equation (13). Figure 2 shows the strongly non-linear behaviour of the function $G(Y_k)$, with a steadily increasing positive value to $G(Y_k)$ as $\bar{y}_k$ increases up to the point where it is one standard deviation s_k short of the grade hurdle ϑ_{g_k}, and the associated values of Y_k and $G(Y_k)$ are minus one and +0.241971 respectively. $G(Y_k)$ then steadily declines from a positive to a negative value as the gap between ϑ_{g_k} and $\bar{y}_k$ passes through zero, with $G(Y_k)$ reaching a minimum of minus 0.241971 when $\bar{y}_k$ exceeds the grade hurdle ϑ_{g_k} by one standard deviation s_k, and the associated value of Y_k is plus one, before $G(Y_k)$ steadily increases in Figure 2 to approach zero from below asymptotically.

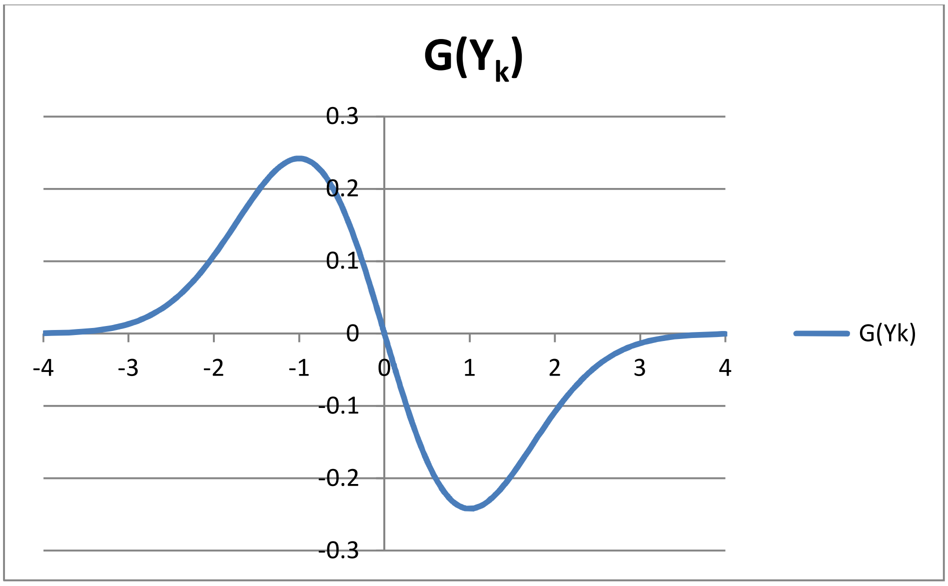

Figure 2 $G(Y_k)$ as a function of Y_k

As in equation (13), the *rate of change of the marginal productivity* of increases in the mean value , $\bar{y}_k$, of the underlying quality from additional expenditure in equation (9), in raising the *mean assessed quality score*, q_k, varies with $G(Y_k)$, and hence non-linearly with how close $\bar{y}_k$ is to the relevant quality hurdle. If the expected underlying quality $\bar{y}_k$ of the EDMU is just below the grade hurdle ϑ_{g_k} by close to one standard deviation s_k in both of the directions $k = 1,2$, and there is only one hurdle in each case for which $\Delta w_k(g_k) > 0$, we will have $q_1'' > 0$ and $q_2'' > 0$ in equations (13) and (16), and hence the convexity condition $\chi_{12} \geq 0$ broken. Again this emphasises the importance of allocative efficiency, rather than simply technical efficiency, under such non-convexity. If there are insufficient slack resources to increase both $\bar{y}_1$ and $\bar{y}_2$ by more than one standard deviation s_k in each case, the EDMU would boost its assessed quality performance by choosing a more uneven policy of switching resources to boost $\bar{y}_1$ at the expense of $\bar{y}_2$, or vice versa, so that its expected underlying quality exceeds the relevant quality hurdle in at least one direction. Which one it should choose depends upon the relative payoffs from $\Delta w_1(g_1) > 0$ and $\Delta w_2(g_2) > 0$, with the extent of the improvement in its allocative efficiency dependent upon these relative payoffs and the non-proportional changes which it makes.

If there is only one grade hurdle in each direction and the EDMU reallocates its resources between x_1 and x_2 to further increase $\bar{y}_1$ and further reduce $\bar{y}_2$, while holding total costs constant, $G(Y_1)$, and hence q_1'', will at some point become negative in Figure 1 and in equations (13) and (16); similarly $G(Y_2)$, and hence q_2'', will decline towards zero in its positive value. As a result, the necessary convexity condition $\chi_{12} \geq 0$ in equation (16) may not be breached locally at all such points along the iso-cost frontier. This emphasises that while the associated output possibility set is here non-convex, as in Figure 1, its frontier is not everywhere concave from above. The existence of some concave sections to the frontier, as in Figure 1, will however make the use of DEA to assess the efficiency of individual EDMUs inappropriate when assessed quality variables are included in their outputs.

4. Multiple quality hurdles and the influence of league tables

The existence of possible multiple grade hurdles is illustrated by the case of the recent Research Excellence Framework (REF, 2014) exercise in UK universities and by its predecessor, the Research Assessment Exercise (RAE, 2008). The summation across these multiple grade hurdles that is involved in equations (12) and (13) may well involve the expected underlying research quality $\bar{y}_1$ being above some lower quality hurdles, implying negative values to the associated $G(Y_1)$ terms in Figure 2, but below one or more higher quality hurdles, implying positive values to the associated $G(Y_1)$ terms in Figure 2. Whether or not the overall weighted sum for q_1'' in equation (13) is positive or negative will depend upon how far away $\bar{y}_1$ is from each such hurdle, and on the relative values of each respective $\Delta w_1(g_1)$ term. One notable feature of the relative weight that was placed upon achievements in successive assessed research quality grades from 1* through to 4* in the RAE, in the determination of the associated QR research funding for each individual assessed EDMU, was that each successive increase $\Delta w_1(g_1)$ was itself strictly increasing (see HEFCE, 2010), with $\Delta w_1(2^*) = 1$, $\Delta w_1(3^*) = 2$, and $\Delta w_1(4^*) = 6$. This itself implies that positive values to $G(Y_1)$ from the underlying expected research quality $\bar{y}_1$ being below a higher quality hurdle will be given more weight in (13) than a negative value to $G(Y_1)$ that arises from $\bar{y}_1$ being at the same time above a lower quality hurdle, so that we may have $q_1'' > 0$ in equations (13) and (16) over some range of values of $\bar{y}_1$.

When assessed teaching quality is included in the efficiency analysis, a notable feature of the relative weight that is placed upon successive grades from 1 to 5 in the NSS in published reports and performance indicators on the percentage of students who are "satisfied" (see e.g. HEFCE, 2014) is that $\Delta w_2(2) = \Delta w_1(3) = 0 = \Delta w_2(5)$ and $\Delta w_1(4) = 1$. The inclusion of only the NSS grades 4 and 5 as indicating that the student is "satisfied" means that when the assessed measure of teaching quality is the percentage of students who are "satisfied", there is no increase in this performance measure if a grade 3 rather than a grade 2 or a grade 1, or a grade 5 rather than a grade 4, is achieved. It is therefore the degree and sign of the difference between the expected underlying teaching quality y_2 and the quality hurdle associated with the boundary between grades 3 and 4 which determines the strength and sign of the relevant $G(Y_2)$ in equation (13). If $\bar{y}_2$ falls short of this hurdle, we will have $q_2'' > 0$, with again the convexity condition $\chi_{12} \geq 0$ broken locally in (16) whenever $q_1'' > 0$ also holds.

Again issues of allocative efficiency become of considerable importance since the EDMU needs here to decide whether to boost their expected underlying teaching quality in order to increase the probability of grade 4 or 5 assessments, or to focus their available resources on boosting their expected underlying research quality to increase the probability of securing a higher research rating. Simply increasing both expected underlying qualities proportionately, to achieve increases in technical efficiency, in contrast may well prove to be a sub-optimal policy given the non-linearities which are involved in equations (11) – (13), and the associated Figure 2. If their existing expected underlying teaching quality is between already well below the grade 4 hurdle, moderate improvements in it will unfortunately have little impact upon its overall expected assessed teaching quality score under the above weighting system. There is then more incentive for the EDMU to sacrifice even more teaching quality by concentrating its available resources more on improving its assessed research quality.

The powerful effect which the weighting system can have on an EDMU's management and policy choices in the presence of non-convexities is reflected also in the widespread use which has been made of the performance indicator of the percentage of pupils who achieve 5 or more A* - C grades to assess the performance of individual secondary schools at Key Stage 4 (KS4), for the GCSE national examinations which pupils in England typically take at the age of 16. This percentage similarly fails to give any additional positive credit for achieving higher grades within the A* - C range, and instead has given an incentive to schools to 'manage the margin' by focussing their resources upon pupils who are close to the grade C hurdle, rather than upon pupils who might excel towards grade A* performance. Fortunately some progress is being made through a recent proposal for a more refined point score system for 8 different grades at KS4 (DFE, 2014), though with pupil discreet grade improvements still playing a major part in the proposed Progress 8 performance measure from 2016 onwards.

An important additional pressure on the management of individual EDMUs can be their position in published national league tables of their assessed quality scores. The growth of *managerialism* within educational institutions (see e.g. Deem *et al*, 2007 and UNESCO, 2004) and of *greater competition* between EDMUs for able students based upon these published scores (see e.g. DBIS, 2011) tends to significantly reinforce these pressures within individual EDMUs. The management objective of the EDMU may be described here as seeking to maximise a valuation function of the form:

$$V(R_1(q_1), R_2(q_2)) \text{ where } R_k(q_\ell) = n\Psi_k(q_k) \text{ for } k = 1,2 \tag{17}$$

where $R_k(q_k)$ is their rank from the bottom of the cumulative distribution $\Psi_k(q_k)$ of q_k across a given total number n of EDMUs in the comparison set. When we consider the possibility frontier facing the EDMU for changing its ranking in different quality directions, the slope of the relevant PPF between $R_1(q_1)$ and $R_2(q_2)$ holding total cost constant is given by:

$$\Gamma_{12}^o = -dR_1 / dR_2 = (dR_1 / dq_1)(-dq_1 / dq_2) / (dR_2 / dq_2) = \xi_1(q_1)\Gamma_{12} / \xi_2(q_2) \tag{18}$$

where $\xi_k(q_k)$ is the density function associated with $\Psi_k(q_k)$. We then have:

$$\chi_{12}^o \equiv d\Gamma_{12}^o / dq_2 = (\xi_1(q_1) / \xi_2(q_2))[-(\xi_{22}(q_2) / \xi_2(q_2))\Gamma_{12} - (\xi_{11}(q_1) / \xi_1(q_1))\Gamma_{12}^2 + \chi_{12}] \tag{19}$$

If $\xi_k(q_k)$ is unimodal with a mode at q_k^o and a positive value to its slope $\xi_{kk} = \partial\xi_k / \partial q_k$ for $q_k < q_k^o$ and a negative value to its slope for $q_k > q_k^o$ for each $k = 1,2$, a given unit improvement in its assessed research quality score q_1 will give a potentially *much greater boost* to the EDMU's research ranking R_1 in (18) than otherwise, for any given reduction in its teaching ranking along the possibility frontier, if the EDMU is currently *close to the mode* of the associated population distribution for q_1 but distant from the mode of the population distribution for q_2. This in turn introduces another important source of non-linearity into the scope for increases in allocative efficiency, here with respect to the position of the EDMU in the relevant league tables. Moreover non-convexity of the associated possibility frontier may again arise here, with the convexity condition $\chi_{12}^o \geq 0$ not guaranteed to be fulfilled in (19) when $q_1 < q_1^o$ and/or $q_2 < q_2^o$, and hence $\xi_{11} > 0$ and/or $\xi_{22} > 0$, even if the local convexity condition $\chi_{12} \geq 0$ holds along the frontier between the q_k directly.

5. Non-parametric frontier analysis for non-convex feasible sets

A non-parametric technique that is consistent with the existence of both convex and non-convex regions of the efficient frontier is provided by a modification of the Free Disposal Hull (FDH) model of Duprin *et al* (1984). Duprin's own FDH model seeks efficiency improvements by identifying the largest proportionate reduction in j's inputs that still involves at least as much of each input as an actual producer in the sample with which it is compared and which has achieved no less of each output as producer j, with the associated mixed-integer program (see Cooper *et al*, 2007):

$$min\ \tau_j\ \ s.t.\ \mu X \leq \tau_j X_j, \mu Q \geq Q_j, \mu u = 1, \mu_r \in \{0,1\}\ for\ each\ \ r = 1,...,n \tag{20}$$

When inputs and outputs are both positive in value, we can transform (20) from an *input*- into an *output-orientated FDH model* by substituting the reciprocal of the output variables as inputs and the reciprocal of input variables as outputs in (20) and in the associated available software, to yield:

$$min\ \tau_j\ \ s.t.\ (1/Q_{rk}) \leq \tau_j(1/Q_{jk}), (1/X_{r\ell}) \geq (1/X_{j\ell})\ for\ each\ \ k = 1,...,K\ and\ \ell = 1,...,L \tag{21}$$

for an appropriate choice of the comparator producer r. (21) in turn is equivalent to:

$$max\,\zeta_j\ s.t.\ \mu X \leq X_j, \zeta_j Q_j - \mu Q \leq 0, \mu u = 1, \mu_r \in \{0,1\}\ for\ each\ r = 1,...,n\,,\ with\ \ \tau_j = 1/\zeta_j \tag{22}$$

that generates an output-orientated alternative to the DEA program (1) that avoids DEA's convexity requirement. (22) results instead in a step-function form of the fitted efficient frontier through all the efficient points, such as A,B, E, F and H in Figures 1 and 3, that are actually achieved by individual producers, rather than the convex hull which DEA considers and which may include non-feasible convex combinations, such as point N in Figure 1. The value of τ_j in (22) provides a measure of technical efficiency of the EDMU j that equals the ratio OJ / OE' in Figure 3, and which has the property that its product with the corresponding measure of allocative efficiency OE' / OM equals the true overall measure of efficiency $\varepsilon_{OT} = OJ / OM$.

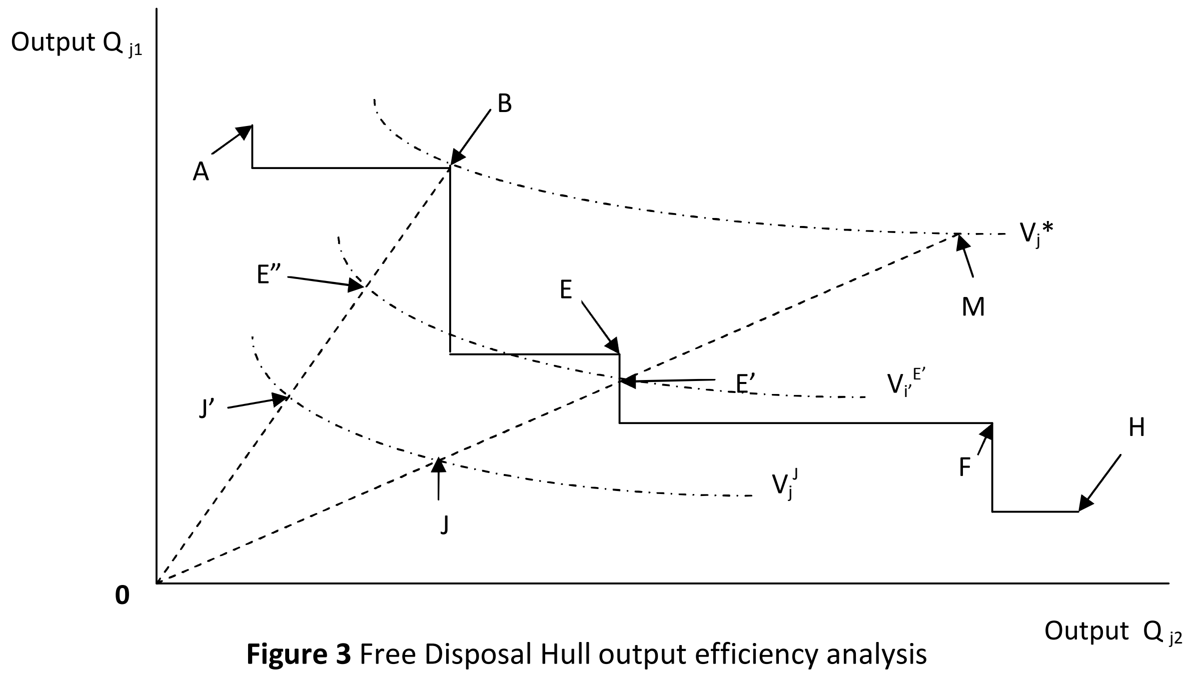

Figure 3 Free Disposal Hull output efficiency analysis

An alternative non-radial measure of technical efficiency suggested by Portela *et al* (2003) in another context would include here the additional slack $E'E$ that a comparison with the efficient point E indicates can be achieved in output Q_{j1}. However, once there are additional slacks in several directions, defining a single such non-radial measure of technical efficiency raises the issue of the weight to be placed upon output increases in different directions. Use of an explicit valuation function would demonstrate that further movements along the efficiency frontier beyond point E, such as to point B in Figure 3, may indeed be desirable. The reciprocal of the associated measure of allocative efficiency $\varepsilon_{Aj} = OE' / OM$ indicates the extent of the additional beneficial gains which can be made by changing the educational output mix from that at points J and E' to that at the optimal point B, with

$$\varepsilon_{Aj} = (OE' / OM) = V_j^{E'} / V_j^* = (OE'' / OB) \; and \; \varepsilon_{OT} = (OJ / OM) = V_j^J / V_j^* = (OJ' / OB) \qquad (23)$$

under a homothetic valuation function which is homogeneous of degree one in the elements of the educational output vector Q_j.

6. Conclusions

Particularly when published measures of the assessed output quality of educational organisations, such as schools and universities, are included in the efficiency analysis, DEA's underlying assumption of convexity of the associated feasible set may not hold. DEA may then not only overstate the scope for efficiency improvements from improving technical efficiency by proportional improvements in these outputs, but also significantly understate, or completely neglect, the scope for improvements in allocative efficiency from non-proportional changes in the educational output mix. The widespread use of league tables for educational assessed output quality measures increases the scope for such non-convexities and associated non-linearities, further boosting the importance of allocative efficiency when educational institutions are subject to resource constraints and under managerial pressure to perform well according to these measures. An assessment of allocative efficiency itself requires a clarification of the valuation function which the institution places upon the volume and assessed quality of their educational outputs in different directions. This becomes more urgent in the presence of non-convexities when more stark choices may need to be made by individual EDMUs between different assessed output quality variables. The nature of the output possibility frontier which maps out the trade-offs that an EDMU may face between the different assessed output quality variables at different points on the frontier may be better revealed by relaxing the convexity assumption, as in the above output-orientated FDH model. At the same time, national policy makers need to re-assess the extent to which the grading and weighting systems which result in the management choices which individual EDMUs face along this frontier of assessed outcomes are out of line with wider educational goals.

References

Arrow K and Enthoven A (1961). Quasi-concave programming. *Econometrica* **29**(4): 779 – 800.

Athanassopoulos A and Shale E (1997). Assessing the comparative efficiency of higher education institutions in the UK by means of data envelopment analysis. *Education Economics* **5**(2)**:** 117-134.

Banker R, Charnes A and Cooper W (1984). Some models for estimating technical and scale inefficiencies in data envelopment analysis. *Management Science* **30**(9): 1078-1092.

Baumol W, Panzar J and Willig R (1982). *Contestable Markets and the Theory of Industry Structure*. Harcourt Brace Jovanovich: New York, NY.

Cooper W, Seiford L and Tone K (2007). *Data Envelopment Analysis*, 2nd edn. Springer: New York, NY.

Deem R, Hillyard S and Reed M (2007). *Knowledge, Higher Education, and the New Managerialism*. Oxford University Press: Oxford.

Department for Education (2014). *Progress 8 School Performance Measure*. DFE: London.

Department of Business, Innovation and Skills (DBIS) (2011*). Higher Education: Students at the Heart of the System*. Cm 8122. HMSO: London.

Duprin D, Simar L and Tulkens H (1984). Measuring labor-efficiency in post offices. In: Marchand M, Pestieau P and Tulkens H (eds) *The Performance of Public Enterprises - Concepts and Measurement*, North-Holland: Amsterdam, pp. 243-267.

Emrouznejad A, Parker B and Tavares G (2008). Evaluation of research in efficiency and productivity: a survey and analysis of the first 30 years of scholarly literature in DEA. *Socio-Economic Planning Sciences* **42**(3): 151-157.

Farrell M (1957). The measurement of productive efficiency. *Journal of the Royal Statistical Society* Series A **120**(3): 253-290.

Henderson J and Quandt R (1980). *Microeconomics Theory – A Mathematical Approach*, 3rd edn. McGraw-Hill: New York, NY.

Higher Education Funding Council for England (2010). *How HEFCE Allocates Its Funding*. HEFCE: Bristol.

Higher Education Funding Council for England (2014). *National Student Survey results and trend analysis 2005-2013.* http://www.hefce.ac.uk/whatwedo/lt/publicinfo/nss/nsstrend/, accessed 25 November 2014.

Izadi H, Johnes G, Oskrochi R and Crouchley R (2002). Stochastic frontier estimation of a CES cost function: the case of higher education in Britain. *Economics of Education Review* **21**(1): 63-71.

Jesson D, Mayston DJ and Smith P (1987). Performance assessment in the education sector: educational and economic perspectives. *Oxford Review of Education* **13**(3): 249-266.

Johnes J and Johnes G (1995). Research funding and performance in UK university departments of economics: a frontier analysis. *Economics of Education Review* **14**(3): 301-314.

NSS (2014). *National Student Survey*. http://www.thestudentsurvey.com/content/nss2012_questionnaire_english.pdf, accessed 25 November 2014.

Portela MCS, Borges P and Thanassoulis E (2003). Finding closest targets in non-oriented DEA models: the case of convex and non-convex technologies. *Journal of Productivity Analysis* **19**(2-3): 251-269.

RAE (2008). *Research Assessment Exercise*. http://www.rae.ac.uk/aboutus/quality.asp, accessed 25 November 2014.

Ramsden P and Moses I (1992). Associations between research and teaching in Australian higher education. *Higher Education* **23**(3): 273-295.

REF (2014). *Research Excellence Framework*. http://www.ref.ac.uk/panels/assessmentcriteriaandlevel definitions/, accessed 25 November 2014.

Thanassoulis E and Dunstan P (1994). Guiding schools to improved performance using DEA: an illustration with data from a local education authority. *Journal of the Operational Research Society* **45** (11): 1247 – 1262.

UNESCO (2004). *Managerialism and Evaluation in Higher Education.* UNESCO Forum Occasional Paper Series Paper No. 7. UNESCO: Paris.