THE UNIVERSITY *of York*

Discussion Papers in Economics

No. 2007/11

The Asymmetric Effect of the Business Cycle on the Equity Premium

By

Peter N Smith, Steffen Sorensen and Mike Wickens

Department of Economics and Related Studies
University of York
Heslington
York, YO10 5DD

The asymmetric effect of the business cycle on the equity premium

P.N. Smith, S. Sorensen and M.R. Wickens[1]

University of York

May 2007

Abstract

We examine the relation between US stock market returns and the US business cycle for the period 1960 - 2003 using a new methodology that allows us to estimate a time-varying equity premium. We identify two channels in the transmission mechanism. One is through the mean of stock returns via the equity risk premium, and the other is through the volatility of returns. We provide support for previous findings based on simple correlation analysis that the relation is asymmetric with downturns in the business cycle having a greater negative impact on stock returns than the positive effect of upturns. We also obtain a new result, that demand and supply shocks affect stock returns differently. We find that negative supply shocks are a very important source of increases in the risk premium. Our model of the relation between returns and their volatility encompasses the CAPM and the results demonstrate the importance of allowing for a time-varying price of volatility risk. The model is implemented using a multi-variate GARCH-in-mean model with an asymmetric time-varying conditional heteroskedasticity and correlation structure.

---

[1] University of York, University of York and University of York and CEPR, respectively.
The authors would like to thank Dick van Dijk, Athanasios Orphanides, Christoph Schleicher and Enrique Sentana for helpful comments.

JEL Classification: G12, C32, C51, E44
Keywords: Equity Returns, Risk Premium, Asymmetry.

1 Introduction

The key to understanding how an asset is priced is the relation between its return and its volatility. This relation lies at the centre of most modern theories of asset pricing and much of the associated empirical work. Intuitively, the larger the uncertainty about the future price of an asset, which increases with its volatility, the greater is the required return to compensate for risk. The problem is to specify exactly what the relation between the return and its volatility is.

Both ad hoc and formal models have been used in the literature. They may be linear or non-linear. The model may seek to explain the asset's return, or its excess return per unit of volatility - the Sharpe ratio. Since it is future (or conditional) volatility that is relevant, a volatility forecasting model is required. This may take the form of predicting future volatility from its past, or by the use of additional, possibly macroeconomic, variables. This has become a common way to link asset-price movements with the macro-economy.

In this paper we use a model with stochastic discount factors (SDFs) that encompasses most of the empirical models used in the literature, including CAPM which is the most widely-used approach. The advantage of this approach is that it enables us to examine the effect of the business cycle on the stock market within a no-arbitrage framework. Our econometric model builds on Kroner and Ng (1998)'s multivariate GARCH model by extending it to include in the equation for equity returns not just "in-mean" conditional volatility but, in addition, "in-mean" conditional covariances (to capture risk premia) and asymmetries to reflect the impact of the business cycle. In order to include these effects, a further extension is required, namely, we must consider the joint distribution of stock returns and the macroeconomic sources of risk. The "in-mean" effects are excluded from the equations for the macroeconomic variables, but for the equity premium to be time-varying, the covariance matrix of the disturbances to the macroeconomic variables is assumed to display conditional heteroskedasticity.

In this way, we obtain new evidence on an old result, and a new result. We show that there are two channels by which macroeconomic shocks affect stock returns: one is their effect on the mean via the equity risk premium, the other is through the volatility of returns. We find that these effects are asymmetric, with downturns in the business cycle having a larger negative impact on stock returns than the positive effect of upturns. This is consistent with previous results based on simple correlation analysis, but puts the whole analysis within a formal no-arbitrage framework.

The new result follows from an identification scheme for the macroeconomic shocks into demand and supply shocks. We find that demand shocks have a different effect on stock returns and the equity risk premium than supply shocks. In a single factor model, we find that nominal returns are *negatively* related to the conditional covariance between inflation and nominal returns. But in a multi-factor model that also includes output, the conditional covariance between inflation and nominal returns has a *positive* effect on nominal returns. The reason for the switch in sign is that the conditional covariance between inflation and output is predominantly negative. In other words, positive inflation shocks are associated with negative output shocks. This has an interesting interpretation. Whereas a positive demand shock tends to increase both inflation and output, a negative supply shock tends to increase inflation, but reduce output. This intuition carries over to formal identification of aggregate demand and supply shocks. We find that negative aggregate supply shocks are an important source of increases in the risk premium in recessions. Aggregate demand shocks, by contrast, appear to be much less important for risk premium variation.

The paper is set out as follows. In Section 2 we review the literature on the relation between stock market returns and volatility. In Section 3 we discuss alternative models of the risk premium to CAPM that may explain the impact of the business cycle on stock returns. In Section 4 we consider econometric issues, including how to model macroeconomic effects and asymmetries in the volatility structure in a way that satisfies the condition of no arbitrage. Our results, based on monthly data for the US stock market, are reported in Section 5. We present our identification of the structural aggregate supply and demand shocks in Section 6. We discuss their impact on the risk premium in that section. The relationship between our results and the CAPM is discussed in Section 7. Our conclusions are presented in Section 8.

2 Stock market returns and volatility

Many papers have examined the effect of stock market volatility of stock returns, most notably, French, Schwert and Stambaugh (1987), Campbell (1987), Harvey (1989), Turner, Startz and Nelson (1989), Baillie and DeGennero (1990) and Glosten, Jagannathan and Runkle (1993). The theoretical basis of these studies is the capital asset pricing model (CAPM) of Sharpe (1964) and Lintner (1965). This can be written as a simple linear relation between the conditional mean and the conditional variance of equity market returns:

$$E_t(R^M_{t+1} - R^f_{t+1}) = \alpha + \beta V_t(R^M_{t+1}) \tag{1}$$

where R^M_{t+1} is the real return on the market and R^f_t is the real return on a risk-free asset. Under CAPM, $\alpha = 0$ and β is the coefficient of relative risk aversion, which may be time-varying. A survey of the results for the US stock market obtained in these studies is provided by Scruggs (1998). He reports that the estimates of β have varied from significantly positive to significantly negative, depending on the measure of market returns, the model of the conditional variance and the method of estimation used.

Broadly, the research into this relation has followed two routes. One involves using increasingly general ways to model conditional volatility. The other employs a more general model of asset pricing than CAPM. Glosten, Jagannathan and Runkle (1993), confirmed by Scruggs (1998), show that an EGARCH-based implementation of equation (1) allowing for asymmetry can produce estimates of β which are negative. Scruggs (1998) then goes on propose a joint model of stock and bond returns where the the estimated partial relation between the stock market return and it's conditional variance is positive and significant. However, Scruggs and Glabadanis (2003) find that this result is not robust. A variant is to include additional variables in the conditional volatility process such as the nominal interest rate. Scruggs (1998) shows that this may change the sign of the estimate of β.

A number of explanations for the asymmetry in the volatility of equity returns have been proposed. Examples are the leverage and volatility-feedback hypotheses of Black (1976) and Campbell and Hentschel (1992)[2] and incompleteness of the information set proposed by Lettau and Ludvigson (2006). None of these papers consider the possibility that it is the absence of a strongly time-varying risk premium that may explain the observed asymmetry. This explanation is central to our results.

3 The equity risk premium and the business cycle

In this paper we adopt a new approach based on taking into account the presence of a time-varying equity premium. We examine whether an explanation both for the different results concerning the relation between the mean return and the volatility of returns, and for the finding of asymmetries in the conditional variance, is that a more general theory of asset pricing than CAPM is required. The theory we propose admits the influence of the business cycle on returns, and allows this to be asymmetric, whilst satisfying the condition of no-arbitrage.

In Smith and Wickens (2002) we review various alternative empirical asset-pricing models to CAPM. In other papers we have applied this methodology to the stock market (Smith, Sorensen and Wickens (2005)), to the term structure of interest rates (Balfoussia and Wickens (2004)) and to the FOREX market (Smith, Sorensen and Wickens (2006)). We now explain this approach and how it can be modified to incorporate business cycle effects and asymmetries in the response of stock returns to shocks arising both from the stock market and from macroeconomic variables.

[2] Bekaert and Wu (2000), Campbell, Lo and MacKinlay (1997) and Schwert (1989) provide evidence against the leverage hypothesis.

3.1 Modelling returns and volatility using stochastic discount factors

The inability to hedge against much of the risk arising from the business cycle implies that this risk will be priced in stock market returns, see Shiller (1993). Further, as long horizon returns are partly forecastable, the equity risk premium must be time-varying. And since risk premia arise from conditional variation between returns and economic factors, this suggests that we should study the effects of the business cycle on stock market returns and volatility. It also implies that we should focus on modelling the risk premium. Our model of the relation between returns and volatility is based on the use of stochastic discount factors

The stochastic discount factor M_t satisfies

$$1 = E_t[M_{t+1}(1 + R_{t+1})] \tag{2}$$

where R_{t+1} is the real asset return. If $r_t = \ln(1 + R_t)$ and $m_t = \ln M_t$ are jointly normally distributed then it can be shown that the expected excess return is

$$E_t(r_{t+1} - r_t^f) + \frac{1}{2}V_t(r_{t+1} - r_t^f) = -Cov_t(m_{t+1}, r_{t+1} - r_t^f) \tag{3}$$

where $r_t^f = \ln(1 + R_t^f)$ is assumed to be known at time t. The conditional volatility term on the left-hand side is the Jensen effect, and the term on the right hand-side is the risk premium which must satisfy $Cov_t(m_{t+1}, r_{t+1}) < 0$ for the risk premium to be positive, see Cochrane (2005). It is common to represent m_t as a linear function of n factors z_{it}

$$m_t = -\textstyle\sum_{i=1}^{n} \beta_i z_{it}$$

implying that the SDF pricing equation is[3]

$$E_t(r_{t+1} - r_t^f) + \frac{1}{2}V_t(r_{t+1}) = \textstyle\sum_{i=1}^{n} \beta_i Cov_t(z_{i,t+1}, r_{t+1}) \tag{4}$$

We refer to this as the SDF model. Most asset pricing models can be shown to be special cases of this model. They differ mainly due to the choice of factors and the restrictions imposed on the coefficients.

The SDF model may be written in a number of different ways. For example, equation (3) can be re-written as

$$\begin{aligned} E_t(r_{t+1} - r_t^f) + \frac{1}{2}V_t(r_{t+1} - r_t^f) &= -SD_t(r_{t+1} - r_t^f)SD_t(m_{t+1})Cor_t(m_{t+1}, r_{t+1} - r_t^f) \qquad (5) \\ &= SD_t(r_{t+1}) \textstyle\sum_{i=1}^{n} \beta_i SD_t(z_{i,t+1}) Cor_t(z_{i,t+1}, r_{t+1}) \end{aligned}$$

[3] Note that as r_t^f is known at time t, $V_t(r_{t+1} - r_t^f) = V_t(r_{t+1})$ and $Cov_t(m_{t+1}, r_{t+1} - r_t^f) = Cov_t(m_{t+1}, r_{t+1})$.

where $SD_t(.)$ denotes the conditional standard deviation and $Cor_t(.)$ the conditional correlation. This is a non-linear relation between an asset's return and its volatility which is attentuated by the volatility of the factors and their conditional correlations with the asset return. The SDF model satisfies the principle of no-arbitrage. It also shows the form in which additional variables should be included in the asset pricing equation, i.e. as terms involving their conditional covariances with the asset return. We also note that the coefficients β_i are unrestricted; they can be positive or negative as long as the overall risk premium is positive.

A different way of expressing the SDF model is in terms of the excess return per unit of volatility, the Sharpe ratio:

$$\frac{E_t(r_{t+1} - r_t^f)}{SD_t(r_{t+1})} = -\frac{1}{2}SD_t(r_{t+1}) + \sum_{i=1}^{n} \beta_i SD_t(z_{i,t+1})Cor_t(z_{i,t+1}, r_{t+1}) \tag{6}$$

This is the form of model used by Lettau and Ludvigson (2006). It is clear that the Sharpe ratio will be small when macroeconomic volatility is low, the correlations between macroeconomic variables and stock returns are close to zero, or the macroeconomic variables are not significantly priced in the stock market. In the special case where the conditional correlations are constant the Sharpe ratio becomes a linear function in the conditional standard deviations. The model coefficients then measure the effect on the Sharpe ratio of a unit of volatility in the factors.

Another way of writing the SDF model brings out its connections with CAPM. This is

$$\begin{aligned} E_t(r_{t+1} - r_t^f) &= [-\frac{1}{2} + \sum_{i=1}^{n} \beta_i \gamma_{i,t}]V_t(r_{t+1}) \qquad (7) \\ \gamma_{i,t} &= \frac{Cov_t(z_{i,t+1}, r_{t+1})}{V_t(r_{t+1})} \end{aligned}$$

This shows that CAPM is a special case of the SDF model in which the coefficient on the conditional variance is constrained to be constant, rather than time varying and non-linearly dependent on the factors. Moreover, in general, this coefficient cannot be interpreted as the coefficient of relative risk aversion.

We conclude that a general representation of the market return that encompasses all of the models above is given by

$$E_t(r_{t+1}^M - r_t^f) = \beta_0 V_t(r_{t+1}^M) + \sum_{i=1}^{n} \beta_i Cov_t(z_{i,t+1}, r_{t+1}^M) \tag{8}$$

and this can be written in several different ways. With two further modifications, this is the model that we shall use in this paper.

The first modification is required because all of the models above assume the existence of a real risk-free asset whereas, in practice, only a nominal risk-free asset is available.[4] The SDF

[4] The nearest to a real risk-free return is the one-period return on an index-linked bond. In the US, index linked bonds are not available for one period (month) and are not perfectly indexed for inflation.

pricing equation, equation (2), can be re-written using nominal returns as

$$1 = E_t[M_{t+1}(1 + I^M_{t+1})\frac{P^c_t}{P^c_{t+1}}]$$

where I^M_t is the nominal market rate of return and P^c_t is the consumer price index. It can be shown that the SDF asset-pricing equation for nominal returns is

$$E_t(i^M_{t+1} - i^f_t) + \frac{1}{2}V_t(i^M_{t+1}) = -Cov_t(m_{t+1}, i^M_{t+1}) + Cov_t(\pi_{t+1}, i^M_{t+1}).$$

where $i^M_t = \ln(1 + I^M_t)$, i^f_t is the nominal risk-free rate and $\pi_{t+1} = \ln(P^c_{t+1}/P^c_t)$ defines the inflation rate. Thus, if we work with nominal returns, we must also include inflation as a factor. Our general model then becomes

$$E_t(i^M_{t+1} - i^f_t) = \phi_0 V_t(i^M_{t+1}) + \phi_1 Cov_t(\pi_{t+1}, i^M_{t+1}) + \textstyle\sum_2^n \phi_i Cov_t(z_{i,t+1}, i^M_{t+1}) \qquad (9)$$

3.2 Including business cycle effects

Schwert (1989) conducted one of the first detailed studies of the effects of the business cycle on stock returns.[5] He investigated whether the volatility of real economic activity is a determinant of stock return volatility on the grounds that common stocks reflect claims on the future profits of corporations. The findings were, however, that the volatility of industrial production growth did not help to predict stock market volatility; on the contrary, stock market volatility was able to predict output volatility. Schwert concluded that stock market volatility and the volatility of industrial production is higher during recessions. He also examined the relation between the stock market return and inflation using Producer Price Index (PPI) inflation as a factor but found that inflation volatility does not help predict future stock return volatility as it is not much affected by recessions. In addition, he considered the effect of volatility in the rate of growth of money. He found this to be a little more volatile during recessions, but it too was unable to predict stock market volatility. Taken together, Schwert's results do not resolve the puzzle of why stock prices are so highly volatile when macroeconomic variables are not. These results followed the analysis of Chen, Roll and Ross (1986) who identify significantly priced effects from industrial production growth, inflation and the term structure of interest rates. These resulst are produced from a Fama-McBeth style analysis of a cross-section of portfolio returns and do not impose any restrictions imposing no arbitrage.

[5] Earlier work by Campbell and Shiller (1988a, 1988b) showed that the log stock price reflects the expectation of future cash flows, future interest rates and the future excess return. If macroeconomic data contains information about expected future cash flows or expected future discount rates, potentially it can explain the time-variation in monthly stock market returns.

Much of the emphasis since Schwert's work has been to examine stock market behaviour using C-CAPM in which consumption is the sole factor of production. The focus of most of this research has been the equity premium puzzle, see Campbell (2003) for a survey and also Smith, Sorensen and Wickens (2005). The general finding, whether calibration analysis or conventional econometric estimation is used, is that consumption does not vary enough to explain stock market volatility and so requires an implausibly large value of the coefficient of relative risk aversion to match the volatility of the equity premium. Among the new findings in Smith, Sorensen and Wickens (2005) were the significance of conditional covariances of both inflation and a real macro variable with the stock return, implying that both are priced sources of risk.

A separate literature based on simple correlation analysis has also examined the relation between stock returns and the business cycle. Erb, Harvey and Viskanta (1994) find that there is a higher correlation between stock returns and the US business cycle during recessions than in periods of boom. They also show that foreign stock markets are more highly correlated with the US stock market when US returns are negative than when they are positive. Further, they find that the correlation between foreign stock returns and the US business cycle is higher in US recessions than booms. These results add to the weight of evidence on the asymmetry of the business cycle effects on stock markets; they also suggest that co-movements in stock markets in different countries are affected by the business cycle.

In this paper we re-consider Schwert's analysis of business cycle effects within a no-arbitrage framework using a generalised SDF model involving macroeconomic variables as factors. This generates two channels through which the business cycle may affect stock returns. First, if the mean return is dependent on the conditional volatility of returns, as in CAPM and ICAPM, and returns and the factors have a joint conditional distribution, then the conditional covariance between returns and the factors allows volatility in the factors to affect volatility in the returns and hence the returns themselves. Second, conditional covariation between the returns and the factors affects returns through the risk premium. Asymmetries in the transmission mechanism may also impact through these two channels.

We consider three macroeconomic factors: industrial production, inflation and money growth. The risk premum is greatest when returns are expected to be low. Low returns occur during recessions, hence we expect returns to have a positive correlation with output. This correlation may also be time varying. The relation between returns and inflation is less clear-cut. Through the Phillips curve relation, macroeconomic theory tends to associate recession with lower inflation. This implies a positive correlation between returns and inflation. However, this is true only when the recession is due to a demand shock. A recession due to a supply shock is more likely to have higher than lower inflation, implying a negative correlation between returns and inflation. This suggests that the correlation between returns and inflation is very likely to be time varying. This is exactly what we find. Our third macroeconomic variable is the rate of growth of narrow money which we expect to also have a negative correlation with returns.

In addition to helping determine the risk premum, time-varying volatility in the macroeconomic variables may have an impact on the volatility of returns. Higher output, inflation and money growth volatility is likely to be associated with higher volatility in returns.

4 The econometric framework

We wish to estimate the joint distribution of the stock market return and the macroeconomic factors subject to the restriction that the conditional mean of the returns equation satisfies the no-arbitrage condition, equation (9)and to allowing business-cycle shocks to impact asymmetrically.

Consider first the multivariate GARCH-in-mean (MGM) model. The advantage of a multivariate over the univariate GARCH model used by, for example, Glosten, Jagannathan and Runkle (1993), is that the variance of each of the dependent variables can be predicted by lagged values of conditional variances of all the variables and lagged covariances between all variables, and lagged squared residuals and cross products of residuals (variance and covariance news). A disadvantage of multivariate GARCH models is that they are highly parameterised. In an attempt to reduce the number of parameters more restrictive formulations have been proposed. One of these is the constant correlation model of Bollerslev (1990). Assuming a constant correlation structure over time is, however, a strong assumption and is normally unwarranted in asset pricing. A second simpler alternative is the dynamic conditional correlation model of Engle (2002) which allows for time-variation in the conditional correlations. This model is, however, less well suited to multivariate GARCH-in-mean models due to estimation problems arising from the "in-mean" effect. Moreover, the assumption of a constant conditional correlation does not seem plausible for asset-pricing models. A third alternative is the Factor ARCH model of Engle and Ng (1990) which allows the factors to drive the conditional covariance matrix.

Rather than use any of these more restictive models, we prefer the more general BEKK model proposed by Engle and Kroner (1995). This allows unrestricted time-varying variances and correlations, and the inclusion of observable macroeconomic factors, see Smith and Wickens (2002) and Smith, Sorensen and Wickens (2005). The BEKK model can also be modified to include asymmetries - see Kroner and Ng (1998) - and allows second moment in-mean effects to represent the risk premium. As a result we obtain the econometric model

$$\mathbf{Y}_{t+1} \ = \ \mathbf{A} \ + \ \sum_{i=1}^{p} \mathbf{B}_i \mathbf{Y}_{t+1-i} \ + \ \sum_{j=1}^{N_1} \mathbf{\Phi}_j \mathbf{H}_{[1:N,j],t+1} \ + \mathbf{\Theta} \Upsilon_{k,t+1} + \ \boldsymbol{\epsilon}_{t+1}, \tag{10}$$

where $\mathbf{Y}_{t+1}$ is an $N \times 1$ vector of dependent variables in which the first N_1 elements are assumed to be the excess returns, $\mathbf{A}$ is an $N \times 1$ vector, the $\mathbf{B}_i$ and $\mathbf{\Phi}_j$ and $\mathbf{\Psi}$ matrices are $N \times N$, $H_{[1:N,j],t+1}$ is the $N \times 1$ j^{th} column of the conditional variance covariance matrix. The first N_1 equations satisfy the restrictions imposed by no arbitrage. The risk premia are given by the first N_1 columns of $\sum_{j=1}^{N_1} \mathbf{\Phi}_j H_{[1:N,j],t+1}$. Thus, the associated $\mathbf{\Phi}_j$ matrices are unrestricted except for the j^{th} element which is $-\frac{1}{2}$. The corresponding rows of $\mathbf{B}_j$ are restricted to zero. The remaining equations have no "in-mean" effect but otherwise are unrestricted. $\boldsymbol{\Upsilon}_{k,t+1}$ is an indicator variable taking the value of 1 in specified periods and zero otherwise.

We define $Y_{t+1} \ = \ \left\{ i^e_{s,t+1} \ \pi_{t+1} \ \Delta m_{t+1} \ \Delta y_{t+1} \right\}$. Thus, there is a single risky return $i^e_{s,t+1}$ (the log excess return of the stock market), and there are three macroeconomic factors: π_{t+1} is the log inflation rate, Δm_{t+1} is the log first difference in narrow money M1 and Δy_{t+1} is the log first difference of industrial production. Consequently, the first row of $\mathbf{\Phi}_1$ appears in the equation for the risky stock return and must satisfy the no-arbitrage condition. The other elements of $\mathbf{\Phi}_1$ appear in the equations for the macro variables and are therefore restricted to equal zero. We use a vector auto-regression of order 1 ($p = 1$) implying that the model can be written

$$\mathbf{Y}_{t+1} \ = \ \mathbf{A} \ + \ \mathbf{B}\mathbf{Y}_t \ + \ \mathbf{\Phi}\mathbf{H}_{[1:N,1],t+1} \ + \mathbf{\Theta}\Upsilon_{1987:10,t+1} + \ \boldsymbol{\epsilon}_{t+1} \tag{11}$$

Only the first row of $\mathbf{B}$ is restricted to be zero; the remaining elements of $\mathbf{B}$ are unrestricted.

$\Upsilon_{1987:10,t+1}$ is a dummy variable which is included to take account of the stock market crash of October 1987. The excess return in this month is clearly an outlier and is almost certainly not explicable by our theory of asset pricing, see Schwert (1998). Thus it takes the value of 1 for $t+1$ corresponding to October 1987 and zero otherwise.

We examine whether business-cycle shocks impact on stock returns asymmetrically through the specification of the error term $\boldsymbol{\epsilon}_{t+1}$. We assume that the error term displays conditional heteroskedasticity. In other words, the covariance matrix of $\boldsymbol{\epsilon}_{t+1}$, and hence the volatility of returns, is partly forecastable and may respond differently to positive and negative business cycle shocks.

We specify the error term as

$$\boldsymbol{\epsilon}_{t+1} \ = \ \mathbf{H}^{\frac{1}{2}}_{t+1} \mathbf{u}_{t+1}, \qquad \mathbf{u}_{t+1} \ \sim \ \mathcal{D}(0, \mathbf{I}_4)$$

where, in order to allow for excess kurtosis in the error term, we assume the data have a joint t-distribution (see, for instance, Hafner (2001)). I_4 is the identity matrix of dimension four. We assume that the conditional covariance matrix $\mathbf{H}_{t+1}$ is an asymmetric version of the BEKK model (ABEKK) defined by

$$\mathbf{H}_{t+1} = \mathbf{CC}^{\intercal} + \mathbf{D}(\mathbf{H}_t - \mathbf{CC}^{\intercal})\mathbf{D}^{\intercal} + \mathbf{E}(\boldsymbol{\epsilon}_t\boldsymbol{\epsilon}_t^{\intercal} - \mathbf{CC}^{\intercal})\mathbf{E}^{\intercal} + \mathbf{G}(\boldsymbol{\eta}_t\boldsymbol{\eta}_t^{\intercal} - \overline{\mathbf{CC}^{\intercal}})\mathbf{G}^{\intercal}, \tag{12}$$

where the asymmetry is due to the term in $\eta_t = \min[\epsilon_t, 0]$. The bar over $CC^{\intercal}$ indicates that the appropriate correction is made since $\mathrm{E}_t(\eta_t\eta_t^{\intercal}) \neq CC^{\intercal}$.[6] The eigenvalues of

$$(\mathbf{D} \otimes \mathbf{D}) + (\mathbf{E} \otimes \mathbf{E}) + (\overline{\mathbf{G} \otimes \mathbf{G}}), \tag{13}$$

must lie inside the unit circle for the BEKK system to be stationary. $\otimes$ is the Kronecker product.

Equation (11) is estimated using the Quasi-Maximum Likelihood estimator proposed by Bollerslev and Wooldridge (1992). For numerical reasons, we may want to scale our variables so that the variables have the same sample variances. The scaled version can be written

$$\mathbf{Y}_{t+1}^{*} = \mathbf{A}^{*} + \mathbf{B}^{*}\mathbf{Y}_t^{*} + \mathbf{\Phi}^{*}\mathbf{H}_{[1:N,1],t+1}^{*} + \mathbf{\Theta}^{*}\Upsilon_{t+1} + \boldsymbol{\epsilon}_{t+1}^{*}, \tag{14}$$

with $Y_{t+1}^{*} = \mathbf{\Gamma}Y_{t+1}$, $\epsilon_{t+1}^{*} = \mathbf{\Gamma}\epsilon_{t+1}$ and $\mathbf{H}_{t+1}^{*} = \mathbf{\Gamma}\mathbf{H}_{t+1}\mathbf{\Gamma}^{\intercal}$. The original coefficient matrices can be recovered as $\mathbf{A} = \mathbf{\Gamma}^{-1}\mathbf{A}^{*}$ and $\mathbf{B} = \mathbf{\Gamma}^{-1}\mathbf{B}^{*}\mathbf{\Gamma}.$ Since we are interested in matching the variances of the data, Γ will be diagonal. For example, the first dependent variable is the excess return on the stock market and we scale inflation so that it has the same variance. As result, the element in the diagonal of Γ takes the value $\sqrt{\frac{\mathrm{Var}(i_{s,t+1}^{e})}{\mathrm{Var}(\pi_{t+1})}}$, where $\mathrm{Var}(\cdot)$ is the sample variance.[7] The conditional covariance matrix in the scaled model can be written

$$\mathbf{H}_{t+1}^{*} = \mathbf{C}^{*}\mathbf{C}^{*\intercal} + \mathbf{D}^{*}(\mathbf{H}_t^{*} - \mathbf{C}^{*}\mathbf{C}^{*\intercal})\mathbf{D}^{*\intercal} + \mathbf{E}^{*}(\boldsymbol{\epsilon}_t^{*}\boldsymbol{\epsilon}_t^{*\intercal} - \mathbf{C}^{*}\mathbf{C}^{*\intercal})\mathbf{E}^{*\intercal} + \mathbf{G}^{*}(\boldsymbol{\eta}_t^{*}\boldsymbol{\eta}_t^{*\intercal} - \overline{\mathbf{C}^{*}\mathbf{C}^{*\intercal}})\mathbf{G}^{*\intercal}$$

where $\eta_t^{*} = \min[\epsilon_t^{*}, 0]$. It follows directly that $\mathbf{C} = \mathbf{\Gamma}^{-1}\mathbf{C}^{*}$, $\mathbf{D} = \mathbf{\Gamma}^{-1}\mathbf{D}^{*}\mathbf{\Gamma}$, $\mathbf{E} = \mathbf{\Gamma}^{-1}\mathbf{E}^{*}\mathbf{\Gamma}$ and $\mathbf{G} = \mathbf{\Gamma}^{-1}\mathbf{G}^{*}\mathbf{\Gamma}$. All of the results reported below are the original coefficients obtained by transforming back to the unscaled model.

The risk premium is given by the first row of

$$\phi_t = \mathbf{\Phi}\mathbf{H}_{[1:N,1],t+1}$$

This can be decomposed in different ways. One decomposition is into the components associated with each of the factors. Thus we can write the total risk premium as

$$\phi_t = \phi_{excess\ return,t} + \phi_{inflation,t} + \phi_{money,t} + \phi_{output,t} \tag{15}$$

[6] $\overline{\mathbf{CC}^{\intercal}}$ is obtained by multiplying the diagonal elements of $\mathbf{CC}^{\intercal}$ by $\frac{1}{2}$ and the off-diagonal elements by $\frac{1}{4}$.

[7] Note that scaling the variables may affect the correction terms. We do not scale the excess return and so the Jensen term should still equal $\frac{1}{2}\mathrm{V}_t(i_{s,t+1}^{e})$.However,we scale inflation and so the correction for working with nominal returns should not be $\mathrm{Cov}_t(i_{s,t+1}^{e}, \pi_{t+1})$ but $\frac{\sqrt{\mathrm{Var}(\pi_{t+1})}}{\sqrt{\mathrm{Var}(i_{s,t+1}^{e})}}\mathrm{Cov}_t(i_{s,t+1}^{e}, \pi_{t+1})$.

A second decomposition allows us to determine the importance of asymmetries. $\mathbf{H}_{t+1}$, as defined by equation (12), has four components, and hence can be re-written as

$$\mathbf{H}_{t+1} = \mathbf{H}_0 \ + \ \mathbf{H}_{1,t+1} \ + \ \mathbf{H}_{2,t+1} \ + \ \mathbf{H}_{3,t+1}$$

Pre-multiplying by $\mathbf{\Phi}$ gives the decomposition

$$\phi_t = \phi_0 + \phi_{1t} + \phi_{2t} + \phi_{3t} \tag{16}$$

where ϕ_{3t} is the component of the risk premium due to asymmetries. ϕ_{1t} is the component due to autoregressive effects and ϕ_{2t} is the component due to ARCH effects.

In estimating this model we make an assumption regarding the initial value of the conditional covariance matrix. One possibility is to set the starting value equal to the unconditional covariance matrix of the dependent variables. Another is to perform the unrestricted vector auto-regression from equation (14) and use the estimated covariance matrix of the residuals. A third possibility is to estimate the starting values, noting from equation (12), that $\mathrm{E}(\mathbf{H}_{t+1}) = \mathbf{CC}^{\intercal}$.[8] All estimations were carried out using each of the starting values, but the final values were virtually identical.

5 The results

5.1 The models estimated

The general model to be estimated can be written as

$$E_t(i^e_{s,t+1}) \ + \ \frac{1}{2}V_t(i^e_{s,t+1}) \ = \ \mathbf{b}^{\intercal} Cov_t(i^e_{s,t+1}, \mathbf{Y}_{t+1}) + \theta \Upsilon_{1987:10,t+1}$$

The individual models differ in their choice of $\mathbf{Y}_{t+1}$. Model 1 is CAPM and takes the form

$$E_t(i^e_{s,t+1}) \ + \ \frac{1}{2}V_t(i^e_{s,t+1}) \ = \ \gamma V_t(i^e_{s,t+1}) + Cov_t(\pi_{t+1}, i^e_{s,t+1}) + \theta \Upsilon_{1987:10,t+1}$$

Model 7 removes the restriction that the conditional covariance with inflation has a unit coefficient and is used to test CAPM. Model 2 and Model 3 are more general than Model 1 but are not associated with any particular theory. Model 2 is a version of ICAPM with three macroeconomic variables. If any of these macroeconomic variables are significantly priced then this would serve as a rejection of CAPM. Model 3 prices only the macroeconomic variables and excludes the conditional variance of the market return. Model 4, Model 5 and Model 6 price each of the macroeconomic variables individually and enable us to evaluate the total contribution of each individual macroeconomic variable.

[8] This starting value is consistent.

5.2 The data

The data are monthly for the US over the period 1960:01 to 2003:12. The stock market returns are the log value-weighted return on all NYSE, AMEX and NASDAQ stocks. The risk-free rate is the one-month US Treasury Bill rate.[9] The macroeconomic data are the log first difference of the index of real industrial production, log CPI inflation and the log first difference of M1. These data are obtained from the Federal Reserve Bank of St. Louis.

In Table 1 we report descriptive statistics for these data. The excess stock market return has little autocorrelation but displays negative skewness, excess kurtosis, non-normality and autocorrelation both in the squared returns and in the absolute returns. This indicates that the volatility of returns is partly predictable and there is evidence of asymmetries in the volatility process. It suggests that an ARCH process with asymmetries may be able to represent these data.

Inflation has substantial autocorrelation, has positive skewness, does not show excess kurtosis, but is non-normal. There is autocorrelation in squared inflation and in the absolute value of inflation. Money growth is very like inflation except that its absolute values have less autocorrrelation. Industrial production closely resembles stock-market returns except that it has stronger first-order autocorrelation in its squares and absolute values. This uni-variate evidence supports the use of a multi-variate asymmetric ARCH model.

5.3 The estimates

The estimates of the various no-arbitrage models with asymmetric effects are reported in Table 2. Model 1 (CAPM) has the lowest explanatory power as measured both by the log-likelihood and by the percentage of the variation in the excess return (adjusted for the Jensen effect and 1987 outlier) explained by variations in the risk premium. The mean residual is significantly different from zero. CAPM constrains the coefficient of the conditional covariance with inflation to be unity. Model 7 shows that this restriction is invalid and suggests that inflation has a stronger impact on returns than CAPM allows.

Model 2 (ICAPM/Epstein-Zin) and Model 3 (SDF) fit almost equally well as Models 1 and 7. In Model 2 three variables are significantly priced: the market return, and two macroeconomic variables, inflation and industrial production. The variability of the implied risk premium for Model 2 is more than 11 times higher than that of Model 1, moreover, its residuals are considerably closer to zero than those of Model 1. The 1987 dummy is only significant in Model 1.

In Model 3 (SDF) all three macroeconomic variables are significantly priced. The significance of money growth in Model 3 but not in Model 2 is a reflection of the effects of correlation between the

[9] This is available from the homepage of Kenneth French, http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/.

explanatory variables, the conditional covariance terms. The unconditional correlation between the conditional covariance of money growth with the market return and the conditional variance of the market return is 0.65. This suggests that money growth may only be significant due to omitting the market return, which is a more significant variable. Nonetheless, Model 3 explains a larger share of the variation in the excess return than Model 2 and its mean residual is closer to zero.

Models 4-6 are SDF models with only a single macroeconomic factor. Inflation is the most significantly priced, followed by industrial production; money growth on its own is not significantly priced. This is a further sign of the effects of correlation between the conditional covariance terms.

These results support previous findings that the volatility of the US stock market return significantly explains the return - or, put another way, the market return is a priced factor. They also show clearly that CAPM can be rejected in favour of a more general asset-pricing model that includes additional macroeconomic factors. It appears that both inflation and output growth are significantly priced, but money growth does not seem to have further useful information. We therefore omit the asset-pricing models involving money growth from our subsequent analysis and concentrate mainly on Model 2. Money growth is not, however, eliminated entirely from the model; it is retained as part of the information set and so has its own equation. In this way, money is still a conditioning variable and so is able to help forecast the conditional covariance matrix of the other variables. This is justified by the significance of money in the multivariate GARCH process.

5.4 Estimates for Model 2

The full set of estimates of Model 2 are reported in Table 3. There are four equations in the model. The first equation is for the excess return and is restricted to satisfy the condition of no-arbitrage. The other three equations have no "in-mean" effects, but do have VAR effects. These are captured in the matrix **B**. Apart from significant own lags, the lagged excess return is strongly significant in the money equation, and lagged inflation is significant in the output equation.

Turning to the GARCH process, the matrices **D** and **E** are highly significant. Although the diagonal terms are the most significant, there are significant off-diagonal effects too so that each variable seems to significantly explain all of the others.[10] For example, an increase in the variance of output growth in the previous period predicts there will be an increase in the variance of the excess return on the stock market in the following period, and vice-versa. There is, therefore, a strong interaction between the stock market and business cycle volatility. There

[10] The restrictions provided by the diagonal BEKK model were tested and rejected in favour of our more general model.

are similar interactions between the stock market and inflation and, interestingly, between output and inflation. A higher inflation variance predicts higher future output variability.

We are particularly interested in the results on asymmetry, which is captured by matrix **G** and we measure the the additional effects on conditional variances and covariances due to negative shocks. We find that the conditional variances of stock market returns and industrial production growth show stong asymmetries. The negative sign on the variance of stock returns implies that negative own shocks have a lower impact on the volatility of returns than positive shocks. In contrast, the positive sign of the output variance implies that negative output shocks have a greater impact on business cycle volatility than positive shocks. In addition we notice that there are strong asymmetries from the off-diagonal terms of **G**. These are more difficult to interpret as they involve cross-effects with other elenments of **G** and the elements of $\eta_t \eta_t' - \overline{\mathbf{CC}'}$. A likelihood ratio test of the joint significance of the elements of the **G** matrix suggests that they are significant at less than the 0.1% significance level ($103.0 \sim \chi^2(16)$).

5.5 The equity risk premium

A number of authors have examined the relation between expected returns and the cycle defined by the NBER. Counter-cyclical expected returns have also been documented by Harrison and Zhang (1999) in the context of models relating returns to their volatility. Chauvet and Potter (2001) show that expected returns are related to the cycle in a non-linear way where the risk-return relationship is driven by a Markov process. Expected returns rise towards the end of downturns in their models. In these papers, however, expected returns are defined by a measure of risk based on the volatility of returns and not by any independent sources of risk as in the present work. Our estimates enable us to examine the relation between expected equity returns and the equity premium. First we consider the estimated equity premia implied by the different models. We then relate the equity premium to the business cycle.

5.5.1 Estimates

In Figure 1 we plot the risk premia for Models 1 and 2, together with the excess stock market return. The shaded areas are recessions as defined by the NBER. The risk premium for Model 2 clearly varies over time much more than that for Model 1 which is positive in each period because in CAPM the risk premium is proportional to the conditional variance of the market return. Whilst the risk premium for Model 2 is mainly positive, from time to time it is negative. We note from Figure 2 that the risk premia for Models 4-6 display many more periods when the risk premium is negative. The fact that the risk premium for Model 2 is less prone to being negative indicates that the conditional covariances are negatively correlated and offset each other. This demonstrates that the simplifying assumption of a constant correlation over time is not appropriate for modelling the joint distribution of the excess return and the macroeconomic variables.

Table 4, which reports the autocorrelation coefficients of the risk premia for the six models, shows that Model 1 has the most persistent risk premium, and Models 4-6 have the least persistent and most volatile. The persistence of the risk premia for Models 2 and 3 are similar, and similar to that for Model 6 for which output is the sole factor. This suggests that the business cycle is the dominant factor determining equity risk.

For most of the time the periods when the risk premium in Model 2 is largest tend to be periods of recession and Model 3 is similar. This is further support for the importance of asymmetries. In Figure 3 we plot the risk premium for Model 2 together with the conditional volatilities for returns and the three macroeconomic factors. The highest correlation is that between the risk premium and the volatility of output. This is evidence in support of the importance of the business cycle in explaining the equity risk premium. We also note that the correlation between the risk premium and the volatility of returns is much lower, suggesting that a simple model relating returns to their volatility does not perform well.

The asymmetric effects on the risk pemium of good and bad news may be judged from Figure 4 where the three time-varying components of the risk premium ϕ_{1t}, ϕ_{2t} and ϕ_{3t} are plotted. The contribution of asymmetries to the risk premium is given by ϕ_{3t} in equation (16). It is clear that ϕ_{1t}, the autoregressive component, is the most important, but next most important is ϕ_{3t}. Moreover, asymmetries seem to have their greatest effect on the risk premium in recessions. This is consistent with the notion that risk attaches more to recessions than booms.

5.5.2 Macroeconomic sources of risk

In Table 5 we report recession dates according to the NBER dating committee. Table 6 provides summary statistics for NBER recession and non-recesssion periods. The top panel provides summary statistics for the variables in NBER recession and non-recession periods as identified in Table 5. The lower panel presents the means of the covariances of the variables with the excess return multiplied by the estimated coefficient, i.e. the contribution of each macroeconomic variable to the equity premium. These results show a striking difference between the mean stock returns and output growth rates during recession and non-recession. This suggests that the business cycle has a strong effect on stock returns. In contrast, we note that the correlations between returns and the macroeconomic variables are not very different between recessions and non-recessions. The interest in this result is that we find that the conditional correlation coefficient of returns and output is strongly time varying, suggesting that this is masked using unconditional correlations.

Table 6 also shows that the model generates an equity premium which is higher in recessions than non-recessions and that this is due not only to higher uncertainty associated with increased volatility of returns, but also to greater covariances associated with higher inflation and lower output growth.

A necessary condition for the macroeconomic factors to be priced sources of time-varying risk is that they display time-varying volatility. In Figure 3 the conditional volatility of each factor is plotted together with the risk total premium. The volatility of the macroeconomic variables clearly varies through time and tends to be greatest during recessions when the risk premium is also at its height. Inflation and output volatility seem to have been lower in the last twenty years than in the more turbulent 1970's whereas, after a period of tranquility, money growth volatility has recently returned to the high levels of 1970's. Maccini and Pagan (2003) have suggested that the decline in output volatility reflects that it follows a square root process. A contributing factor is that there has been a reduction in negative shocks to output in the most recent period.

Another necessary condition for the macroeconomic factors to be priced is that they are correlated with the excess return. In Figure 5 we plot the time-varying correlations between the market excess return and the macroeconomic factors. This shows the strength of the correlations and the fact that they vary over time. The conditional correlation between the excess market return and inflation is predominantly negative, unlike the correlations with output and money growth. These are also cyclical.

Combining this information gives the contribution of each factor to the total risk premium. This is plotted in Figure 6. We find that the contribution of the market return is positive, but that of inflation is nearly always negative, whilst the contribution of output fluctuates in sign, being largely negative in the 1970's and positive in the 1980's, but becoming negative again during the late 1990's recession. To gain more understanding of what is happening, in Figure 7 we show for Model 2 the time-varying correlations between certain macroeconomic factors and the correlation with the risk premium. For example in the 1974/5 recession a large negative cyclical shock is associated with a substantial rise in the risk premium. Figure 7 also reveals that during this recession, the correlation between inflation and output are strongly negative, reflecting the fact that the recession was caused by a supply shock - the rise in oil and other commodity prices - and not a demand shock. There is another strong negative correlation in 1979 when there was a second oil price shock. During later recessions inflation and output are positively correlated which is consistent with these recessions being due instead to negative demand shocks.

We investigate these issues further by estimating model 2 in an alternative format. We re-write equation (9) as

$$E_t(i^M_{t+1} - i^f_t) = \phi_0 V_t(i^M_{t+1}) + \sum_2^3 \phi_i Cov_t(z_{i,t+1}, i^M_{t+1}) + \sum_4^5 \phi_i Cov_t(z_{it+1}, \pi_{t+1}) \tag{17}$$

where the covariance of inflation with the risky return has been replaced by the covariance between inflation and the macro factors. Equation (17) allows the impact of the covariances between inflation and the macro factors to be identified directly. Two estimates of this model are presented as models 8 and 9 in Table 7. Compared with Model 2, in Model 8 the covariance between inflation and output growth is very significant whilst that of inflation and the equity return becomes insignificant as expected from equation (17). Model 9 confirms that the covariance between money growth and inflation is insignificant. The covariance of inflation and output growth has a large positive coefficient in Model 8. The average value of this covariance is negative as is it's impact on the risk premium. In recessions the covariance becomes more negative suggesting that supply shocks are the dominant cause of recessions over the whole sample period. From the contribution of the inflation-output growth covariance to the risk premium we find that there is a difference in the risk premium between recessions and non-recessions of, on average, 3.5% at an annualised rate. The impact of the return-output growth covariance is negative in Model 8 as it is in Model 2. In recessions, this positive covariance becomes smaller thus increasing the size of the risk premium.

These findings reveal many other things too. For example, periods with high risk premia are associated with periods of very low correlation between money and output suggesting that negative correlation between money and output shocks coincide with more risky stock market returns. At the end of the recessions and shortly after, the risk premium tends to decline implying more favourable economic conditions that make the stock market less risky. The recessions of 1974/5 and 1980 had a negative correlation between inflation and output and so were heavily affected by a supply shock, but were followed by a strong positive correlation between inflation and output, suggesting a demand stimulus was given to the economy to counteract the recession.

6 Assessing the impact of structural macroeconomic shocks on the risk premium

6.1 Identifying the macroeconomic shocks

Our analysis of the relation between the equity premium and macroeconomic variables over the business cycle has focused so far on their covariance structure. Moreover, we used an externally defined measure of the business cycle. In this section we assess the impact on the equity premium of business cycle shocks that are internal to the model and structural to the macro economy.

Following from our findings that the effect of the business cycle on the equity premium appears to differ between recession and non-recession and between recessions caused by supply and demand shocks, we seek to identify the shocks into supply and demand by using an extension of the modification of the Blanchard and Quah (1989) method due to Robertson and Wickens (1997).

Consider the model:

$$
\begin{aligned}
\mathbf{Y}_{t+1} &= \mathbf{A} + \mathbf{B}\mathbf{Y}_t + \mathbf{\Phi}\mathbf{H}_{[1:N,1],t+1} + \mathbf{\Theta}\Upsilon_{t+1} + \boldsymbol{\epsilon}_{t+1} \\
\boldsymbol{\epsilon}'_{t+1} &= [\boldsymbol{\epsilon}_{it+1}, \boldsymbol{\epsilon}_{\pi t+1}, \boldsymbol{\epsilon}_{\Delta mt+1}, \boldsymbol{\epsilon}_{\Delta yt+1}]'
\end{aligned}
$$

with estimation errors $\boldsymbol{\epsilon}_{t+1}$. We wish to identify the structural shocks $\mathbf{u}_{t+1}$ from the observed shocks $\boldsymbol{\epsilon}_{t+1}$ where the two are related through

$$\boldsymbol{\epsilon}_{t+1} = \mathbf{G}\mathbf{u}_{t+1}$$

Identification entails restrictions on the matrix $\mathbf{G}$. The key identifying assumption is that a demand shock has no long-run effect on output whereas the effect of a supply shock on output may be permanent. Both demand and supply shocks may have immediate impacts on output and inflation.

The identifying restrictions are given by

$$
\begin{bmatrix} \boldsymbol{\epsilon}_{it+1} \\ \boldsymbol{\epsilon}_{\pi t+1} \\ \boldsymbol{\epsilon}_{\Delta mt+1} \\ \boldsymbol{\epsilon}_{\Delta yt+1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ g_{21} & g_{22} & g_{23} & g_{24} \\ 0 & 0 & 1 & 0 \\ g_{41} & g_{42} & g_{43} & g_{44} \end{bmatrix} \cdot \begin{bmatrix} u_{rt} \\ u_{\sup t} \\ u_{\Delta mt} \\ u_{\mathrm{dem} t} \end{bmatrix} \tag{18}
$$

where the demand and supply shocks are $u_{\mathrm{dem}t}$ and $u_{\sup t}$, respectively. These two structural shocks have a unit variance and are contemporaneously uncorrelated so that $\sigma^2_{u_{\sup t}} = 1$ and $\sigma^2_{u_{\mathrm{dem}\, t}} = 1$. The required additional identifying restriction on $\mathbf{G}$ is

$$g_{44}[1 - b_{22}] + g_{24}b_{42} = 0 \tag{19}$$

where b_{ij} is the ij^{th} element of the estimate of $\mathbf{B}$. The other two shocks, the excess return and money growth shocks, u_{rt} and $u_{\Delta mt}$, are set equal to their corresponding reduced form errors.

6.2 Structural Shocks, the Business Cycle and the Risk Premium

The supply and demand shocks that have been generated by this identification scheme can be related to the NBER recession periods that are listed in Table 5. In NBER recession periods the aggregate supply shock has a mean value of -8.45% on an annualised basis. In the non-recession periods the mean supply shock is 4.26%. The demand shock has a mean of 0.71% in recession periods and 2.49% in non-recession periods. The two shocks are plotted in Figure 8 just for the NBER recession periods. Negative supply shocks appear to be more closely related to the NBER recession periods than negative demand shocks, especially prior to the mid 1908's. Figure 8 also reveals that demand shocks are mostly smaller than supply shocks.

The consequences of business cycle shocks for the equity premium, and therefore expected excess returns, can be examined in more detail through the impulse response functions for the supply and demand shocks. These are depicted in Figures 9 - 12. We show negative and positive 1% impulses to aggregate supply and demand along with 95% confidence intervals computed from 1,000 bootstrap simulations. Comparison of these figures shows that the supply shocks have a greater impact on output than on inflation whilst demand shocks have a greater impact on inflation than output. These findings echo those in the business cycle literature.

The difference between the impacts of negative and positive supply shocks on the equity premium is striking. Figures 9 and 10 show that a 1% negative supply shock raises the risk premium by more than 1% in the second month of the shock.[11] This effect is significantly positive and persistent. In contrast, a positive supply shock has a small impact, with a negative peak on the equity premium which is not very significant. Asymmetries in the covariance function are a major contributory factor in producing these contrasting results. In comparison to supply shocks, Figures 11 and 12 show that demand shocks, whether positive and negative, cause the risk premium to fall, although not significantly so demand shocks. To reinforce the point that asymmetries matter strongly affect the findings of this paper, and to emphasise that demand shocks are relatively unimportant for the risk premium, Figures 13 and 14 provide simulations of the impact of small negative and positive demand shocks. In both cases the impact of the shocks on the equity premium is close to zero, and certainly not significantly different from zero for any period of the reponse.

In all of the simulations the response of money growth to either aggregate supply or demand shocks is insignificant (see Figures 9 - 14). However, the point estimates of the negative (positive) response of money growth to positive supply (demand) shocks are consistent with more developed multivariate structural VAR models such as those in Keating (2000).

We now have a more complete picture of the relation between the equity premium and the business cycle as viewed within a no-arbitrage framework. The key features are the covariances between returns and the macro sources of risk, and the asymmetric behaviour of the covariance

[11] In each case we show the deviation of the risk premium from it's steady-state value.

function linking equity returns to the macro variables.

7 The implications for CAPM

We began this study by observing that the key to understanding how an asset is priced is the relation between its return and its volatility. As a result, it seems obvious to use CAPM to study the relation. We have shown, however, that the standard formulation of CAPM is not general enough and that the evidence provides strong support for the SDF model. We have also noted in equation (7) that we can interpret the SDF model as a more general version of CAPM in which the coefficient on the conditional volatility of returns is time varying.

In Figure 15 we plot this coefficient, unsmoothed and smoothed. It is very striking how volatile the coefficient is. This reveals how inadequate standard CAPM is in explaining the relation between returns and volatility. We also note that although the coefficient is positive most of the time, in the 1970's it is highly negative for a period. This shows once more the problem that standard asset pricing models have in explaining the behaviour of stock returns during that period.

8 Conclusions

The main findings in this paper are of a strong asymmetric relation between the US business cycle and the US stock market over the period 1960 to 2003, and that downturns in the business cycle have a greater negative impact on stock returns than the positive effect of upturns.

In contrast to the pioneering work of Schwert and later purely empirically-based approaches, including simple correlation analysis, our analysis was conducted within an explicit no-arbitrage framework of the relation between returns and their volatility based on several models of asset pricing involving stochastic disciunt factors. This enabled us to derive a formal relation between returns and the business cycle via the equity risk premium. This model is capable of encompassing a number of different asset-pricing theories, including CAPM. An advantage of this model ois that we can then relate the equity risk premium to the business cycle. We are also able to investigate the potential effects of other macroeconomic variables such as inflation and money growth. Our results support the use of three priced factors: output, inflation and the stock market return.

Another feature of our analysis is that we model the joint distribution of stock returns and observable macroeconomic variables using an asymmetric multivariate GARCH model with conditional covariance "in-mean" effects to represent the risk premium. This is a more general approach than that used hitherto in the literature as it neither excludes conditional covariance effects in the mean, nor does it restrict the conditional correlation structure to be constant over time. Further, the conditional covariances are not restricted to be linear functions of the factors as in the Vasicek model. These generalisations strongly influence our new findings. In addition to the three priced factors, we find that money growth should also be included in the joint distribution.

In our model, there are two channels through which the business cycle may affect stock returns. There is a mean effect coming via the equity risk premium, and there is a volatility effect coming through the conditional covariance matrix. All three macroeconomic variables operate significantly through the volatility of returns, but only output and inflation have a significant effect on the mean return.

As a result of allowing for time-varying correlation we discovered a difference in the effects on stock returns between a recession caused by negative supply shocks and one caused by negative demand shocks. We found that the correlation between output and inflation was negative during the recessions caused by the two oil price shocks of the 1970's. Formal identification of the shocks confirms that they were caused by negative supply shocks. In contrast, the earlier and later recessions were associated with a positive correlation between output and inflation, suggesting that these recessions were caused by negative demand shocks.

We began this study by observing that the key to understanding how an asset is priced is the relation between its return and its volatility. As a result, it may appear obvious to use CAPM to study the relation. We have shown, however, that the standard formulation of CAPM is not general enough and that the evidence provides strong support for the SDF model. We have also shown that we can interpret the SDF model as a more general version of CAPM in which the coefficient on the conditional volatility of returns is time varying.

In Figure 15 we plot this coefficient, unsmoothed and smoothed. It is very striking how volatile the coefficient is and that, although the coefficient is positive most of the time, in the 1970's it is highly negative for a period. This reveals how inadequate the standard unconditional CAPM is in explaining the relation between returns and volatility. Our results show the importance of time variation in the coefficient on the conditional volatility of returns and how this may be explained by macroeconomic factors within a more general SDF framework.

References

Baillie, R. and R.P. Degennaro, *"Stock Returns and Volatility"*, Journal of Financial and Quantitative Analysis, Vol. 25, No. 2, June, 1990

Balfoussia, C and M.R. Wickens, *"Macroeconomic Sources of Risk in the Term Structure"*, CESifo Working Paper 1329, November 2004

Bekaert, G. and G. Wu, *"Asymmetric Volatility and Risk in Equity Markets"*, The Review of Financial Studies, Vol. 13, No. 1, p. 1-42, Spring, 2000

Blanchard, O. and D. Quah, "*The Dynamic Effect of Demand and Supply Disturbances*", American Economics Review, Vol. 79, p 655-673, 1989.

Breeden, D., "An Intertemporal Asset Pricing Model with Stochastic Consumption and Investment Opportunities", Journal of Financial Economics, Vol. 7, Issue 3, p. 265-296, 1979

Black, F., *"Studies of Stock Price Volatility Changes"*, In Proceedings of the 1976 Meetings of the Business and Economic Statistics Section

Bollerslev, T., *"Modelling the Coherence in Short-Run Nominal Exchange Rates: A Multivariate Generalised ARCH Model"*, The Review of Economics and Statistics, Vol. 74, No. 3, p. 498-505, 1990

Bollerslev, T. and J.M. Wooldridge, *"Quasi-Maximum Likelihood Estimation and Inference in Dynamic Models with Time-Varying Covariances"*, Econometric Reviews, Vol. 11, No. 2, p. 143-172, 1992

Campbell, J., *"Stock Returns and the Term Structure"*, Journal of Financial Economics, Vol. 18, p. 373-399, 1987

Campbell, J.Y. and L. Hentschel, *"No News is Good News"*, Journal of Financial Economics, Vol. 31, Issue 3, p. 281-318, June, 1992

Campbell, J.Y. and R. Shiller,*"The Dividend-Price Ratio and Expectations of Future Dividends and Discount Factors"*, Review of Financial Studies, Vol. 1, Issue 3, p. 195-227, 1988a

Campbell, J.Y. and R. Shiller, *"Stock Prices, Earnings and Expected Dividends"*, The Journal of Finance, Vol. 43, No. 3, p. 661-676, June, 1988b

Campbell, J.Y., Lo, A.W. and A.C. MacKinlay, **The Econometrics of Financial Markets**, 1997, Princeton University Press, Second printing

Capiello, L., Engle R.F. and K. Sheppard, *"Asymmetric Dynamics in the Correlations of Global Equity and Bond Returns"*, manuscript, New York University, 2004.

Chauvet, M. and S. Potter, "*Nonlinear Risk*", Macroeconomics Dynamics, 5, pp 621-646, 2001.

Chen, N., Roll, R. and S. Ross (1986), "Economic Forces and the Stock Market", Journal of Business, 59, 3, 383-403.

Cochrane, J., **Asset Pricing**, Princeton Univeristy Press, 2nd Edition, 2005.

Erb, C., C.C. Harvey and T. Viskanta, "Forecasting international equity correlations", Financial Analysts Journal, Nov., 32-45, 1994.

Engle, R.F., *"Dynamic Conditional Correlation - A Simple Class of Multivariate GARCH Models"*, Journal of Business and Economic Statistics, Vol. 17, No. 5, 2002

Engle, R.F. and K.K. Kroner, *"Multivariate Simultaneous Generalised ARCH"*, Econometric Theory, Vol. 11, No. 1, p. 122-150, 1995

Engle, R.F. and V.K. Ng, *"Asset Pricing with a Factor-ARCH Covariance Structure - Empirical Evidence for Treasury Bills"*, Journal of Econometrics, Vol. 45, p. 213-237, 1990

Epstein, L.G. and S.E. Zin, *"Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework"*, Econometrica, Vol. 57, No. 4, p. 937-969, 1989

French, K.R., Schwert, G.W. and R.F. Stambaugh, *"Expected Stock Return and Volatility"*, Journal of Financial Economics, Vol. 19, p. 3-30, 1987

Glosten, L.R., Jagannathan R. and D.E. Runkle, *"On the Relation between the expected value and the volatility of the nominal excess returns on stocks",* Journal of Finance, Vol. 48, p. 1779-1801, 1993

Harrison, P. and H. Zhang, *"An Investigation of the Risk and Return Relation at Long Horizons"*, Review of Economics and Statistics, 81(3), p399-408, 1999

Harvey, C., *"Time-Varying Conditional Covariances in Tests of Asset Pricing Models"*, Journal of Financial Economics, Vol. 24, p. 289-317, 1989

Keating, J., "*Macroeconomic Modeling with Asymmetric Vector Autoregressions*", Journal of Macroeconomics 22, 1, 1-28, 2000.

Kroner, K.F. and V.K. Ng, *"Modelling Asymmetric Comovements of Asset Returns"*, The Review of Financial Studies, Vol. 11, No. 4, p. 817-844, 1998

Lettau, M. and S.C. Ludvigson, *"Measuring and Modeling Variation in The Risk-Return Tradeoff "*, in **Handbook of Financial Econometrics**, Y. Ait-Sahalia and L.P. Hansen (eds), 2006 (forthcoming).

Lintner, J., *"The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets"*, Review of Economics and Statistics, Vol. 47, p. 13-37, 1965

Maccini, L.J. and A. Pagan, *"Exploring the Role of Inventories in the Business Cycle"*, Manuscript, Australian National University, 2003

Merton, R.,*"An Intertemporal Capital Asset Pricing Model"*, Econometrica, Vol. 41, No. 5, September, p. 867-887, 1973

Robertson, D. and M.R.Wickens, *"Measuring real and nominal shocks and their transmission mechanism under different monetary systems"*, Oxford Bulletin of Economics and Statistics, 59, p. 5-28, 1997.

Schwert, G. W., *"Why Does Stock Market Volatility Change Over Time ?"*, Journal of Finance, Vol. 44, No. 5, p. 1115-1153, December, 1989

Schwert, G. W., *"Stock Market Volatility: Ten Years after the Crash"*, For Brooking-Wharton Conference on Financial Institutions, Available from http://schwert.ssb.rochester.edu/gws.htm, 1998

Scruggs, J.T., *"Resolving the Puzzling Intertemporal Relation between the Market Riskpremium and Conditional Market Variance: A Two-Factor Approach"*, Journal of Finance, Vol. 53, No. 2, p. 575-603, April, 1998

Scruggs, J.T. and P. Glabadanidis, "*Risk Premia and the Dynamic Covariance Between Stock and Bond Returns*", Journal of Financial and Quantitative Analyis, 38, 295-316, 2003.

Sharpe, W.,*"Capital Asset Prices: A theory of Market Equilibrium under Conditions of Risk"*, Journal of Finance, Vol. 19, No. 3, September, p. 425-442, 1964

Shiller, R, **Macromarkets**, Oxford University Press, 1st Edition, 1993

Smith, P.N. and M.R. Wickens,*"Asset pricing with observable stochastic discount factors"*, Journal of Economic Surveys, Vol. 16, p. 397-446, 2002

Smith, P.N., Sorensen, S. and M.R. Wickens, *"General Equilibrium Theories of the Equity Risk Premium: Estimates and Tests"*, Manuscript, University of York, 2005.

Smith, P.N., Sorensen, S. and M.R. Wickens, *"Macroeconomic Sources of FOREX Risk"*, Manuscript, University of York, 2006.

Smith, P.N., Sorensen, S. and M.R. Wickens, *"An Asset Market Integration Test Based on Observable Macroeconomic Stochastic Discount Factors"*, in P. Sinclair (ed), **Exchange Rates, Capital Flows and Policy**, Routledge, 2004

Turner, C.M., Startz, R. and C.R. Nelson, *"A Markov Model of Heteroskedasticity, Risk, and Learning in the Stock Market"*, NBER Working Paper 2818, 1989.

Table1 : Descriptive Statistics

	$i^e_{s,t+1}$	π_{t+1}	Δm_{t+1}	Δy_{t+1}
Mean	4.46	4.27	5.02	3.04
Std. Dev	53.46	3.62	6.02	9.00
Skewness	-0.71	1.03	0.12	-0.59
Kurtosis	5.79	4.70	4.08	5.92
Normality	59.91**	90.10**	21.52**	72.36**
$\rho(x_t, x_{t-1})$	0.07	0.66	0.52	0.37
$\rho(x_t, x_{t-2})$	-0.05	0.60	0.33	0.29
$\rho(x_t, x_{t-3})$	-0.01	0.56	0.33	0.26
$\rho(x_t, x_{t-4})$	-0.01	0.54	0.31	0.21
$\rho(x_t, x_{t-5})$	0.07	0.54	0.33	0.08
$\rho(x_t, x_{t-6})$	-0.03	0.54	0.34	0.10
$\rho(x_t^2, x_{t-1}^2)$	0.05	0.66	0.53	0.27
$\rho(x_t^2, x_{t-2}^2)$	0.12	0.62	0.34	0.14
$\rho(x_t^2, x_{t-3}^2)$	0.15	0.59	0.31	0.14
$\rho(x_t^2, x_{t-4}^2)$	0.08	0.56	0.23	0.05
$\rho(x_t^2, x_{t-5}^2)$	0.10	0.57	0.22	-0.04
$\rho(x_t^2, x_{t-6}^2)$	0.09	0.58	0.26	0.07
$\rho(\|x\|_t, \|x\|_{t-1})$	0.05	0.63	0.44	0.31
$\rho(\|x\|_t, \|x\|_{t-2})$	0.06	0.61	0.26	0.13
$\rho(\|x\|_t, \|x\|_{t-3})$	0.07	0.54	0.22	0.10
$\rho(\|x\|_t, \|x\|_{t-4})$	0.03	0.52	0.23	0.05
$\rho(\|x\|_t, \|x\|_{t-5})$	0.02	0.55	0.20	-0.04
$\rho(\|x\|_t, \|x\|_{t-6})$	0.02	0.52	0.21	0.04

$\rho(.)$ is the correlation and x_t is the relevant column variable

Note: Two stars as superscipt indicates that normality is rejected using 0.99 CV. x refers to variable in first row of table.

Table 2. Estimates of Models 1 to 7

	M1	M2	M3	M4	M5	M6	M7
$V_t(i^e_{t+1})$	3.57	11.14					10.49
	(3.75)	(2.53)					(3.15)
$Cov_t(i^e_{t+1}, \pi_{t+1})$	1	780.28	533.99	-663.67	1	1	1093.17
		(2.98)	(2.10)	(4.10)			(3.95)
$Cov_t(i^e_{t+1}, \Delta m_{t+1})$		-18.49	496.42		1342.50		
		(0.16)	(2.51)		(2.82)		
$Cov_t(i^e_{t+1}, \Delta y_{t+1})$		-312.09	-341.34			-353.60	
		(3.71)	(3.38)			(4.33)	
$\Upsilon_{1987:10,t+1}$	-0.27	-0.27	-0.2791	-0.29	-0.29	-0.26	-0.28
	(2.39)	(0.82)	(0.84)	(1.13)	(1.90)	(1.28)	(1.00)
ν	10.83	9.90	10.05	9.61	9.47	9.84	8.92
	(4.83)	(5.36)	(5.38)	(5.27)	(5.24)	(5.36)	(5.57)
Log Likelihood	-2130.5	-2109.2	-2112.0	-2120.5	-2124.2	-2117.5	-2118.8
LR Risk Premium	8.60	7.54	6.79	9.02	7.58	5.69	8.79
Average Residual	-2.27	-1.34	-0.60	-2.74	-1.41	0.46	-2.60
Risk Share (%)	0.59	11.90	12.11	8.70	11.60	11.80	11.20

Note: Share of risk = $100 \cdot Var(\phi_t)/Var(i^e_{t+1} + \frac{1}{2}V_t(i^e_{t+1}) - \widehat{\theta}\Upsilon_{1987:10,t+1})$

ν = degrees of freedom. LR: Long Run or average. Absolute t-statistics in parenthesis.

Table 3. Estimates of Model 2

$$\mathbf{Y}_{t+1} = \mathbf{A} + \mathbf{B}\mathbf{Y}_t + \mathbf{\Phi}\mathbf{H}_{[1:N,1],t+1} + \mathbf{\Theta}\Upsilon_{1987:10,t+1} + \boldsymbol{\epsilon}_{t+1}$$

$$\boldsymbol{\epsilon}_{t+1} = \mathbf{H}_{t+1}^{\frac{1}{2}}\mathbf{u}_{t+1}, \qquad \mathbf{u}_{t+1} \sim \mathcal{D}(0, \mathbf{I}_4)$$

$$\mathbf{H}_{t+1} = \mathbf{C}\mathbf{C}^\intercal + \mathbf{D}(\mathbf{H}_t - \mathbf{C}\mathbf{C}^\intercal)\mathbf{D}^\intercal + \mathbf{E}(\boldsymbol{\epsilon}_t\boldsymbol{\epsilon}_t^\intercal - \mathbf{C}\mathbf{C}^\intercal)\mathbf{E}^\intercal + \mathbf{G}(\boldsymbol{\eta}_t\boldsymbol{\eta}_t^\intercal - \overline{\mathbf{C}\mathbf{C}}^\intercal)\mathbf{G}^\intercal$$

$$\widehat{A} = \begin{bmatrix} 0 \\ \underset{(5.85)}{0.0967} \\ \underset{(4.42)}{0.1306} \\ \underset{(5.80)}{0.2792} \end{bmatrix}, \quad \widehat{\Phi} = \begin{bmatrix} \underset{(2.53)}{11.14} & \underset{(3.02)}{780.28} & \underset{(0.16)}{-18.49} & \underset{(3.71)}{-312.09} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\widehat{B} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \underset{(0.70)}{0.0016} & \underset{(20.44)}{0.6621} & \underset{(1.19)}{0.0193} & \underset{(0.87)}{-0.0107} \\ \underset{(4.22)}{0.0142} & \underset{(0.86)}{0.0535} & \underset{(15.78)}{0.6007} & \underset{(0.26)}{-0.0062} \\ \underset{(0.16)}{0.0009} & \underset{(2.24)}{-0.2150} & \underset{(0.92)}{0.0475} & \underset{(6.83)}{0.2793} \end{bmatrix}$$

$$\widehat{D} = \begin{bmatrix} \underset{(4.46)}{0.5589} & \underset{(1.67)}{2.4030} & \underset{(3.29)}{3.5705} & \underset{(3.87)}{2.7721} \\ \underset{(3.12)}{0.0233} & \underset{(8.58)}{0.7501} & \underset{(2.08)}{-0.1550} & \underset{(2.50)}{-0.1349} \\ \underset{(1.88)}{0.0362} & \underset{(0.32)}{-0.1029} & \underset{(6.07)}{-0.5781} & \underset{(1.95)}{0.1246} \\ \underset{(4.26)}{0.1483} & \underset{(2.57)}{-1.1940} & \underset{(1.68)}{0.2977} & \underset{(3.07)}{-0.3907} \end{bmatrix}$$

$$\widehat{E} = \begin{bmatrix} \underset{(0.24)}{-0.0085} & \underset{(1.13)}{-0.4953} & \underset{(1.49)}{-0.7441} & \underset{(0.15)}{0.0315} \\ \underset{(1.09)}{-0.0029} & \underset{(6.55)}{0.3002} & \underset{(0.30)}{-0.0100} & \underset{(1.85)}{-0.0290} \\ \underset{(0.72)}{-0.0035} & \underset{(0.53)}{-0.0568} & \underset{(7.27)}{0.5315} & \underset{(2.65)}{-0.1017} \\ \underset{(2.12)}{-0.0170} & \underset{(0.33)}{-0.0489} & \underset{(2.46)}{-0.2386} & \underset{(0.20)}{0.0131} \end{bmatrix}$$

$$\widehat{G} = \begin{bmatrix} \underset{(1.91)}{-0.0661} & \underset{(1.66)}{-1.5521} & \underset{(3.24)}{-1.3589} & \underset{(0.34)}{0.0762} \\ \underset{(3.76)}{-0.0119} & \underset{(0.80)}{0.0736} & \underset{(0.80)}{-0.0202} & \underset{(2.27)}{0.0575} \\ \underset{(0.19)}{-0.0021} & \underset{(0.97)}{-0.2081} & \underset{(0.04)}{0.0060} & \underset{(0.55)}{-0.0347} \\ \underset{(0.92)}{-0.0117} & \underset{(0.74)}{0.2254} & \underset{(0.96)}{-0.1171} & \underset{(6.66)}{0.5432} \end{bmatrix}$$

$$100 \cdot \widehat{C} = \begin{bmatrix} \underset{(6.74)}{4.8236} & 0 & 0 & 0 \\ \underset{(0.07)}{0.0017} & \underset{(8.30)}{0.2369} & 0 & 0 \\ \underset{(1.08)}{0.0435} & \underset{(1.15)}{0.0469} & \underset{(6.88)}{0.4657} & 0 \\ \underset{(1.57)}{0.1404} & \underset{(0.58)}{0.0458} & \underset{(0.64)}{-0.0382} & \underset{(6.99)}{0.7290} \end{bmatrix}$$

$$1200^2 \cdot \widehat{C}\widehat{C}' = \begin{bmatrix} 3350.40 & 1.18 & 30.24 & 97.50 \\ 1.18 & 8.08 & 1.61 & 1.58 \\ 30.24 & 1.61 & 31.82 & -1.38 \\ 97.50 & 1.58 & -1.38 & 79.87 \end{bmatrix}$$

Table 4. Autocorrelation coefficients for risk premia

	ρ_1	ρ_2	ρ_3	ρ_4	ρ_5	ρ_6	ρ_{12}
$\phi_t^{Model\ 1}$	0.98	0.96	0.93	0.91	0.88	0.85	0.65
$\phi_t^{Model\ 2}$	0.43	0.59	0.31	0.30	0.09	0.09	−0.10
$\phi_t^{Model\ 3}$	0.33	0.58	0.26	0.27	0.11	0.07	−0.12
$\phi_t^{Model\ 4}$	0.73	0.61	0.42	0.23	0.09	−0.03	−0.23
$\phi_t^{Model\ 5}$	0.57	0.58	0.41	0.32	0.20	0.17	−0.13
$\phi_t^{Model\ 6}$	0.26	0.60	0.18	0.35	0.05	0.20	−0.07
$\phi_t^{Model\ 7}$	0.64	0.70	0.50	0.47	0.28	0.26	−0.09

Note: Absolute t-statistics in parenthesis.

Table 5: Recession dates and number of observations

60:03-61:05	69:11-70:10	73:11-75:03	80:01-80:07	81:07-82:11	90:07-91:03	01:03-01:11
15	12	17	7	17	9	9

Total no. obs. = 86

Table 6. Summary statistics comparing periods of recession with other periods

Model 2	log return	inflation	money	ind. prod.	risk premium
Mean in recessions	-6.5147	5.9593	5.0598	-7.3957	10.88
Mean elsewhere	6.4664	3.8649	5.0537	4.9132	7.038
Correlation with log returns in recessions	1	-0.1417	0.0920	0.0190	
Correlation with log returns elsewhere	1	-0.1394	0.0723	0.0480	
Mean conditional SD during recessions	54.9312	3.4322	5.7111	10.6250	
Mean conditional SD deviation elsewhere	48.9321	2.4689	4.9801	7.4919	
Mean contributions to risk prem. in recessions	28.6471	-16.7822	-0.2983	-0.6827	1
Mean contributions to risk prem. elsewhere	22.7963	-9.2757	-0.1772	-6.3054	1

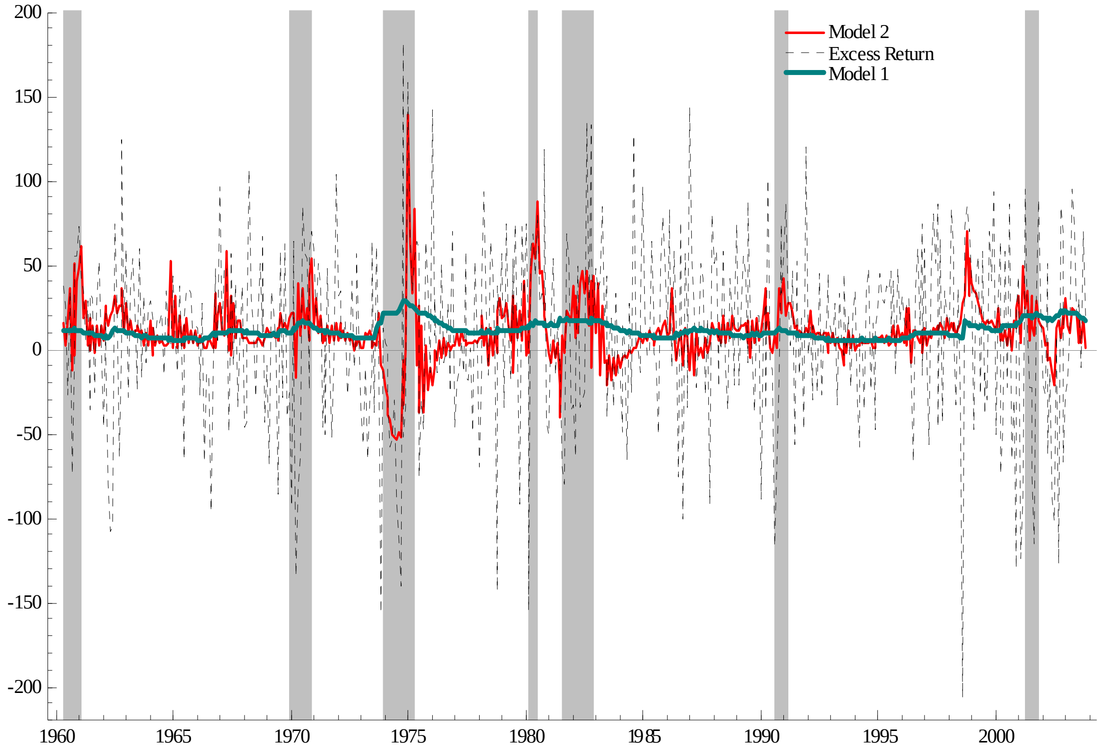
Table 7. Alternative risk premium representations

	Model 2	Model 8	Model 9
$\mathrm{V}_t(i^e_{t+1})$	11.15 (2.53)	5.58 (2.11)	11.45 (2.38)
$\mathrm{Cov}_t(i^e_{t+1}, \pi_{t+1})$	780.28 (2.98)	13.46 (0.12)	876.0 (2.78)
$\mathrm{Cov}_t(i^e_{t+1}, \Delta m_{t+1})$	-18.49 (0.16)	177.15 (2.30)	0.332 (0.00)
$\mathrm{Cov}_t(i^e_{t+1}, \Delta y_{t+1})$	-312.09 (3.71)	-136.45 (2.51)	-337.06 (3.61)
$\mathrm{Cov}_t(\pi_{t+1}, \Delta m_{t+1})$			960.96 (0.70)
$\mathrm{Cov}_t(\pi_{t+1}, \Delta y_{t+1})$		4534.8 (3.96)	
$\Upsilon_{1987:10,t+1}$	-0.27 (0.82)	-0.278 (0.48)	-0.27 (0.67)
ν	9.90 (5.36)	9.58 (5.64)	9.80 (5.38)
Log Likelihood	-2109.2	-2105.0	-2109.0
LR Risk Premium	7.54	6.61	7.54
Average residual	-1.3402	-0.3737	-1.3501
Risk Share (%)	11.90	12.9	12.0

Note: Share of risk = $100 \cdot Var(\phi_t)/Var(i^e_{t+1} + \frac{1}{2}V_t(i^e_{t+1}) - \widehat{\theta}\Upsilon_{1987:10,t+1})$

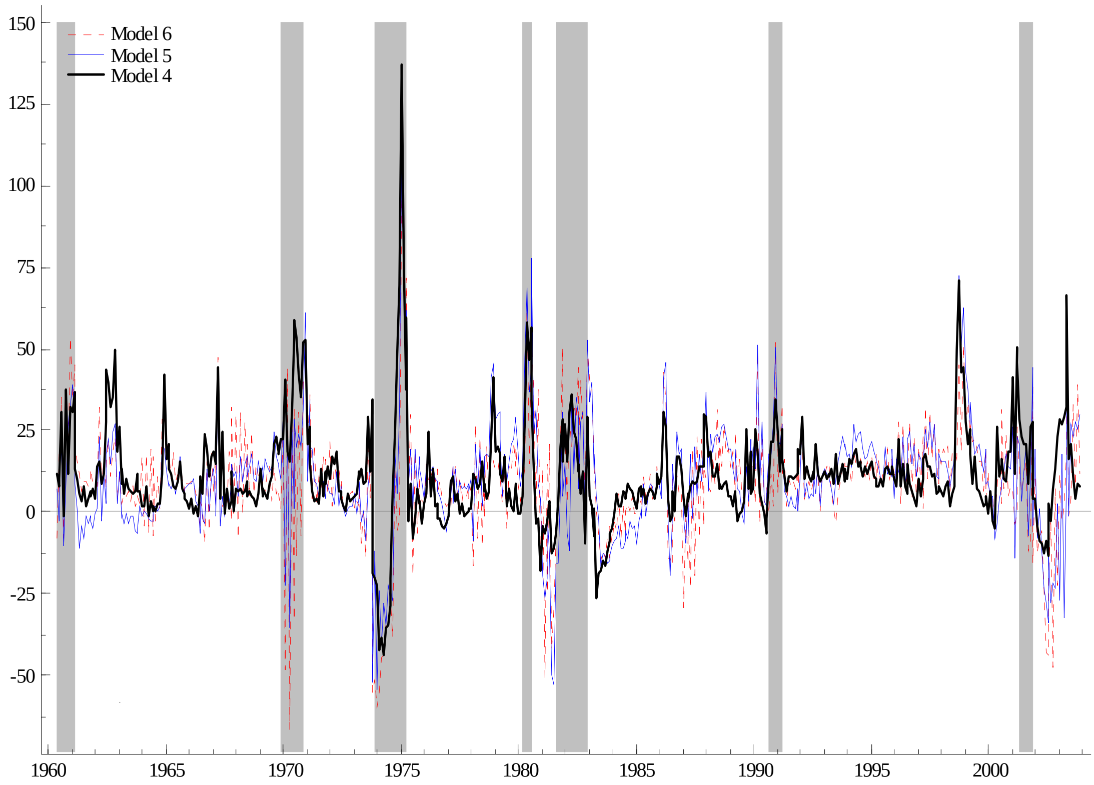
ν = degrees of freedom. LR: Long Run or average. Absolute t-statistics in parenthesis.

Figure 1: Risk premia for Models 1-2 and excess return



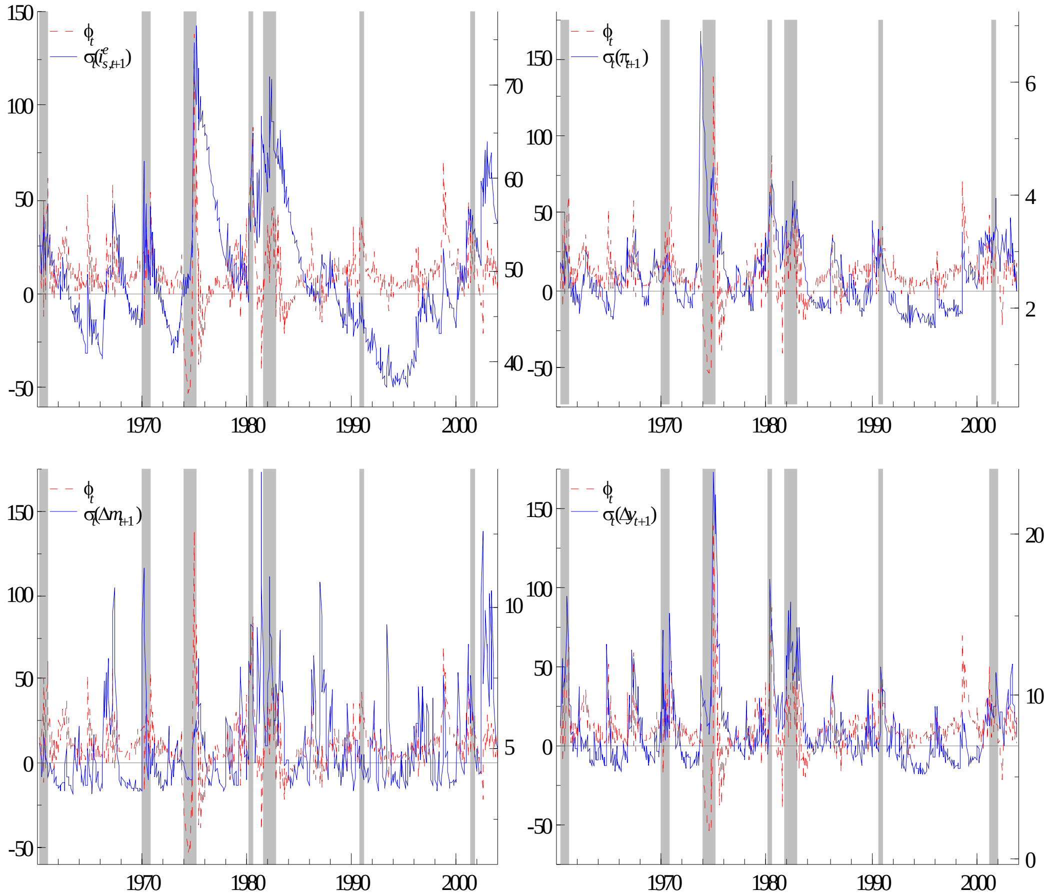
Notes: The excess return is net of the Jensen effect and the October 1987 dummy. The data are measured in annualised percentages. Shaded areas are recessions as defined by the NBER.

Figure 2: Risk premia for Models 4-6



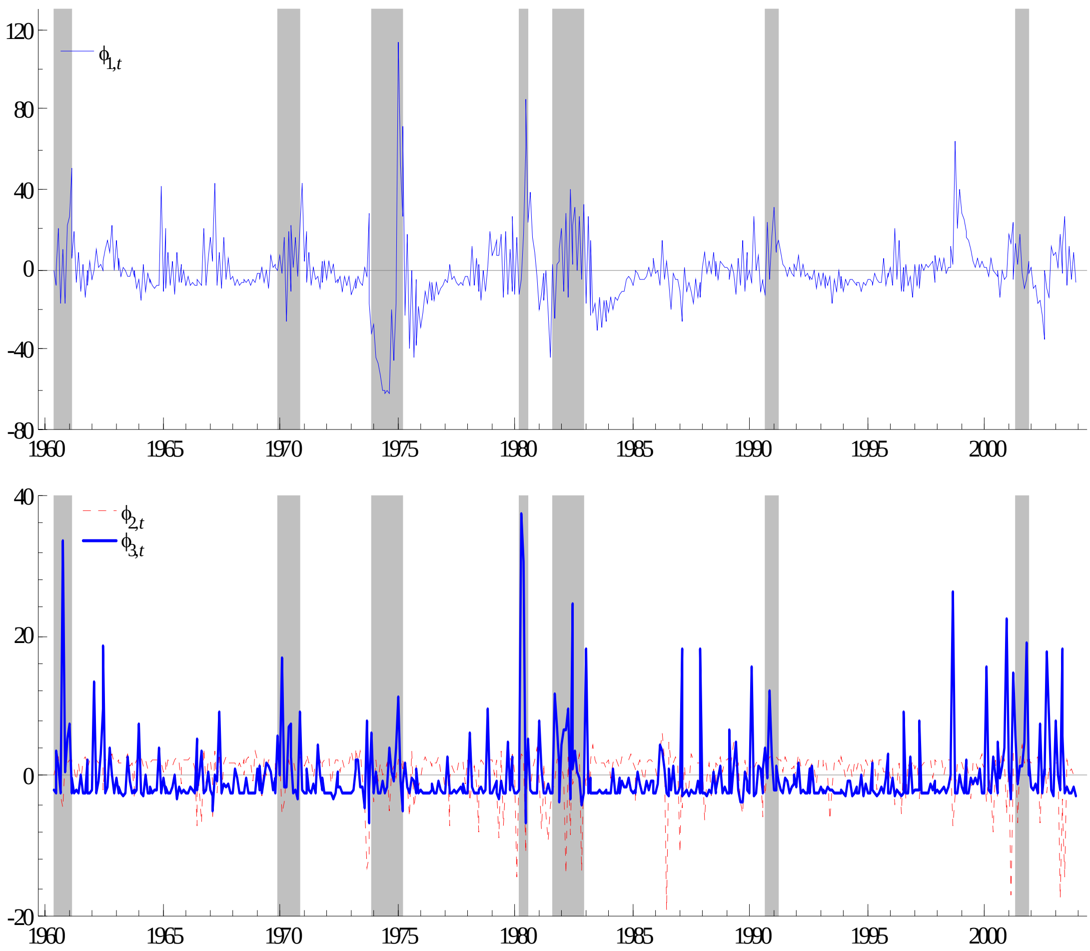
Notes: see Figure 1.

Figure 3: The risk premium and the conditional variances of the factors



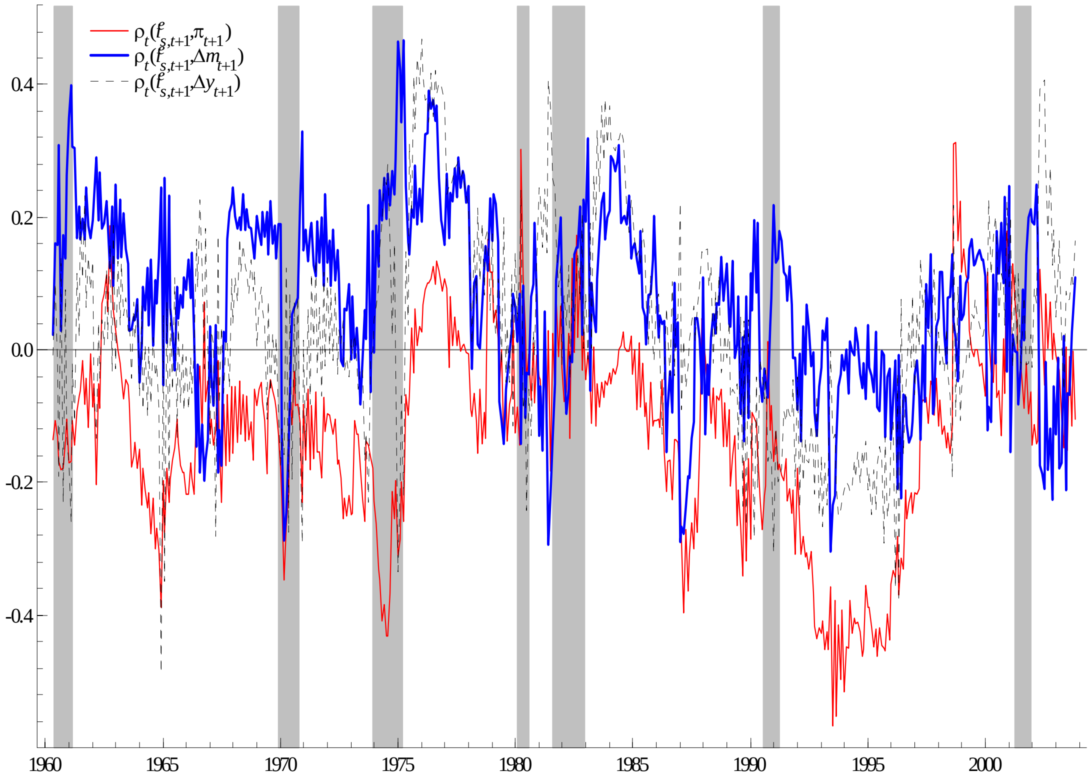
Notes: the scale for the risk premium is on the left axis and that for the correlations is on the right. All are measured in annualised percentages. The unconditional correlations are $\rho(\phi_t, \sigma_t(i^e_{t+1})) = 0.19$, $\rho(\phi_t, \sigma_t(\pi_{t+1})) = 0.04$, $\rho(\phi_t, \sigma_t(\Delta m_{t+1})) = 0.07$, $\rho(\phi_t, \sigma_t(\Delta y_{t+1})) = 0.31$. Shaded are recessions as defined by the NBER.

Figure 4: The contribution to risk of asymmetries



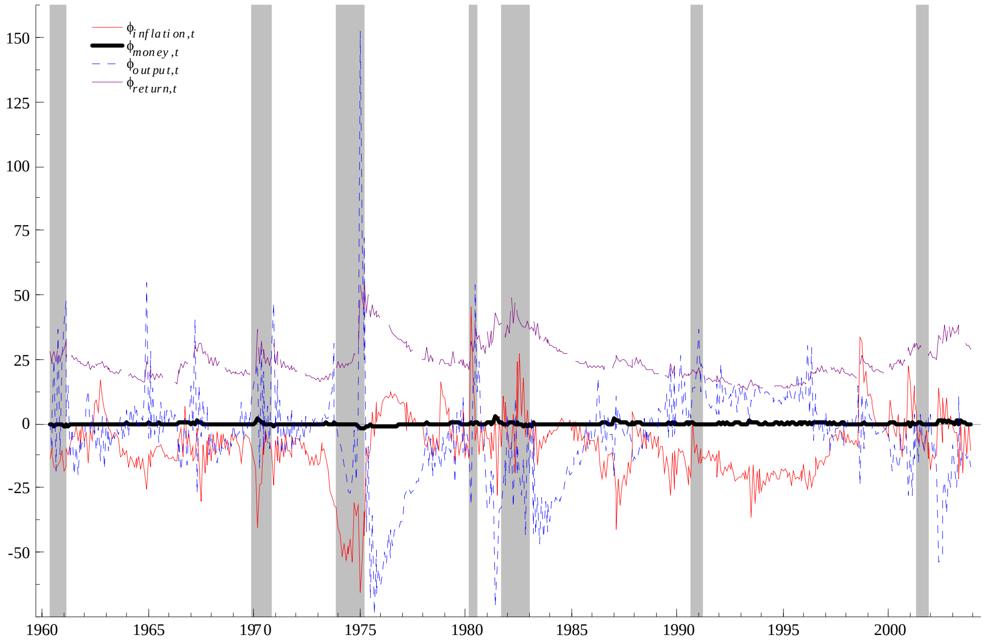
Notes: See figure 1.

Figure 5: Time-varying correlations between the excess return and the factors



Notes: see Figure 1.

Figure 6: The contribution to risk of the macroeconomic factors



Notes: see Figure 1.

Figure 7: The risk premium and time-varying correlation between the factors

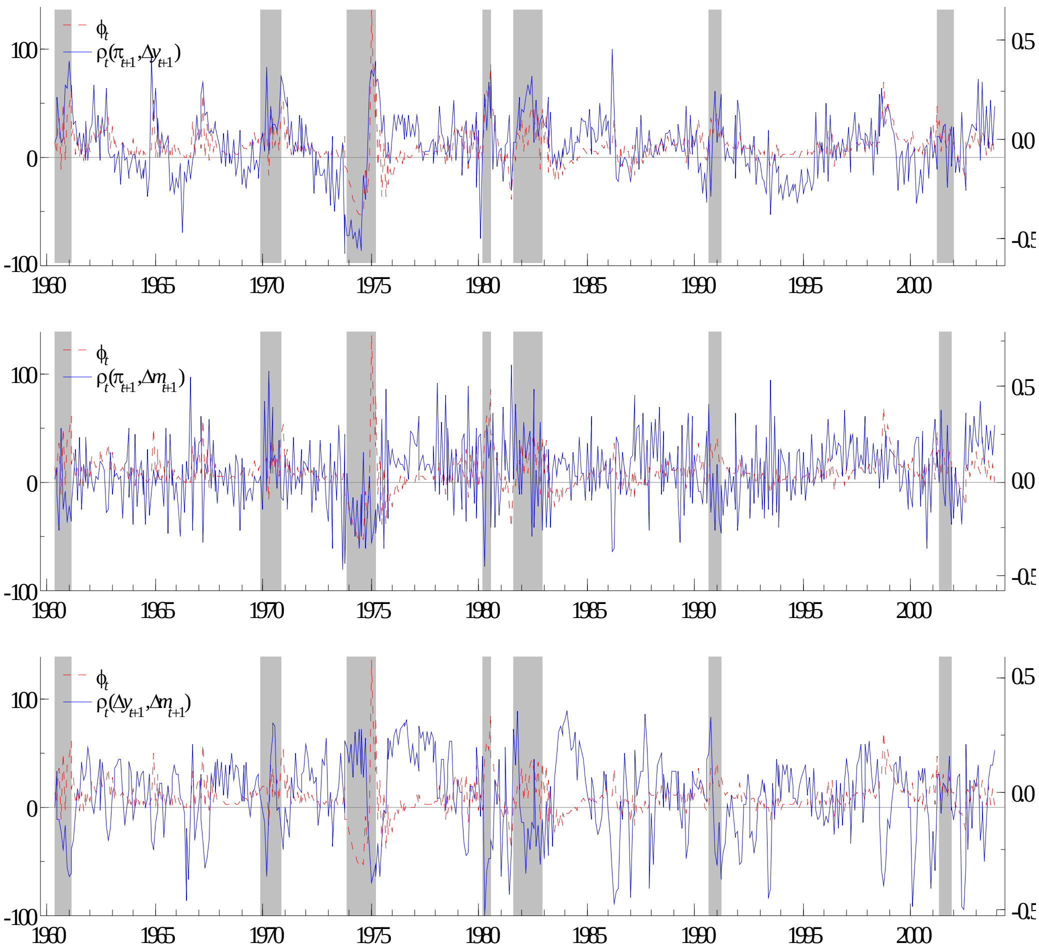


Figure 8: Structural Supply and Demand Shocks in NBER Recession Periods

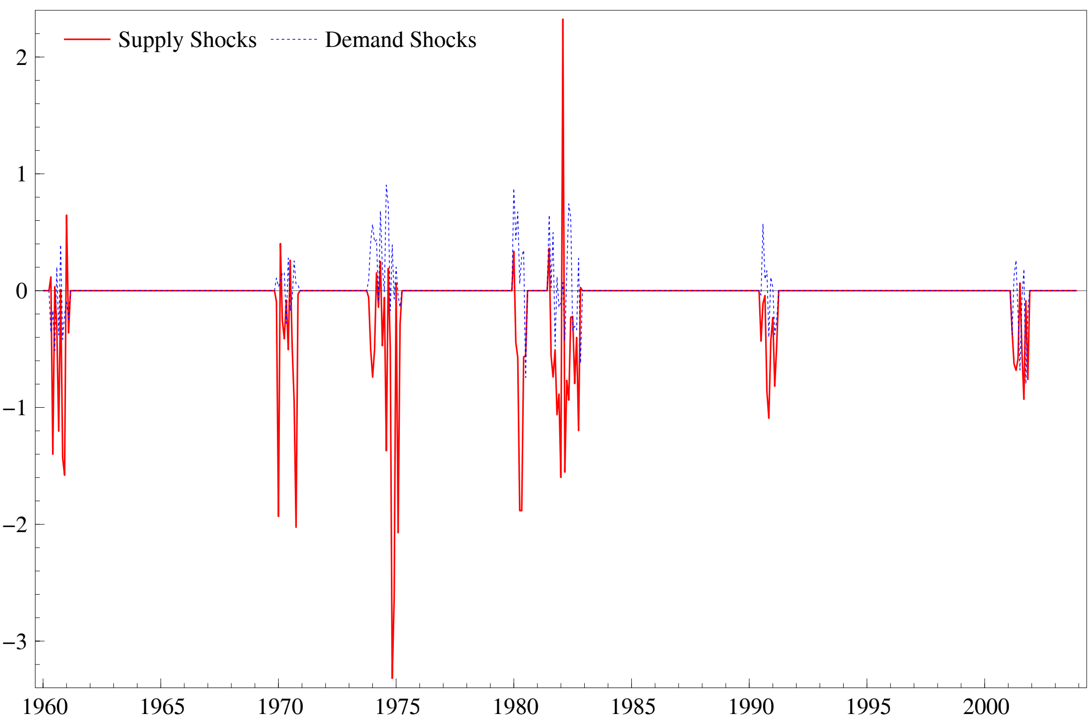
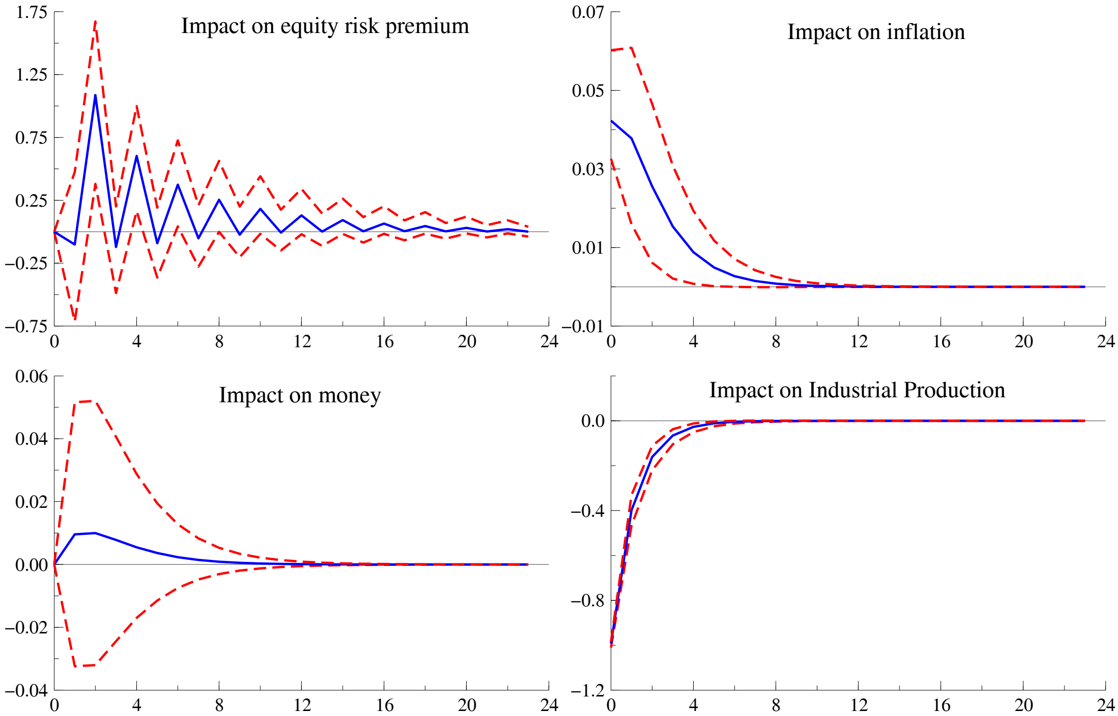


Figure 9: Negative Aggregate Supply Shock (-1%)



Impact on inflation

Figure 10: Positive Aggregate Supply Shock (+1%)

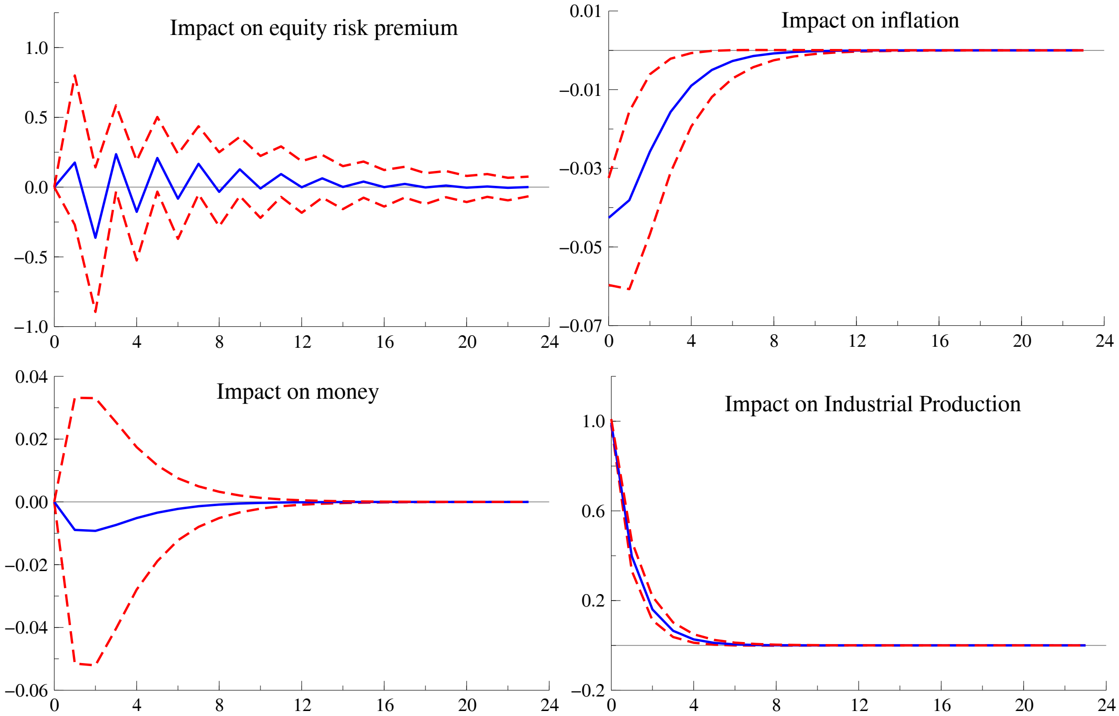


Figure 11: Negative Aggregate Demand Shock (-1%)

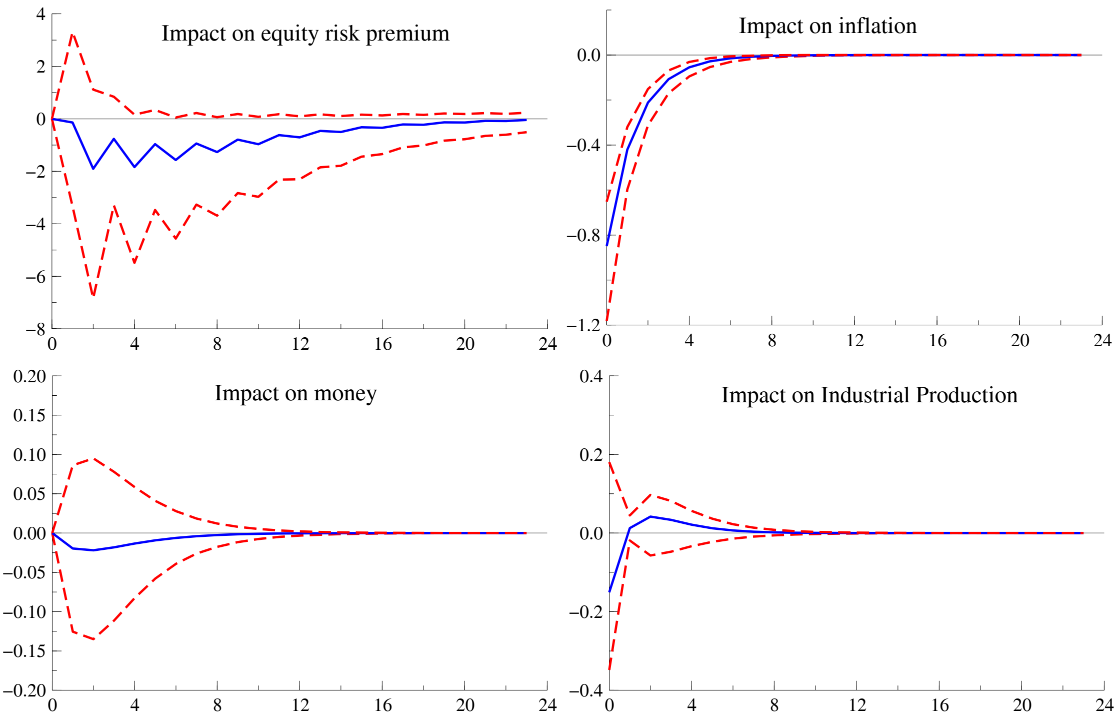
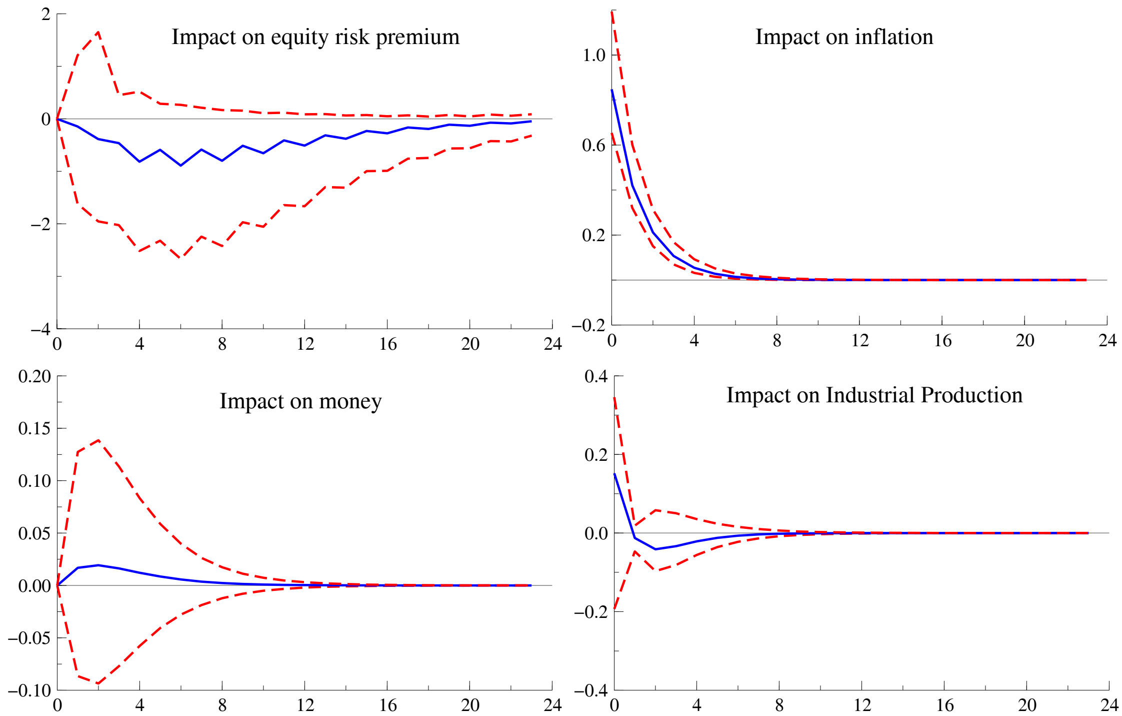
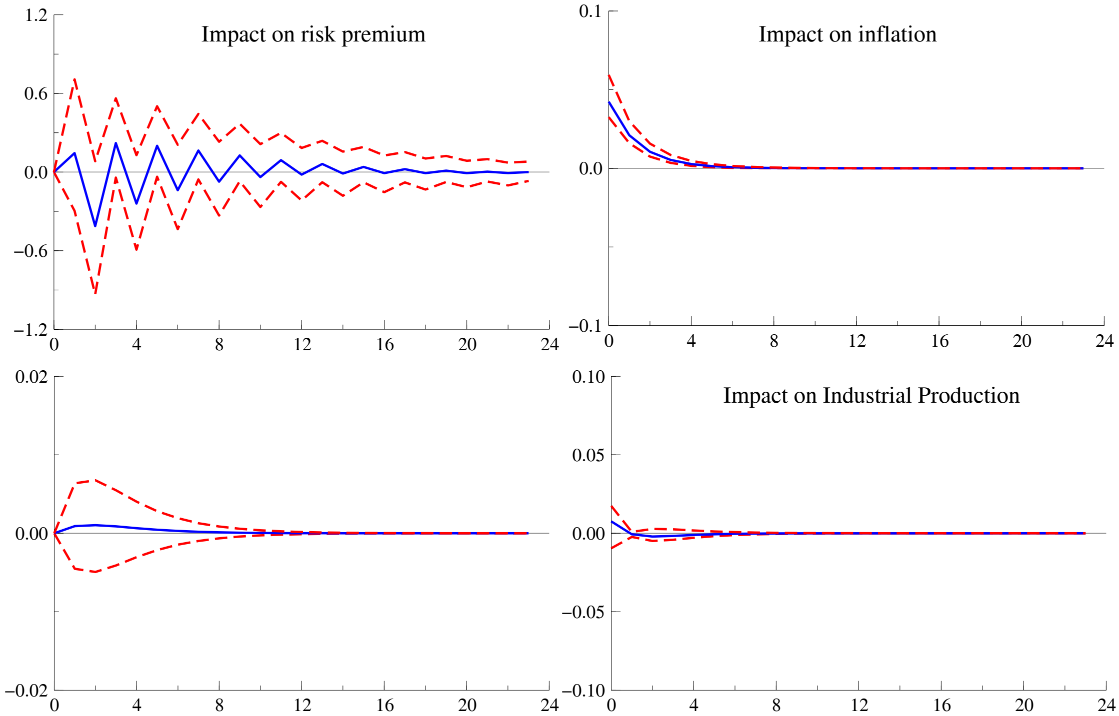


Figure 12: Positive Aggregate Demand Shock (+1%)



Impact on equity risk premium

Figure 13: Small Positive Aggregate Demand Shock (+0.05%)



Impact on equity risk premium

Figure 14: Small Negative Aggregate Demand Shock (-0.05%)

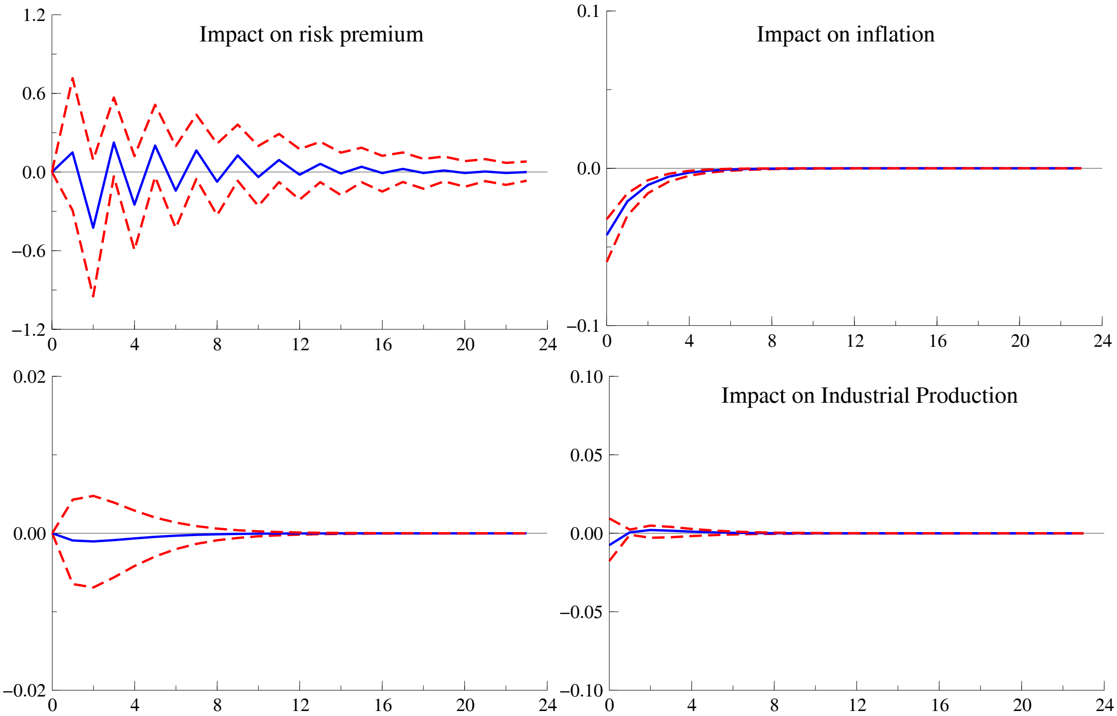


Figure 15: The risk premium per unit of variance

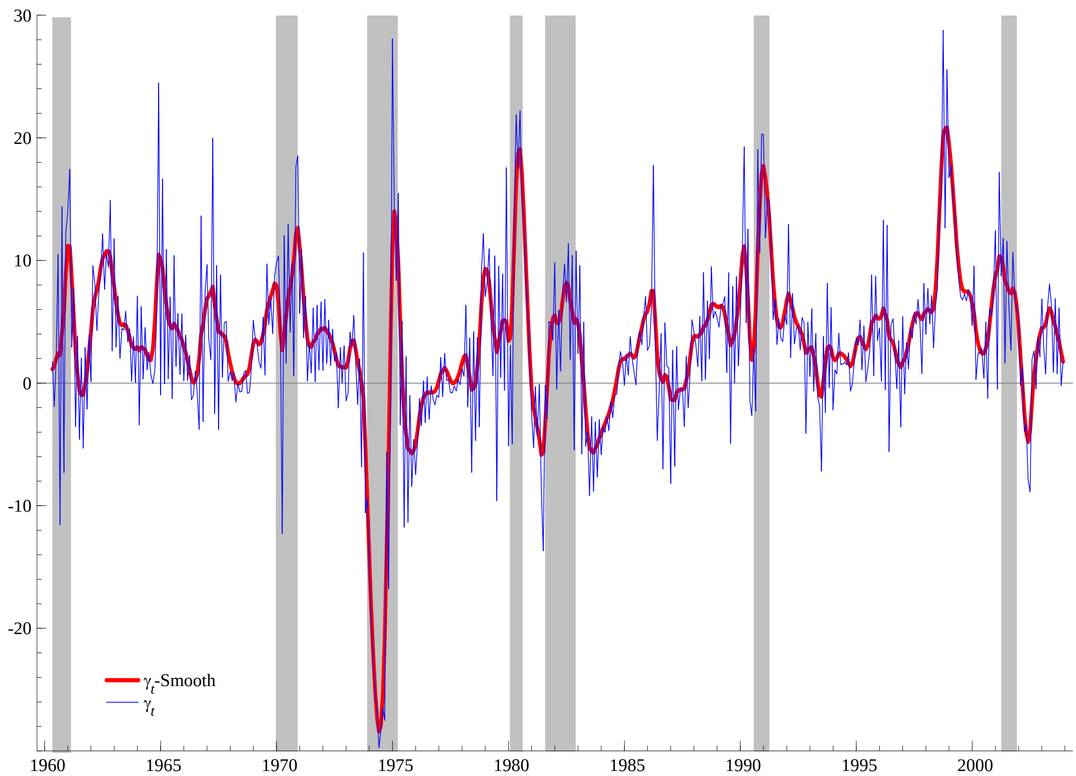


Figure 16: Notes: see Figure 1 γ_t is the risk premium divided by the conditional variance of stock returns in Model 2.