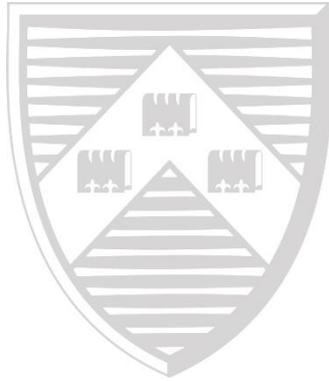


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**Towards a quantitative theory of automatic stabilizers: the role of demographics**

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# Towards a quantitative theory of automatic stabilizers: the role of demographics\*

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## Abstract

Employment volatility is larger for young and old workers than for prime aged. At the same time, in economies with high tax rates, the share of total hours supplied by the young/old workers is smaller. These two observations imply a negative correlation between government size (measured by the share of taxes in total output) and aggregate output volatility. This paper assesses in a calibrated heterogenous agent, overlapping generations model the quantitative importance of these two facts to account for the empirical relation between government size and macroeconomic stability. The baseline calibration accounts correctly for the quantitative relation between output volatility and government size observed in the data.

JEL classification codes: E32; E62; H30; J10; J21.

Keywords: Automatic Stabilizers; Distortionary Taxes; Demographics.

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# 1 Introduction

The motivation for this paper consists of two simple observations. The first is that there is substantial evidence that countries or regions with large governments (measured by the share of taxes in total output) display business cycle fluctuations that are less volatile, as shown in Galí (1994), Rodrik (1998) and Fatás and Mihov (2001). The second observation, which is documented by Clark and Summers (1981), Ríos-Rull (1996), and Gomme et al. (2005) is that fluctuations in hours of market work over the business cycle vary quite dramatically across different demographic groups of the population. In particular, the young experience much greater volatility of employment and total hours worked than the prime aged over the business cycle. Moreover, in a recent paper Jaimovich and Siu (2009) find that changes in the age composition of the labor force account for a significant fraction of the variation in business cycle volatility observed in the US and other G7 economies. Hence, in this article we pose the following question: can the relationship between government size and macroeconomic stability be explained by changes in the demographic composition of the workforce resulting from distortionary taxation?

The hypothesis we put forward is that large governments stabilize output fluctuations because the share of total market hours supplied by young and older workers is smaller in economies with high tax rates. In turn, these differences in the demographic composition of the workforce reduce the *aggregate* labor supply elasticity. Thus, in the tax-distorted real business cycle model we analyze, a relationship emerges between the size of the government (measured by the share of taxes in total output) and the volatility of the cyclical component of aggregate output, consistent with the notion of automatic stabilizers.<sup>1</sup>

The suggestion that time devoted to market work is affected by changes in tax and in transfer policies is one which has received considerable attention. For instance, recent work by Prescott (2004), Rogerson (2006, 2008), Krusell et al. (2008, 2010), Ohanian et al. (2008) and Berger and Heylen (2011) argue that differences in tax and transfer policies can account for a large share of the difference in the amount of hours spent working in Europe and in the US. Moreover, Rogerson and Wallenius (2009) document that the differences in employment rates

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<sup>1</sup>So-called ‘built-in stabilizers’ are features of the tax structure that make tax liabilities automatically respond to current economic conditions (for instance, distortionary labor and capital income taxes) and reduce aggregate volatility. The stabilizing effect of the income tax is traditionally thought to operate via an assumed sensitivity of consumption demand to changes in current tax liabilities. But, according to the Ricardian proposition, this sensitivity is zero. Thus, Christiano (1984) concludes that under a strict version of the Ricardian proposition, the income tax cannot play a role as an automatic stabilizer. Nonetheless, distortionary taxes may affect macroeconomic stability by affecting the aggregate supply and, in particular, the aggregate labor supply elasticity.

between Europe and the US are due almost exclusively to differences in employment rates for young and old workers. Thus, these authors argue that differences in market hours which result from variation in tax and transfer policies are dominated by differences among young and old individuals. This observation offers further motivation for the work we develop in this paper.

Our paper aims at providing a quantitative evaluation of the strength of the automatic stabilizers in an equilibrium business cycle model, based on the relationship between the tax system and the aggregate labor supply elasticity.<sup>2</sup> We develop a fully calibrated stochastic overlapping generation model along the lines of Ríos-Rull (1996) and we model labor supply choices in the extensive margin by way of a non-linear production function of labor services, as in Prescott et al. (2009).<sup>3</sup> The model also includes heterogeneous preferences and, in particular, labor supply elasticities that change over the life-cycle. These changes are calibrated to match differences in the relative cyclical volatility of employment and also differences in employment rates in high tax rate and low tax rate countries, observed for each demographic group.<sup>4</sup> A related calibration strategy has recently been explored by Dyrda et al. (2012) who also generate age differences in hours volatility through differences in preferences.<sup>5</sup> In doing this Dyrda et al. (2012) provide a measurement of the aggregate labor supply elasticity that is consistent with micro estimates, yet yields a much higher value. In our paper we establish a similar result in the context of a large overlapping generation economy. In particular, the labor supply elasticity of all prime aged individuals is small, as implied by the meta-analysis of quasi-experimental studies presented in Chetty et al. (2012) but, given the heterogeneity in preferences, the aggregate labor supply elasticity under the baseline calibration is equal to

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<sup>2</sup>To be sure, our paper is not suitable to study the welfare impact of automatic stabilizers, which in certain contexts relates to income stabilization (Blanchard, 1984). Taxation distorts the consumption-savings decisions and the labor supply choices. Optimal taxation must balance distortions versus insurance. However, in our OLG framework, as in Ríos-Rull (1996), markets are sequentially complete (individuals face complete markets conditionally on the state of the economy at their birth) and the gains from automatic stabilizers are negligible. For the government to have a potential insurance role, agents must be unable to enter private insurance contracts by assuming incomplete risk sharing because of private information and moral hazard considerations. See McKay and Reis (2013) for a detailed study of the insurance role of automatic stabilizers in an incomplete markets DSGE model.

<sup>3</sup>In recent work Erosa et al. (2012) rely on a nonlinear mapping from the workweek length to the wage rate to activate an extensive margin.

<sup>4</sup>Chetty et al. (2012) show that estimates of steady-state elasticities of the response of employment to taxes are similar whether one relies on macro or micro data, although they may differ when one estimates intertemporal substitution elasticities.

<sup>5</sup>In their model, young individuals have more volatile total hours because they are assumed to have different preferences (higher labor supply elasticity) and also the option to “move in with their parents” in recessions making their hours worked even more procyclical.

0.84, a value recommended by Chetty et al. (2012) to calibrate stand-in agent RBC models.<sup>6</sup>

We represent preferences using the GHH utility function which eliminates wealth effects in the labor supply choices.<sup>7</sup> Although the use of GHH preferences has the drawback of being inconsistent with a balanced growth path, it offers two important advantages: first, it has the attractive implication that the changes in the equilibrium levels of employment resulting from distortionary taxes are robust to changes in the assumptions concerning the use of the tax revenue and the nature of transfer programs;<sup>8</sup> second, the use of these preferences together with the calibration that attributes a large labor supply elasticity to young and old workers relative to prime aged, implies that employment differences resulting from distortionary taxes are largely due to differences in employment rates among the young and the old, consistent with empirical evidence.<sup>9</sup>

An important aspect that differentiates this paper from the literature that examines the relationship between government size and aggregate volatility is that we study if the model is *quantitatively* consistent with the observed strength of the automatic stabilizers, while earlier contributions mostly focus on the sign of the relationship between government size and macroeconomic stability.<sup>10</sup> To do so, we first calibrate the model to the US economy to match cross-sectional information on the wage profile and the relative level and volatility of market hours across age groups. We then apply standard development accounting methods. We feed the theoretical economy with different fiscal policy parameters that mimic the fiscal profile of OECD countries. This allows us to generate a sample of simulated OECD economies. These economies differ from the benchmark calibrated economy only in their fiscal policy

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<sup>6</sup>Mennuni (2013) in the context of a general equilibrium model of the business cycle with overlapping generations similar to ours, also explores the possibility that changes in the composition of labor affect the evolution of aggregate volatility, but focuses on differences across gender and schooling.

<sup>7</sup>See Greenwood et al. (1988).

<sup>8</sup>See Ljungqvist and Sargent (2007) for a discussion of the implications of changing the explicit details of tax and transfer programs in the context of the balanced growth path representative agent model and Erosa et al. (2012) in the context of the life-cycle model.

<sup>9</sup>Excluding intertemporal substitution in labor supply is also consistent with the findings of Jaimovich and Rebelo (2009) that over the business cycle wealth effects are weak.

<sup>10</sup>Galí (1994) examines whether income taxes and government purchases behave as automatic stabilizers in the basic, technology shock-driven, real business cycle (RBC) model. He finds that the relationship between government size and macroeconomic stability implied by the standard model is qualitatively counterfactual. The model in Greenwood and Huffman (1991) also generates a positive correlation between aggregate volatility and taxes. Guo and Harrison (2006) discuss the robustness of the results in Galí (1994). Andrés et al. (2008) extend the analysis in Galí (1994) and study how alternative models of the business cycle can replicate the relationship between government size and macroeconomic stability. Their analysis shows that adding nominal rigidities and costs of capital adjustment to the standard model can generate a negative correlation between government size and the volatility of output.

parameters. The quantitative assessment of the model requires comparing the responsiveness of aggregate volatility to changes in government size implied by the model and observed in the data. The model-implied government size emerges as an endogenous outcome resulting from mimicking the fiscal profile of the OECD countries in our sample.<sup>11</sup> Under the baseline calibration our model accounts for 80% of the relationship between output volatility and government size. The strength of the automatic stabilizers is explained by the changes in the aggregate labor supply elasticity implied by the workforce demographic composition.

The remainder of the paper is organized as follows. In Section 2 we provide empirical evidence about the relationship between the workforce demographic composition, government size and macroeconomic stability. We introduce the model in Section 3. In Section 4 we establish three results concerning the relationship between government size and the composition of the workforce implied by the model. In Section 5 we describe our calibration procedure and in Section 6 we examine the quantitative implications of the baseline economy. In Section 7 we study the relationship between government size and macroeconomic stability implied by the model and compare it to the data. In Section 8 we consider two additional quantitative experiments. Finally, Section 9 offers concluding remarks.

## 2 Motivating evidence

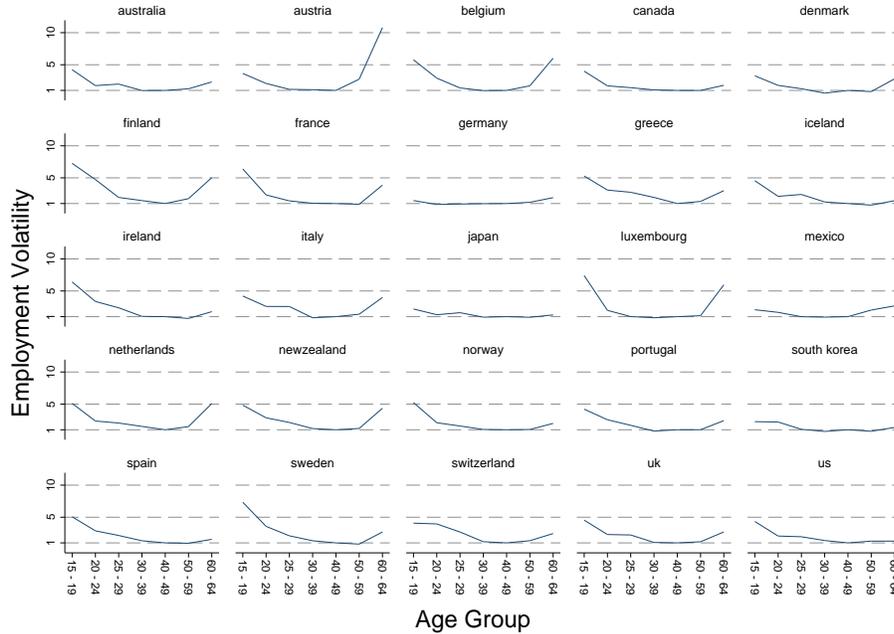
The hypothesis put forward in this paper is that large governments stabilize fluctuations in output because they encourage the demographic groups exhibiting high labor supply volatility to work relatively fewer hours. In this Section we document some empirical evidence that motivates this mechanism. We start by showing that the differences in cyclical volatility of employment across demographic groups are a general feature of the OECD economies: in all the countries, the cyclical volatility of employment exhibits a u-shaped profile over the life cycle, with young and older workers exhibiting the highest cyclical volatility. Second, we show that the employment share of the young and old in total employment is lower in countries with large governments. Third, we show that accounting for the demographic composition of the labor force is important to explain the differences in hours and output volatility.<sup>12</sup>

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<sup>11</sup>A fiscal profile is a set of taxes (labor income tax, capital income tax and consumption tax) and the share of government spending in GDP.

<sup>12</sup>We also show that the relevant margin of adjustment is the extensive margin (relative employment rates). Although young employed workers also work fewer hours in countries with large governments, this effect is not responsible for the negative correlation between government size and aggregate hours volatility.

Figure 1: volatility of employment by demographic group, OECD



Note: The data is annual and the source is the OECD Labour Force Statistics. All variables are reported in logs as deviations from an HP trend with smoothing parameter 6.25. The volatility is expressed relative to the 40 – 49 age group.

## 2.1 The employment volatility profile over the life cycle

We begin by documenting a well established relationship between employment volatility and age: the employment volatility of young and old workers is larger than the employment volatility of prime-age workers. Jaimovich and Siu (2009) show that in all G7 countries young workers experience much greater volatility of employment and hours worked than the prime-aged over the business cycle; this is also true for those closer to retirement. We show that this empirical relationship is true in a large cross-section of OECD countries.<sup>13</sup> To illustrate this fact, we follow the approach of Gomme et al. (2005) and Jaimovich and Siu (2009), who report cyclical employment volatilities for various age groups. We use annual

<sup>13</sup>Several studies have illustrated that the labor market behavior of the young and the old differs from the behavior of prime-aged workers. For instance, Pencavel (1987), Killingsworth and Heckman (1987) and Blundell and Macurdy (1999) provide microeconomic evidence that the elasticity of labor supply is larger for the younger and the older workers. Blanchard and Diamond (1990) and Janiak and Wasmer (2008) show that employment impulse responses for young and old workers are larger in magnitude than middle-age workers.

Table 1: employment volatility over the life-cycle

	(1)
	Employment Volatility
15 – 19	3.609*** (12.47)
20 – 24	1.448*** (5.00)
25 – 29	0.798*** (2.76)
30 – 39	0.101 (0.35)
50 – 59	0.228 (0.79)
60 – 64	2.223*** (7.68)
Observations	175
$R^2$	0.680

*t* statistics in parentheses

\*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$

Note: Each observation corresponds to a country and one of the following age groups: 15 – 19, 20 – 24, 25 – 29, 30 – 39, 40 – 49, 50 – 59 and 60 – 64. The listed regressors are dummy variables for each age group except the reference group (40 – 49). Country dummies and the intercept are included but not listed. See Appendix A for details about the data.

data on employment by age group from the OECD for an unbalanced panel of 25 countries from 1970 to 2009.

We build seven categories: individuals aged between 15 and 19 years old, 20 – 24, 25 – 29, 30 – 39, 40 – 49, 50 – 59 and 60 – 64 years old. For each of these categories, we extract the business cycle component of employment by applying the Hodrick-Prescott (HP) filter to the logged series with smoothing parameter equal to 6.25 as suggested by Ravn and Uhlig (2002), and we calculate the standard deviation. We report the relative volatility, given by the standard deviation of each age group relative to the standard deviation of the group aged between 40 and 49.

Figure 1 displays the results for a large cross-section of OECD countries.<sup>14</sup> The figure shows an ubiquitous u-shaped relationship between age and employment volatility at business cycle frequencies. In all countries the volatility of employment is the highest either for the workers aged 15 to 19 or for the workers aged 60 to 64. The employment volatility of the youngest workers is on average nearly five times that of the workers aged 40 to 49. The workers aged 60 to 64 also display large employment volatility, on average more than three times that of the workers aged 40 to 49. Finally, in all the countries the prime-age workers (aged 40 to 49) have the most stable labor supply. Table 1 shows that the differences in employment volatility over the life-cycle are statistically significant.<sup>15</sup>

## 2.2 Demographic composition of employment and government size

The second fact we document concerns the relationship between the demographic composition of the workforce and government size (measured by the ratio between total tax revenue and Gross Domestic Product). In particular, we are interested in the correlation between government size and what Jaimovich and Siu (2009) call the *volatile-aged* employment share (defined as the ratio between the employment of individuals aged 15 to 29 and 60 to 64 and the employment of individuals aged 15 to 64). Panel (a) of Figure 2 shows the relationship between the volatile-aged employment share and government size. Each observation in the sample corresponds to an OECD country over one of the following time intervals: 1970 – 1979, 1980 – 1989, 1990 – 1999 and 2000 – 2009. The scatter plot shows a strong negative correlation between the volatile-aged employment share and government size. The first column of Table 2 shows that the relationship is statistically significant.

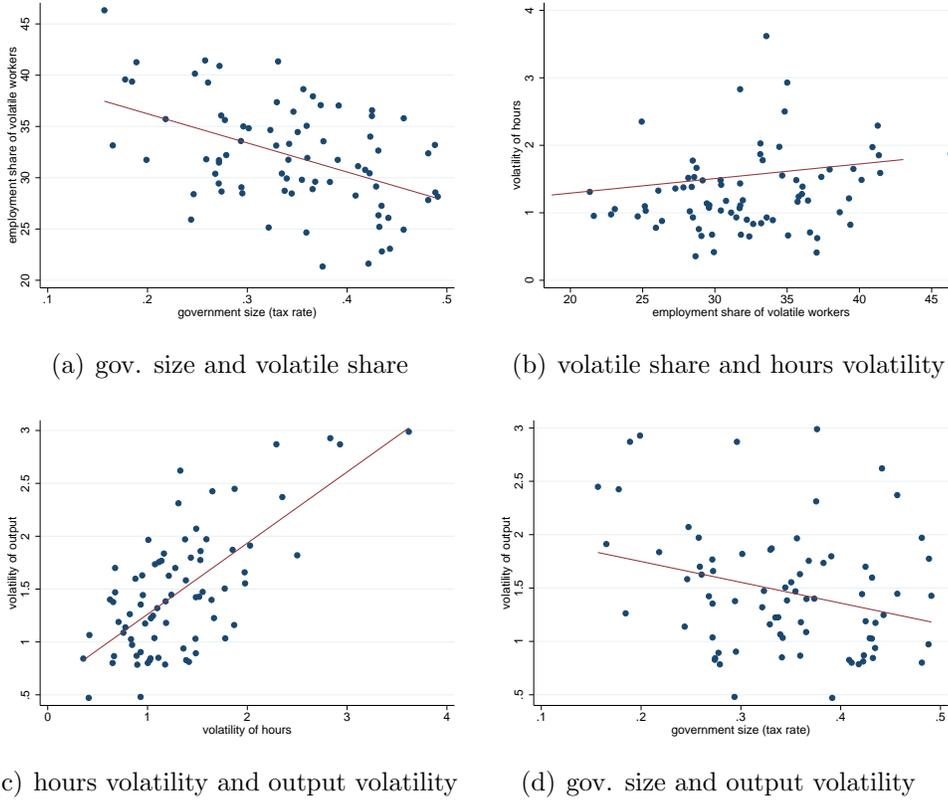
One could argue that countries with a large share of old individuals in the population need large government. For example, large governments help provide the old with social security and healthcare. At the same time, in countries with a large share of old individuals in the

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<sup>14</sup>Not reported here, we also used data at the US state level (for both employment and hours volatility), which we constructed from the Current Population Survey. Results are qualitatively similar. Quantitatively, the volatility ratio of the 15 – 19 years old is lower with an average equal to 2. The 60 – 64 age group displays similar volatility. For the US state level data, the identity of the group displaying the lowest volatility is more heterogeneous. The lowest volatility age group is either the 30 – 39, the 40 – 49 or the 50 – 59 group.

<sup>15</sup>Here we have focused on employment. However, the cyclical volatility of involuntary unemployment also varies between age groups. For example, Elsby et al. (2010) show that the young were the most affected during the Great Recession and in earlier recessions. However, Elsby et al. (2010) also show that there are no differences in the cyclical volatility of unemployment transition rates between these demographic groups: the probability of finding or retaining a job is not more cyclical for the young compared to the prime aged workers. Moreover, the cross-country correlation between unemployment volatility and government size is negligible. For these reasons, we henceforth focus only on employment fluctuations.

Figure 2: government size and aggregate volatility (OECD)



Note: annual data on Tax to GDP ratios and GDP are from the OECD outlook database, while data on hours worked are from the Conference Board Total Economy database. The sample includes the following countries: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Iceland, Ireland, Italy, Japan, Luxembourg, Mexico, Netherlands, New Zealand, Norway, Portugal, South Korea, Spain, Sweden, Switzerland, the United Kingdom and the United States. The Volatile Share of employment corresponds to the ratio between the employment of the population aged 15 to 29 and 60 to 64 and the employment of the population aged 15 to 64. The Output and Hours Volatility corresponds to the standard deviation of the cyclical components, given by the log deviations from the HP trends with smoothing parameter 6.25. Each observation corresponds to a country and one of the following time periods: 1970 – 1979, 1980 – 1989, 1990 – 1999, and 2000 – 2009.

Table 2: government size, demographic structure and hours volatility

	(1)	(2)	(3)	(4)	(5)
	vol. share	vol. share	young share	young share	vol. of hours
Gov. Size (tax rate)	-20.180*** (5.226)	-16.250*** (5.256)	-20.145*** (5.234)	-16.217*** (5.263)	
Share of 60+ in pop.		-20.584** (8.009)		-20.576** (8.019)	
Volatile Share					0.056*** (0.016)
Constant	38.463*** (2.057)	42.284*** (2.456)	38.397*** ( 2.059)	42.216*** (2.479)	-0.026 (0.405)
Observations	75	75	75	75	75
$R^2$	0.472	0.518	0.471	0.517	0.169

Standard errors in parentheses

\*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$ 

Note: Each observation corresponds to a country and one of the following time periods: 1970 – 1979, 1980 – 1989, 1990 – 1999, and 2000 – 2009. Time dummies are included but not listed. Volatility of hours is the standard deviation of the respective cyclical component (calculated using the HP filter with smoothing parameter 6.25). The volatile share corresponds to the share of employment of the population aged 15 to 29 and 60 to 64 in the total employment of the population aged 15 to 64. Gov size is the ratio between total tax revenue and GDP. See Appendix A for details about the data.

population, it is natural to have a lower share of young workers in aggregate employment. Hence, the negative correlation between government size and the volatile-aged employment share could be spurious and due to the varying share of old individuals in the population.<sup>16</sup> To address this issue, the second column in Table 2 also controls for the share of individuals aged 60 or more in the population. The results indicate that countries with a large share of individuals aged more than 60 indeed have a lower volatile-aged employment share. This implies that the coefficient for government size is also lower than in the regression of column (1), but it is still significant and suggests that countries with large government have a lower volatile-aged employment share for reasons that are not related to the demographic structure of the population. Columns (3) and (4) report the same regressions as in columns (1) and (2), by excluding individuals aged 60 to 64 from the volatile-aged employment share. The estimated coefficients are very similar. Thus, although the demographic structure of the population simultaneously affects the workforce composition and the size of the government, the latter exerts an independent effect on the share of young workers in employment.

We argue that, as a result of the negative correlation between the volatile-aged share of

<sup>16</sup>We thank a referee for this comment.

Table 3: government size and aggregate volatility

	(1) vol. output	(2) vol. output
Volatility of Hours	0.713*** (0.078)	
Gov. Size (tax rate)		-1.819** (0.794)
Constant	0.848*** (0.156)	1.621*** (0.299)
Observations	77	77
$R^2$	0.565	0.099

Standard errors in parentheses  
 \*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$

Note: Each observation corresponds to a country and one of the following time periods: 1970 – 1979, 1980 – 1989, 1990 – 1999, and 2000 – 2009. Time dummies are included but not listed. Volatility of output and volatility of hours are the standard deviation of the respective cyclical component (calculated using the HP filter with smoothing parameter 6.25). Gov Size is the ratio between total tax revenue and GDP. See Appendix A for details about the data.

employment and government size, total employment should be less volatile in countries with large governments. Panel (b) of Figure 2 shows that the correlation between the volatile-aged share of employment and hours volatility is positive. The last column of Table 2 shows that the relationship is statistically significant. This positive correlation follows from the life-cycle profile of employment volatility documented earlier. Finally, as Panel (c) of Figure 2 illustrates, the volatility of hours is positively associated with the volatility of aggregate output. The upshot is that countries with large governments are associated with more stable output fluctuations, as illustrated in Panel (d) of Figure 2. Table 3 confirms that the relations represented in the Panels (c) and (d) of Figure 2 are statistically significant.

### 2.3 Government size and stabilization: the role of demographics

Table 4 reports results on the relationship between hours and output volatility, government size and the demographic structure of the workforce. The columns (1) and (3), show the relationship between hours volatility and government size, and between output volatility and government size, respectively. Both columns illustrate the stabilization role of the government

Table 4: demographic structure and aggregate volatility

	(1) vol. hours	(2) vol. hours	(3) vol. output	(4) vol. output
Gov. Size (tax rate)	-1.819** (0.794)	-0.868 (0.848)	-1.909** (0.775)	-1.275 (0.841)
Volatile Share		0.049*** (0.018)		0.035** (0.017)
Constant	1.621*** (0.299)	0.466 (0.628)	2.182*** (0.290)	1.236** (0.623)
% Change in Fiscal Coefficient		-52%		-33%
Observations	77	75	77	75
$R^2$	0.099	0.182	0.134	0.180

Standard errors in parentheses

\*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$ 

Note: Each observation corresponds to a country and one of the following time periods: 1970 – 1979, 1980 – 1989, 1990 – 1999, and 2000 – 2009. Time dummies are included but not listed. Volatility Hours and Volatility Output are, respectively, the standard deviation of the cyclical component of aggregate hours and of output (calculated using the HP filter with smoothing parameter 6.25). The Volatile Share corresponds to the ratio between the employment of the population aged 15 to 29 and 60 to 64 and the employment of the population aged 15 to 64. Gov Size is the ratio between total tax revenue and GDP. See Appendix A for details about the data.

sector: large governments (measured by total tax revenue as a fraction of GDP) are associated with lower volatility of aggregate hours, and with lower volatility of output. In turn, columns (2) and (4) concern directly the key argument advanced in this paper. It expands the list of regressors included in the regressions (1) and (3) with the volatile-aged employment share as a control variable. In both regressions, once the demographic control variable is included, the magnitude of the coefficient associated with government size is smaller and it is no longer statistically significant. In the regression concerning the volatility of hours the slope coefficient falls by 52%, while in the regression concerning output volatility the slope coefficient falls by 33%. This findings support the hypothesis that the stabilization role of the government is in part explained by the changes in the demographic structure of the workforce associated with changes in tax rates.

## 2.4 Evidence on the intensive margin of adjustment

The evidence discussed so far is about how taxes influence the demographic composition of employment and how this in turn impacts the volatility of aggregate hours worked and output. Variation in total hours worked in the intensive margin (hours worked by those in employment) have so far been ignored. This component is quantitatively unimportant for the fluctuation of hours worked in the case of the United States as most of the variation of aggregate hours worked is brought by the extensive margin of adjustment (Hansen, 1985). However, it has a greater significance in more regulated economies such as in France for example (Ohanian and Raffo, 2012).<sup>17</sup> We now document that there is also a negative correlation between government size and the hours worked by young employed workers, relative to the hours worked by the prime aged employed workers. However, the variation in the intensive margin of hours worked across countries is not an aspect that helps explain the negative correlation between government size and macroeconomic volatility.<sup>18</sup>

The OECD provides data on the supply of hours worked at the intensive margin for three age categories (15 – 24, 25 – 54 and 55 – 64). The variable is labelled “*Average usual weekly hours worked on the main job*”. Because we are interested in the impact of taxes on the demographic composition of hours worked, we consider the ratio of hours supplied by the employed individuals aged 15 to 24 relative to the hours supplied by the employed individuals aged 25 to 54 in order to understand how taxes may affect the relative supply of hours by the young. To understand the impact on the relative supply by older workers, we consider the ratio of the 55 – 64 to the 25 – 54. The first two columns of Table 5 regress each ratio on our measure of government size. Notice that the number of observations decreases comparing to the previous regressions because the variable is not available for all countries. It is shown that government size has a significant negative impact on the relative supply of hours worked by young employees. However, the table does not suggest any effect on the relative supply by employees aged 55 to 64. These results indicate that government size also influences the demographic composition of hours worked at the intensive margin, especially by decreasing the relative hours worked by employees aged 15 to 24.<sup>19</sup>

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<sup>17</sup>The data examined in Ohanian and Raffo (2012) is at quarterly frequencies. Instead, we consider annual data and calibrate our model to the annual frequency. The importance of the extensive margin (employment) to explain cyclical variations in total hours is even more predominant at annual frequencies, as business cycle fluctuations in hours conditional on employment only account for about 1/6 of the fluctuations in aggregate hours at an annual frequency (Heckman, 1984).

<sup>18</sup>We thank the editor for suggesting to study this additional channel.

<sup>19</sup>There is also evidence of a u-shaped pattern for volatility over the life cycle: the average volatility of hours worked by employees across a set of 22 OECD countries for the 25-54 years old is 0.4%, while it is 1.2% and 0.9% for employees aged 15-24 and 55-64 respectively.

Table 5: government size and hours worked at the intensive margin

	(1) rel. hours by 15-24 (int.)	(2) rel. hours by 55-64 (int.)	(3) hours vol. (total)	(4) hours vol. (total)	(5) hours vol. (total)	(6) hours vol. (total)	(7) hours vol. (int.)	(8) hours vol. (int.)
Gov. Size (tax rate)	-0.453*** (0.162)	0.053 (0.064)	-2.578** (1.084)	-1.902* (1.031)				
Rel. hours 15-24 (int.)			-1.251 (-0.856)		-.520 (.834)		.903* (0.498)	
Rel. hours 55-64 (int.)				-2.038 (2.176)		-2.490 (2.210)		-0.509 (1.369)
Constant	1.110*** (0.062)	1.006*** (0.025)	3.306*** (1.026)	3.967* (2.223)	1.724** (0.816)	3.778 (2.270)	-0.328 (0.486)	1.056 (1.406)
Observations	57	57	57	57	57	57	57	57
$R^2$	0.210	0.212	0.123	0.102	0.027	0.043	0.083	0.080

Standard errors in parentheses

\*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$ 

Note: Each observation corresponds to a country and one of the following time periods: 1970 – 1979, 1980 – 1989, 1990 – 1999, and 2000 – 2009. Time dummies are included but not listed. The volatility of hours is the standard deviation of the respective cyclical component (calculated using the HP filter with smoothing parameter 6.25). Gov Size is the ratio between total tax revenue and GDP.

In the vein of the exercise done in Table 4, in columns (3) and (4) we examine if the relation between government size and macroeconomic volatility survives controlling for the relative hours worked by employed workers aged 15 – 24 (third column) or 55 – 64 (fourth column). The exercise shows that the relation between government size and the volatility of total hours remains significant and of the same magnitude after controlling for the demographic composition of hours worked at the intensive margin. Moreover, the demographic composition of hours worked by those in employment is not significant in the regression. Thus the channel generating the negative correlation between government size and aggregate volatility operates through the extensive margin.<sup>20</sup> Columns (5) and (6) provide an explanation for the weak contribution of the intensive margin: the demographic composition of hours worked at the intensive margin is not significant to explain the aggregate volatility of total hours worked even when government size is not considered as a regressor. Column (7) suggests that the demographic structure may have an effect on the volatility of total hours worked at the intensive margin, but this relation is too weak to have an impact on the volatility of total hours.

<sup>20</sup>Similar results are obtained if one considers the volatility of output instead of the volatility of hours.

In summary, in this Section we have documented the four following facts: i) the employment of young/old individuals fluctuates much more over the business cycle than that of prime-age individuals; ii) across the OECD countries, the volatile-aged share of employment declines as the size of the government increases, even after controlling for the demographic structure of the population; iii) there is a negative relation between the size of the government and the cyclical volatility of aggregate hours and output, however, controlling for the demographic composition of the workforce attenuates substantially this relationship; iv) this mechanism operates along the extensive margin. In what follows we propose a theoretical model based on these four facts. Our objective is to investigate if an overlapping-generation real business cycle model that accommodates workforce heterogeneity implies a stabilizing role for the government sector qualitatively and quantitatively consistent with the data

### 3 The model

In this Section, we present a model that features age specific differences in cyclical volatility of hours worked. The framework is that of an overlapping generations economy as in Ríos-Rull (1996), and we model labor supply choices in the extensive margin by way of a non-linear production function of labor services, as in Prescott et al. (2009). Time is discrete and each date  $t$  corresponds to a year. Each year a continuum (with measure  $\mu_1$ ) of individuals is born. Individuals live a maximum of  $T$  periods, but face random lifespans. We denote an individual's age by  $i \in \mathcal{I} \equiv \{1, \dots, T\}$ . The conditional probability of surviving from age  $i$  to  $i + 1$  is  $\zeta_i$ , with  $\zeta_0 = 1$  and  $\zeta_T = 0$ . Thus, the mass of individuals alive at age  $i$  is  $\mu_i = \mu_1 \prod_{j=1}^i \zeta_{j-1}$ .<sup>21</sup> All individuals must retire at age  $M < T$ . The other features of the economy are those of the standard RBC model featuring capital adjustment costs, with competitive labor and capital markets.

#### 3.1 Preferences and labor supply

Preferences of an individual aged  $i$  are specified over yearly consumption and total hours worked and take the form introduced in Greenwood et al. (1988), given by

$$u(c, n; i) = \frac{1}{1 - \sigma} \left( c - \frac{\lambda_i n^{1+1/\eta_i}}{1 + 1/\eta_i} \right)^{1-\sigma}, \quad (1)$$

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<sup>21</sup>The mass of newborns  $\mu_1$  is chosen so that the total population  $\sum_{i=1}^T \mu_i$  has unit size.

with  $\sigma > 0$  and where  $c$  denotes consumption and  $n$  the total hours worked in the year.<sup>22</sup> The preference parameters  $\lambda_i$  and  $\eta_i$  are age dependent and, in particular,  $\eta_i$  is the wage-elasticity of labor supply.<sup>23</sup> Following, for example, Bils and Cho (1994) and Cho and Cooley (1994) we distinguish between the number of hours worked per unit of time (say a week), denoted  $h \in [0, \underline{h}]$ , and the number of weeks the individual works in the year, denoted  $e \in [0, \underline{e}]$ .<sup>24</sup> Hence, total hours worked in a year are  $n = eh$ . Without loss of generality, we normalize the total “number of weeks” in the year,  $\underline{e}$ , to unity and interpret  $e$  as the employment rate as in Cho and Cooley (1994).

Preferences are specified over total hours worked in the year and, hence, the number of hours worked per week and the work weeks per year are perfect substitutes. However, in the spirit of Prescott et al. (2009) and Rogerson and Wallenius (2009) we assume that the mapping from hours worked per week and labor services per week is non-linear. Thus, the individual faces a meaningful choice between hours worked per week (work week length) and work weeks per year. In particular, for an individual aged  $i$ , the mapping from weekly hours worked to labor services per week is given by

$$\ell_i = g(h; i), \tag{2}$$

where the function  $g(h; i)$  is assumed to have the shape drawn in Figure 3. In particular, for each age group  $i$ ,  $g(h; i)$  is increasing in  $h$ , it is equal to zero at the origin and, over the domain  $[0, 1]$ , the function  $g(h, i)$  is first convex and then becomes concave. Thus, the function  $g(h; i)$  captures two key features: first, that over some domain of hours, a part-time worker is often less productive than a full-time worker; second, that after some point working longer hours leads to fatigue and, hence, lower returns to work.<sup>25</sup>

The length of the workweek which is optimally chosen by individuals is the level  $\bar{h}_i$  at which

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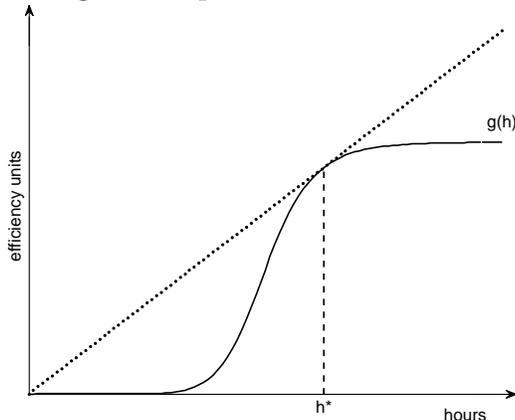
<sup>22</sup>If  $\sigma = 1$ , the utility function specializes to  $u(c, n; i) = \ln\left(c - \frac{\lambda_i n^{1+1/\eta_i}}{1+1/\eta_i}\right)$ .

<sup>23</sup>There are several factors that may explain why different age groups may be characterized by different labor supply elasticities. These factors are discussed for instance in Choi et al. (2014). Reasons are related to family formation, human capital accumulation and depreciation, savings, the retirement age, among others. These factors are not modeled explicitly in our model. Hence, we interpret the assumed heterogeneity in labor supply elasticities as a reduced form capturing all these factors. The recent model by Dyrda et al. (2012) also allows labor supply elasticities to depend upon age, for similar reasons.

<sup>24</sup>Cho and Cooley (1994) specify preferences over instantaneous consumption and hours worked. Instead, we specify preferences over consumption and total hours worked over a discrete period (say, a year).

<sup>25</sup>The function  $g(h; i)$  is age specific to allow matching the changes in the number of hours worked per employed worker and the changes in earnings that occur over the life-cycle, as observed in the data. In Section 5 we specify the parametrization of the function  $g(\bullet)$  in detail.

Figure 3: optimal choice of hours



the following condition holds

$$\frac{g(\bar{h}_i; i)}{\bar{h}_i} = g'(\bar{h}_i; i), \quad (3)$$

with the interpretation that the average productivity of work in the week is equal to the marginal productivity of an additional hour of work that week.<sup>26</sup>

This point is illustrated in Figure 3. Past that point, increasing labor services can be done more efficiently in the extensive margin, by raising the number of weeks of work. It is noteworthy that, despite being endogenous, the optimal length of the workweek does not depend on wealth or the wage rate. Thus, adjustments in total hours take place along the extensive margin which is consistent with the evidence from the US that most of the cyclical (and secular) variation in hours is on the employment margin.<sup>27</sup>

Individuals born at date  $t$  seek to maximize their life-time expected utility, given by

$$E_t \left[ \sum_{i=1}^T \beta^{i-1} \left( \prod_{j=1}^i \zeta_{j-1} \right) u(c_{i,t+i-1}, n_{i,t+i-1}; i) \right], \quad (4)$$

and where  $c_{i,t} \geq 0$  and  $n_{i,t} \in (0, \underline{h} \times \underline{e})$ .

<sup>26</sup>In the sequel, we always assume interior solutions for both  $h$  and  $e$ . See the Appendix B.1 for a detailed derivation of condition (3) and the remaining first-order necessary conditions to solve the individual problem.

<sup>27</sup>See for example Heckman (1984) and Hansen (1985).

### 3.2 Financial markets

As in Ríos-Rull (1996), we assume that markets are sequentially complete and the full set of Arrow securities can be traded. In addition, two outside assets are traded: one-period government bonds and shares of the representative firm (the volume of outstanding equity shares is normalized to unity). Individuals also have access to actuarially fair contracts for annuities. These contracts are arrangements whereby all the individuals of the same cohort sign a contract in which survivors share the assets (or debts) of the agents that die. Thus, the following period's asset are the current period savings divided by the probability of surviving. The resulting budget constraint faced by an individual aged  $i$  at date  $t$  is

$$(1 + \tau_c) c_{i,t} + p_t s_{i+1,t+1} + d_t b_{i+1,t+1} + \sum_{z \in \mathcal{Z}} q_t^z x_{i+1,t+1}^z = (1 - \tau_h) w_t g(\bar{h}_i; i) e_{i,t} + a_{i,t} + L_t, \quad (5)$$

where  $\tau_c$  and  $\tau_h$  are the consumption and labor income tax rates,  $x_{i+1,t+1}^z$  constitutes the amount of state-contingent Arrow securities, for each event  $z \in \mathcal{Z}$ , bought by individuals aged  $i$ , at price  $q_t^z$ ;  $b_{i+1,t+1}$  are the government bonds bought by individuals aged  $i$ , at discount price  $d_t$ ; and  $s_{i+1,t+1}$  are the shares in the representative firm owned by an individual aged  $i$  at the end of the period and  $p_t$  is the ex-dividend price of those shares. The taxable labor income of an individual aged  $i$  is  $w_t g(\bar{h}_i; i) e_{i,t}$ , where  $\bar{h}_i$  denotes the hours worked by an individual aged  $i$  who chooses optimally the length of the work week. Finally, the individual's resources include lump-sum transfers received from the government  $L_t$  and her start of period wealth, given by

$$a_{i,t} = \frac{(\pi_t + p_t) s_{i,t} + b_{i,t} + x_{i,t}}{\zeta_{i-1}}, \quad (6)$$

where  $x_{i-1,t}$  and  $b_{i-1,t}$  are the payments from the Arrow securities and the government bonds, respectively, and  $(\pi_t + p_t) s_{i-1,t}$  is the income from the shares owned in the representative firm, with  $\pi_t$  the after-tax profits distributed to the shareholders.

### 3.3 Firms

We consider a one-sector model economy where the single good produced serves two purposes: consumption and investment. Output is produced by a representative firm that combines capital and labor services via a constant returns to scale Cobb-Douglas production function

$$Y_t = \exp(\epsilon_t^1) ((u_t/\bar{u}) K_t)^\alpha H_t^{1-\alpha}, \quad (7)$$

where the capital services are the product of the stock of capital  $K_t$  and the rate of capital utilization,  $(u_t/\bar{u})$ ;

$$H_t \equiv \sum_{i=1}^M \mu_i g(\bar{h}_i; i) e_{it} \quad (8)$$

are the efficiency units of labor services in period  $t$ . Fluctuations in log total factor productivity  $\epsilon_t^1$ , follow an exogenous Markov process with strictly positive transition matrix.

Increases in the utilization rate of capital are costly because higher utilization rates imply faster depreciation rates; the depreciation function is

$$\delta(u_t) = \delta_0 + \delta_1 (u_t/\bar{u})^{1+\varsigma}, \quad (9)$$

with  $\varsigma > 0$  and  $\bar{\delta} \equiv \delta_0 + \delta_1 \in (0, 1)$  the steady state depreciation rate.<sup>28</sup> The representative firm faces adjustment costs in investment, represented by the following equation for capital accumulation

$$K_{t+1} - K_t = \Phi\left(\frac{I_t}{K_t}\right) K_t - \delta_t K_t, \quad (10)$$

with  $\delta_t \equiv \delta(u_t)$  the capital depreciation rate at date  $t$ . Following King and Watson (1996) and Basu and Kimball (1997), the capital adjustment cost function  $\Phi(\bullet)$  is increasing, concave, and satisfies  $\Phi(\bar{\delta}) = \bar{\delta}$  and  $\Phi'(1) = 1$ .

The representative firm seeks to maximize its value to shareholders, given by

$$J(K_t; \epsilon_t^1) = \max_{I_t, H_t} \left\{ \pi_t + E_t \left[ \Lambda_{t+1} J(K_{t+1}; \epsilon_{t+1}^1) \right] \right\}, \quad (11)$$

subject to the motion equation (10), and where  $\Lambda_{t+1}$  is the stochastic discount factor of the firm's shareholders.<sup>29</sup> The after-tax profits,  $\pi_t$ , are given by<sup>30</sup>

$$\pi_t = (1 - \tau_k) \left[ \exp(\epsilon_t^1) K_t^\alpha H_t^{1-\alpha} - w_t H_t - I_t \right], \quad (12)$$

<sup>28</sup>This functional form for the depreciation function has been proposed in Basu and Kimball (1997). The parameter  $\bar{u}$ , the steady state utilization, is normalized to 1 without loss of generality.

<sup>29</sup>Because markets are sequentially complete, all individuals who own shares in the representative firm have the same stochastic discount factor,  $\Lambda_{t+1}$  for all  $i = 1, \dots, T-1$  (See Appendix for details). This makes it possible to price the firm's shares. Instead, with incomplete financial markets the objective of the firm is not well defined (for a detailed treatment see Grossman and Hart 1979).

<sup>30</sup>We are assuming dividend taxation is the only form of tax on capital and, hence, ignoring corporate taxes. In the United States, both dividend taxes and corporate taxes are levied, yielding a "double taxation" of capital income that we are abstracting from. See Santoro and Wei (2011) for a detailed analysis of the impact of dividend and corporate income taxes on investment and asset returns in a stochastic general equilibrium model.

where the wage rate  $w_t$  is taken as given by the firm. The optimality condition solving the firm's problem are given in Appendix B.2.

### 3.4 Government

The government taxes capital income (profits), labor income and consumption expenditure, at the rates  $\tau_k$ ,  $\tau_h$  and  $\tau_c$ , respectively. From the expenditure side, the government spends  $G_t$  as government consumption, provides lump-sum transfers denoted  $L_t$  and services its debt obligations. Hence, the government budget constraint reads

$$d_t B_{t+1} = G_t + L_t + B_t - \tau_k (Y_t - w_t H_t - I_t) - \tau_h w_t H_t - \tau_c C_t. \quad (13)$$

There is a simple feedback rule relating lump-sum transfers to the level of debt, while the government spending in log-deviation from steady state  $\tilde{G}_t$  is the sum of two components, a stochastic disturbance and a predetermined component.

The dynamics of  $L_t$  and  $G_t$  are described by the following two equations

$$\hat{L}_t = -\varphi_L \hat{B}_t, \quad (14)$$

$$\tilde{G}_t = \rho_G \tilde{G}_{t-1} - \varphi_G \hat{B}_t + \sigma_g \epsilon_t^2, \quad (15)$$

where  $\epsilon_t^2$  is an i.i.d. exogenous stochastic shock;  $\hat{L}_t \equiv (L_t - \bar{L}) / \bar{Y}$  and  $\hat{B}_t \equiv (B_t - \bar{B}) / \bar{Y}$  are, respectively, lump-sum transfers and debt in deviation from steady-state as percentage of the steady state output. The parameters  $\varphi_L$ ,  $\rho_G$  and  $\varphi_G$  are positive constants, consistent with the transversality condition of the government sector, namely

$$E_t \left[ \lim_{z \rightarrow \infty} (\prod_{i=t}^z d_i) B_{z+1} \right] = 0. \quad (16)$$

The purpose of the fiscal rules introduced in equations (15) and (14) is quantitative. It allows us to obtain realistic government and budget deficit dynamics. Moreover, they are empirically motivated by the fact that in many OECD countries successful fiscal consolidation is ensured through expenditure adjustments.<sup>31</sup>

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<sup>31</sup>See e.g. Alesina and Perotti (1995), McDermott and Wescott (1996), Andres and Domenech (2006). In our baseline calibration, we choose values for the parameters  $\varphi_L$ ,  $\rho_G$ ,  $\varphi_G$  and  $\sigma_g$  based on estimates from a vector autoregression in reduced form that determine the joint dynamics of government spending, public debt and lump-sum transfers.

### 3.5 Equilibrium

A history of shocks  $z_t \equiv (\epsilon_t^1, \epsilon_t^2) \in \mathcal{Z}$  up to time  $t$  is denoted by  $\mathbf{h}_t \equiv \{z_0, \dots, z_t\}$ , with  $\mathbf{h}_{t+1} = \{\mathbf{h}_t, z_{t+1}\}$ . The set of possible histories up to time  $t$ , denoted  $\mathcal{H}_t$ , is a finite set, and  $\mathcal{H} \equiv \{\mathcal{H}_t\}_{t=0}^\infty$  is a countable set. A *competitive equilibrium* is a set of stochastic processes for individual allocations,  $c_i(\mathbf{h}_t)$ ,  $n_i(\mathbf{h}_t)$ ,  $e_i(\mathbf{h}_t)$ ,  $s_{i+1}(\mathbf{h}_t)$ ,  $b_{i+1}(\mathbf{h}_t)$ ,  $x_{i+1}^z(\mathbf{h}_t)$  for all  $z \in \mathcal{Z}$ , and individual wealth holdings,  $a_i(\mathbf{h}_t)$ , for all  $i \in M$ ; aggregate inputs,  $K(\mathbf{h}_t)$  and  $H(\mathbf{h}_t)$ ; government fiscal policy  $G(\mathbf{h}_t)$ ,  $L(\mathbf{h}_t)$  and  $B(\mathbf{h}_t)$ ; security prices,  $p(\mathbf{h}_t)$ ,  $d(\mathbf{h}_t)$  and  $q^z(\mathbf{h}_t)$  for all  $z \in \mathcal{Z}$ ; wage rate  $W(\mathbf{h}_t)$  and shadow price of capital  $\mathcal{Q}(\mathbf{h}_t)$ , for all  $\mathbf{h}_t \in \mathcal{H}_t$  and  $\mathcal{H}_t \in \mathcal{H}$ , such that: the allocations are feasible; all individuals maximize (4) subject to (5); the stand-in firm maximizes its value to shareholders, given by (11); the government fiscal rules and constraints, described by equations (13), (14), (15) and (16) are satisfied. As is standard, aggregate capital at date  $t$  is measurable with respect to the history up to  $t - 1$ , and the feasibility conditions reads

$$C(\mathbf{h}_t) + I(\mathbf{h}_t) + G(\mathbf{h}_t) = Y(\mathbf{h}_t), \quad (17)$$

where the aggregate consumption and efficient units of labor are given by

$$C(\mathbf{h}_t) = \sum_{i=1}^T \mu_i c_i(\mathbf{h}_t), \quad (18)$$

$$H(\mathbf{h}_t) = \sum_{i=1}^T \mu_i g(\bar{h}_i) e_i(\mathbf{h}_t). \quad (19)$$

Finally, the security markets must clear, implying the following conditions

$$\sum_{i=1}^T \mu_i s_i(\mathbf{h}_t) = 1, \quad (20)$$

$$\sum_{i=1}^T \mu_i b_i(\mathbf{h}_t) = B(\mathbf{h}_t), \quad (21)$$

$$\sum_{i=1}^T \mu_i x_i^z(\mathbf{h}_t) = 0, \quad \text{for all } z \in \mathcal{Z}. \quad (22)$$

As in Ríos-Rull (1996), the computation of equilibrium is based on linear decision rules. Following standard steps, the firm's and the individual optimality conditions, and the market clearing conditions are log-linearized around steady state and combined so as to characterize

the equilibrium dynamics. We represent a variable  $X$  in log-deviation from steady state by  $\tilde{X}$ , and we denote the steady state of  $X$  by  $\bar{X}$  and the expectation at  $t$  of  $\tilde{X}_{t+1}$  by  $\tilde{X}_{t+1}^t$ . The system of equations describing the model aggregate dynamics is given by

$$\mathbf{AZ}_{t+1}^t = \mathbf{BZ}_t + \mathcal{E}_t \quad (23)$$

where  $\mathbf{Z}_t$  is the vector of endogenous variables and  $\mathcal{E}_t$  is the vector of exogenous stochastic disturbances. A detailed derivation of the equilibrium conditions is collected in Appendix B.3, while Appendix B.4 includes a detailed description of the algorithm to find the steady state equilibrium. The log-linear model is described in Appendix B.5.

## 4 Government size and aggregate labor supply elasticity

In this Section, we examine three important aspects of the model. First, we illustrate the differences in the cyclical volatility of employment and hours across the different demographic groups in the model. Second, we focus on the steady-state of the economy and ask how the share of employment and hours worked by each demographic group varies as the size of the government is changed. Third, we show that the aggregate labor supply elasticity is increasing in the share of volatile workers in the economy, justifying the stabilizing role of distortionary taxation.

### 4.1 Cyclical properties of hours over the life-cycle

We start by considering the cyclical properties of hours worked by the different demographic groups. For each age group, we consider the cyclical properties of the work week length  $h_{i,t}$ , the employment rate  $e_{i,t}$  and total hours worked  $n_{i,t} = e_{i,t}h_{i,t}$ . The optimality condition for the choice of total hours worked and work week length for an individual aged  $i = 1, \dots, M$ , are given by

$$n_{i,t} = \left[ \left( \frac{1 - \tau_h}{1 + \tau_c} \right) \frac{g(h_{i,t}; i) h_{i,t}^{-1} w_t}{\lambda_i} \right]^{\eta_i}, \quad (24)$$

$$\frac{g(h_{i,t}; i)}{h_{i,t}} = g'(h_{i,t}; i). \quad (25)$$

From (25) it follows that  $h_{i,t} = \bar{h}_i$ , so that the work week length is acyclical and all the cyclical fluctuation in total hours worked occur in the extensive margin. Thus, the cyclical properties of the employment rate and the total hours worked are identical.<sup>32</sup>

The result that follows compares total hours volatility across each demographic group and, of course, also applies to the cyclical properties of the employment rates.

**Lemma 1.** *Denote by  $\sigma_i$  the standard deviation of the logarithm of total hours worked by individuals aged  $i$  and  $\sigma_w$  the standard deviation of the logarithm of the wage rate. It follows that*

$$\sigma_i = \eta_i \sigma_w, \tag{26}$$

where  $\eta_i$  is the Frisch labor supply elasticity of individuals aged  $i$ .

Lemma 1 follows immediately from equation (24) and the result that  $h_{i,t}$  is acyclical. It implies that demographic groups with large labor supply elasticity display more volatile employment rates (and, hence, total hours worked) over the business cycle. These are the volatile workers. This simple result is the main element of the mechanism explaining the relation between the government size and macroeconomic stability in the model that we study. If the share of total hours worked by the volatile workers decreases, the volatility of aggregate hours worked also decreases because of the change in the composition of the labor force. As larger tax rates raise the employment rate of the more stable demographic groups, the share of total hours worked by these groups increases and the cyclical volatility of aggregate hours worked decreases.

## 4.2 Steady state: taxation and labor force composition

Next, we focus on the steady-state of the economy and ask how the share of employment and total hours worked by each demographic group varies as the size of the government is changed. We consider the steady state of an economy without aggregate uncertainty and without government debt. It is not an entirely deterministic steady-state since individuals face an uncertain lifespan. But, this is idiosyncratic risk which is shared efficiently through the annuities market and, hence, plays no aggregate role. The detailed characterization of the steady state is described in Appendix B.4.

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<sup>32</sup>This is consistent with the evidence from the US that most of the cyclical (and secular) variation in hours is on the employment margin. This is particularly true at annual frequencies (the one relevant for our calibration), as business cycle fluctuations in hours conditional on employment only account for about 1/6 of the fluctuations in aggregate hours at an annual frequency (Heckman, 1984).

Because of the form chosen for the utility function, each individual's labor effort is determined independently of the intertemporal consumption/saving choice. By combining (24) and (25), we obtain that total hours worked by individuals aged  $i$  are

$$\bar{n}_i = \left[ \left( \frac{1 - \tau_h}{1 + \tau_c} \right) \frac{\bar{w}g(\bar{h}_i; i) \bar{h}_i^{-1}}{\lambda_i} \right]^{\eta_i} \quad (27)$$

Moreover, it follows from (25) that the intensive margin (work week length) is unaffected by changes in tax and transfer policies. The upshot is that the changes in total hours worked by each demographic group are entirely explained by changes in their employment rates. In particular, as the size of the government increases, the employment rate of individuals with high labor supply elasticity falls relatively to that of individuals with low labor supply elasticity. These relative changes alter the workforce composition toward individuals with less elastic labor supplies.

In particular, consider the steady state employment rates of the individuals aged  $i$  in two countries with different fiscal profiles, denoted  $\bar{e}_i$  and  $\bar{e}'_i$ . Now consider the employment rates for a different demographic group  $j$  in the same two countries, denoted  $\bar{e}_j$  and  $\bar{e}'_j$ . Making use of equation (27), noticing that  $\bar{e}_i = \bar{n}_i/\bar{h}_i$  and taking logs we have that

$$\frac{\ln(\bar{e}_i/\bar{e}'_i)}{\ln(\bar{e}_j/\bar{e}'_j)} = \frac{\eta_i}{\eta_j} \quad (28)$$

Thus, the percentage difference in employment rates in the two countries for individuals aged  $i$ , relative to the percentage difference in employment rates in the two countries for individuals aged  $j$ , is greater, the greater the labor supply elasticity of individuals aged  $i$  relative to the labor supply elasticity of individuals aged  $j$ .

**Lemma 2.** *Consider the steady-state equilibrium of alternative economies that have different fiscal policy profiles. The percentage difference in employment rates in the two countries for individuals aged  $i$ , relative to the percentage difference in employment rates in the two countries for individuals aged  $j$ , is given by  $(\eta_i/\eta_j)$ .*

The proof of Lemma 2 follows immediately from the inspection of equation (27), and is shown in Appendix C.

The upshot is that an increase in the tax rate that lowers the aggregate employment rate, also changes the composition of the aggregate labor supply towards the less volatile individuals and, from Lemma 1, to a decrease in the aggregate labor supply volatility. This is because

the percentage fall in the employment rate of the individuals with low labor supply elasticity is less than the percentage fall in the employment rate of the individuals with high labor supply elasticity.<sup>33</sup>

### 4.3 The aggregate labor supply elasticity

The third result we obtain concerns the relationship between the aggregate labor supply elasticity and taxes. We establish the Proposition that follows.

**Proposition 1.** *Around the steady state equilibrium the aggregate labor supply elasticity is given by the following expression*

$$\frac{d \ln H_t}{d \ln w_t} \equiv \mathcal{E}_n = \sum_{i=1}^M \bar{s}_{hi} \eta_i, \quad (29)$$

where  $\bar{s}_{hi} \equiv \mu_i g(\bar{h}_i; i) \bar{e}_i / \bar{H}$  is the share of efficient units of labor supplied by individuals aged  $i$  in steady state. Moreover,

$$\frac{d \mathcal{E}_n}{d \tau_j} = \frac{\mathfrak{J}_j}{\tau_j} \sigma_\eta, \quad \forall j = \{h, k, c\}, \quad (30)$$

where  $\sigma_\eta \equiv \sum_{i=1}^M \bar{s}_{hi} \eta_i^2 - (\sum_{i=1}^M \bar{s}_{hi} \eta_i)^2$  is the cross-sectional variance of the Frisch elasticities with

$$\mathfrak{J}_j = \begin{cases} \frac{d \ln \bar{w}}{d \ln \tau_h} - \frac{\tau_h}{1 - \tau_h} & \text{if } j = h, \\ \frac{d \ln \bar{w}}{d \ln \tau_c} - \frac{\tau_c}{1 + \tau_c} & \text{if } j = c \text{ and} \\ \frac{d \ln \bar{w}}{d \ln \tau_k} & \text{if } j = k. \end{cases} \quad (31)$$

Thus, the sensitivity of the aggregate labor supply elasticity to changes in tax rates is increasing in the dispersion of the individual elasticities  $\eta_i$  across demographic groups.

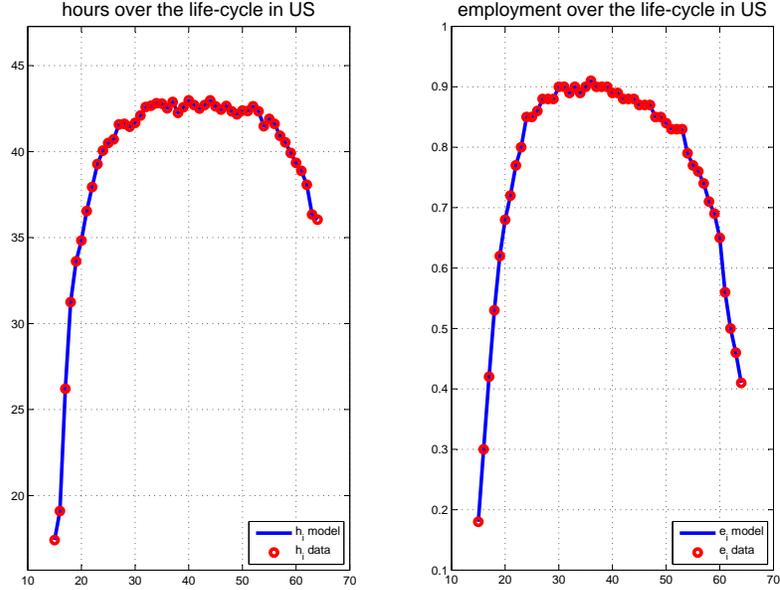
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<sup>33</sup>The elasticity of the employment rate to changes in the labor income tax is given by

$$\frac{d \bar{e}_i}{d \tau_h} \frac{\tau_h}{\bar{e}_i} = - \left( \frac{\tau_h}{1 - \tau_h} - \frac{d \bar{w}}{d \tau_h} \frac{\tau_h}{\bar{w}} \right) \eta_i$$

The steady state wage rate is given by  $\bar{w} = (1 - \alpha) (\bar{K} / \bar{Y})^{\alpha / (1 - \alpha)}$ . In our large scale OLG model, a closed form solution for the capital-output ratio  $(\bar{K} / \bar{Y})$  and, thus, for the elasticities  $\frac{d \bar{w}}{d \tau_h} \frac{\tau_h}{\bar{w}}$  is not available. But the elasticities  $\frac{d \bar{e}_i}{d \tau_h} \frac{\tau_h}{\bar{e}_i}$  are always found to be negative. See Appendix B.4 for a detailed description of the algorithm to find the steady state equilibrium.

Figure 4: employment and hours over the life-cycle



Data source: Blundell, Bozio, and Laroque (2013).

The proof of Proposition 1 is in Appendix D. A simple corollary of Proposition 1 is that a change in the fiscal profile that lowers aggregate employment will also lower the aggregate labor supply elasticity, as Lemma 2 implies that the share of efficient units of labor supplied by the individuals with low labor supply elasticity is increased.

In what follows, we examine the quantitative properties of the model and, in particular, we investigate whether the model is capable of replicating the stabilizing role of the government that features in the empirical data.

## 5 Calibration

We set a period length to be one year to match the frequency of the OECD data on hours fluctuations. We calibrate the model to the US economy for the period 1970 – 2009, making use of three types of data: i) data on the fiscal profile of the economy, ii) cross-sectional information on the wage profile and on the relative level and volatility of employment across age groups, iii) available estimates of the extensive margin Frisch elasticity of labor supply documented in Chetty et al. (2012) and iv) aggregate annual time-series.

## 5.1 Parametrization of the $g(\bullet)$ function

We begin by describing the calibration of the  $g(\bullet)$  function that controls the life-cycle profile of hours worked by those in employment. Assuming an interior solution, the first order conditions for optimal choice of hours is given by condition (3). In the sequel, we assume the parametrization of  $g(\bullet)$  given by

$$g(h; i) \equiv \frac{1}{1 + \kappa_i h^{-\varrho_i}}, \quad (32)$$

where  $\kappa_i > 0$  and  $\varrho_i > 1$  are age-specific parameters. This function is equal to zero at the origin, increasing and is first convex and then becomes concave. By choosing this functional form and making use of condition (3), the optimal number of hours by those in employment is given by

$$\bar{h}_i = [\kappa_i (\varrho_i - 1)]^{1/\varrho_i}. \quad (33)$$

Now, using (33) to substitute in (32), we find that

$$g(\bar{h}_i; i) = 1 - \frac{1}{\varrho_i}. \quad (34)$$

Thus, the optimal labor services produced per week are dependent only on the parameters  $\varrho_i$ , and the upshot is that the parameters  $\varrho_i$  can be calibrated to match the life-cycle profile of weakly earnings, obtained from the PSID. In turn, the parameters  $\kappa_i$  are calibrated to match the life-cycle profile of hours worked by the individuals in employment (intensive margin) in the US, obtained from Blundell et al. (2013).<sup>34</sup> Finally, from equations (27), (33) and (34), and the fact that  $\bar{e}_i = \bar{n}_i/\bar{h}_i$ , we have that the employment rates for each demographic group are given by

$$\bar{e}_i = \left[ \left( \frac{1 - \tau_h}{1 + \tau_c} \right) \frac{\bar{w} (1 - 1/\varrho_i)}{\lambda_i} \right]^{\eta_i} \left[ \frac{1}{\kappa_i (\varrho_i - 1)} \right]^{1 + \eta_i/\varrho_i}. \quad (35)$$

Thus, given the choice of values for the parameters  $\varrho_i$ ,  $\kappa_i$  and  $\eta_i$ , a target for the capital-output ratio that determines the wage rate  $\bar{w}$ , and the tax rates  $\tau_h$  and  $\tau_c$ , the values for each  $\lambda_i$  are chosen to match the employment rates over the life-cycle (extensive margin) in the US, also obtained from Blundell et al. (2013). As illustrated in Figure 4, this calibration strategy allows us to match exactly the life-cycle profile of hours worked by the employed workers and

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<sup>34</sup>This paper provides a new analysis of the main stylized facts underlying the evolution of labor supply at the extensive and intensive margins in three countries: the United States, the United Kingdom and France. They propose a definition of the extensive and intensive margins corresponding respectively to the employment rate and to hours when employed. We thank Antoine Bozio for kindly providing us with their data.

Table 6: baseline calibration (summary)

parameter	target/source
$\beta$	0.985 Investment/GDP ratio of 14%
$\sigma$	2.000 Greenwood et al. (1988)
$\alpha$	0.283 Capital income share
$\bar{\delta}$	0.050 5% capital depreciation (Cooley and Prescott, 1995)
$\phi$	2.500 Basu and Kimball (1997)
$\varsigma$	0.560 Burnside and Eichenbaum (1996)
$\rho$	0.847 Solow residuals autocorrelation
$\sigma_{\epsilon^1}$	0.016 US output volatility
$\bar{g}_y$	0.224 Government spending as a fraction of GDP of 22%
$\rho_G$	0.919 VAR estimation
$\sigma_{\epsilon^2}$	0.015 VAR estimation, standard deviation of residuals
$\varphi_G$	0.110 VAR estimation
$\varphi_L$	0.180 VAR estimation
$\tau_h$	0.256 Tax rate on labor income (Carey and Rabesona, 2002)
$\tau_k$	0.371 Tax rate on capital income (Carey and Rabesona, 2002)
$\tau_c$	0.053 Tax rate on consumption (Carey and Rabesona, 2002)

Note: target/source indicates the target or source informing the choice or parameter value.

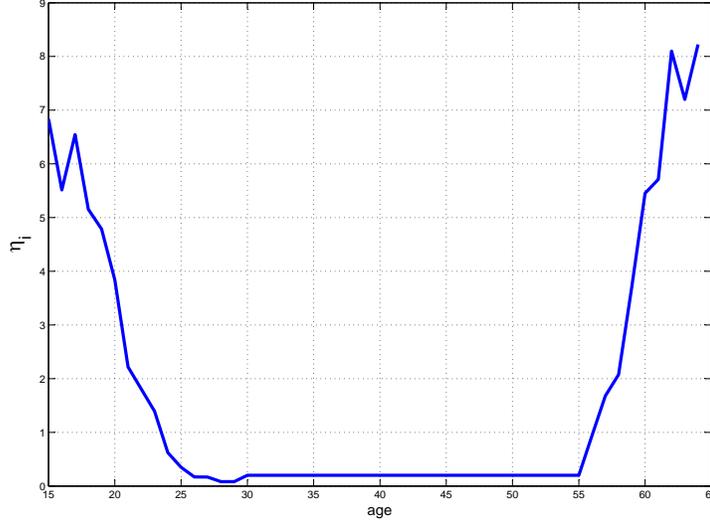
the employment rates observed in the data.

## 5.2 Demographic structure and labor supply elasticities

We now describe the aspects of the calibration which have to do with the demographic structure of the workforce and setting values for the labor supply elasticity over the life-cycle. This is the most important part of the calibration because it determines the relation between the demographic composition of the workforce and aggregate volatility. The main target we use for the calibration of the labor supply elasticities is the relative cyclical volatility of the total hours worked by the individuals aged between 15 and 29 (the young demographic group) relative to the total hours worked by the remaining individuals. We use  $\sigma_{15-29}^n$  to denote the standard deviation of the log total hours worked by the young and  $\sigma_{30-64}^n$  that by those aged between 30 and 64. In Appendix E we show that

$$\frac{\sigma_{15-29}^n}{\sigma_{30-64}^n} = \left[ \eta_1 \sum_{i=1}^{15} \frac{\mu_i \bar{n}_i (\eta_i / \eta_1)}{\bar{N}_{15-29}} \right] \left[ \eta_{16} \sum_{i=16}^{40} \frac{\mu_i \bar{n}_i (\eta_i / \eta_{16})}{\bar{N}_{30-64}} + \eta_{40} \sum_{i=41}^{50} \frac{\mu_i \bar{n}_i (\eta_i / \eta_{40})}{\bar{N}_{30-64}} \right]^{-1}, \quad (36)$$

Figure 5: employment Frisch elasticity by age ( $\eta_i$ )



where

$$\bar{N}_{15-29} = \sum_{i=1}^{15} \mu_i \bar{n}_i \quad \text{and} \quad \bar{N}_{30-64} = \sum_{i=16}^{50} \mu_i \bar{n}_i, \quad (37)$$

are, respectively, the total hours worked by the individuals aged 15 to 29 and by those aged 30 to 64, in steady-state. Using equation (36) to solve for  $\eta_1$  (the labor supply elasticity of individuals aged 15) yields<sup>35</sup>

$$\eta_1 = \left[ \sum_{i=1}^{15} \frac{\mu_i \bar{n}_i (\eta_i / \eta_1)}{\bar{N}_{15-29}} \right]^{-1} \left[ \eta_{16} \sum_{i=16}^{40} \frac{\mu_i \bar{n}_i (\eta_i / \eta_{16})}{\bar{N}_{30-64}} + \eta_{40} \sum_{i=41}^{50} \frac{\mu_i \bar{n}_i (\eta_i / \eta_{40})}{\bar{N}_{30-64}} \right] \left( \frac{\sigma_{15-29}^n}{\sigma_{30-64}^n} \right). \quad (38)$$

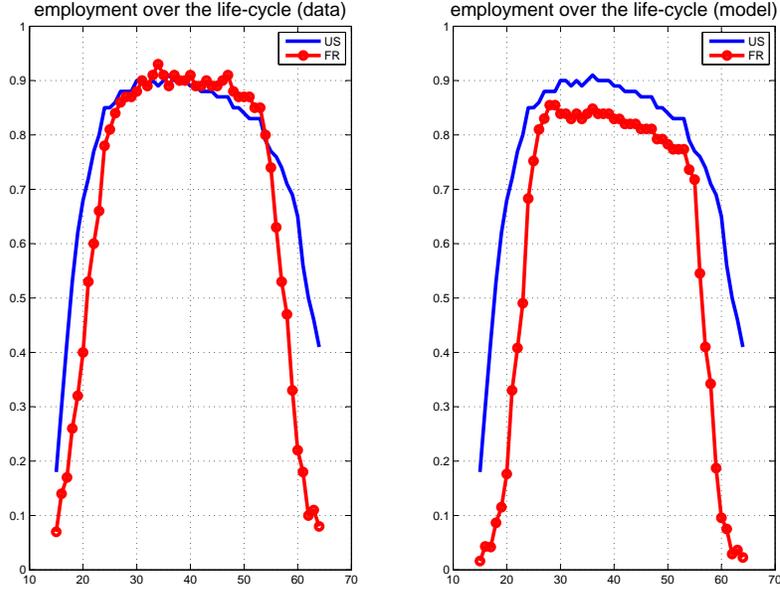
Our calibration strategy assumes that all prime aged individuals (those aged between 30 and 54) have the same labor supply elasticity  $\eta_{\text{prime age}} = 0.20$ . This parameter is chosen based on micro-econometric evidence and, in particular, the meta-analysis of quasi-experimental studies presented in Chetty et al. (2012). They report elasticity estimates ranging from 0.13 to 0.43, with an overall mean across the studies of 0.28. Using  $\eta_{\text{prime age}} = 0.20$  to substitute for  $\eta_i$  for all  $i = 16 \dots, 40$ , (the prime aged individuals) in (38) yields<sup>36</sup>

$$\eta_1 = \left[ \sum_{i=1}^{15} \frac{\mu_i \bar{n}_i (\eta_i / \eta_1)}{\bar{N}_{15-29}} \right]^{-1} \left[ 0.20 \left( \frac{\bar{N}_{30-54}}{\bar{N}_{30-64}} \right) + 0.20 \sum_{i=41}^{50} \frac{\mu_i \bar{n}_i (\eta_i / \eta_{40})}{\bar{N}_{30-64}} \right] \left( \frac{\sigma_{15-29}^n}{\sigma_{30-64}^n} \right). \quad (39)$$

<sup>35</sup>Recall that an individual aged 15 is indexed by  $i=1$  in the model and, similarly,  $i = 16$  and  $i = 40$  correspond to individuals aged 30 and 54, respectively.

<sup>36</sup>See Appendix E for a detailed derivation.

Figure 6: employment rates in two countries (model and data)



Data source: Blundell, Bozio, and Laroque (2013).

Most parameters in equation (39) correspond to targets that we observe and that the model matches exactly. Specifically, the relative volatility ( $\sigma_{15-29}^n/\sigma_{30-64}^n$ ) is obtained from the OECD time series data on employment by age groups. The parameters  $\mu_i$  (the population of individuals aged  $i$ ) are obtained from the OECD population statistics. The parameters  $\bar{n}_i$  correspond to the steady state targets for the level of total hours worked by the individuals over their life-cycle obtained from Blundell et al. (2013), and are, therefore, also matched exactly. Once we have the values for  $\mu_i$  and  $\bar{n}_i$ , the aggregates  $\bar{N}_{15-29}$  and  $\bar{N}_{30-64}$  are computed.

The only parameters that still need to be obtained in (39) are  $(\eta_i/\eta_1)$  for  $i = 1, \dots, 15$ , and  $(\eta_i/\eta_{40})$  for  $i = 41, \dots, 50$ .<sup>37</sup> These ratios are calibrated to match the relative life-cycle profile of employment in two countries with different fiscal profiles. We use data on the employment rates over the life-cycle in the US (a country with low tax rates) and in France (a country with

<sup>37</sup>The partition of the population into these three demographic groups: the young (aged 15 to 29), the prime aged (aged 30 to 54) and the old (aged 55 to 64) is not arbitrary. The profile of employment is almost flat for the prime aged individuals, and also similar across countries, suggesting fairly homogeneous preferences. Thus, we attribute the same labor supply elasticity to all prime aged individuals. Instead, for the young and old individuals, the employment rates vary a lot across countries and over the life-cycle. To capture this variation across countries and over the life-cycle, the preferences must vary with age within these demographic groups. This is very similar to the decomposition suggested in Blundell et al. (2011), who document that individuals near retirement exhibit the largest differences in employment rates across countries with different tax systems. Rogerson and Wallenius (2009) make a similar point.

high tax rates), from Blundell et al. (2013). This approach is consistent with the findings in Chetty et al. (2012), who show that estimates of steady-state elasticities of the response of employment to taxes are similar whether one relies on macro or micro data, although they may differ when one estimates intertemporal substitution elasticities. In particular, consider the steady state employment rates of individuals aged  $i$  in the US and France, denoted  $\bar{e}_i^{\text{US}}$  and  $\bar{e}_i^{\text{FR}}$ , respectively. From equation (28) and Lemma 2 we have that

$$\frac{\eta_i}{\eta_1} = \frac{\ln(\bar{e}_i^{\text{US}}/\bar{e}_i^{\text{FR}})}{\ln(\bar{e}_1^{\text{US}}/\bar{e}_1^{\text{FR}})}, \quad \text{and} \quad \frac{\eta_i}{\eta_{40}} = \frac{\ln(\bar{e}_i^{\text{US}}/\bar{e}_i^{\text{FR}})}{\ln(\bar{e}_{40}^{\text{US}}/\bar{e}_{40}^{\text{FR}})}. \quad (40)$$

Using (40) to substitute in (39) for  $(\eta_i/\eta_1)$  and  $(\eta_i/\eta_{40})$ , we obtain  $\eta_1$  and the entire life-cycle profile for the labor supply elasticity parameters. This profile is shown in Figure 5. For all prime aged individuals, aged between 30 and 54, the labor supply elasticity is set at 0.2. Instead, for young and old individuals, the Frisch elasticities are allowed to vary, reaching a maximum of around 8.22.

The implied aggregate labor supply elasticity,  $\mathcal{E}_n$ , given by equation (29), is equal to 0.84. Chetty et al. (2012) recommend, based on micro-econometric studies, that representative agent equilibrium macro models should be calibrated with a Frisch elasticity of aggregate hours of 0.86. Our aggregate labor supply elasticity is remarkably close to their proposed Frisch elasticity. Thus, heterogeneity in labor supply elasticities is important to reconcile the micro-econometric evidence on labor elasticities and the calibration required in representative agent RBC models.<sup>38</sup>

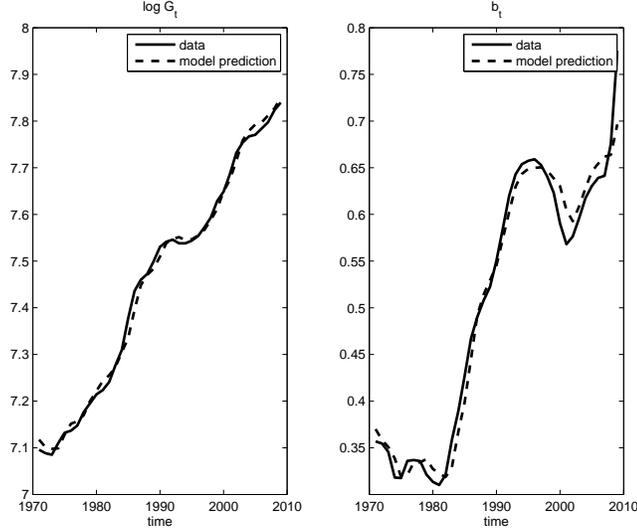
Figure 6 illustrates some of the implications of our calibration strategy. The figure contrasts the employment rates for the US and for France implied by the model and in the data. Of course we match exactly the employment rates over the life-cycle in the US, as this is one of our calibration targets. But, remarkably, the model also matches very well the empirical employment rates, in particular of the individuals aged 15 to 24 and those older than 55, in France.<sup>39</sup> Both the empirical employment rates of young and old workers in France and their theoretical counterparts are well below the same employment rates in the US. This is important, as the aggregate labor supply elasticity depends crucially on the composition of the labor force. The upshot is that countries with high tax rates have substantially lower aggregate labor supply elasticities, as implied by Proposition 1. In turn, the low aggregate

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<sup>38</sup>This argument is pursued in recent work by Dyrda et al. (2012).

<sup>39</sup>Only if our model is exactly correct and differences in taxes are the only explanation for differences in employment rates across countries over the life-cycle, would we match the employment rates in France exactly. The fact that we match quite well these employment rates provides an encouraging measure of the model's goodness of fit.

Figure 7: VAR Estimation of the  $G$  and  $B$  processes



Data source: BEA, CEA, CBO and authors' calculations.

labor supply elasticity implies lower aggregate volatility.

### 5.3 Government sector

We choose the tax rates on capital income, labor income and consumption based on evidence documented in Carey and Rabesona (2002), who have produced series for the average effective tax rates on capital income, labor income and consumption for the OECD countries based on the methodology proposed by Mendoza et al. (1994). In Section 7 we make use of these cross-country data for examining the relation between government size and aggregate volatility across OECD economies. For the purpose of the calibration, we use the tax rates which are reported by these authors for the US economy. The values chosen for each tax rate are  $\tau_k = 0.3712$ ,  $\tau_c = 0.0526$  and  $\tau_h = 0.2567$ , as reported in Table 6.

We set values for the parameters  $\varphi_L$ ,  $\rho_G$ ,  $\varphi_G$  and  $\sigma_g$  based on the estimates of a vector autoregression (VAR) in reduced form that models the joint dynamics of transfers, government spending and public debt. To measure  $G_t$  we use data on real government consumption expenditures and gross investment from the Bureau of Economic Analysis, and to measure public debt in percentage of steady state output we use the ratio between gross federal debt held by the public from the Council of Economic Advisors and the Congressional Budget

Table 7: VAR Estimation for  $G$  and  $B$ 

	$\ln G_{t-1}$	$\widehat{B}_{t-1}$	$R^2$	Linear Trend
$\ln G_t$	0.9129*** (0.0709)	-0.1101*** (0.0443)	0.999	Yes
$\widehat{B}_t$	0.2777** (0.0981)	0.8423*** (0.0613)	0.979	Yes

Standard errors in parentheses.

\*\* significant at 5%, \*\*\* significant at 1%.

Office's estimate of potential output. The system of equations to be estimated is

$$\begin{pmatrix} \ln G_t \\ \widehat{B}_t \end{pmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{pmatrix} \log G_{t-1} \\ \widehat{B}_{t-1} \end{pmatrix} + \Gamma_t + \begin{pmatrix} e_t^g \\ e_t^b \end{pmatrix}, \quad (41)$$

where the same notation for  $G_t$  and  $b_t$  are used to refer to their empirical counterparts, the matrix  $A$  is the AR(1) coefficients of the VAR,  $\Gamma_t$  represents a linear time trend and  $e_t^g$  and  $e_t^b$  are residuals.

Table 7 reports the estimation results. We find that both government spending and public debt are persistent processes, with partial autocorrelation coefficients equal to 0.91 and 0.84 respectively. The results indicate that increases in past debt tend to reduce current spending (the estimate of  $A_{12}$  is  $-0.110$ ). This finding is consistent with the prevalence of expenditure based fiscal consolidation. Increases in past spending raise current debt (the estimate for  $A_{21}$  is 0.278), implying deficit-financed expenditure. The  $R^2$ 's in Table 7, together with the displayed prediction of the estimated VAR in Figure 7, indicate a good fit of the estimation. This is mostly a consequence of the large persistence of the process and of the presence of a deterministic time trend.

In the Appendix B.5, we show that in the model economy the joint dynamics of government spending and the debt in percentage of steady-state output are described by the following system of equations

$$\begin{pmatrix} \widetilde{G}_t \\ \widetilde{B}_t \end{pmatrix} = \begin{bmatrix} \rho_G & -\varphi_G \\ \beta^{-1}\bar{g}_y & \beta^{-1}(1 - \varphi_L) \end{bmatrix} \begin{pmatrix} \widetilde{G}_{t-1} \\ \widetilde{B}_{t-1} \end{pmatrix} + \begin{pmatrix} \epsilon_t^g \\ v_t \end{pmatrix}, \quad (42)$$

where  $v_t \equiv -\beta^{-1} [\tau_y \tilde{Y}_t + \bar{c}_y \tau_c \tilde{C}_t]$  and  $\bar{g}_y$  and  $\bar{c}_y$  are the steady-state shares of government spending and aggregate consumption in output. Hence, we use the estimates of  $A_{11}$ ,  $A_{12}$  and  $A_{22}$  to deduce the implied values of  $\rho_G$ ,  $\varphi_G$  and  $\varphi_L$ , while the estimate of  $A_{21}$  can be used as a non-restricted moment to evaluate the model. This exercise produces values for  $\rho_G = 0.913$ ,  $\varphi_G = 0.110$  and  $\varphi_L = 0.180$ . We obtain a value for  $\sigma_g$  by calculating the standard deviation of the estimated residuals  $\epsilon_t^2$ . This gives  $\sigma_{\epsilon^2} = 0.015$ .

Using data from the Bureau of Economic Analysis, the steady-state ratio of government consumption to output  $\bar{g}_y$  is calculated to be 22%, which corresponds to the average share of government spending in output over the period 1970 – 2009. Given the calibrated value for the discount factor (see below), we can compare the estimation of the coefficient  $A_{21}$  with the calibrated value for  $\beta^{-1}\bar{g}_y$ . The latter is equal to 0.226, while the estimate of  $A_{21}$  is 0.278, with the difference not statistically significant. Finally, the value of  $\bar{g}_y$  implies a steady-state value for  $L$  which is 10.3% of output .

## 5.4 Technology and preferences

The calibration of the technology parameters requires setting values for the parameters of the capital adjustment costs function and the capital depreciation function ( $\phi$ ,  $\bar{\delta}$  and  $\varsigma$ ) and the stochastic process for the technology shock ( $\rho$  and  $\sigma_{\epsilon^1}$ ). Our methodology here follows ideas developed in Basu et al. (2006), King and Rebelo (1999) and Basu and Kimball (1997). Moreover, two preference parameters remain to be fixed: The discount factor  $\beta$  and the inverse of the elasticity of intertemporal substitution  $\sigma$ .

Basu and Kimball (1997) estimate Solow residuals in a model characterized by variable capital utilization and convex adjustment costs for capital. They use annual data for a panel of US firms from 21 manufacturing industries for the period 1949 — 1985. Our calibration of  $\phi$  considers their estimate of convex adjustment costs, and allows us to replicate the volatility of investment. The fixed value for  $\phi$  is 2.5. We set  $\bar{\delta} = 0.05$ , implying a steady-state annual depreciation rate of 5%, following Cooley and Prescott (1995). The capital income share,  $\alpha$ , is set equal to 0.283 based on the value implied by the National Income and Product Accounts (NIPA). The investment to output ratio is measured at 14% using the NIPA. We set  $\varsigma = 0.560$  following Burnside and Eichenbaum (1996) who estimate this parameter using aggregate data. This value also falls in the range of estimates of Basu and Kimball (1997). Our target for the investment-output ratio implies a value for the discount factor  $\beta$  equal to 0.985. In addition, we set the inverse elasticity of intertemporal substitution  $\sigma = 2$  as in Greenwood et al. (1988).

We use data provided by FRED and obtained in Feenstra et al. (2013) to determine the process for  $\epsilon_t^1$ . In particular, these data include information on capital services. However, because the rate of capital utilization is not observable, Solow residuals cannot be directly calculated. For this reason, we use model-based proxies for utilization as in Basu et al. (2006) and King and Rebelo (1999). Specifically, we use the first-order conditions of the individual problem and the representative firm to substitute out  $u_t$  in the production function and then calculate residuals. In the Appendix F, we show that this exercise allows to express  $\epsilon_t^1$  as follows

$$\epsilon_t^1 + \Gamma_t \approx \left(1 - \frac{\alpha \bar{\delta} \phi}{\nu}\right) \ln Y_t - \alpha \left(1 + \frac{1 - \bar{\delta} \phi}{\nu}\right) \ln K_t + \frac{\alpha}{\nu} \ln K_{t+1} - (1 - \alpha) \ln H_t, \quad (43)$$

where  $\Gamma_t$  is a trend component and  $\nu = (1 + \phi)(1 + \varsigma)\bar{\delta}$ .<sup>40</sup> We calculate the residuals from this equation and use them to estimate an AR(1) process with a linear trend and obtain the value for  $\rho = 0.847$ . We set  $\sigma_{\epsilon^1} = 0.016$  to match the volatility of US output.

## 6 Properties of the baseline economy

In this Section we study the behavior of the model under the benchmark calibration, before analyzing how aggregate volatilities are affected by changes in the size of the government. We look at the implications of the model for the aggregate business cycle statistics and for the relative employment volatilities of different demographic groups.

### 6.1 Aggregate business cycle statistics

Table 8 displays relevant aggregate statistics for the theoretical economy under the baseline calibration. The fiscal profile of the theoretical economy corresponds to that of the US, so that we may compare the US business cycle statistics with those implied by the model. The table shows the properties of output, consumption, investment, government spending and total hours worked in both the data and the model, as described by the volatility of their cyclical components and the correlation with the cyclical component of output. The annual data on total hours worked is from the Conference Board Total Economy Database for the sample period 1970 – 2009, while Output, Private Consumption and Private Investment

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<sup>40</sup>The resulting correlation between the estimated  $\epsilon_t^1$  and  $\epsilon_t^2$  is  $-0.04$  and not significantly different from zero, justifying the assumption that the two shocks are uncorrelated.

Table 8: US business cycle statistics (model and data)

variable	std. dev.		correlation		output share	
	data	model	data	model	data	model
output	1.45	1.45	1.00	1.00	–	–
consumption	1.20	1.36	0.90	0.98	0.67	0.64
investment	5.13	4.23	0.94	0.96	0.14	0.14
government spending	0.93	0.96	-0.13	0.04	0.22	0.22
total hours	1.28	0.78	0.92	1.00	–	–
employment rate 15 – 64	1.04	0.78	0.89	1.00	–	–

Note: Data on GDP, consumption, investment and government spending is from the NIPA tables. Inventories are excluded from the measure of investment. Data on hours worked is from the Conference Board Total Economy Database. Employment rate 15 – 64 is from the OECD and corresponds to the employment/population ratio among the individuals aged 15 to 64. Cyclical component is the log in deviations from an HP trend with smoothing parameter 6.25. The model’s reported statistics are calculated under the US fiscal profile.

(fixed capital formation) are taken from the NIPA Tables. The cyclical components are found by applying the HP filter to the logged series with smoothing parameter equal to 6.25, as recommended for annual data in Ravn and Uhlig (2002).

The baseline model matches the volatility of aggregate variables at least as well as the standard RBC model. Volatilities of consumption and investment are comparable to their empirical counterparts. The model suffers from the same drawback as the standard RBC model: the volatility of total hours is about half that of output but in the data the relative volatility of total hours is close to 90%. However, our model attributes a labor supply elasticity to the majority of the population (the prime aged individuals) that is very low, consistent with the micro-econometric evidence. So the fact that the model performs at least as well as the representative agent RBC model (typically calibrated with labor supply elasticities around 1) is significant.<sup>41</sup> Moreover, since in the theoretical economy movements in total hours occur only through fluctuations in employment (as workers optimally choose not to change hours) it

<sup>41</sup>Recall that the aggregate labor supply elasticity,  $\mathcal{E}_n$ , implied by our calibration is equal to 0.84, consistent with Chetty et al. (2012) recommendation, that representative agent equilibrium macro models should be calibrated with a Frisch elasticity of aggregate hours of 0.86.

Table 9: volatility of employment by age group (model and data)

age group	std.dev		$\frac{\text{std. dev.}}{\sigma_{15-64}}$	
	data	model	data	model
15 – 29	1.63	1.27	1.57	1.64
30 – 54	0.78	0.16	0.75	0.20
55 – 64	0.70	2.73	0.66	3.52
30 – 64	0.74	0.58	0.70	0.74
15 – 64	1.04	0.78	1.00	1.00

Note: standard deviation of the logarithm of the employment rates for each age group, computed based on HP filtered data with smoothing parameter 6.25. Source: OECD Labour Force Statistics by Sex and Age, 1970 – 2009. The model’s reported statistics are calculated under the US fiscal profile.

is perhaps preferable to look at the volatility of employment in the data. The model accounts for 75% of the volatility of employment.

Total hours and output in the theoretical economy are perfectly correlated because of the choice of utility function that excludes intertemporal substitution in the individual’s labor supply decisions. The high correlations between output and the private components of aggregate expenditure are the result of the RBC structure of the model. Finally, the shares of consumption, investment and government spending in output are the same as the ones found in the data because of the restrictions imposed by our calibration strategy.

## 6.2 Employment fluctuations by age group

One of our calibration targets was the volatility of the young relative to that of those aged 30 to 64. This relative volatility,  $(\sigma_{15-29}/\sigma_{30-64})$ , is equal to 2.20 and is exactly matched by the model. But to judge the goodness of fit of the model, it is useful to look at the relative volatilities that were not used as targets for the calibration. Table 9 looks at the model’s ability to match the cyclical volatility of employment for specific demographic groups, the young (15 to 29), the prime aged (30 - 44) and those aged 54 to 64. We report the volatility of each age group relative to the total hours volatility, (std. dev./ $\sigma_{15-64}$ ).

The right-hand panel of the table shows the relative volatility for each group. The model matches very well the relative volatility of the young (1.57 in the data and 1.64 in the model)

and also the relative volatility of the individuals aged 30 to 64 (0.70 in the data and 0.74 in the model). This is a consequence of our choice of calibration target (the relative volatility of these two groups). However, the model produces a relative volatility for the prime aged individuals that is too low compared to the data (0.75 in the data and 0.20 in the model) and, as a consequence, a relative volatility for those aged 55 to 64 that is too high. This is a consequence of the low Frisch labor supply elasticity attributed to the prime aged workers.<sup>42</sup> In Section 8, we show results from an alternative calibration of the elasticity parameters  $\eta_i$ . The alternative calibration matches exactly the relative volatility of the young workers, 15 – 29 and that of the older workers, 55 – 64. This alternative calibration requires a higher labor supply elasticity for prime aged workers  $\eta_{30-54}$ . But, we choose this parameter to maintain the aggregate labor supply elasticity of total hours  $\mathcal{E}_n$  equal to 0.84, consistent with microeconomic evidence.

## 7 Government size and automatic stabilizers

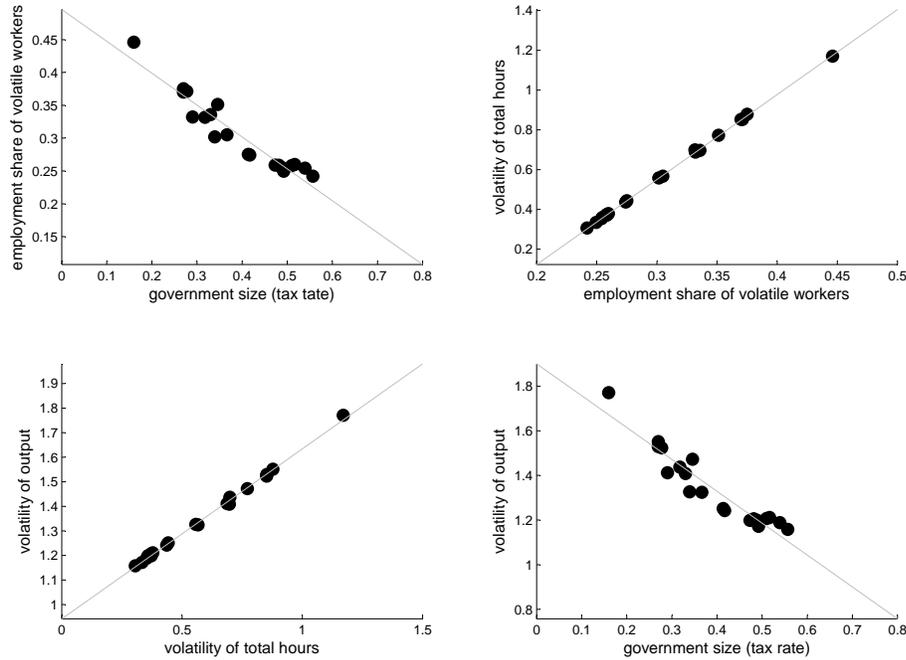
We now investigate if the model is able to reproduce the negative correlation between the size of the government (measured by the tax revenue to output ratio) and aggregate volatility. Beyond the simple qualitative relation between the fiscal profile and macroeconomic stability, we are interested in the quantitative implications of the model. In particular, we compare the strength of the automatic stabilization in the model and in the data. To do this, we feed to the model different combinations of values for the tax rates and for the government spending as a share of GDP, with each combination chosen to mimic the fiscal profile of a particular OECD country. By following this procedure, we make sure that the size of the government is varied endogenously, in a way which is dictated by the changes in fiscal policy parameters across OECD countries. This allows us to investigate whether we are able to replicate quantitatively the relation between government size and macroeconomic stability across OECD countries.

The fiscal parameters that need to be chosen for each artificial economy are the three tax rates, and the steady-state government spending to output ratio. Each set of tax rates are chosen based on the tax rates of a given country, as estimated by Carey and Rabesona (2002).

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<sup>42</sup>This shows that allowing for heterogeneous labor supply elasticities is not a panacea. Although, the model aggregate labor supply elasticity is consistent with the calibration of a representative agent RBC model, the low labor supply elasticity attributed to the prime aged produces too stable employment for this group. The same point is made by Chetty et al. (2012), who argue that it remains a challenge to formulate models that fit the fluctuations in employment rates of prime aged individuals when calibrated to match extensive margin labor supply elasticities around 0.2.

Figure 8: government size and aggregate volatility (model)



Notes: Volatility of output and hours from the model are the standard deviation of the HP filtered output and hours implied by the model. Each observation corresponds to an economy whose fiscal policy parameters are chosen to mimic the fiscal profile of a specific OECD economy.

In addition, the steady state government spending to output ratio is chosen to match the historical average for the same country as reported in the national accounts. For each fiscal profile mimicking an OECD country, we calculate the model implied size of the government and the standard deviation of the HP filtered output and aggregate hours. The implied size of the government, an endogenous outcome, is measured by the steady state ratio of total tax-revenue to output. We reproduce the cross-country regressions which are performed in Fatás and Mihov (2001) by regressing the volatility of output and aggregate hours on government size.

We turn first to Figure 8. In this figure we reproduce the scatter plots in Figure 2 but this time with the model based artificial economies, each parameterized to reproduce the fiscal profile of an OECD country. The variables defined in Figure 8 correspond exactly to the ones defined in Figure 2. It is apparent from comparing Figures 2 and 8 that the association between government size, the demographic composition of the workforce and macroeconomic stability implied by the theoretical economy is qualitatively consistent with the data. Higher

Table 10: regressions of volatility on government size (model and data)

$\sigma = \beta_0 + \beta_1$ (tax rate)		data	model	ratio ( $\beta_1$ )
output volatility	$\beta_0$	2.182	1.690	75%
	$\beta_1$	-1.909	-1.428	
total hours volatility	$\beta_0$	1.621	1.388	114%
	$\beta_1$	-1.819	-2.074	

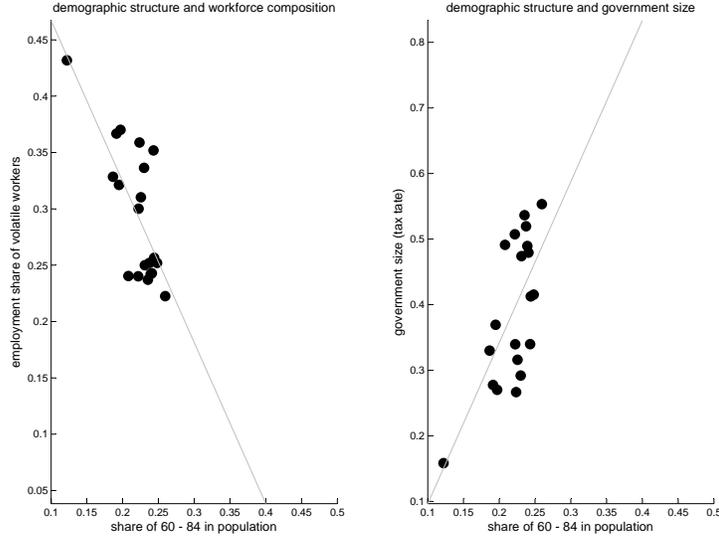
Note: This table gives results for OLS regressions where the dependent variables are, respectively, output volatility and total hours worked volatility and the explanatory variable is the tax-revenue to output ratio. The volatility of output and hours is given by the standard deviation of the series in log deviations from an HP trend with smoothing parameter 6.25. The effective tax rates used to calibrate the fiscal profile of each economy in the simulations are from Carey and Rabesona (2002). Concerning the empirical regressions, each observation corresponds to a country and one of the following time periods: 1970 – 1979, 1980 – 1989, 1990 – 1999, and 2000 – 2009. Time dummies are included but not listed.

taxes imply a low employment share of volatile workers in the workforce; The smaller the employment share of volatile workers, the lower the volatility of aggregate total hours worked and the lower the volatility of output.

In Table 10 we investigate to what extent the model reproduces quantitatively the negative relationship between government size and the volatility of aggregate output and total hours fluctuations. The table reports the estimates from a linear regression between aggregate volatility and government size using the empirical OECD data and the same moments implied by the model (both for hours and output volatility). The slope coefficient measures the strength of the automatic stabilizers. This exercise allows us to interpret our results from a quantitative perspective: the last row of Table 10 shows the ratio between the slope coefficient estimated using the moments implied by the theoretical model and the slope coefficient estimated using the empirical data.

The model is remarkably successful at explaining the stabilizing role of the government. Our baseline calibration implies a slope coefficient in the regression of output volatility on government size that is 75% of the coefficient resulting from the empirical regression. The findings concerning the relationship between hours volatility and government size are equally encouraging: the slope associated with the regression of hours volatility on government size corresponds to 114% of the slope observed in the data. Hence, the baseline model can almost

Figure 9: exogenous variation in demographic structure (model)



Notes: The left-panel shows in the vertical axis the employment share of the volatile workers, aged 15 – 29 and 60 – 64. The right-panel shows in the vertical axis the ratio of total taxes to output. The horizontal axis shows the share of individuals aged 60 – 84 in the population. Each observation results from setting the fiscal parameters to mimic the fiscal profile of a specific OECD country and the population parameters,  $\mu_i$  for  $i = 1, \dots, 70$ , to mimic the same country’s demographic structure.

exactly replicate the strength of the automatic stabilizers. The changes in the labor force composition may, therefore, be an important element to understand why countries that have high tax rates also have less volatile business cycles.

## 8 Additional experiments

In this Section we consider two additional quantitative experiments. In the first experiment, we see how the quantitative performance of the model is changed when we allow for exogenous changes in the demographic structure of the population as in the OECD data. In the second experiment we consider an alternative calibration strategy for the labor supply elasticity parameters,  $\eta_i$ . The purpose is to match exactly the relative volatility of the young workers, 15 – 29, but also that of the older workers, 55 – 64, whilst preserving the same aggregate labor supply elasticity of total hours  $\mathcal{E}_n$ .

Table 11: variations in demographic structure and automatic stabilizers strength

$\sigma = \beta_0 + \beta_1$ (tax rate)		data	model	ratio ( $\beta_1$ )
output volatility	$\beta_0$	2.182	1.958	83%
	$\beta_1$	-1.909	-1.593	
total hours volatility	$\beta_0$	1.621	1.388	114%
	$\beta_1$	-1.819	-2.074	

Note: This table gives results for OLS regressions where the dependent variables are, respectively, output volatility and total hours worked volatility and the explanatory variable is the tax-revenue to output ratio. The volatility of output and hours is given by the standard deviation of the series in log deviations from an HP trend with smoothing parameter 6.25. The effective tax rates used to calibrate the fiscal profile of each economy in the simulations are from Carey and Rabesona (2002). The population parameters,  $\mu_i$  for  $i = 1, \dots, 70$ , mimic each country’s demographic structure. Concerning the empirical regressions, each observation corresponds to a country and one of the following time periods: 1970 – 1979, 1980 – 1989, 1990 – 1999, and 2000 – 2009. Time dummies are included but not listed.

## 8.1 Experiment 1: exogenous demographic changes

In the quantitative evaluation done above, we have only considered changes in countries’ fiscal profiles, leaving all other parameters at their baseline values. In particular, the population’s demographic structure (the list of parameters  $\mu_i$ , for  $i = 1, \dots, 70$ , corresponding to the population of individuals aged 15 to 85) was, for each country, set at the levels observed for the US. This may be inappropriate, if there are reasons to believe that in the data, changes in fiscal profiles are correlated with exogenous changes in the demographic structure of the population. If, for example, countries with an older population have larger tax rates (maybe driven by greater demand for social security or health care), we would observe that countries with an older work-force have higher government sizes. But, these variations in the work-force composition and in government size would be jointly caused by exogenous changes in the age structure of the population that we have omitted. In fact, in Section 2 we have indeed shown that once we control for the share of old in the population, the relationship between the volatile share of workers and government size is attenuated (although still negative and significant).

To incorporate exogenous changes in the demographic structure of the population in the quantitative evaluation of the model, we once again consider the exercise of feeding the model with different combinations of fiscal parameters. But this time we change the demographic structure of the population by setting values for the list of parameters  $\mu_i$  for  $i = 1, \dots, 85$ ,

Table 12: volatility of employment by age group (alternative calibration)

age group	std.dev		$\frac{\text{std. dev.}}{\sigma_{15-64}}$	
	data	model	data	model
15 – 29	1.63	1.22	1.57	1.61
30 – 54	0.78	0.59	0.75	0.77
55 – 64	0.70	0.52	0.66	0.69
30 – 64	0.74	0.58	0.70	0.76
15 – 64	1.04	0.76	1.00	1.00

Note: standard deviation of the logarithm of the employment rates for each age group, computed based on HP filtered data with smoothing parameter 6.25. Source: OECD Labour Force Statistics by Sex and Age, 1970 – 2009. The model’s reported statistics are calculated under the US fiscal profile.

to also mimic the demographic structure of each OECD country considered. We then look at the relation between government size and macroeconomic volatility implied by the model, but controlling for exogenous variation in demographics. Figure 9 illustrates the implications in the theoretical economy of allowing for exogenous variation in the demographic structure of the population. When we simultaneously mimic the OECD population statistics and the OECD fiscal profiles the model implies a negative correlation between the share of individuals aged 60 – 84 and the employment share of the volatile workers, aged 15 – 29 and 60 – 64 (this is shown in the left-panel of Figure 9). At the same time, the model implies a negative correlation between the share of individuals aged 60 – 84 and the size of the government (this is shown in the right-panel of Figure 9). These two relations are as in the data, and they confirm that it is important to control for exogenous differences in the population demographic structure, to not overestimate the role of government size in changing the composition of the workforce.

The main results from this experiment are shown in Table 11 where we consider the same regression equation as in Table 10, but this time with the demographic structure used to calibrate the model (the parameters  $\mu_i$ ) that change for each country as in the data. The regression coefficient  $\beta_1$  implied by the model in the regression equation for output volatility is equal to  $-1.593$ . This corresponds to 83% of the empirical counterpart ( $-1.909$ ). Thus, once we introduce exogenous demographic changes in the theoretical economy, the model accounts for an additional 8% of the empirical relation between government size and macroeconomic stability. This result, suggests that it is important to control for exogenous changes in the

Table 13: regressions of volatility on government size (alternative calibration)

$\sigma = \beta_0 + \beta_1$ (tax rate)		data	model	ratio ( $\beta_1$ )
output volatility	$\beta_0$	2.182	1.722	43%
	$\beta_1$	-1.909	-0.829	
total hours volatility	$\beta_0$	1.621	1.114	61%
	$\beta_1$	-1.819	-1.118	

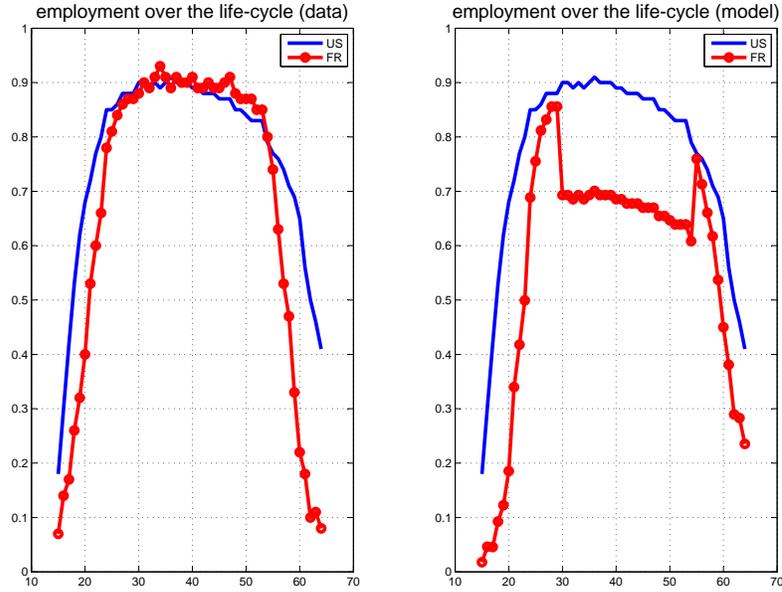
Note: This table gives results for OLS regressions where the dependent variables are, respectively, output volatility and total hours worked volatility and the explanatory variable is the tax-revenue to output ratio. The volatility of output and hours is given by the standard deviation of the series in log deviations from an HP trend with smoothing parameter 6.25. The effective tax rates used to calibrate the fiscal profile of each economy in the simulations are from Carey and Rabesona (2002). Concerning the empirical regressions, each observation corresponds to a country and one of the following time periods: 1970 – 1979, 1980 – 1989, 1990 – 1999, and 2000 – 2009. Time dummies are included but not listed.

structure of the population, to estimate the independent effect that changes in government size exert on macroeconomic stability. Turning to the fluctuation in total hours worked, the size of the estimated stabilization effect of the government is not affected by the change in the calibration.

## 8.2 Experiment 2: an alternative calibration for $\eta_i$

Our baseline calibration used as target to set values for the labor supply elasticities, the volatility of employment of fluctuations of young workers, relative to those aged 30 to 64 and a labor supply elasticity of  $\eta_{30-54} = 0.2$  for the prime aged workers, based on the meta-analysis of quasi-experimental studies in Chetty et al. (2012). We have shown in Section 6 that, the baseline calibration fitted well the effect of taxes on the life-cycle profile of employment and also the relative volatility of the young workers. But, the baseline calibration also attributes too little volatility to the prime aged workers and too much to the older workers, aged 55 to 64. Here, we consider an alternative calibration of the elasticity parameters  $\eta_i$ . The alternative calibration matches exactly the relative volatility of the young workers, 15 – 29 and that of the older workers, 55 – 64. This alternative calibration requires a higher labor supply elasticity for prime aged workers  $\eta_{30-54}$ . But, we choose this parameter to maintain the aggregate labor supply elasticity of total hours  $\mathcal{E}_n$  equal to 0.84, consistent with the implications of the

Figure 10: employment rates in two countries (alternative calibration)



Data source: Blundell, Bozio, and Laroque (2013).

microeconomic evidence for the calibration of representative agent macro models.

The implications of the alternative strategy for the cyclical employment volatilities of the various demographic groups are shown in Table 12. In particular, the important feature of the alternative calibration is that the model is now able to match very well the volatility of the three relevant groups, the young, prime aged and older workers, relative to the volatility of total hours worked. In turn, the implications of the alternative calibration for the model's ability to replicate the strength of the automatic stabilizers are shown in Table 13. Although the model's ability to explain the negative relation between the size of the government and macroeconomic stability is weakened, the model's performance is still good. In particular, the model explains 43% of the relation between government size and output volatility and 61% for the volatility of aggregate total hours worked.

The alternative calibration yields a labor supply elasticity for the prime aged workers of  $\eta_{30-54} = 0.76$ . This number is about three times as large as the mean extensive margin Frisch labor supply elasticity recommended by Chetty et al. (2012), based on the microeconomic evidence. The upshot of setting the labor supply elasticity of the prime aged workers to this higher value, is that we overestimate the variation in employment rates of the prime aged workers across countries with different fiscal profiles. This is shown in Figure 10, that compares the employment rates over the life-cycle in the data and implied by the model,

in two countries with different fiscal profiles (France and the US). Thus, a trade-off emerges, between matching the cyclical volatility of employment of the prime aged individuals and matching the secular (steady state) effect of changes in taxation on the employment rates of the same demographic group. This finding is strikingly consistent with that of Chetty et al. (2012), who argue that the micro-econometric estimates of the labor supply elasticity are consistent with the macro steady state estimates obtained from comparisons across countries with different tax regimes, but are too low to explain the fluctuations in employment of the prime aged workers at business cycle frequencies.

The latter finding also confirms the importance of matching changes in the composition of the work-force to explain the quantitative importance of automatic stabilizers. The alternative calibration, overestimates the steady state reduction in employment among the prime aged workers and underestimates the reduction in steady state employment for the older workers caused by higher tax rates. Thus, it implies a smaller tilt of the workforce towards the more stable workers caused by higher taxation and, consequently, implies weaker automatic stabilizers.

## 9 Conclusion

Two empirical facts serve as the principal motivation for this paper. The first is that there is a strong negative correlation between government size and the volatility of business cycles across OECD countries. This feature of the data is difficult to explain using the standard real business cycle model. The second empirical fact is the substantial heterogeneity across demographic groups in terms of the cyclical volatility of employment and total hours worked. Taken together, these two empirical facts suggest a mechanism whereby changes in the size of the government are associated with changes in the demographic composition of the workforce. An increase in tax rates tilts the workforce composition towards the prime aged workers lowering the aggregate labor supply elasticity and business cycle volatility.

We develop a fully calibrated stochastic overlapping generation model with heterogeneous preferences and, in particular, labor supply elasticities that change over the life-cycle. We calibrate the model to match the relative volatility of employment of the different demographic groups and informed by the micro-econometric evidence on the extensive margin labor supply elasticity. We use the theoretical economy to investigate the relationship between the size of the government and the volatility over the business cycle of total hours worked and output. We find that the model is able to explain a substantial part of the negative correlation

between government size and business cycle volatility. Our results suggest that modeling labor force heterogeneity and, in particular, differences across demographic groups is important to explain quantitatively some important features of the business cycle.

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## APPENDIX

### A Data

In Section 2 we consider an unbalanced panel of 25 OECD countries composed of Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Iceland, Ireland, Italy, Japan, Luxembourg, Mexico, Netherlands, New Zealand, Norway, Portugal, South Korea, Spain, Sweden, Switzerland, United Kingdom and United States, over the period 1970–2009. The variable definitions and data sources are as follows:

**Employment Volatility by Age Group:** This variable is constructed from annual data on the employment by age group obtained from the OECD Labour Force Statistics by Sex and Age. The age groups categories are 15 – 19, 20 – 24, 25 – 29, 30 – 39, 40 – 49, 50 – 59 and 60 – 64 years old. For each country we obtain the cyclical component by removing the HP trend using the smoothing parameter 6.25 and the volatility measures correspond to the standard deviations of the cyclical components.

**Employment Share of Young:** This variable is obtained from the same OECD labor force statistics. We split the sample period in four subperiods: Each observation corresponds to a country and one of the following time periods: 1970 – 1979, 1980 – 1989, 1990 – 1999, and 2000 – 2009. For a country/subperiod to be included we require to have data for that country at least in 5 occasions in that subperiod (yielding a sample of 77 country/subperiod observations). The variable is defined as the ratio between the employment of individuals aged 15 to 29 and the employment of individuals aged 15 to 64, averaged over the subperiod and in percentage.

**Government Size:** This variable corresponds to total tax revenue as percentage of GDP from the OECD Revenue Statistics, an annual database that presents detailed and internationally comparable tax data.

**Hours Volatility:** The data on total annual hours comes from the Conference Board Total Economy Database. From this database we obtain a balanced panel for total hours worked in each OECD country between 1970 and 2009. The variable **Hours Volatility** is the standard deviation (in each subperiod) of the cyclical component obtained using the HP filter over the entire sample period.

**Output Volatility:** This variable is the standard deviation (in each subperiod) of the cyclical component of real output. Real output is obtained from the OECD national accounts.

The US business cycle statistics concerning GDP, consumption, investment and government spending is from the NIPA tables. Inventories are excluded from the measure of investment. Public Debt in Percentage of Output is the ratio between gross federal debt held by the public from the Council of Economic Advisors and the Congressional Budget Offices estimate of potential output.

Concerning the fiscal policy variables, we choose the tax rates on capital income, labor income and consumption based on evidence documented in Carey and Rabesona (2002) who have produced series for the average effective tax rates on capital income, labor income and consumption for the OECD countries based on the methodology proposed by Mendoza et al. (1994).

Finally, we use data on the extensive margin and intensive margin labor supply for the US and for France from Blundell et al. (2013). This paper provides a new analysis of the main stylized facts underlying the evolution of labor supply at the extensive and intensive margins in three countries: the United States, the United Kingdom and France. They propose a definition of the extensive and intensive margins corresponding respectively to the employment rate and to hours when employed.

## B Steady state and equilibrium dynamics

### B.1 First order conditions (individual problem)

For a given number of total hours worked  $n$ , individuals must choose how many hours to work each week  $h$  and how many weeks to work  $e$ , to maximize the labor services produced  $\ell$ . Thus, the individual solves the problem

$$\begin{aligned} \max_{h,e} \quad & \ell = g(h; i) e \\ \text{s.t. :} \quad & he = n \\ & e \in [0, 1] \\ & h \in [0, 1] \end{aligned} \tag{B.1}$$

Assuming an interior solution, the first order conditions for optimal choice of hours is

$$g'(h; i) = \frac{g(h; i)}{h}, \tag{B.2}$$

The remaining first order conditions solving the individual's problem are the following

$$n_{i,t} = \left[ \left( \frac{1 - \tau_h}{1 + \tau_c} \right) \frac{g'(\bar{h}_i; i) w_t}{\lambda_i} \right]^{\eta_i}, \quad \text{for all } i = 1, \dots, M, \quad (\text{B.3})$$

$$d_t = E_t(\Lambda_{t+1}), \quad (\text{B.4})$$

$$p_t = E_t[(\pi_{t+1} + p_{t+1}) \Lambda_{t+1}] \quad (\text{B.5})$$

with,

$$\Lambda_{t+1} \equiv \beta \left( \frac{f_{i,t}}{f_{i+1,t+1}} \right)^\sigma, \quad \text{for all } i = 1, \dots, T-1,$$

$$f_{i,t} \equiv c_{i,t} - \frac{\lambda_i n_{i,t}^{1+1/\eta_i}}{1 + 1/\eta_i}, \quad \text{for all } i = 1, \dots, M,$$

$$f_{i,t} \equiv c_{i,t}, \quad \text{for all } i = M+1, \dots, T.$$

## B.2 First order conditions (firm's problem)

The first order conditions solving the firm's problem are

$$0 = -(1 - \tau_k) \mathcal{Q}_t + E_t \Lambda_{t+1} J'_{t+1}, \quad (\text{B.6})$$

$$w_t = (1 - \alpha) Y_t / H_t, \quad (\text{B.7})$$

$$\alpha (Y_t / K_t) = \mathcal{Q}_t (1 + \varsigma) \delta_t, \quad (\text{B.8})$$

$$J'_t = (1 - \tau_k) \alpha (Y_t / K_t) - (1 - \tau_k) (I_t / K_t) + (1 - \tau_k) \mathcal{Q}_t (1 - \delta_t + \Phi_t), \quad (\text{B.9})$$

$$K_{t+1} = (1 - \delta_t) K_t + \Phi_t K_t, \quad (\text{B.10})$$

with  $\mathcal{Q}_t = \Phi'(I_t / K_t)^{-1}$  the shadow price of capital (Tobin's  $\mathcal{Q}$ ),  $\delta_t = \bar{\delta} (u_t / \bar{u})^{1+\varsigma}$  and  $\Phi_t = \Phi(I_t / K_t)$ .

Finally, combining (B.6)—(B.10), yields the equilibrium condition

$$\mathcal{Q}_t K_{t+1} = E_t \left[ \Lambda_{t+1} \left( \alpha Y_{t+1} - I_{t+1} + \mathcal{Q}_{t+1} K_{t+2} \right) \right]. \quad (\text{B.11})$$

### B.3 Equilibrium conditions

The equilibrium of the model is characterized by the following conditions

$$\begin{aligned} & (1 + \tau_c) c_{i,t} + p_t s_{i+1,t+1} + d_t b_{i+1,t+1} + \sum_{z \in \mathcal{Z}} q_t^z x_{i+1,t+1}^z \\ & = (1 - \tau_h) w_t g'(\bar{h}_i; i) n_{i,t} + \frac{(\pi_t + p_t) s_{i,t} + b_{i,t} + x_{i,t}}{\zeta_{i-1}} + L_t, \quad \text{for all } i = 1, \dots, T-1, \end{aligned} \quad (\text{B.12})$$

$$n_{i,t} = \left[ \left( \frac{1 - \tau_h}{1 + \tau_c} \right) \frac{g'(\bar{h}_i; i) w_t}{\lambda_i} \right]^{\eta_i}, \quad \text{for all } i = 1, \dots, M, \quad (\text{B.13})$$

$$d_t = E_t (\Lambda_{t+1}), \quad (\text{B.14})$$

$$\Lambda_{t+1} = \beta \left( \frac{f_{i,t}}{f_{i+1,t+1}} \right)^\sigma, \quad \text{for all } i = 1, \dots, T-1, \quad (\text{B.15})$$

$$f_{i,t} = c_{i,t} - \frac{\lambda_i n_{i,t}^{1+1/\eta_i}}{1 + 1/\eta_i}, \quad \text{for all } i = 1, \dots, M, \quad (\text{B.16})$$

$$f_{i,t} = c_{i,t}, \quad \text{for all } i = M+1, \dots, T, \quad (\text{B.17})$$

$$p_t = E_t [(\pi_{t+1} + p_{t+1}) \Lambda_{t+1}], \quad (\text{B.18})$$

$$\pi_t = (1 - \tau_k) (\alpha Y_t - I_t) \quad (\text{B.19})$$

$$K_{t+1} = \Phi(I_t/K_t) K_t + (1 - \delta_t) K_t, \quad (\text{B.20})$$

$$\delta_t = \delta_0 + \delta_1 (u_t/\bar{u})^{1+\varsigma}, \quad (\text{B.21})$$

$$w_t = (1 - \alpha) (Y_t/H_t), \quad (\text{B.22})$$

$$\alpha (Y_t/K_t) = \mathcal{Q}_t (1 + \varsigma) \delta_1 (u_t/\bar{u})^{1+\varsigma}, \quad (\text{B.23})$$

$$\mathcal{Q}_t K_{t+1} = E_t \left[ \Lambda_{t+1} \left( \alpha Y_{t+1} - I_{t+1} + \mathcal{Q}_{t+1} K_{t+2} \right) \right], \quad (\text{B.24})$$

$$\mathcal{Q}_t = \Phi' (I_t/K_t)^{-1}, \quad (\text{B.25})$$

$$Y_t = \exp(\epsilon_t^1) ((u_t/\bar{u}) K_t)^\alpha H_t^{1-\alpha}, \quad (\text{B.26})$$

$$H_t = \sum_{i=1}^M \mu_i g'(\bar{h}_i; i) n_{i,t}, \quad (\text{B.27})$$

$$C_t = \sum_{i=1}^T \mu_i c_{i,t}, \quad (\text{B.28})$$

$$Y_t = C_t + I_t + G_t. \quad (\text{B.29})$$

where the capital adjustment cost function  $\Phi(\bullet)$  is increasing and concave, and with  $z_t$  that follows a first order autoregressive process. In addition, the following parameter restrictions are assumed

$$\Phi(\bar{\delta}) = \bar{\delta}, \quad (\text{B.30})$$

$$\Phi'(\bar{\delta}) = 1, \quad (\text{B.31})$$

$$-\frac{\Phi'(\bar{\delta})}{\Phi''(\bar{\delta})\bar{\delta}} = \phi. \quad (\text{B.32})$$

Finally, the government sector is characterized by a budget constraint

$$G_t + L_t + B_t = d_t B_{t+1} + \tau_k \alpha Y_t - \tau_k I_t + \tau_h (1 - \alpha) Y_t + \tau_c C_t, \quad (\text{B.33})$$

and fiscal rules that represent the dynamics of  $L_t$  and  $G_t$ , given by

$$\widehat{L}_t = -\varphi_L \widehat{B}_t, \quad (\text{B.34})$$

$$\widetilde{G}_t = \rho_G \widetilde{G}_{t-1} - \varphi_G \widehat{B}_t + \sigma_g \epsilon_t^2, \quad (\text{B.35})$$

with  $\epsilon_t^2$  an i.i.d. exogenous stochastic shock, and where  $\widehat{L}_t \equiv (L_t - \bar{L}) / \bar{Y}$ ,  $\widehat{B}_t \equiv (B_t - \bar{B}) / \bar{Y}$  and  $\widetilde{G}_t$  denotes government spending in log-deviation from steady state.

In what follows we first characterize the steady state equilibrium and next the equilibrium dynamics of the log-linearized model. We represent a variable  $X$  in log-deviation from steady state by  $\widetilde{X}$ , and we denote the steady state of  $X$  by  $\bar{X}$ .

## B.4 Steady state equilibrium

We now characterize the steady state of the economy without aggregate uncertainty and without government debt. It is not an entirely deterministic steady-state since individuals face an uncertain lifespan. But, this is idiosyncratic risk which is shared efficiently through the annuities market and, hence, plays no aggregate role.

In the steady state of the economy without aggregate uncertainty and without government debt, we have that  $\bar{b}_i = 0$  for all  $i$  and the Arrow securities play no role. There is a single outside asset, which are shares in the stand-in firm,  $\bar{s}_i$ . Thus, the budget constraint of an individual aged  $i$  is given by

$$(1 + \tau_c) \bar{c}_i = \left[ (1 - \tau_h) \bar{W}_i \bar{n}_i + \frac{(\bar{\pi} + \bar{p}) \bar{s}_i}{\zeta_{i-1}} + \bar{L} - \bar{p} \bar{s}_{i+1} \right], \quad (\text{B.36})$$

with  $\bar{W}_i = \bar{w}_i g(\bar{h}_i; i) \bar{h}^{-1}$ . Using (B.16) to substitute for  $\bar{c}_i$  yields

$$\bar{f}_i + \frac{\lambda_i \bar{n}_i^{1+1/\eta_i}}{1 + 1/\eta_i} = (1 + \tau_c)^{-1} \left[ (1 - \tau_h) \bar{W}_i \bar{n}_i + \frac{(\bar{\pi} + \bar{p}) \bar{s}_i}{\zeta_{i-1}} + \bar{L} - \bar{p} \bar{s}_{i+1} \right]. \quad (\text{B.37})$$

Using the Euler equation (B.14) and equation (B.15), we write  $\bar{f}_i$  in terms of  $\bar{f}_1$

$$\bar{f}_1 (\beta/\bar{d})^{(i-1)/\sigma} + \frac{\lambda_i \bar{n}_i^{1+1/\eta_i}}{1 + 1/\eta_i} = (1 + \tau_c)^{-1} \left[ (1 - \tau_h) \bar{W}_i \bar{n}_i + \frac{(\bar{\pi} + \bar{p}) \bar{s}_i}{\zeta_{i-1}} + \bar{L} - \bar{p} \bar{s}_{i+1} \right]. \quad (\text{B.38})$$

Each  $\bar{n}_i$  is a function of the wage rate  $\bar{w}$ , given by (B.13), for  $i = 1, \dots, M$ , and is 0 for  $i > M$ . Using (B.22) and the fact that, with Cobb-Douglas production,  $(\bar{H}/\bar{Y}) = (\bar{K}/\bar{Y})^{\alpha/(\alpha-1)}$ , the wage rate  $\bar{w}$  is given by the following function of the economy's capital-output ratio

$$\bar{w} = (1 - \alpha) (\bar{K}/\bar{Y})^{\alpha/(1-\alpha)}. \quad (\text{B.39})$$

From (B.14) and (B.18) it follows that in steady state, the price of the stand-in firm satisfies

$$\bar{p} = (\bar{\pi} + \bar{p}) \bar{d} \quad (\text{B.40})$$

and, hence, the stand-in firm's rate of return is given by  $\bar{R} \equiv (\bar{\pi} + \bar{p})/\bar{p} = 1/\bar{d}$ . From combining the equations (B.14), (B.19) and (B.24) we have that in steady state

$$\bar{d}\bar{\pi} = (1 - \bar{d})(1 - \tau_k)\bar{K}. \quad (\text{B.41})$$

Combining (B.40) and (B.41) we find that  $\bar{p} = (1 - \tau_k)\bar{K}$ . That is, the market value of the stand-in firm is equal to the quantity of capital adjusted for dividends' taxation. The upshot is that the return on capital is also a function of the capital-output ratio, given by

$$\bar{R} \equiv \frac{\bar{\pi} + \bar{p}}{\bar{p}} = \frac{\bar{\pi} + (1 - \tau_k)\bar{K}}{(1 - \tau_k)\bar{K}} = \alpha (\bar{K}/\bar{Y})^{-1} + (1 - \delta). \quad (\text{B.42})$$

Thus, for a given capital-output ratio combined with the boundary conditions  $\bar{s}_1 = 0$  and  $\bar{s}_{M+1} = 0$ , we obtain a solution for  $\bar{f}_1$ , given by

$$\bar{f}_1 \sum_{i=1}^T Z_i \bar{d}^{(i-1)(1-1/\sigma)} \beta^{(i-1)/\sigma} = \sum_{i=1}^T Z_i \bar{d}^{(i-1)} \left[ \left( \frac{1 - \tau_h}{1 + \tau_c} \right) \bar{W}_i \bar{n}_i + \frac{\bar{L}}{1 + \tau_c} - \frac{\lambda_i \bar{n}_i^{1+1/\eta_i}}{1 + 1/\eta_i} \right], \quad (\text{B.43})$$

with  $Z_i = \left( \prod_{j=0}^{i-1} \zeta_j \right)$ . Having a solution for  $\bar{f}_1$ , it is immediate to obtain the sequence for consumption over the life-cycle,  $\{\bar{c}_i\}_{i=1}^T$ , using the Euler equation (B.14), together with equations (B.15), (B.16) and (B.17), as follows

$$\bar{c}_i = (\beta/\bar{d})^{(i-1)/\sigma} \bar{f}_1 + \frac{\lambda_i \bar{n}_i^{1+1/\eta_i}}{1 + 1/\eta_i}, \quad \text{for all } i = 1, \dots, M, \quad (\text{B.44})$$

$$\bar{c}_i = (\beta/\bar{d})^{(i-1)/\sigma} \bar{f}_1, \quad \text{for all } i = M + 1, \dots, T$$

Thus, given a guess for the aggregate capital-output ratio in steady state  $(\bar{K}/\bar{Y})$ , we obtain the life-cycle sequence for consumption.

We only need to find the equilibrium capital-output ratio. Given (B.43) and (B.44), the algorithm to compute the equilibrium capital-output ratio is as in Ríos-Rull (1996):

1. In round  $j$ , guess an aggregate capital-output ratio,  $(\bar{K}/\bar{Y})_j$ ;
2. Given this guess, solve for the consumption sequence over the life-cycle using (B.44);
3. Using the budget constraint (B.36) and the initial condition  $\bar{s}_1 = 0$  obtain the life-cycle sequence for asset holdings  $\{\bar{s}_{i+1}\}_{i=1}^T$ , implied by the consumption sequence;
4. Verify if the financial markets clear:  $\sum_{i=1}^T \mu_i \bar{s}_{i+1} = 1$ ; If it does, stop, else update the guess for the capital output-ratio,  $(\bar{K}/\bar{Y})_{j+1}$  and return to step 1.

## B.5 Equilibrium conditions in log-linear form

Following standard steps, the individuals and the stand-in firm optimality conditions, and the market clearing conditions are log-linearized and combined so as to characterize the equilibrium. The equilibrium conditions in log-linear form are

$$\begin{aligned} & (1 + \tau_c) \bar{c}_i \tilde{c}_{i,t} + \bar{p} \bar{s}_{i+1} (\tilde{p}_t + \tilde{s}_{i+1,t+1}) \\ & = (1 - \tau_h) \bar{w} g'(\bar{h}_i; i) \bar{n}_i (\tilde{w}_t + \tilde{n}_{i,t}) + \frac{(\bar{\pi} + \bar{p}) \bar{s}_i \tilde{s}_{i,t} + \bar{p} \bar{s}_i \tilde{p}_t + \bar{\pi} \bar{s}_i \tilde{\pi}_t}{\xi_{i-1}} - \varphi_L \bar{Y} \hat{B}_t, \end{aligned} \quad (\text{B.45})$$

$$\tilde{n}_{i,t} = \eta_i \tilde{w}_t, \quad \text{for all } i = 1, \dots, M, \quad (\text{B.46})$$

$$\tilde{d}_t = \sigma \left( \tilde{f}_{i,t} - \tilde{f}_{i+1,t+1} \right), \quad \text{for all } i = 1, \dots, T-1, \quad (\text{B.47})$$

$$\left( \bar{c}_i - \frac{\lambda_i \bar{n}_i^{1+1/\eta_i}}{1 + 1/\eta_i} \right) \tilde{f}_{i,t} = \bar{c}_i \tilde{c}_{i,t} - \lambda_i \bar{n}_i^{1+1/\eta_i} \tilde{n}_{i,t}, \quad \text{for all } i = 1, \dots, M, \quad (\text{B.48})$$

$$\tilde{f}_{i,t} = \tilde{c}_{i,t}, \quad \text{for all } i = M+1, \dots, T, \quad (\text{B.49})$$

$$\tilde{p}_t = (1 - \bar{d}) E_t [\tilde{\pi}_{t+1}] + \bar{d} E_t [\tilde{p}_{t+1}] + \tilde{d}_t, \quad (\text{B.50})$$

$$(1 - \bar{d}) \tilde{\pi}_t = \bar{d} \left[ \alpha (\bar{Y}/\bar{K}) \tilde{Y}_t - \bar{\delta} \tilde{I}_t \right], \quad (\text{B.51})$$

$$\tilde{K}_{t+1} - \tilde{K}_t = \bar{\delta} \left( \tilde{I}_t - \tilde{K}_t - \tilde{\delta}_t \right), \quad (\text{B.52})$$

$$\tilde{\delta}_t = (1 + \varsigma) \tilde{u}_t, \quad (\text{B.53})$$

$$\tilde{w}_t = \tilde{Y}_t - \tilde{H}_t, \quad (\text{B.54})$$

$$\tilde{Y}_t - \tilde{K}_t = \tilde{Q}_t + \tilde{\delta}_t, \quad (\text{B.55})$$

$$\tilde{Q}_t + \tilde{K}_{t+1} - \tilde{d}_t = E_t \left[ (1 - \bar{d}) \tilde{\pi}_{t+1} + \bar{d} (\tilde{Q}_{t+1} + \tilde{K}_{t+2}) \right], \quad (\text{B.56})$$

$$\tilde{Q}_t = (1/\phi) (\tilde{I}_t - \tilde{K}_t), \quad (\text{B.57})$$

$$\tilde{Y}_t = \epsilon_t^1 + \alpha (\tilde{u}_t + \tilde{K}_t) + (1 - \alpha) \tilde{H}_t, \quad (\text{B.58})$$

$$\tilde{H}_t = \sum_{i=1}^M (\mu_i g'(\bar{h}_i; i) \bar{n}_i / \bar{H}) \tilde{n}_{i,t}, \quad (\text{B.59})$$

$$\tilde{C}_t = \sum_{i=1}^T (\mu_i \bar{c}_i / \bar{C}) \tilde{c}_{i,t}, \quad (\text{B.60})$$

$$\tilde{Y}_t = (\bar{C}/\bar{Y}) \tilde{C}_t + \bar{\delta} (\bar{K}/\bar{Y}) \tilde{I}_t + (\bar{G}/\bar{Y}) \tilde{G}_t, \quad (\text{B.61})$$

together with the individual boundary conditions  $\tilde{s}_{1,t} = 0$  and  $\tilde{s}_{M+1,t} = 0$ .

Finally, by linearizing the government budget constraint (B.33) around a steady state with zero debt and a balanced primary budget, and making use of the fiscal feedback rules (B.34) and (B.35) we obtain the following description of the debt dynamics

$$\hat{B}_{t+1} = \bar{d}^{-1} \left[ (1 - \varphi_L) \hat{B}_t - \tau_y \tilde{Y}_t + \tau_k (\bar{I}/\bar{Y}) \tilde{I}_t - (\bar{C}/\bar{Y}) \tau_c \tilde{C}_t + (\bar{G}/\bar{Y}) \tilde{G}_t \right], \quad (\text{B.62})$$

$$\tilde{G}_t = \rho_G \tilde{G}_{t-1} - \varphi_G \hat{B}_t + \sigma_g \epsilon_t^2, \quad (\text{B.63})$$

where  $\tau_y \equiv \alpha \tau_k + (1 - \alpha) \tau_h$ .

## C Proof of Lemma 2

From Equation (35) it follows that the steady state employment rate of individuals aged  $i$  in logs is

$$\ln(\bar{e}_i) = \eta_i \ln \left[ \left( \frac{1 - \tau_h}{1 + \tau_c} \right) \bar{w} \right] + \eta_i \ln \left[ \frac{(1 - 1/\varrho_i)}{\lambda_i} \right] + (1 + \eta_i/\varrho_i) \left[ \frac{1}{\kappa_i (\varrho_i - 1)} \right]. \quad (\text{C.1})$$

Therefore, comparing employment in to different countries,  $\bar{e}_i$  and  $\bar{e}'_i$ , we have that

$$\begin{aligned} \ln(\bar{e}_i/\bar{e}'_i) &= \ln(\bar{e}_i) - \ln(\bar{e}'_i) \\ &= \eta_i \left\{ \ln \left[ \left( \frac{1 - \tau_h}{1 + \tau_c} \right) \bar{w} \right] - \ln \left[ \left( \frac{1 - \tau'_h}{1 + \tau'_c} \right) \bar{w}' \right] \right\}. \end{aligned} \tag{C.2}$$

From (C.2) it follows that

$$\frac{\ln(\bar{e}_i/\bar{e}'_i)}{\ln(\bar{e}_j/\bar{e}'_j)} = \frac{\eta_i}{\eta_j}, \tag{C.3}$$

which is equation (28) in the main text.

## D Proof of Propostion 1

Combining equations (B.46) and (B.59) together with the the condition (3), and using the fact that  $\bar{e}_i = \bar{n}_i/\bar{h}_i$ , we obtain

$$\tilde{H}_t = \sum_{i=1}^M \left[ \frac{\mu_i g(\bar{h}_i; i) \bar{e}_i}{\bar{H}} \right] \eta_i \tilde{w}_t, \tag{D.1}$$

From equation (D.1) the aggregate labor supply elasticity is given by the following expression

$$\mathcal{E}_n \equiv \frac{d \ln H}{d \ln w} = \sum_{i=1}^M \bar{s}_{hi} \eta_i, \tag{D.2}$$

where  $\bar{s}_{hi} \equiv \mu_i g(\bar{h}_i; i) \bar{e}_i/\bar{H}$  is the share of efficient units of labor supplied by individuals aged  $i$  in steady state.

The next part of the proposition, the impact of taxes on the aggregate labor supply elasticity, is obtained by simply deriving the expression above with respect to each tax rate  $\tau_j$ ,  $j = \{h, c, k\}$ . To calculate this derivative, first notice that the elasticity of labor supplied by each demographic group can be written as

$$\frac{d \ln \bar{e}_i}{d \ln \tau_j} = \eta_i \mathfrak{J}_j, \tag{D.3}$$

where  $\mathfrak{J}_j$  is defined in Propostion 1. Equation (D.3) is obtained by deriving equation (27)

and noticing that  $\bar{h}_i$  is independent from  $\tau_j$ . Then, use the definition of  $H_t$  in equation (8), to get

$$\frac{d\bar{H}}{d\tau_j} = \sum_{i=1}^M \mu_i g(\bar{h}_i; i) \frac{d\bar{e}_i}{d\tau_j}. \quad (\text{D.4})$$

By replacing (D.3) in (D.4), we have that

$$\frac{d\bar{H}}{d\tau_j} = \sum_{i=1}^M \mu_i g(\bar{h}_i; i) \eta_i \mathfrak{J}_j \frac{\bar{e}_i}{\tau_j}, \quad (\text{D.5})$$

or that

$$\frac{d\bar{H}}{d\tau_j} = \mathcal{E}_n \frac{\bar{H}}{\tau_j} \mathfrak{J}_j \quad (\text{D.6})$$

from the definition in equation (D.2). By rearranging the equation above we obtain the following elasticity:

$$\frac{d \ln \bar{H}}{d \ln \tau_j} = \mathcal{E}_n \mathfrak{J}_j. \quad (\text{D.7})$$

With equations (D.3) and (D.7), we can now calculate the impact of each tax rate on the aggregate labor supply elasticity as from (D.2) one notices that

$$\frac{d\mathcal{E}_n}{d\tau_j} = \sum_{i=1}^M \eta_i \mu_i g(\bar{h}_i; i) \left( \frac{d \ln \bar{e}_i}{d \ln \tau_j} - \frac{d \ln \bar{H}}{d \ln \tau_j} \right) \frac{\bar{e}_i}{\bar{H}} \frac{1}{\tau_j}. \quad (\text{D.8})$$

Hence,

$$\frac{d\mathcal{E}_n}{d\tau_j} = \sum_{i=1}^M \eta_i \mu_i g(\bar{h}_i; i) (\eta_i - \mathcal{E}_n) \frac{\bar{e}_i}{\bar{H}} \frac{\mathfrak{J}_j}{\tau_j}. \quad (\text{D.9})$$

From the definition in (D.2), this equation can be rewritten as

$$\frac{d\mathcal{E}_n}{d\tau_j} = \left[ \sum_{i=1}^M \bar{s}_{hi} \eta_i^2 - \left( \sum_{i=1}^M \bar{s}_{hi} \eta_i \right)^2 \right] \frac{\mathfrak{J}_j}{\tau_j}, \quad (\text{D.10})$$

completing the proof of Propostion 1.

## E Calibration of the $\eta_i$

We first derive equation (36) in the main text. From equation (37), we have that

$$\bar{N}_{15-29,t} = \sum_{i=1}^{15} \mu_i \bar{n}_{i,t}. \quad (\text{E.1})$$

In log-linear form the same equations take the form

$$\bar{N}_{15-29,t} = \sum_{i=1}^{15} \frac{\mu_i \bar{n}_i \tilde{n}_{i,t}}{\bar{N}_{15-29}} = \left( \sum_{i=1}^{15} \frac{\mu_i \bar{n}_i \eta_i}{\bar{N}_{15-29}} \right) \tilde{w}_t, \quad (\text{E.2})$$

where the final equality follows from the fact that  $\tilde{n}_{i,t} = \eta_i \tilde{w}_t$ , and  $\bar{N}_{15-29}$  denotes the total hours worked by the individuals aged 15 to 29, in steady state. From (E.2), it follows that

$$\sigma_{15-29} = \left( \sum_{i=1}^{15} \frac{\mu_i \bar{n}_i \eta_i}{\bar{N}_{15-29}} \right) \sigma_w. \quad (\text{E.3})$$

By multiplying and dividing (E.3) by  $\eta_1$ , we obtain

$$\sigma_{15-29} = \eta_1 \left[ \sum_{i=1}^{15} \frac{\mu_i \bar{n}_i (\eta_i / \eta_1)}{\bar{N}_{15-29}} \right] \sigma_w. \quad (\text{E.4})$$

In an analogous way, we have that

$$\begin{aligned} \sigma_{30-64} &= \left[ \sum_{i=16}^{50} \frac{\mu_i \bar{n}_i \eta_i}{\bar{N}_{30-64}} \right] \sigma_w, \\ &= \left[ \sum_{i=16}^{40} \frac{\mu_i \bar{n}_i \eta_i}{\bar{N}_{30-64}} + \sum_{i=41}^{50} \frac{\mu_i \bar{n}_i \eta_i}{\bar{N}_{30-64}} \right] \sigma_w, \\ &= \left[ \eta_{16} \sum_{i=16}^{40} \frac{\mu_i \bar{n}_i (\eta_i / \eta_{16})}{\bar{N}_{30-64}} + \eta_{40} \sum_{i=41}^{50} \frac{\mu_i \bar{n}_i (\eta_i / \eta_{40})}{\bar{N}_{30-64}} \right] \sigma_w, \end{aligned} \quad (\text{E.5})$$

Combining (E.4) and (E.5) we obtain

$$\frac{\sigma_{15-29}}{\sigma_{30-64}} = \eta_1 \left[ \sum_{i=1}^{15} \frac{\mu_i \bar{n}_i (\eta_i / \eta_1)}{\bar{N}_{15-29}} \right] \left[ \eta_{16} \sum_{i=16}^{40} \frac{\mu_i \bar{n}_i (\eta_i / \eta_{16})}{\bar{N}_{30-64}} + \eta_{40} \sum_{i=41}^{50} \frac{\mu_i \bar{n}_i (\eta_i / \eta_{40})}{\bar{N}_{30-64}} \right]^{-1}, \quad (\text{E.6})$$

which corresponds to equation (36) in the main text.

Next, we assume that all prime aged individuals have the same labor supply elasticity:  $\eta_i = 0.20$  for all  $i = 16 \dots 40$ . The upshot is that (E.6) becomes

$$\frac{\sigma_{15-29}}{\sigma_{30-64}} = \eta_1 \left[ \sum_{i=1}^{15} \frac{\mu_i \bar{n}_i (\eta_i / \eta_1)}{\bar{N}_{15-29}} \right] \left[ 0.20 \left( \frac{\bar{N}_{30-54}}{\bar{N}_{30-64}} \right) + 0.20 \sum_{i=41}^{50} \frac{\mu_i \bar{n}_i (\eta_i / \eta_{40})}{\bar{N}_{30-64}} \right]^{-1}, \quad (\text{E.7})$$

where we used the fact that  $\sum_{i=16}^{40} \mu_i \bar{n}_i = \bar{N}_{30-54}$ .

Solving for  $\eta_1$  yields

$$\eta_1 = \left[ \sum_{i=1}^{15} \frac{\mu_i \bar{n}_i (\eta_i / \eta_1)}{\bar{N}_{15-29}} \right]^{-1} \left[ 0.20 \left( \frac{\bar{N}_{30-54}}{\bar{N}_{30-64}} \right) + 0.20 \sum_{i=41}^{50} \frac{\mu_i \bar{n}_i (\eta_i / \eta_{40})}{\bar{N}_{30-64}} \right] \frac{\sigma_{15-29}}{\sigma_{30-64}}, \quad (\text{E.8})$$

which is equation (39) in the main text.

Finally, the calibration of the relative labor supply elasticities ( $\eta_i / \eta_1$ ) for  $i = 1, \dots, 15$ , and ( $\eta_i / \eta_{40}$ ) for  $i = 41, \dots, 50$ , are based on the Lemma 2 and in particular equation (28). In particular, comparing employment in two different countries, say US and FR, we have that

$$\frac{\ln(\bar{e}_i^{\text{US}} / \bar{e}_i^{\text{FR}})}{\ln(\bar{e}_j^{\text{US}} / \bar{e}_j^{\text{FR}})} = \frac{\eta_i}{\eta_j}, \quad (\text{E.9})$$

which is equation (40) in the main text.

## F Calculating the Solow residuals

The following expression for capital utilization in terms of output and capital, in deviation from steady state, can be obtained from equations (B.53), (B.55) and (B.57)

$$(1 + \varsigma) \tilde{u}_t = \tilde{Y}_t - \left( \frac{\phi - 1}{\phi} \right) \tilde{K}_t - \frac{\tilde{I}_t}{\phi}. \quad (\text{F.1})$$

After replacing for investment using (B.52) and making use of (B.53), one obtains

$$(1 + \phi)(1 + \varsigma) \bar{\delta} \tilde{u}_t = \bar{\delta} \phi \tilde{Y}_t + (1 - \bar{\delta} \phi) \tilde{K}_t - \tilde{K}_{t+1}. \quad (\text{F.2})$$

Replacing the expression for  $\tilde{u}_t$  in the production function in logs we obtain the following approximation

$$\epsilon_t^1 + \Gamma_t \approx \left(1 - \frac{\alpha \bar{\delta} \phi}{\nu}\right) \ln Y_t - \alpha \left(1 + \frac{1 - \bar{\delta} \phi}{\nu}\right) \ln K_t + \frac{\alpha}{\nu} \ln K_{t+1} - (1 - \alpha) \ln H_t, \quad (\text{F.3})$$

where  $\nu = (1 + \phi)(1 + \varsigma)\bar{\delta}$  and  $\Gamma_t$  is a trend component for TFP.