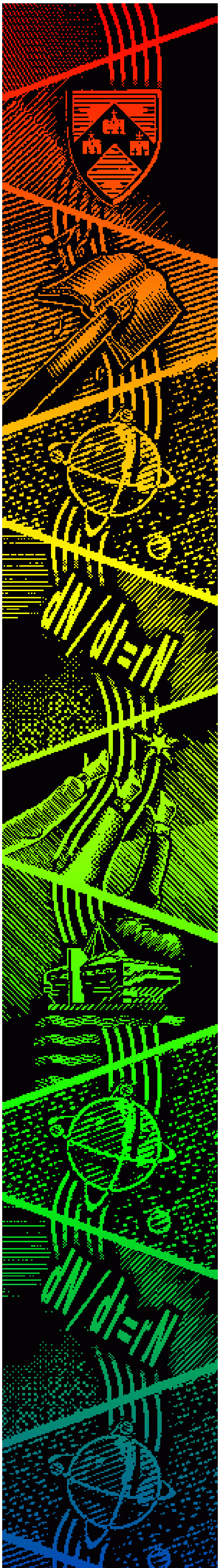

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To Have and to Hold: A Dynamic Cost-Benefit Analysis of Temporary Unemployment Measures

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To Have and to Hold: A Dynamic Cost-Benefit Analysis of Temporary Unemployment Measures

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Abstract

This paper analyses the temporary unemployment regulations that several governments of European countries have introduced during the recent recession. We view these measures as a collection of real options that governments provide to firms and value these options. We study, in particular, the effect of such measures on the liquidation decision of the firm. In addition, we study the effect of government limitations on the duration of the programme. We find that temporary unemployment measures delay a firm's liquidation. However, since the costs of the programme are incurred before the benefits, the discounted benefit-to-cost ratio does not necessarily indicate the programme is good for welfare.

Keywords: Temporary unemployment, Real options, Dynamic cost-benefit analysis

JEL classification: G33, G38, H53

1 Introduction

During recent recessions, following the 2007 credit crunch, many Western governments used temporary unemployment measures to soften the blow of reduced economy-wide demand on the labour force.[1] The basic idea of all these measures is that firms reduce the time worked by each worker (or a fraction of workers) while the government makes up (part of) the lost wage income to individual employees. The philosophy is that it is better to keep as many people in work rather than making them redundant. This is confirmed by, for example, Davis and Von Wachter (2011), who estimate that the sunk cost of redundancy is some 11% of life-time earnings. It has been estimated by Hijzen and Venn (2011) that temporary unemployment measures have saved 5,000–6,000 jobs in the Netherlands alone. However, it is also pointed out that such measures also support jobs that would have survived the recession anyway. That indicates that firms tend to continue to benefit from temporary unemployment measures beyond their usefulness from a social point of view.

In order to investigate these issues we develop a model of temporary unemployment measures that views them as a real option to the firm. The firm has the option to enter the programme and reduce its wage bill, at the cost of reducing its production and, hence, its revenues. In its least regulated form, this option is, in fact a compound option: the firm has an option to enter the programme, but after entry it obtains a new option to leave the programme. This latter option is two-sided as the firm can decide to either leave the programme and liquidate or leave the programme and return back to normal production levels. We characterize the (unique) solution to this compound option.

In addition, we solve the firm's optimal timing problem if the government specifies an upper boundary on profits beyond which the firm is forced to leave the programme. This makes the option the firm gets after entering the programme less valuable, because it provides less flexibility.[2] We show, numerically, that the optimal policy to enter the programme is, as a consequence, non-monotonic in the upper bound set by the government. This indicates that there is room for government policy to maximize social welfare.

The importance of a dynamic model to analyse policy programmes like temporary unemployment becomes obvious when comparing their benefits and costs by means of a benefit-to-cost ratio (BCR). In the context of temporary unemployment the benefits are the reduction of expected sunk costs of redundancy in the case of liquidation. Under temporary unemployment measures the firm's optimal time to liquidate is later than without, which means that the present value of these sunk costs is lower. The costs of the measure are the expected wages that the government has to pay while the firm is in the programme. It is, however, not enough to compute expected benefits and costs and take their ratio. A crucial feature of this kind of programme is that the costs are incurred *before* the full benefits are realized. Therefore, the (expected) discount factor that needs to be applied to the benefits is smaller than the expected discount factor that

[1] Examples are *Deeltijd WW* in the Netherlands, *Kurzarbeit* in Germany, *Tijdelijke werkloosheid* in Belgium, *Chômage partiel* in France.

[2] The firm can now only decide on liquidation, not on when to revert to normal production levels.

should be applied to the costs. In order to find these correct expected discount factors, a dynamic approach is crucial.

Using this approach we show through numerical examples that the BCR is highly sensitive to the current state of the economy and the upper boundary that the government sets for the programme. Through this exercise we, again numerically, determine the optimal (in the sense of maximizing the BCR) upper bound. This bound is always lower than the firm's optimal exit policy if it could choose this freely.

The policy implication of the paper is that temporary unemployment programmes can be welfare enhancing. However, the government should carefully choose the upper bound below which firms are allowed to be in the programme. From numerical calculations it follows that this optimal boundary depends on the current state of the economy. In fact, the boundary should be set lower if the economy is in a worse state. This result is counter-intuitive because it makes the option to enter the programme less valuable to the firm. However, if the upper boundary is set high, then the firm stays in the programme longer, which increases the costs and pushes the benefits farther into the future.

The paper is organized as follows. In Section 2 we set up the model and derive the firm's optimal liquidation policy in the absence of any government measures. In Section 3 the model is augmented with a temporary unemployment measure and the firm's optimal programme entry and exit policies are derived. The effect of an exogenously determined programme upper exit bound on the firm's policy is studied in Section 4, whereas the welfare effects of the temporary unemployment are analysed in Section 5. Some concluding remarks are given in Section 6.

2 The Standard Liquidation Decision

In this section we will introduce the basic liquidation decision of a firm that is currently active in a market. It is assumed that the evolution of the firm's revenues is subject to uncertainty. Uncertainty is modeled on a measurable space $(\Omega, \mathscr{F})$, endowed with a filtration $(\mathscr{F}_t)_{t\geq 0}$. We consider a family of probability measures P_y, $y \in \mathbb{R}_+$, on $(\Omega, \mathscr{F})$. A particular firm is assumed to have a cash inflow that is given by $Q_N Y$, where Q_N is the production level of the firm and Y is the stochastically evolving price level, which under P_y, Y evolves according to the geometric Brownian motion,

$$dY_t = \mu Y_t dt + \sigma Y_t dz_t, \quad Y_0 = y,\ \mathsf{P}_y\text{-a.s.}$$

It is assumed that the firm produces a quantity Q_N under normal conditions at a cost c_N and that it discounts profits at a constant rate $r > \mu$.

The present value (under P_y) of an operational firm without the liquidation option is

$$F_N(y) = \mathsf{E}_y \left[\int_0^\infty e^{-rt} \left(Q_N Y_t - c_N \right) dt \right] = \frac{Q_N y}{r - \mu} - \frac{c_N}{r}.$$

The firm's value with the liquidation option then is the solution to the optimal stopping problem

$$F_N^*(y) = F_N(y) + \sup_\tau \mathsf{E}_y \left[e^{-r\tau} (-F_N(Y_\tau)) \right]. \tag{1}$$

Because the planning horizon is infinite and $(Y_t)_{t\geq 0}$ is strongly Markovian with continuous sample paths (a.s.) the optimal policy will be to liquidate at the first hitting time of an endogenously determined trigger Y_N^*, i.e. at the Markov time $\check{\tau}(Y_N^*) := \inf\{t \geq 0 | Y_t \leq Y_0^*\}$.[3] Since $(Y_t)_{t\geq 0}$ has continuous sample paths, the optimal stopping problem (1) can, therefore, be formulated as a maximization problem over the threshold:[4]

$$\begin{aligned} F_N^*(y) &= F_N(y) + \sup_{Y^*} \mathsf{E}_y \left[e^{-r\tau(Y^*)}(-F_N(Y_{\tau(Y^*)})) \right] \\ &= F_N(y) + \sup_{Y^*} \mathsf{E}_y \left[e^{-r\check{\tau}(Y^*)} \right] (-F_N(Y^*)). \end{aligned}$$

Since the Laplace transform of GBM can easily be computed via Dynkin's formula (see, for example, Øksendal, 2000) as

$$\mathsf{E}_y \left[e^{-r\check{\tau}(Y^*)} \right] = \left(\frac{y}{Y^*} \right)^{\beta_2},$$

where $\beta_2 < 0$ is the negative root of the quadratic equation

$$\frac{1}{2}\sigma^2\beta(\beta - 1) + \mu\beta - r = 0, \tag{2}$$

the optimal stopping problem reduces to

$$F_N^*(y) = F_N(y) + \sup_{Y^*} \left(\frac{y}{Y^*} \right)^{\beta_2} (-F_N(Y^*)).$$

The objective function is continuous and concave so that a global maximum is attained on $[0, \infty]$, which we denote by Y_N^*. It can easily be established using standard techniques that the optimal liquidation trigger is given by

$$Y_N^* = \frac{\beta_2}{\beta_2 - 1} \frac{r - \mu}{Q_N} \frac{c_N}{r}.$$

Therefore,

$$F_N^*(y) = \begin{cases} \frac{Q_N y}{r-\mu} - \frac{c_N}{r} + \left(\frac{y}{Y_N^*} \right)^{\beta_2} \left[\frac{c_N}{r} - \frac{Q_N Y_N^*}{r-\mu} \right] & \text{if } y > Y_N^*, \\ 0 & \text{if } y \leq Y_N^*. \end{cases}$$

3 Temporary Unemployment Regulation

Suppose that the government of the country in which the firm operates introduces a temporary unemployment regulation that can affect the firm as follows. The programme is characterized by a reduction in costs due to a lower wage bill. The new costs are assumed to be constant and equal to $c_P < c_N$ per period. As a result the firm will produce less, say $Q_P < Q_N$.

As is the case in most countries where such measures exist, a firm can only partake in the programme once. In the Netherlands, for example, a firm could[5] reduce its number of FTEs for 13 weeks, which could be renewed at most three times. In order to simplify the analysis we assume that the firm has an infinitely lived

[3] See, for example, Dixit and Pindyck (1994).

[4] For a formal proof see, for example, Thijssen (2010).

[5] The temporary unemployment regulation was stopped by the Dutch government on July 1st, 2011.

option to stop the programme. Once the firm decides to stop the programme it either returns to its original production level or it decides to leave the industry. Again for simplicity, it is assumed that liquidation is an irreversible decision.

In order to write down the value of the firm while it is in the programme we need some more notation. The present value of never leaving the programme is

$$F_P(y) = \mathsf{E}_y \left[\int_0^\infty e^{-rt}(Q_P Y_t - c_P)dt \right] = \frac{Q_P y}{r-\mu} - \frac{c_P}{r}.$$

If the firm does not have the option to go back to its normal production level, but only has the option to liquidate, then, by analogy with Section 2, the value of the firm is where

$$Y_P^* = \frac{\beta_2}{\beta_2 - 1} \frac{r-\mu}{Q_P} \frac{c_P}{r},$$

is the optimal liquidation trigger.

However, when the firm decides to exit the programme it actually has to choose whether to go back to operating at original capacity or to exit the market. It stands to reason that the firm decides to exit the programme and operate under normal conditions once the process $(Y_t)_{t\geq 0}$ hits an endogenously determined trigger Y_H from below, i.e. at the Markov time $\hat{\tau}(Y_H) := \inf\{t \geq 0 | Y_t \geq Y_H\}$, or to exit once $(Y_t)_{t\geq 0}$ hits an endogenously determined trigger $Y_L < Y_H$ from above, i.e. at time $\check{\tau}(Y_L)$. The triggers Y_L and Y_H need to be determined simultaneously. In order to do so, define, for $Y_L \leq y \leq Y_H$,

$$\begin{aligned}
\hat{\nu}_y(Y_L, Y_H) &:= \mathsf{E}_y \left[e^{-r\hat{\tau}(Y_H)} \middle| \hat{\tau}(Y_H) < \check{\tau}(Y_L) \right] P_y(\hat{\tau}(Y_H) < \check{\tau}(Y_L)), \quad \text{and} \\
\check{\nu}_y(Y_L, Y_H) &:= \mathsf{E}_y \left[e^{-r\check{\tau}(Y_L)} \middle| \hat{\tau}(Y_H) > \check{\tau}(Y_L) \right] P_y(\hat{\tau}(Y_H) > \check{\tau}(Y_L)).
\end{aligned}$$

It can be shown (see, for example, Stokey, 2009), that

$$\hat{\nu}_y(Y_L, Y_H) = \frac{y^{\beta_1} Y_L^{\beta_2} - Y_L^{\beta_1} y^{\beta_2}}{Y_H^{\beta_1} Y_L^{\beta_2} - Y_L^{\beta_1} Y_H^{\beta_2}}, \quad \text{and} \quad \check{\nu}_y(Y_L, Y_H) = \frac{Y_H^{\beta_1} y^{\beta_2} - y^{\beta_1} Y_H^{\beta_2}}{Y_H^{\beta_1} Y_L^{\beta_2} - Y_L^{\beta_1} Y_H^{\beta_2}},$$

where $\beta_1 > 1$ is the positive solution to (2).

The firm's value can then be written as the solution to an optimal stopping problem:

$$F_P^*(y) := F_P(y) + \max_{Y_L, Y_H} \left\{ \hat{\nu}_y(Y_L, Y_H) \left[F_N^*(Y_H) - F_P(Y_H) \right] + \check{\nu}_y(Y_L, Y_H) [-F_P(Y_L)] \right\}.$$

In the continuation region the solution is given by $F_P^*(y) = Ay^{\beta_1} + By^{\beta_2} + F_P\,(y)$. This problem can not be solved analytically, but the first order conditions for optimum can be solved numerically. That is, (Y_L, Y_H) should satisfy

$$\begin{cases} \hat{\nu}_y(\cdot) \frac{\partial F_N^*(\cdot) - F_P(\cdot)}{\partial Y_H} + \frac{\partial \hat{\nu}_y(\cdot)}{\partial Y_H} [F_N^*(Y_H) - F_P(Y_H)] + \frac{\partial \check{\nu}_y(\cdot)}{\partial Y_H} [-F_P(Y_L)] = 0, \\ -\check{\nu}_y(Y_L, Y_H) \frac{\partial F_P(\cdot)}{\partial Y_L} + \frac{\partial \hat{\nu}_y(\cdot)}{\partial Y_L} [F_N^*(Y_H) - F_P(Y_H)] + \frac{\partial \check{\nu}_y(\cdot)}{\partial Y_L} [-F_P(Y_L)] = 0. \end{cases} \tag{3}$$

This system of first order conditions should be satisfied for every $y \in [Y_L, Y_H]$. In particular, (3) should be satisfied at Y_L and Y_H. This reduces (3) to

$$\begin{aligned}
\beta_1 A Y_L^{\beta_1} + \beta_2 B Y_L^{\beta_2} &= -Y_L F_P'(Y_L), \\
\beta_1 A Y_H^{\beta_1} + \beta_2 B Y_H^{\beta_2} &= Y_H [(F_N^*)'(Y_H) - F_P'(Y_H)],
\end{aligned} \tag{4}$$

where

$$
\begin{aligned}
A &= \frac{Y_L^{\beta_2}[F_N^*(Y_H) - F_P(Y_H)] - Y_H^{\beta_2}[-F_P(Y_L)]}{Y_H^{\beta_1}Y_L^{\beta_2} - Y_L^{\beta_1}Y_H^{\beta_2}} > 0, \\
B &= \frac{Y_H^{\beta_1}[-F_P(Y_L)] - Y_L^{\beta_1}[F_N^*(Y_H) - F_P(Y_H)]}{Y_H^{\beta_1}Y_L^{\beta_2} - Y_L^{\beta_1}Y_H^{\beta_2}} > 0.
\end{aligned}
$$

Solving (4) gives the triggers for returning to the original production level, Y_H, and exiting the market, Y_L, respectively. These triggers, in turn, determine the value of the firm while it is in the programme, $F_P^*(\cdot)$. A first observation is that the firm liquidates later if it has the option to return to normal production.[6]

Lemma 1 *It holds that $Y_L < Y_P^*$.*

Using the triggers Y_L and Y_H we can now establish the value of a firm with the option to enter the temporary unemployment programme. Again, the optimal policy will be to enter the programme once economic conditions have deteriorated sufficiently, i.e. to enter the programme at time $\check{\tau}(Y^*)$, for some trigger Y^*. Since the value of a firm in the programme is always at least as large as the value of exiting immediately, it is intuitively clear that it should hold that $Y^* > Y_N^*$. In other words, it should never be optimal to liquidate immediately and not to make use of the temporary unemployment programme. The optimal stopping problem that the firm needs to solve is

$$
\begin{aligned}
F^*(y) &= F_N(y) + \sup_{\tau} \mathsf{E}_y\left[e^{-r\tau}(F_P^*(Y_\tau) - F_N(Y_\tau))\right] \\
&= F_N(y) + \max_{Y^*}\left(\frac{y}{Y^*}\right)^{\beta_2}[F_P^*(Y^*) - F_N(Y^*)].
\end{aligned} \tag{5}
$$

The (non-linear) first order condition of this maximization problem can be solved numerically to find the optimal entry trigger Y^*:

$$
\frac{\partial}{\partial Y^*}[F_P^*(\cdot) - F_N(\cdot)] = \beta_2 \frac{F_P^*(Y^*) - F_N(Y^*)}{Y^*}.
$$

In order to prove the existence of a programme entry trigger $Y^* \in (Y_L, Y_H)$, we need to make an additional assumption that ensures that the expected revenue of the programme is sufficiently large. In particular, there must exist economic situations (i.e. values of y) for which the expected present value of the revenues being in the programme as opposed to operate normally fall sufficiently short of the expected present value of the cost benefits.

Assumption 1 *Define the threshold*

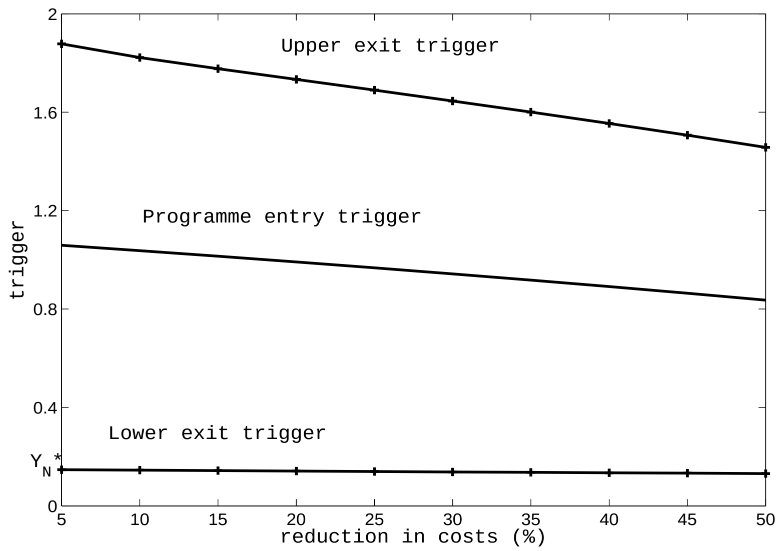
$$
\check{Y} = \left[\frac{1-\beta_2}{\beta_1(\beta_1-\beta_2)A}\frac{Q_N - Q_P}{r-\mu}\right]^{\frac{1}{\beta_1-1}}.
$$

At the threshold $\check{Y}$ it holds that

$$
\check{Y}\frac{Q_N - Q_P}{r-\mu} > \frac{\beta_1\beta_2}{(\beta_1-1)(\beta_2-1)}\frac{c_N - c_P}{r}.
$$

[6] All proofs are in the appendix.

Figure 1: Triggers for programme entry, programme exit, and default. Here it is assumed that for a cost reduction α, $c_P = (1-\alpha)c_N$ and $Q_P = (1-\alpha)^\gamma Q_N$, where we take $\gamma = .2$. The base case parameters are $Q_N = 10$, $c_N = 8$, $r = .04$, $\mu = .03$, and $\sigma = .15$.



Proposition 1 *Under Assumption 1 there exists a unique trigger Y^* at which it is optimal to enter the programme if and only if $Y_L < Y_N^*$.*

A sufficient condition for $Y_L < Y_N^*$ is that $Y_P^* < Y_N^*$, which is satisfied if and only if

$$\frac{c_P}{c_N} < \frac{Q_P}{Q_N}.$$

That is, the relative cost reduction should not exceed the relative revenue reduction. If this condition is not satisfied then if the firm did not have the option to revert to normal production the firm would liquidate sooner when running at lower production level Q_P than at the higher production level Q_N. If then, in addition, the option value of reverting back to normal production is not high enough it is possible that $Y_L > Y_N^*$ and the optimal stopping problem (5) has no solution. If, however, the problem has a solution, then it is optimal to enter the programme before it is optimal to liquidate.

Lemma 2 *If $Y_L < Y_N^*$, then $Y^* > Y_N^*$.*

In Figure 1, the triggers Y_L, Y_H, and Y^* are plotted for various values of cost reduction. It looks like the liquidation threshold is only marginally influenced by the temporary unemployment measure. We will see later, however, that the quantitative effect in terms of benefits can be quite large.

4 Government-Determined Programme Exit Trigger

Suppose that the government does not allow the firm to choose it own upper exit trigger Y_H. This could be because the government fears that the firm will stay in the programme for too long, which increases the costs of running it. Suppose, then, that the government fixes a level $\hat{Y}_H \in (Y_N^*, Y_H)$ at which the firm is forced to leave the programme. The optimal liquidation threshold while in the programme now is the solution to the optimal stopping programme

$$\widehat{F}_P^*(y) := F_P(y) + \max_{Y_L} \Big\{ \hat{\nu}_y(Y_L, \hat{Y}_H) \Big[F_N^*(\hat{Y}_H) - F_P(\hat{Y}_H) \Big] + \check{\nu}_y(Y_L, \hat{Y}_H)[-F_P(Y_L)] \Big\}.$$

In the continuation region, this function equals $\widehat{F}_P^*(y) = \widehat{A} y^{\beta_1} + \widehat{B} y^{\beta_2} + F_P\,(y)$, for constants $\widehat{A}$ and $\widehat{B}$ which need to be determined, together with the threshold Y_L. These three unknowns need to solve a value-matching condition at $\hat{Y}_H$ and a value-matching and smooth-pasting condition at Y_L, i.e.

$$\begin{cases} \widehat{A}\hat{Y}_H^{\beta_1} + \widehat{B}\hat{Y}_H^{\beta_2} = F_N^*(\hat{Y}_H) - F_P(\hat{Y}_H) \\ \widehat{A}Y_L^{\beta_1} + \widehat{B}Y_L^{\beta_2} = -F_P(Y_L) \\ \beta_1\widehat{A}Y_L^{\beta_1-1} + \beta_2\widehat{B}Y_L^{\beta_2-1} = -F_P'(Y_L) \end{cases}.$$

Once the threshold $Y_L(\hat{Y}_H)$ has been determined, the firm can find the entry threshold $Y^*(\hat{Y}_H)$, by solving

$$\widehat{F}^*(y) = F_N(y) + \max_{Y^*(\hat{Y}_H)} \left(\frac{y}{Y^*(\hat{Y}_H)} \right)^{\beta_2} [\widehat{F}_P^*(Y^*(\hat{Y}_H)) - F_N(Y^*(\hat{Y}_H))].$$

In Figure 2, the triggers $Y_L(\hat{Y}_H)$ and $Y^*(\hat{Y}_H)$ are plotted for various values of $\hat{Y}_H$, where $\hat{Y}_H$ is varied from Y_N^* to the optimal programme exit trigger (for the firm), Y_H.

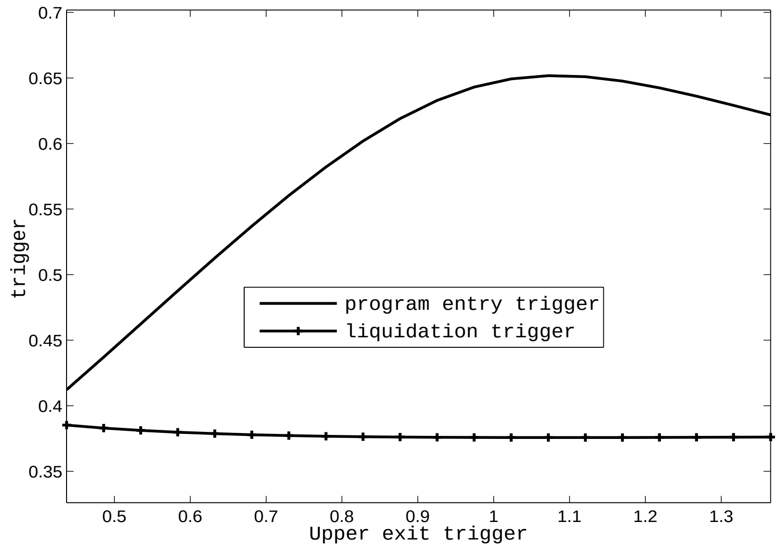
The non-monotonicity of the entry trigger can be explained through the influence of two, opposing forces. On the one hand, an increase in the upper exit threshold just simply makes it *possible* to enter the programme earlier and benefit from the cut in the wage costs. However, on the other hand, because the firm can only enter the programme once, it will not want to enter too soon. For higher upper exit thresholds the latter effect dominates.

Note that the firm itself would choose an upper exit trigger that is not lead to the highest possible entry trigger. This is because in choosing the triggers the firm values the compound option, not just the option to enter the programme.

5 Welfare Effects of Temporary Unemployment Regulation

In this section we compute the welfare effects of the temporary unemployment programme through a benefit-to-cost ratio (BCR). In order to do so we need to specify more clearly which part of the firm's costs are due to labour.

Figure 2: Triggers for programme entry and liquidation. The base case parameters are $Q_N = 10$, $c_N = 8$, $Q_P = 8.85$, $c_P = 7.07$, $r = .04$, $\mu = .01$, and $\sigma = .15$.



Assume that the firm produces its output using fixed capital and labour stocks according to the Cobb-Douglas production function

$$Q_N = \bar{K}^{1-\gamma}\bar{L}^{\gamma}.$$

The unit prices of capital and labour are given by $r > \mu$ and w, respectively. So, every period the firm's cash outflow equals $c_N = r\bar{K} + w\bar{L}$.

The programme specifies the fraction of productive labour that can be entered, denoted by α, and the fraction of the wage bill of this fraction that will be paid for by the government, denoted by η. For simplicity it is assumed that workers do not experience a drop in nominal wages, so that the firm pays a fraction $1 - \eta$ of the wage bill of unproductive labour. As a result, the cash outflow drops to

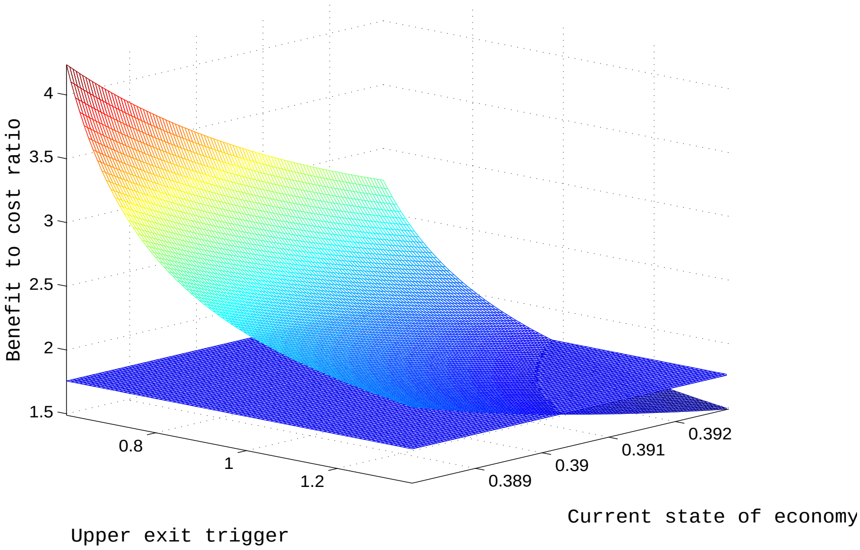
$$c_P = r\bar{K} + w(1-\alpha)\bar{L} + w\alpha(1-\eta)\bar{L} = c_N - w\eta\alpha\bar{L},$$

whereas the cash inflow drops to $Q_P Y$, where

$$Q_P = \bar{K}^{1-\gamma}[(1-\alpha)\bar{L}]^{\gamma} = (1-\alpha)^{\gamma} Q_N.$$

The benefits of the programme are taken to be the reduction in losses over life-time labour income that accrue from being laid-off. These can be seen as sunk costs of liquidation that accrue not to the shareholders, but to society as a whole and are, thus, not taken into account by the firm's management, which has a legal obligation to maximize shareholder value. Let this reduction be denoted by χ. It has been estimated by Davis and Von Wachter (2011) that being laid off costs a typical male worker 11% of his future earnings. During a recession this rises to 19%. The benefit-to-cost ratio (BCR) of the programme is established in the following proposition.

Figure 3: BCR of temporary unemployment programme. The base case parameters are $Q_N = 10$, $c_N = 8$, $r\bar{K} = w\bar{L}$, $r = .04$, $\mu = .01$, $\sigma = .15$, $\eta = .7$, and $\gamma = .3$. The level of the hyperplane is the static BCR.



Proposition 2 *Suppose that $Y_L < Y_N^*$ and let $Y_N^* \leq y \leq Y_H$. Assume that the government unemployment benefits consist of the full salary. Then*

$$BCR(y) = DFR(y)\frac{\chi + 1 - \eta}{\alpha\eta},$$

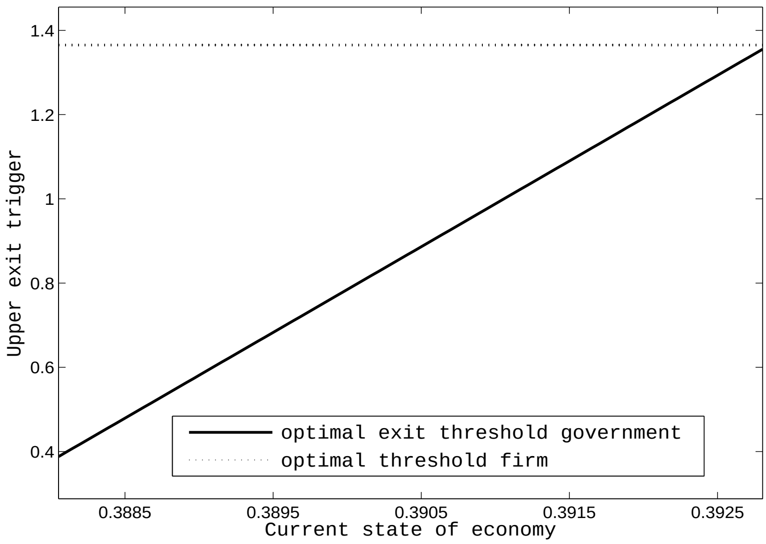
where $DFR(y)$, the discount factor ratio*, is*

$$DFR(y) := \begin{cases} \frac{(y/Y_N^*)^{\beta_2} - \check{\nu}_y(Y_L, Y_H) - \hat{\nu}_y(Y_L, Y_H)(Y_H/Y_N^*)^{\beta_2}}{1 - \check{\nu}_y(Y_L, Y_H) - \hat{\nu}_y(Y_L, Y_H)} & \text{if } y \in (Y_N^*, Y^*], \\ \frac{(y/Y_N^*)^{\beta_2} - (y/Y^*)^{\beta_2}[\check{\nu}_{Y^*}(Y_L, Y_H) + \hat{\nu}_{Y^*}(Y_L, Y_H)(Y_H/Y_N^*)^{\beta_2}]}{(y/Y^*)^{\beta_2}[1 - \check{\nu}_{Y^*}(Y_L, Y_H) - \hat{\nu}_{Y^*}(Y_L, Y_H)]} & \text{if } y \in (Y^*, Y_H). \end{cases}$$

The full salary assumption creates a positive bias in the benefits of the programme. For the same parameters as used for previous figures, Figure 3 plots the BCR as a function of the chosen upper threshold trigger (by the government) and the current state of the economy, represented by y. As can be seen, the benefits always outweigh the costs, leading to a BCR that exceeds unity. In addition, there is a region of the parameter space (i.e. for relatively low y and $\hat{Y}_H$) where the dynamic BCR does not exceed the naive NPV of the programme. This is the region where $DFR < 1$. This implies that the static BCR can both overestimate and underestimate the dynamic BCR.

In Figure 4 we plot the optimal upper exit threshold that the government should choose given the current state of the economy. That is, for each y we choose the exit trigger $\hat{Y}_H$ that maximizes the BCR. It can be seen that, the worse the state of the economy, the lower the government should set the upper exit threshold. This makes intuitive sense, since if y is low and $\hat{Y}_H$ is high, then the firm is expected to stay in the programme for a long time. This (i) increases the costs of the programme and (ii) reduces the present value of the benefits.

Figure 4: Optimal upper exit threshold of programme. The base case parameters are $Q_N = 10$, $c_N = 8$, $r\bar{K} = w\bar{L}$, $r = .04$, $\mu = .01$, $\sigma = .15$, $\eta = .7$, and $\gamma = .2$.



6 Concluding Remarks

In this paper we studied a firm that can make use of temporary unemployment regulations. Using a real options approach we derived the optimal thresholds for the firm to enter such a programme and, once entered, to exit and revert to normal production levels, or to liquidate. The liquidation threshold is computed both when the firm is using the programme and when it is not. We show that a firm that is using the regulation will default later than a firm that is not using it.

Furthermore, we take the point of view of a government and calculate the benefits for society of such regulations by a dynamic benefit-to-cost ratio. To maximize welfare, governments should impose an upper exit threshold for the regulation to the firms. This is because firms want to stay longer in the regulation than is optimal from a welfare perspective. It turns out that in worse states of the economy the government should set the exit threshold lower.

We list three possible extensions of this research. First, it would be interesting to make the duration that the firms stay in the regulation time dependent. With such a model one could calculate the optimal duration of the programme both from a firm's and society perspective. Second, one could investigate what the effect would be from the possibility to make use of the programme more than once. Would firms enter the programme earlier? Would they exit earlier? Would this be beneficial for society? Third, it could be interesting to study the effect of the regulation in a competitive setting. How is the entry threshold of one firm affected by actions of other firms? What if a firm is in competition with another firm that is based in a country where these regulations do not exist?

A Proofs

Proof of Lemma 1 Recall that Y_P^* solves the first order condition

$$F_P'(Y_P^*) + \beta_2 \frac{F_P(Y_P^*)}{Y_P^*} = 0,$$

whereas Y_L solves

$$\begin{cases} AY_L^{\beta_1} + BY_L^{\beta_2} = -F_P(Y_L) \\ \beta_1 AY_L^{\beta_1 - 1} + \beta_2 BY_L^{\beta_2 - 1} = -F_P'(Y_L). \end{cases}$$

Using the value-matching condition to solve for $BY_L^{\beta_2}$ and substituting this into the smooth pasting condition gives

$$(\beta_1 - \beta_2)AY_L^{\beta_1 - 1} - \beta_2 \frac{F_P(Y_L)}{Y_L} = -F_P'(Y_L).$$

Defining $f(y) = -F_P'(y) + \beta_2[F_P(y)/y]$, we now get that

$$\begin{cases} f(Y_L) = (\beta_1 - \beta_2)AY_L^{\beta_1 - 1} > 0 \\ f(Y_P^*) = 0, \end{cases}$$

so that $Y_L < Y_P^*$, because Y_P^* is a maximum location of $F_P^*(\cdot)$. ■

Proof of Proposition 1 First note that for $y \in [Y_L, Y_H]$ we can write

$$F_P^*(y) = F_P(y) + Ay^{\beta_1} + By^{\beta_2}.$$

This implies that the foc for maximizing $F^*(\cdot)$ can be written as $g(y) = 0$, where

$$\begin{aligned} g(y) &= -\beta_2[F_P(y) - F_N(y) + Ay^{\beta_1} + By^{\beta_2}] + y[F_P'(y) - F_N'(y) + \beta_1 Ay^{\beta_1 - 1} + \beta_2 By^{\beta_2 - 1}] \\ &= -\beta_2[F_P(y) - F_N(y)] + y[F_P'(y) - F_N'(y)] + (\beta_1 - \beta_2)Ay^{\beta_1}. \end{aligned}$$

Since

$$g'(y) = (1 - \beta_2)\frac{Q_P - Q_N}{r - \mu} + \beta_1(\beta_1 - \beta_2)Ay^{\beta_1 - 1},$$

and

$$g''(y) = \beta_1(\beta_1 - 1)(\beta_1 - \beta_2)Ay^{\beta_1 - 2} > 0,$$

$g(\cdot)$ is a strictly convex function, which can, therefore, have at most two zeros.

The minimum location of $g(\cdot)$ on $[Y_L, Y_H]$ can be found analytically:

$$\begin{aligned} g'(y) = 0 \iff y^{\beta_1 - 1} &= \frac{1 - \beta_2}{\beta_1(\beta_1 - \beta_2)A} \frac{Q_N - Q_P}{r - \mu} = 0 \\ \iff y &= \left[\frac{1 - \beta_2}{\beta_1(\beta_1 - \beta_2)A} \frac{Q_N - Q_P}{r - \mu}\right]^{\frac{1}{\beta_1 - 1}} \equiv \check{Y}. \end{aligned}$$

We then find that

$$\begin{aligned}
g(\check{Y}) &= (1-\beta_2)\frac{Q_P - Q_N}{r-\mu}\check{Y} + (\beta_1-\beta_2)A\check{Y}^{\beta_1} + \beta_2\frac{c_P - c_N}{r} \\
&= \check{Y}\left[(1-\beta_2)\frac{Q_P - Q_N}{r-\mu} + (\beta_1-\beta_2)A\check{Y}^{\beta_1-1}\right] + \beta_2\frac{c_P - c_N}{r} \\
&= \check{Y}\left[(1-\beta_2)\frac{Q_P - Q_N}{r-\mu} + (\beta_1-\beta_2)A\frac{1-\beta_2}{\beta_1(\beta_1-\beta_2)A}\frac{Q_N - Q_P}{r-\mu}\right] + \beta_2\frac{c_P - c_N}{r} \\
&= \check{Y}\frac{Q_N - Q_P}{r-\mu}\left(-(1-\beta_2) + \frac{1-\beta_2}{\beta_1}\right) + \beta_2\frac{c_P - c_N}{r} \\
&= \check{Y}\frac{Q_N - Q_P}{r-\mu}\frac{(1-\beta_1)(1-\beta_2)}{\beta_1} + \beta_2\frac{c_P - c_N}{r} < 0,
\end{aligned}$$

where the last inequality follows from Assumption 1.

Define

$$f(y) = F_P(y) - F_N(y) + Ay^{\beta_1} + By^{\beta_2}.$$

Since $AY_H^{\beta_1} + BY_H^{\beta_2} = F_N^*(Y_H) - F_P(Y_H)$, it holds that

$$\begin{aligned}
f(Y_H) &= F_P(Y_H) - F_N(Y_H) + AY_H^{\beta_1} + BY_H^{\beta_2} \\
&= F_N^*(Y_H) - F_N(Y_H) = \left(\frac{Y_H}{Y_N^*}\right)^{\beta_2}[-F_N(Y_N^*)],
\end{aligned}$$

and

$$f'(Y_H) = \beta_2\left(\frac{Y_H}{Y_N^*}\right)^{\beta_2}\frac{1}{Y_H}[-F_N(Y_N^*)].$$

We, therefore, find that

$$\begin{aligned}
g(Y_H) &= -\beta_2 f(Y_H) + Y_H f'(Y_H) \\
&= (1-\beta_2)\left(\frac{Y_H}{Y_N^*}\right)^{\beta_2}[-F_N(Y_N^*)] > 0.
\end{aligned}$$

So, $g(\cdot)$ has a zero at $Y_* \in (\check{Y}, Y_H)$, which represents a minimum of $F^*(\cdot)$.

The function $F^*(\cdot)$ has a (unique) maximum at some $Y^* \in (Y_L, \check{Y})$ if and only if $g(Y_L) > 0$. Note that

$$\begin{aligned}
f(Y_L) &= F_P(Y_L) - F_N(Y_L) + AY_L^{\beta_1} + BY_L^{\beta_2} \\
&= F_P(Y_L) - F_N(Y_L) + [-F_P(Y_L)] = -F_N(Y_L),
\end{aligned}$$

and

$$f'(Y_L) = -F_N'(Y_L),$$

which implies that

$$g(Y_L) = -\beta_2[-F_N(Y_L)] + Y_L[-F_N'(Y_L)].$$

At Y_N^* it holds that

$$-\beta_2[-F_N(Y_N^*)] + Y_N^*[-F_N'(Y_N^*)] = 0,$$

since Y_N^* is a maximum location of $F_N^*(\cdot)$. It, therefore holds that $g(Y_L) > 0 \iff Y_L < Y_N^*$. ∎

Proof of Lemma 2 At Y^* it holds that $g(Y^*) = 0$, i.e. that

$$\begin{aligned} &\beta_2[F_P(Y^*) - F_N(Y^*)] = Y^*[F_P'(Y^*) - F_N'(Y^*)] + (\beta_1 - \beta_2)A(Y^*)^{\beta_1} \\ \Longleftrightarrow\ &\beta_2[-F_N(Y^*)] - Y^*[-F_N'(Y^*)] = -\left[-\beta_2(-F_P(Y^*)) + Y^*(-F_P'(Y^*))\right] + (\beta_1 - \beta_2)A(Y^*)^{\beta_1}. \end{aligned} \tag{6}$$

Since at Y_P^* it holds that

$$-\beta_2[-F_P(Y_P^*)] + Y_P^*[-F_P'(Y_P^*)] = 0,$$

Y_P^* is a maximum location, and $Y^* > Y_P^*$, we have that the term between square brackets on the right-hand side of (6) is negative. Therefore, the right-hand side of (6) is positive.

This implies that

$$-\beta_2[F_P(Y^*) + F_N(Y^*)] < 0.$$

Since Y_N^* solves

$$-\beta_2[F_P(Y^*) + F_N(Y^*)] = 0,$$

and Y_N^* is a maximum location it, therefore, holds that $Y^* > Y_N^*$. ■

Proof of Proposition 2 The benefits of the programme are given by the costs that are not incurred because of the existence of the programme. These consist of the wage costs that are paid by the government as unemployment benefits and the sunk costs of unemployment. These are fractions $1 - \eta$ and χ, respectively, of total wages. Define

$$\check{\tau}_y(\bar{Y}) = \inf\{t \geq 0 | Y_t \leq \bar{Y}, Y_0 = y, \mathsf{P}_y - a.s.\},$$

for any $\bar{Y}$. Let $y > Y^*$. Then, the total social costs if the programme does not exist are

$$\begin{aligned} C_N(y) :=& \mathsf{E}_y\left[\int_{\check{\tau}_y(Y_N^*)}^{\infty} e^{-rt}(\chi + 1 - \eta)w\bar{L}dt\right] \\ =& \mathsf{E}_y[e^{-r\check{\tau}_y(Y_N^*)}](\chi + 1 - \eta)\frac{w}{r}\bar{L} \\ =& \left(\frac{y}{Y_N^*}\right)^{\beta_2}(\chi + 1 - \eta)\frac{w}{r}\bar{L}. \end{aligned}$$

Next we define

$$\hat{\tau}_y(\bar{Y}) = \inf\{t \geq 0 | Y_t \geq \bar{Y}, Y_0 = y, \mathsf{P}_y - a.s.\},$$

for any $\bar{Y}$. The cost to society with the programme are

$$\begin{aligned} C_P(y) :=& \mathsf{E}_y\Big[\int_{\check{\tau}_y(Y^*) + \check{\tau}_{Y^*}(Y_L)}^{\infty} e^{-rt}(\chi + 1 - \eta)w\bar{L}dt\Big|\check{\tau}_y(Y_L) < \hat{\tau}_y(Y_H)\Big]\mathsf{P}_y(\check{\tau}_y(Y_L) < \hat{\tau}_y(Y_H)) \\ &+ \mathsf{E}_y\Big[\int_{\check{\tau}_y(Y^*) + \check{\tau}_{Y^*}(Y_H) + \check{\tau}_{Y_H}(Y_N^*)}^{\infty} e^{-rt}(\chi + 1 - \eta)w\bar{L}dt\Big|\hat{\tau}_y(Y_H) < \check{\tau}_y(Y_L)\Big]\mathsf{P}_y(\hat{\tau}_y(Y_H) < \check{\tau}_y(Y_L)) \\ =& \Xi(y)(\chi + 1 - \eta)\frac{w}{r}\bar{L}, \end{aligned}$$

where

$$
\begin{aligned}
\Xi(y) =& \mathsf{E}_y\Big[e^{-r(\check{\tau}_y(Y^*)+\check{\tau}_{Y^*}(Y_L))}\Big|\check{\tau}_y(Y_L)<\check{\tau}_y(Y_H)\Big]\mathsf{P}_y(\check{\tau}_y(Y_L)<\check{\tau}_y(Y_H))\\
&+\mathsf{E}_y\Big[e^{-r\left(\check{\tau}_y(Y^*)+\check{\tau}_{Y^*}(Y_H)+\check{\tau}_{Y_H}(Y_N^*)\right)}\Big|\check{\tau}_y(Y_H)<\check{\tau}_y(Y_L)\Big]\mathsf{P}_y(\check{\tau}_y(Y_H)<\check{\tau}_y(Y_L))\\
=&\check{\nu}_y(Y_L,Y_H)+\mathsf{E}_y\Big[e^{-r\check{\tau}_y(Y_H)}\mathsf{E}_{Y_H}\Big[e^{-r\check{\tau}_{Y_H}(Y_N^*)}\Big]\Big|\hat{\tau}_y(Y_H)<\check{\tau}_y(Y_L)\Big]\mathsf{P}_y(\hat{\tau}_y(Y_H)<\check{\tau}_y(Y_L))\\
=&\check{\nu}_y(Y_L,Y_H)+\mathsf{E}_{Y_H}\Big[e^{-r\check{\tau}_{Y_H}(Y_N^*)}\Big]\mathsf{E}_y\Big[e^{-r\hat{\tau}_y(Y_H)}\Big|\hat{\tau}_y(Y_H)<\check{\tau}_y(Y_L)\Big]\mathsf{P}_y(\hat{\tau}_y(Y_H)<\check{\tau}_y(Y_L))\\
=&\left(\frac{y}{Y^*}\right)^{\beta_2}\left[\check{\nu}_{Y^*}(Y_L,Y_H)+\hat{\nu}_{Y^*}(Y_L,Y_H)\left(\frac{Y_H}{Y_N^*}\right)^{\beta_2}\right].
\end{aligned}
$$

The costs of the programme consist of paying a fraction $\alpha\eta$ of the wage bill during the time the firm is in temporary unemployment. These costs equal

$$
\begin{aligned}
C(y) :=& \mathsf{E}_y\Big[\int_{\check{\tau}_y(Y^*)}^{\check{\tau}_y(Y^*)+\check{\tau}_{Y^*}(Y_L)} e^{-rt}\alpha\eta w\bar{L}dt\Big|\check{\tau}_y(Y_L)<\hat{\tau}_y(Y_H)\Big]\mathsf{P}_y(\check{\tau}_y(Y_L)<\hat{\tau}_y(Y_H))\\
&+\mathsf{E}_y\Big[\int_{\check{\tau}_y(Y^*)}^{\check{\tau}_y(Y^*)+\check{\tau}_{Y^*}(Y_H)} e^{-rt}\alpha\eta w\bar{L}dt\Big|\hat{\tau}_y(Y_H)<\check{\tau}_y(Y_L)\Big]\mathsf{P}_y(\hat{\tau}_y(Y_H)<\check{\tau}_y(Y_L))\\
=&\Upsilon(y)\alpha\eta\frac{w}{r}\bar{L},
\end{aligned}
$$

where

$$
\begin{aligned}
\Upsilon(y) =& \mathsf{E}_y\Big[e^{-r\check{\tau}_y(Y^*)}\Big]-\mathsf{E}_y\Big[e^{-r(\check{\tau}_y(Y^*)+\check{\tau}_{Y^*}(Y_L))}\Big|\check{\tau}_y(Y_L)<\hat{\tau}_y(Y_H)\Big]\mathsf{P}_y(\check{\tau}_y(Y_L)<\hat{\tau}_y(Y_H))\\
&-\mathsf{E}_y\Big[e^{-r(\check{\tau}_y(Y^*)+\check{\tau}_{Y^*}(Y_H))}\Big|\hat{\tau}_y(Y_H)<\check{\tau}_y(Y_L)\Big]\mathsf{P}_y(\hat{\tau}_y(Y_H)<\check{\tau}_y(Y_L))\\
=&\left(\frac{y}{Y^*}\right)^{\beta_2}-\mathsf{E}_y\Big[e^{-r\check{\tau}_y(Y^*)}\mathsf{E}_{Y^*}\Big[e^{-r\check{\tau}_{Y^*}(Y_L)}\Big]\Big|\check{\tau}_y(Y_L)<\hat{\tau}_y(Y_H)\Big]\mathsf{P}_{Y^*}(\check{\tau}_y(Y_L)<\hat{\tau}_y(Y_H))\\
&-\mathsf{E}_y\Big[e^{-r\check{\tau}_y(Y^*)}\mathsf{E}_{Y^*}\Big[e^{-r\check{\tau}_{Y^*}(Y_H)}\Big]\Big|\hat{\tau}_y(Y_H)<\check{\tau}_y(Y_L)\Big]\mathsf{P}_{Y^*}(\hat{\tau}_y(Y_H)<\check{\tau}_y(Y_L))\\
=&\left(\frac{y}{Y^*}\right)^{\beta_2}-\mathsf{E}_y\Big[e^{-r\check{\tau}_y(Y^*)}\Big]\mathsf{E}_{Y^*}\Big[e^{-r\check{\tau}_{Y^*}(Y_L)}\Big|\check{\tau}_y(Y_L)<\hat{\tau}_y(Y_H)\Big]\mathsf{P}_{Y^*}(\check{\tau}_y(Y_L)<\hat{\tau}_y(Y_H))\\
&-\mathsf{E}_y\Big[e^{-r\check{\tau}_y(Y^*)}\Big]\mathsf{E}_{Y^*}\Big[e^{-r\check{\tau}_{Y^*}(Y_H)}\Big|\check{\tau}_y(Y_H)<\check{\tau}_y(Y_L)\Big]\mathsf{P}_{Y^*}(\check{\tau}_y(Y_H)<\check{\tau}_y(Y_L))\\
=&\left(\frac{y}{Y^*}\right)^{\beta_2}\left[1-\check{\nu}_{Y^*}(Y_L,Y_H)-\hat{\nu}_{Y^*}(Y_L,Y_H)\right].
\end{aligned}
$$

So, it follows that

$$
BCR(y)=\frac{C_N(y)-C_P(y)}{C(y)}=DFR(y)\frac{1+\eta+\chi}{\eta\alpha}.
$$

The proof for $Y_N^* \leq y < Y^*$ now follows straightforwardly. ■

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