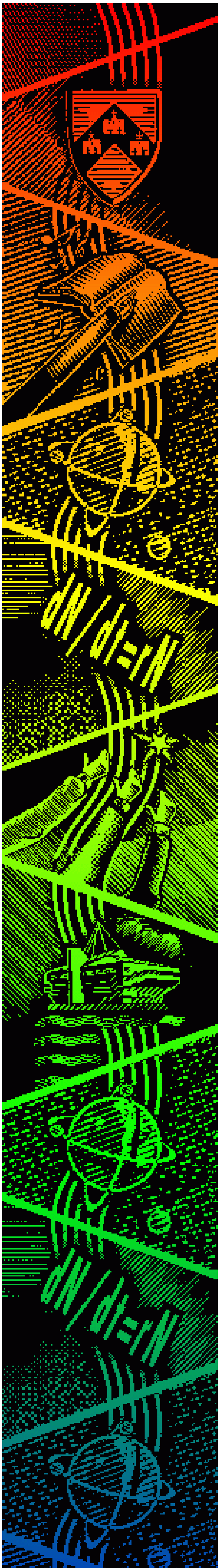

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Endogenized Production Sets in a General Equilibrium Model with Incomplete Markets

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Endogenized Production Sets in a General Equilibrium Model with Incomplete Markets

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Abstract

The paper develops a general equilibrium model with incomplete markets where the asset structure is endogenized. This asset structure allows to consider a new class of objective functions of profit maximizing firms. This class of objective functions is independent of the utilities of the stock holders. Corporate equilibrium properties are studied for this model. It is shown that the organization of production is generally efficient. This result is a consequence of the generalization of the separation theorem of the Arrow-Debreu model to incomplete markets. Finally, the paper shows that corporate equilibria are not independent of the firms financial activities.

1 Introduction

Most of the literature about general equilibrium models with incomplete markets derives equilibrium properties independently of the asset structures. For a sample of the huge literature on exogenous asset formation models see Diamond [1967], Drèze [1974], Grossman and Hart [1979], Geanokopolos [1990], Magill and Quinzii [2002], and others. In these models, fixed production sets are stretched over two periods. A natural interpretation of profits maximization is for the producer to choose inputs of production in period one such that period two outputs maximize profits. It is well known that in this economic scenario, firms need be assigned a rule in order to obtain a closed form equilibrium solution. This extra information is derived from the utilities of the owners of the firm, i.e. Drèze [1974] or Grossman and Hart [1979] criteria. Thus, it follows that firms maximize exogenously assigned average utilities. This has non-trivial economic implications. Assigning utilities to the firms comes at cost of abandoning the decentralization property of the Arrow-Debreu model. Another drawback of this class of exogenous asset formation models is the independency of the production sets from the financial activities of the firms. As a consequence of this asset structure, where the financing of production essentially plays no role, financial policies are indeterminate. For a sample of this literature see Stiglitz [1969], Stiglitz [1974], DeMarzo [1988], Magill and Quinzii [2002], and Duffie and Shafer [1986a]. Finally, assigning utilities to the objective function of the firm introduces a further source of inefficiency (Geanokopolos et al. [1990]). Hence, welfare distortions.

By means of a leading example, the aim of this paper is to improve on the theory of the firm in incomplete markets where the economic scenario is reduced to exogenous

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asset structures. Thus, we add more structure to the economic model by considering the role intermediate goods and financial assets play in the theory of the firm in incomplete markets. This enables us to endogenize the production capacity available to the firm. The paper considers an economic scenario where firms issue stocks in period one, buy intermediate goods, and build up their production capacity. Total capital a firm can purchase is bounded by the cash it can acquire through the stock market. In period two, the firm has a well defined production set in each state of nature. Its short run activity is then to choose inputs of production such that profits are maximized. Short run production inputs are financed with the revenue obtained from selling goods in period two. Long run production factors are financed through the stock market in the first period, and determine the firm's size.

This economic scenario is sufficiently interesting to derive objective functions of the firms which are independent of the utilities of the stock holders. For this class of models we show that the activities of the consumers are independent of the activities of the producers. This generalizes the decentralization property of the Arrow-Debreu model to incomplete markets. This result improves on the models introduced by Drèze [1974], and Grossman and Hart [1979] where the objective function of the firm is not independent of the utility of the owners of the firm. Another property of our model is that firms maximize profits in the long run. This implies that in the long run, firms chose profit maximizing finance quantities, and purchase capital in order to build up their production set. In the short run, they chose profit maximizing inputs of production at competitive prices, and given installed production capacity. This economic interpretation of the activities of the firm is not possible in models with exogenous asset structure. The immediate consequence of this structure is that the organization of productive activities of the firm is efficient. This follows from the independence of the sequential objective function of the firm from the utilities of the stock holders. Finally, we show that, since the level of production capacity of the firm is not independent of the activities of the firm on financial markets, financial policies have real equilibrium effects. This result is a consequence of the more realistic asset structure considered.

Section two introduces the development of the model and its results. Section three is a conclusion. All proofs are in the appendix.

2 The Economic Model

This paper considers a variation of the model of the firm introduced in Stiefenhofer [2009] and Stiefenhofer [2010] for the case that financial activities are explicitly modeled. This requires the introduction of an extensive form model of the firm. We consider a simple model with sufficient structure to highlight the main properties of interest. Let the budget constraints of the single agent as a consumer be

$$\begin{aligned} p(0)\cdot x(0) &= p(0)\cdot \omega(0) - qz \\ p(s)\cdot x(s) &= p(s)\cdot \omega(1) + R(\bar{y}, s)z. \end{aligned}$$

A consumption bundle $x = (x(0), x(s))$ is a collection of vectors defined on the strictly positive orthant $\mathbb{R}^{l(S+1)}_{++}$ with associated strictly positive price system $p = (p(0), p(s))$ in $\mathbb{R}^{l(S+1)}_{++}$. A financial quantity z (number of stocks) is a strictly positive real number $\mathbb{R}_{++}$ with associated price system q in $\mathbb{R}_{++}$. We denote the initial resources of this economy $\omega = (\omega(0), \omega(1))$ in $\mathbb{R}^{2l}_{++}$. Note that there is no aggregate uncertainty in this economy, instead we consider firm specific risk. An uncertain state of nature is an element denoted

s in the exhaustive set of mutually exclusive elements S. $R(\bar{y}, s)z$ denotes the return of investment into the firm. The consumer invests into the firm in order to transfer wealth across time and between uncertain states of nature.

In a one agent model the agent also performs the role of the producer, and therefore, adds following variables to his constraints

$$\begin{array}{c} p(0) \cdot x(0) = p(0) \cdot \omega(0) - qz + qb - p(0) \cdot \bar{k}(0) \\ p(s) \cdot x(s) = p(s) \cdot \omega(1) + R(\bar{y}, s)z + p(s) \cdot y(0) \end{array},$$

where $\bar{k}(0)$ denotes the capital purchased. Let aside the modeling of financing production for a while, therefore, let $\xi = z + \hat{b}$, where $\hat{b}$ is a feasible financial policy of the firm such that $\hat{b} \Rightarrow Y|_{\hat{b}}$. Here, take production set $Y|_{\hat{b}}$ as given. This production set is available to the firm in period two. The consumer's budget set is defined by

$$B_\xi = \left\{ (x; \xi, y) \in \mathbb{R}^{l(S+1)}_{++} \times \mathbb{R} \times \mathbb{R}^{lS} : \begin{array}{l} p(0) \cdot x(0) = p(0) \cdot \omega(0) - q\xi - p(0) \cdot \bar{k}(0) \\ p(s) \cdot x(s) = p(s) \cdot \omega(1) + R(\bar{y}, s)\xi + p(s) \cdot y(0) \end{array} \right\}. \tag{1}$$

The agent 's control problem is to choose $(x; \xi, y)$ such that utility of consumption of goods is maximized. By reduced form, we mean a model where financial policies are not explicitly modeled and decisions of the agents not fully separated. We formally introduce the reduced form model below.

Definition 1 *A reduced form equilibrium $(\bar{p}, \bar{q})$ with associated equilibrium allocations $\left(\bar{x}; \bar{\xi}, \bar{y}\right)$ for generic initial resources $\omega \in \Omega$ satisfies:*

$$\begin{array}{ll} (i) & \left(\bar{x}; \bar{\xi}, \bar{y}\right) \arg\max \{u(x) : x \in B_\xi\} \\ (ii) & \bar{\xi} = 0 \\ & \bar{x}(0) = \omega(0) + \bar{k}(0) \\ & \bar{x}(s) = \omega(1) + \bar{y}(s) \quad \textit{for all } s \in \{1, ..., S\}. \end{array} \tag{2}$$

Theorem (1) although derived within a simple economic framework is non-trivial. At first sight it seems to reproduce the result of the existing literature on production. This is commented on in remarks (1), and (2) below. However, remark (3) enables to interpret this result as not only separating the activities of the agent, but also separating the objective function of the firm from the utility of the owner of the firm. The result suggests that the classical GEI model of production is a special case of the model introduced in this paper.

Theorem 1 *$(\bar{p}, \bar{q})$ is a reduced form equilibrium with associated equilibrium allocations $\left(\bar{x}; \bar{\xi}, \bar{y}\right)$ for generic initial resources $\omega \in \Omega$ separating activities of the agent as a consumer and as a producer if assign the gradient vector $\bar{\beta}$ to the firm and following conditions are satisfied:*

$$\begin{array}{ll} (i) & \left(\bar{x}; \bar{\xi}\right) \arg\max \{u(x) : x \in B_\xi\} \\ (ii) & (\bar{y}) \arg\max \left\{\bar{\beta}\bar{p}(s) \cdot y(s) : y \in B_\xi\right\} \\ (iii) & \bar{\xi} = 0 \\ & \bar{x}(0) = \omega(0) + \bar{k}(0) \\ & \bar{x}(s) = \omega(1) + \bar{y}(s) \qquad \textit{for all } s \in \{1, ..., S\}. \end{array} \tag{3}$$

Remark 1 *Note that this separation result is still dependent on the present value vector of the consumer. Consequently, the objective function of the firm is not decentralized yet, only the activities of the agent as a consumer and as a producer.*

Remark 2 *If one is willing to accept that the consumer assigns his own present value vector to the firm to evaluate future income streams, then this model reproduces the result from the literature (One can alternatively think of this model as an entrepreneurship model).*

Remark 3 *However, if one is willing to think that the single agent is perfectly able to separate his activity as a consumer and as a producer, then this model allows him as a producer not to attach the present value of the consumer to the objective function of the firm, since as a producer, he is not exposed to the no-arbitrage condition. This follows from the different role financial assets and intermediate goods play.*

Remark three becomes even more natural once considering economic scenarios beyond the single agent model.

2.1 Decentralizing the objective function (by assumption of long run profit maximization)

Assume that the producer maximizes long run profits. This implies that at $t = 0$, ξ implicitly finances the production set available to a firm in $t = 1$. Denote the production set available to the firm $Y|_{\hat{b}}$ and assume that it exists. Short run profit maximization then implies that the producer chooses inputs of production, given production capacity at fixed financial policy, such that production of outputs maximizes his profits. The financing of production inputs in $t = 1$ is defined by the sell of production outputs. Long run profit maximization implies that the fixed financial policy of the firm maximizes profits over both periods. The reduced form objective of long run maximization of profits is then to

$$(\bar{y}) \arg\max \left\{ \sum\nolimits_{s=1}^{S} \bar{p}(s) \cdot y(s) : y(s) \in Y|_{\hat{b}}(s), \forall s \in S \right\}. \tag{4}$$

Proposition 1 $(\bar{p}, \bar{q})$ *is a reduced form long run profit maximizing equilibrium with decentralized objective function of the firm and with associated equilibrium allocations* $\left[\left(\bar{x}, \bar{\xi}\right), (\bar{y})\right]$ *for generic initial resources* $\omega \in \Omega$*, if following conditions are satisfied:*

$$\begin{array}{ll} \textit{(i)} & \left(\bar{x}; \bar{\xi}\right) \arg\max \left\{u(x) : x \in B_{\xi}\right\} \\ \textit{(ii)} & (\bar{y}) \arg\max \left\{ \sum_{s=1}^{S} \bar{p}(s) \cdot y(s) : y(s) \in Y|_{\hat{b}}(s), \forall s \in S \right\} \\ \textit{(iii)} & \bar{\xi} = 0 \\ & \bar{x}(0) = \omega(0) + \bar{k}(0) \\ & \bar{x}(s) = \omega(1) + \bar{y}(s) \textit{ for all } s \in \{1, ..., S\}. \end{array}$$

This result follows from remark (3). It shows the independence of the objective function from the present value vector of the agent as a consumer. The result is a consequence of the endogenous asset structure of the model, where the firm builds up production capacity by issuing stocks. This is, although derived in its simplest form, an interesting result. It generalizes, by means of a simple example, the decentralization property of the Arrow-Debreu model to the case of incomplete markets.

2.2 Productive efficiency of the reduced form model with decentralized objective function

The next result shows that the organization of production is generally efficient. This result improves on the existing literature, where the organization of production is generally inefficient. The result follows from the endogenized asset structure for which we showed the independence of the objective function from the utility of the stockholders.

Proposition 2 $(\bar{p}, \bar{q})$ *is a reduced form equilibrium with efficient organization of production and decentralized objective function of the firm (long run profits maximization) with associated equilibrium allocations* $[(\bar{x}, \bar{\xi}), (\bar{y})]$ *for generic initial resources* $\omega \in \Omega$.

The proof of proposition (2) makes use of the fact that the objective function of the firm is independent of any assigned present value vectors of the consumer to it. The problem of the firm in the reduced form model is therefore essentially equivalent to the problem of the firm in the Arrow Debreu model. Hence, the organization of production is efficient. This result also implicitly states that any utility maximizing model of the firm in GEI introduces a further source of inefficiency.

Remark 4 *This model has similar (in)efficiency properties of equilibrium as the classical GEI exchange model. The point here is that at variance with the classical GEI model of production the organization of production does not introduce a further source of inefficiency by attaching some present value* β *to the objective function of the firm.*

Proposition 3 $(\bar{p}, \bar{q})$ *is a reduced form centralized financial markets equilibrium with equilibrium allocations* $[(\bar{x}, \bar{\xi}), (\bar{y})]$ *with inefficient organization of production of the firm for generic initial resources* $\omega \in \Omega$*, if and only if*

$$\bar{\beta}(s) \neq \overrightarrow{e} \text{ for every } s \in \{1, ..., S\} \tag{5}$$

is satisfied.

The condition $\bar{\beta}(s) \neq \overrightarrow{e}$ for every $s \in \{1, ..., S\}$ is generally satisfied for centralized general equilibrium models with incomplete markets. This follows from the no-arbitrage condition. $\overrightarrow{e}$ is a unit vector of appropriate dimension.

Alternatively, consider a two agent model, and assign different average present value vectors to the objective function of the firm. Thus, real equilibrium distortions due to the dependency of the objective function of an average present value vector.

2.3 The extensive form model

We now introduce the extensive form model, where decisions are fully decentralized and financial policies explicitly modeled. Consider the consumer's constraints

$$\begin{aligned} p(0) \cdot x(0) &= p(0) \cdot \omega(0) - qz \\ p(s) \cdot x(s) &= p(s) \cdot \omega(1) + \theta(\bar{z}) R(\bar{y}, s), \end{aligned}$$

where qz is the value the consumer is willing to invest into the firm at expected return R. As a producer, the agent issues stocks b satisfying $qb = qz$, buys capital $k(0)$ such that income from selling stocks is equal to his expenditure on capital consumption, therefore, $qb = p(0) \cdot k(0)$. At $t = 0$, the producer's long run problem is to

$$\left(\bar{k}(0);\bar{b}\right)\arg\max\left\{\bar{q}b:\bar{q}\bar{z}=\bar{q}b=\bar{p}(0)\cdot k(0)\right\}, \tag{6}$$

where the level of capital, $\overline{k}(0)$, implies total production capacity available to the firm, a correspondence $\Phi|_{\hat{b}}$. This correspondence in turn determines the production set available to the firm, denoted $Y|_{\hat{b}}$. Given this production set, and the set of states of nature, the producer's $t = 1$ short run problem is to

$$(\bar{y}(s))\arg\max\left\{\bar{p}(s)\cdot y(s):y(s)\in Y|_{\hat{b}}(s),\forall s\in S\right\}. \tag{7}$$

Inputs of production are financed with sells from outputs. The level of revenue a firm can generate in each state $s \in \{1, ..., S\}$ depends on the available production set determined in the certain state of the world.

Theorem (2) shows the independence of the objective function from any present value vector derived from the owners of the firm for the extensive form model. It also also establishes, through the objective function of the firm, a link between the real and financial sector. This result improves on the GEI literature on production which considers the production and financial sets to be dichotomic sets.

Theorem 2 *$(\bar{p},\bar{q})$ is a decentralized objective function extensive form equilibrium with associated equilibrium allocations $((\bar{x},\bar{z}),(\bar{y},\bar{b}))$ for generic initial resources $\omega\in\Omega$, if for any feasible $\hat{b}\leqslant\bar{z}$ following conditions are satisfied:*

$$\begin{array}{ll} (i) & (\bar{x};\bar{z})\arg\max\{u(x):x\in B_z\} \\ (ii) & \underset{\left(\bar{y},\hat{b};(\bar{k}(0))\right)}{\arg\max}\left\{\bar{q}b+\sum_{s=1}^{S}\bar{p}(s)\cdot y(s):\begin{array}{ll}\bar{q}\bar{z}\geq\bar{q}b=\bar{p}(0)\cdot k(0) & \\ y(s)\in Y|_{\hat{b}}(s) & s\in S\end{array}\right\} \\ (iii) & \bar{z}+\hat{b}=0 \quad \theta(\bar{z})=1 \\ & \bar{x}(0)=\omega(0)+\bar{k}(0) \\ & \bar{x}(s)=\omega(1)+\bar{y}(s) \text{ for all } s\in\{1,..,S\}. \end{array} \tag{8}$$

Next result is a first step towards a study of the Modigliani and Miller theorem for the class of endogenous asset formation models. It shows that, as a consequence of the independency of the production set on the financial activities of the firm, financial policies have real effects. This result follows from the way financial assets and intermediate goods enter the model. In particular, the objective function of the firm links the real with the financial sphere. In the classical GEI model, firms issue stocks in order to finance production inputs in period one, here, firms issue stocks in order to buy capital and to build up their production set. Hence, real effects. Note that this result is established without considering other financial assets. The idea of the proof is to improve on the implicit assumption that financial policy is independent of the production set of the firm at first instance. More work needs to be done in order to proof the full version of the Modigliani and Miller theorem.

Proposition 4 *(i) If $((\bar{p},\bar{q}),(\bar{x},\bar{z}),(\bar{y},\hat{b}))$ is an extensive form equilibrium (EFE) with decentralized objective function for generic initial resources $\omega\in\Omega$ then $((\bar{p},\bar{q}),(\bar{x},\bar{\xi}),(\bar{y}))$ is a reduced form equilibrium (RFE) with decentralized objective function for generic initial resources $\omega\in\Omega$ where*

$$\bar{\xi}=\bar{z}+\hat{b} \tag{9}$$

(ii) If $((\bar{p},\bar{q}),(\bar{x},\bar{\xi}),(\bar{y}))$ is a (RFE) with decentralized objective function for generic initial resources ω then $((\bar{p},\bar{q}),(\bar{x},\bar{z}),(\bar{y},\hat{b}))$ is a (RFE) with decentralized objective function for generic initial resources $\omega \in \Omega$ for any $\hat{b} \leq \bar{z}$ satisfying

$$\bar{z} + \hat{b} = \bar{\xi}. \tag{10}$$

3 Conclusion

This paper considers the leading case of an endogenous asset formation model. In this model intermediate goods and financial assets play an essential role in determining the firms production set. Adding this structure to the economic model has non-trivial implications for the theory of the firm in incomplete markets. (i) It allows to expand the profit maximization criterion of the Arrow-Debreu model to a model with incomplete income transfer space. This is at variance with the literature, where the objective function of the firm is not independent of the utilities of the stock holders. (ii) As a consequence of the sequential structure of the objective function of the firm introduced, the organization of productive activity is efficient. This is welfare improving since the redistribution of endowments is more efficient relative to utility dependent production models. (iii) Since the efficient boundary of the production set is not independent of the financial activities of the firm, it follows that financial policies are determinate. The failure of the Modigliani and Miller theorem follows from the improvement on their implicit assumption that the production set available to a firm is exogenous. Hence, independent of the financial activities of the firm.

Future research shall generalize the results derived in this paper. For example, a more general version of the Modigliani and Miller theorem should be considered. This involves including bonds and other financial assets. Then, it would be natural to consider the possibility of default. This is research in progress.

A Appendix: Mathematical Proofs

Proof 1 (Theorem 1) *Forming the Lagrangean*

$$\begin{aligned} L = \quad u(x) \quad & -\lambda(0)\left[\bar{p}(0)\cdot x(0) - \bar{p}(0)\cdot\omega(0) + \bar{q}\xi + \bar{p}(0)\cdot\bar{k}(0)\right] \\ & - \textstyle\sum_{s=1}^{S}\lambda(s)\left[\bar{p}(s)\cdot x(1) - \bar{p}(s)\cdot\omega(s) + R(\bar{y},s)\xi + \bar{p}(s)\cdot y(s)\right] \\ & - \textstyle\sum_{s=1}^{S}\mu(s)\left[\Phi|_{\hat{b}}\left(\bar{y}(s)\right)\right] \end{aligned}$$

the first order conditions are necessary and sufficient for $(x;\xi,y)$ to be a solution of equilibrium definition (1) if there exists $\lambda \in \mathbb{R}^{S+1}_{++}$ and $\mu \in \mathbb{R}^{S}_{++}$ such that

$$\nabla L\left(\bar{x},\bar{\xi},\bar{y},\bar{\lambda},\bar{\mu}\right) \equiv 0$$

This is equivalent to

$$\nabla u(\bar{x}) = \bar{\lambda}\bar{p} \tag{11}$$

$$\bar{q} = \frac{\sum_{s=1}^{S}\bar{\lambda}(s)}{\bar{\lambda}(0)}R(\bar{y},s) \tag{12}$$

$$\sum\nolimits_{s=1}^{S}\bar{\mu}(s)\nabla\left.\Phi\right|_{Z}\left(\bar{y}(s)\right) = \sum\nolimits_{s=1}^{S}\bar{\lambda}(s)\bar{p}(s) \tag{13}$$

$$\bar{p}\cdot\bar{x} = \bar{p}\cdot\omega + \Pi(\bar{y},\bar{p})\bar{\xi} \tag{14}$$

$$\Phi|_{\hat{b}}\left(\bar{y}(s) = 0\right. \tag{15}$$

Let $\bar{\beta}(s) = \left(\frac{\sum_{s=1}^{S} \bar{\lambda}(s)}{\bar{\lambda}(0)} \right)$, *then* $\bar{q} = \sum_{s=1}^{S} \bar{\beta}(s) R(\bar{y}, s)$ *and from* (11) *and* (13) *have*

$$\sum_{s=1}^{S} \bar{\lambda}(s)\bar{p}(s) = \nabla u(\bar{x}(s)) = \sum_{s=1}^{S} \bar{\mu}(s) \nabla \left. \Phi \right|_{\hat{b}} (\bar{y}(s))$$

using $\bar{\beta}$

$$\sum_{s=1}^{S} \bar{\beta}(s)\bar{p}(s) = \frac{1}{\bar{\lambda}(0)} \nabla u(\bar{x}(s)) = \sum_{s=1}^{S} \frac{\bar{\mu}(s)}{\bar{\lambda}(0)} \nabla \left. \Phi \right|_{Z} (\bar{y}(s)) \tag{16}$$

The first part of the proof shows that equilibrium definition (1) has a solution, and therefore, that (i) of (1) theorem has a solution. It remains to show that part (ii) of theorem (1) has a solution. Now, if assign $\bar{\beta}(s)$ *for each* $s \in S$ *to the optimization problem of the producer, and the producer takes* $\bar{\beta}$ *as given then*

$$(\bar{y}) \arg\max \left\{ \sum_{s=1}^{S} \bar{\beta}(s)\bar{p}(s) \cdot y(s) : y(s) \in \left. Y \right|_{\hat{b}} \right\}. \tag{17}$$

This problem has a solution if there exists $\nu \in \mathbb{R}^{lS}_{++}$ *such that*

$$L(\bar{y}) \equiv 0 \tag{18}$$

This is equivalent to

$$\sum_{s=1}^{S} \bar{\beta}(s)\bar{p}(s) = \sum_{s=1}^{S} \bar{\nu}(s) \nabla \left. \Phi \right|_{\hat{b}} (\bar{y}(s)) \tag{19}$$

The separation result follows from (16) *and* (19).

$$\sum_{s=1}^{S} \frac{\bar{\mu}(s)}{\bar{\lambda}(0)} \nabla \left. \Phi \right|_{\hat{b}} (\bar{y}(s)) = \sum_{s=1}^{S} \bar{\nu}(s) \nabla \left. \Phi \right|_{\hat{b}} (\bar{y}(s)). \tag{20}$$

Proof 2 (Proposition 1) *The result follows from the first order conditions*

$$\sum_{s=1}^{S} \bar{\beta}(s)\bar{p}(s) = \frac{1}{\bar{\lambda}(0)} \nabla u(\bar{x}(s))$$
$$\bar{q} = \sum_{s=1}^{S} \bar{\beta}(s) R(\bar{y}, s)$$

and from

$$(\bar{y}) \arg\max \left\{ \sum_{s=1}^{S} \bar{p}(s) \cdot y(s) : y(s) \in \left. Y \right|_{\hat{b}} \right\}$$

This problem has a solution if there exists $\nu \in \mathbb{R}^{lS}_{++}$ *such that*

$$L(\bar{y}) \equiv 0$$

This is equivalent to

$$\sum_{s=1}^{S} \bar{p}(s) = \sum_{s=1}^{S} \bar{\nu}(s) \nabla \left. \Phi \right|_{\hat{b}} (\bar{y}(s)) \tag{21}$$

Equ. (21) *is independent of the present value vector* $\bar{\beta}$ *of the consumer. This decentralizes the objective function of the firm.*

Proof 3 (Proposition 2) *Consider a reduced form incomplete financial markets equilibrium with decentralized profit maximizing objective function* $(\bar{p}, \bar{q})$ *with associated equilibrium allocations* $(\bar{x}, \bar{\xi}), (\bar{y})$ *for an economy* $\omega \in \Omega$. *Let* (x, y) *not be a constraint productive efficient allocation at price system* $\bar{p}$ *and* $\bar{q}$, *and period one financial trade* $\bar{\xi} = \bar{z} + \hat{b}$ *for implicit feasible* $\hat{b}$. *Then, because* (x, y) *at* $\bar{\xi}$ *is a feasible competitive financial markets equilibrium with production allocation at* $t = 1$, *it satisfies*

$$\begin{aligned} \bar{x}(0) &= \omega(0) + \bar{k}(0) \\ \bar{x}(s) &= \omega(1) + \bar{y}(s) \\ \bar{\xi} &= 0, \end{aligned} \tag{22}$$

Because (x, y) *is not efficient optimal, in the sense that period two allocations are not optimal, given financial constraint* $\bar{\xi}$ *implying production capacity* $\bar{k}(0)$ *and technology* $\Phi|_{\hat{b}}$, *which in turn implies the constraint production set available to the firm in* $t = 1$, *denoted* $Y|_{\hat{b}}$, *there must exist and alternative feasible allocation* $(\hat{x}, \hat{y})$ *within the constraint production set available to the producer* $Y|_{\hat{b}}$ *such that*

$$u(\hat{x}(0), \hat{x}(s); \bar{\xi}) > u(x(0), x(s); \bar{\xi}), \text{ for all } s \in \{1, .., S\} \tag{23}$$

We have that

$$\begin{aligned} &\bar{p}(0) \cdot \hat{x}(0) \geqslant \bar{p}(0) \cdot x(0) \\ &\bar{p}(s) \cdot \hat{x}(s) > \bar{p}(s) \cdot x(s), \text{ for all } s \in \{1, .., S\} \end{aligned} \tag{24}$$

This implies that

$$\begin{aligned} &\bar{p}(0) \cdot \hat{x}(0) \geqslant \bar{p}(0) \cdot x(0) \\ &\bar{p}(s) \cdot \hat{x}(s) > \bar{p}(s) \cdot x(s), \text{ for all } s \in \{1, .., S\} \end{aligned}$$

We then have for feasible $(\hat{x}, \hat{y})$ *that*

$$\begin{gathered} \bar{p}(0) \cdot \omega(0) + \bar{p}(0) \cdot \bar{k}(0) \geqslant \bar{p}(0) \cdot \omega(0) + \bar{p}(0) \cdot \bar{k}(0) \\ \bar{p}(s) \cdot (\hat{y}(s) + \omega(1)) > \bar{p}(s) \cdot (y(s) + \omega(1)), \end{gathered}$$

for all $s \in \{1, .., S\}$ *so that period two long run profits*

$$\bar{p}(s) \cdot \hat{y}(s) > \bar{p}(s) \cdot y(s). \tag{25}$$

However, this implies that $\bar{p} \cdot \hat{y} > \bar{p} \cdot y$, *where* $\hat{y} \in Y|_{\hat{b}}$ *at equilibrium* $\bar{\xi}$. *This is a contradiction to the fact that* $y_j \in Y|_{\hat{b}}$ *is profit maximizing at price system* $\bar{p}$ *and* $\bar{\xi}$.

Proof 4 (Proposition 3) *To see the inefficient organization of production of the first model (similar to the literature). Assign present value vector* $\bar{\beta}$ *to the firm, then for financial constraint* $\bar{\xi}$ *let the firm maximize its present value profits*

$$(\bar{y}) \arg\max \left\{ \sum_{s=1}^{S} \bar{\beta}(s) \bar{p}(s) \cdot y(s) : y(s) \in Y|_{\hat{b}} \right\} \tag{26}$$

Then (26) *is equal to*

$$(\bar{y}) \arg\max \left\{ \sum_{s=1}^{S} \bar{p}(s) \cdot y(s) : y(s) \in Y|_{\hat{b}} \right\} \tag{27}$$

if and only if

$$\bar{\beta}(s) = e \text{ for every } s \in S \in \{1, .., S\} \tag{28}$$

where e is an unit vector. This condition is generally not satisfied for a no-arbitrage equilibrium when $S > n$. For any $\bar{\beta}(s)$

$$\vec{0} < \bar{\beta}(s) < e \tag{29}$$

the centralized model of production is less efficient than the fully decentralized objective function model since

$$\max_{\bar{y}} \Pi|_{\beta} (\bar{\beta}\bar{p}) < \max_{\bar{y}} \Pi(\bar{p}) \tag{30}$$

$y(s)|_{\bar{\beta}} \neq y(s) \in Y|_{\hat{b}}$.

Proof 5 (Theorem 2) *The result follows from the first order conditions*

$$\sum\nolimits_{s=1}^{S} \bar{\beta}(s)p(s) = \frac{1}{\bar{\lambda}(0)} \nabla u(\bar{x}(s))$$
$$\bar{q} = \sum\nolimits_{s=1}^{S} \bar{\beta}(s)R(\bar{y}, s)$$

and from

$$(\bar{y}) \arg\max \left\{ \sum\nolimits_{s=1}^{S} \bar{p}(s) \cdot y(s) : y(s) \in Y|_{\hat{b}} \right\}$$

equivalent to

$$\arg\max_{\left(\bar{y}, \hat{b}; (\bar{k}(0))\right)} \left\{ \bar{q}b + \sum\nolimits_{s=1}^{S} \bar{p}(s) \cdot y(s) : \begin{array}{l} \bar{q}\bar{z} \geq \bar{q}b = \bar{p}(0) \cdot k(0) \\ y(s) \in Y|_{Z} \end{array} \quad s \in S \right\} \tag{31}$$

for feasible $\hat{b}$. This problem has a solution for any feasible $\hat{b}$ equivalent to the solution in the (RFE), where there exists $\nu \in \mathbb{R}^{lS}_{++}$ such that

$$L(y) \equiv 0$$

This is equivalent to

$$\sum\nolimits_{s=1}^{S} \bar{p}(s) = \sum\nolimits_{s=1}^{S} \bar{\nu}(s) \nabla \left. \Phi \right|_{\hat{b}} (\bar{y}(s))$$

Equ. (31) *is independent of the present value $\beta(s)$ of the consumer for any feasible $\hat{b} \leq \bar{z}$. This decentralizes the objective function of the firm.*

Proof 6 (Proposition 4) *Part (i). Let us first show that $((\bar{x}, \bar{\xi}), (\bar{y}))$ satisfy the first order conditions (1) $\tau \in \langle \Pi(\bar{y}, \bar{p}) \rangle$, $\beta \in \langle \Pi(\bar{y}, \bar{p}) \rangle^{\perp}$, and (2) $y \in Y|_{\hat{b}}$, $p \in N_{Y|_Z}(y)$ so that conditions (i) and (ii) in (RFE) are satisfied, where $\tau = \Pi(\bar{y}, \bar{p})z$ is an income vector. The FOC's for the consumer's problem in (EFE) are*

$$p \cdot x = p \cdot \omega + \Pi(\bar{y}, \bar{p}) \begin{bmatrix} \bar{z} \\ \theta(\bar{z}) \end{bmatrix}, \text{ and } \beta \Pi(\bar{y}, \bar{p}) = 0 \tag{32}$$

and can be rewritten as

$$p \cdot x = p \cdot \omega + \Pi(\bar{y}, \bar{p})\xi, \text{ and } \beta\Pi(\bar{y}, \bar{p}) = 0$$

since $\bar{\xi}\big|_{\hat{b}} = \bar{z} + \hat{b}$, *so that (1) above holds for any feasible* $\hat{b} \leqslant \bar{z}$. *The firm's problem in (EFE)*

$$\underset{(\bar{y},(\hat{b};\bar{k}(0)))}{\arg\max} \left\{ \bar{q}b + \sum\nolimits_{s=1}^{S} \bar{p}(s) \cdot y(s) : \begin{array}{ll} \bar{q}\bar{z} \geq \bar{q}b = \bar{p}(0) \cdot k(0) & s \in S \\ y(s) \in Y|_{\hat{b}}\,(s) & \end{array} \right\}$$

for any feasible $\hat{b} \leqslant \bar{z}$ *reduces to*

$$\underset{(\bar{y}(s))}{\arg\max} \left\{ \sum\nolimits_{s=1}^{S} \bar{p}(s) \cdot y(s) : y(s) \in Y|_{\hat{b}}\,(s), s \in S \right\}$$

since feasible $b \Rightarrow \Phi|_{\hat{b}}\,(s) \Longrightarrow Y|_{\hat{b}}$ *for which the first order condition (2) which is equivalent to* $\nu(s)\nabla\, \Phi|_{\hat{b}} = p(s)$ *above holds. The result follows ,since market clearing condition* $\bar{\xi}\big|_{\hat{b}} = \bar{z} + \hat{b} = 0$, *and* $\bar{x}(0) = \omega(0) + \bar{k}(0)$, $\bar{x}(s) = \omega(1) + \bar{y}(s)$ *for all* $s \in \{1, .., S\}$ *hold.*
Part (ii). If $((\bar{x}, \bar{\xi}), (\bar{y}))$ *is an (RFE) for implicit* $\hat{b}$, *then the first order conditions are satisfied. This implies that for any feasible* $\hat{b} \leq z$

$$\underset{(\bar{y}(s))}{\arg\max} \left\{ \sum\nolimits_{s=1}^{S} \bar{p}(s) \cdot y(s) : y(s) \in Y|_{\hat{b}}\,(s), s \in S \right\} \tag{33}$$

expands to

$$\underset{(\bar{y},(\hat{b};\bar{k}(0)))}{\arg\max} \left\{ \bar{q}b + \sum\nolimits_{s=1}^{S} \bar{p}(s) \cdot y(s) : \begin{array}{ll} \bar{q}\bar{z} \geq \bar{q}b = \bar{p}(0) \cdot k(0) & s \in S \\ y(s) \in Y|_{\hat{b}}\,(s) & \end{array} \right\} \tag{34}$$

for which the first order conditions are satisfied, hence $\bar{y}$ *is a solution of (ii) in (EFE) for feasible* $\hat{b}$. *Pick any feasible* $\hat{b}$ *and define*

$$z + \hat{b} = \bar{\xi} \tag{35}$$

such that $\Pi z + \Pi\hat{b} = \Pi\bar{\xi}$ *becomes* $\Pi(z + \hat{b}) = \Pi\bar{\xi}$, *then first order conditions for the consumer of the (EFE)* (32) *is satisfied for* $(\bar{x}, z)$. $(\bar{x}, z)$ *is a solution of (EFE)_(i) and* $(\bar{y}, \hat{b})$ *is a solution of (EFE) (ii). The result follows from* $0 = \xi = z + \hat{b}$ *and* $x\bar{(}0) = \omega(0) + \bar{k}(0)$, $\bar{x}(s) = \omega(1) + \bar{y}(s)$ *for all* $s \in \{1, .., S\}$.

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