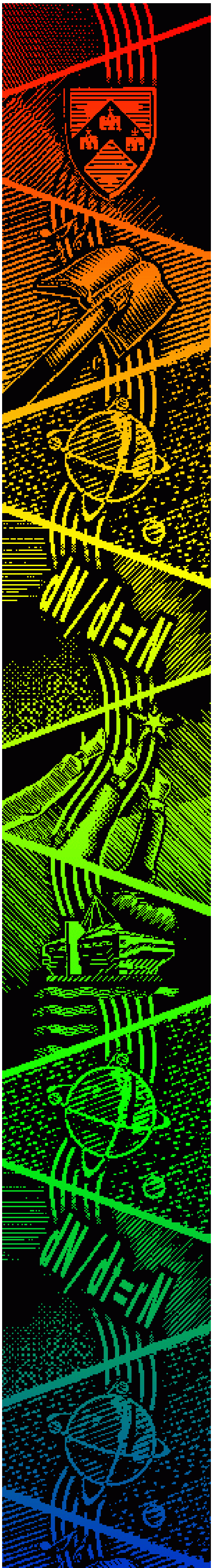

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Risk Sharing, the Cost of Equity and the Optimal Capital Structure of the Regulated Firm

by

Clive J Stones

Department of Economics and Related Studies
University of York
Heslington
York, YO10 5DD

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Clive J Stones
The University of York

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Abstract

This paper considers the relationship between the regulator's pricing decision and the allocation of risk between consumers and shareholders. Consumers are willing to trade-off price variations against a lower expected price. Prices are higher in adverse economic conditions but shareholder returns are not necessarily lower. It might be optimal to insure shareholders against market risk to achieve a lower expected price. The socially optimal capital structure depends on consumers' and shareholders' attitudes to risk. There is only one very special set of conditions where the social optimum is 100% debt finance with the firm operating on a 'not-for-profit' basis.

Key words: Regulation, Gearing, Leverage, Debt finance, Equity finance.
JEL Classification: G320, G380, L510

1 Introduction

Although there has been a considerable amount of theoretical research into incentive schemes for controlling monopoly power, in practice, price controls are generally the only mechanism that is used to regulate privately owned monopolies. The design of such controls, however, involves making a trade-off between allocative and productive efficiency. This arises because a firm's costs are determined by factors outside its control as well as by its own efforts,

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both of which are largely unobservable by a regulator. The theoretical understanding of this trade-off has been explored at length by Laffont and Tirole (1993)

As a result there has been much discussion about the relative merits of the 'price-cap' and the 'rate-of-return' systems of price control. In its extreme form, rate-of-return regulation is simply an arrangement under which the firm's prices are determined by, and continuously adjusted in accordance with, its actual costs. Although this might achieve allocative efficiency and avoid excessive profits it provides no incentive for the firm to reduce costs and achieve productive efficiency. Conversely, under a pure price-cap system, there is a predetermined upper limit on prices. This provides strong incentives to increase productive efficiency but is likely to result in allocative inefficiency since prices can be out of line with costs. In other words, the firm might be able to obtain monopoly rents which create an associated deadweight loss. Consequently, regulatory systems in many parts of the world have evolved into hybrids of both these forms of price control.

While the literature has concentrated on the effect of different price control mechanisms on incentives and the behaviour of the firm, little attention has been given to their effect on consumers and investors. In particular, a rate-of-return system is likely to result in greater variability in prices than a price-cap system implying a higher level of risk for consumers and lower risks for the firm. This in turn suggests that the cost of capital of the regulated firm should be lower under a rate-of-return system if the firm's profits are positively correlated with the return on the market portfolio. From a regulatory policy perspective, therefore, there is a further potential trade-off to consider viz. a trade-off for the consumer between greater price variability and the possibility of a lower expected price.[1]

The capital structure of the regulated firm has also received little attention in the literature. Typically, as Spiegel (1996) notes, an implied assumption is made that the regulated firm is wholly financed by equity with the cost of equity being determined exogenously in the capital markets. Even where the firm's capital structure has been considered, the focus is again on the firm's behaviour and its potential to use high levels of leverage (the debt-capital

[1] Cowan (2004) acknowledges the possibility of a relationship between the form of the price control and the cost of capital but his analysis of the optimal allocation of risk between consumers and the regulated firm takes a different approach. The firm is assumed, instead, to be risk averse with a utility function that is solely dependent on the firm's profits.

ratio) as a means of exerting a price-influence effect on regulatory decisions (e.g. Taggart (1981) and (1985), Dasgupta and Nanda (1993), Spiegel (1994), and Spiegel and Spulber (1994) and (1997)).

In one of the most important early contributions to finance theory, Modigliani and Miller (1958) and (1963) showed that, in the absence of corporate taxation, the value of a firm and its overall cost of capital are unaffected by its capital structure. However, a key assumption in their analysis is that the firm's operating cashflows are determined independently of its capital structure and this might not be applicable to a firm which is subject to price controls. As argued above, the design of the price control mechanism can affect the risks carried by investors. Since this will influence both the amount of equity finance that is needed and, potentially, its cost, the capital structure of the regulated firm may not be irrelevant even in the absence of corporate taxation. It is not surprising, therefore, that risk, the cost of capital and capital structure have been the subject of much debate between regulators and regulated utilities at price-cap reviews in the UK.[2]

The aim of this paper is to investigate the relationship between the form of price control and the allocation of risk between consumers and shareholders. In particular, it considers the implications for the firm's cost of equity and capital structure and examines the conditions for achieving a social optimum.

The question of whether there is a socially optimal capital structure for a regulated firm has previously been considered by De Fraja and Stones (2004). They concluded that, when consumers are risk averse, there is a socially optimal capital structure where consumers carry some risk, in the sense that they are willing to accept a degree of price variability in exchange for a lower expected price. In their model, De Fraja and Stones (2004) assume that the cost of equity is simply a function of the level of debt as a proxy for the relationship between the firm's cost of equity and investment risk. This paper, however, examines the more general case when the level of leverage and the regulator's decision on prices are allowed to have separate and independent effects on the firm's cost of equity.

A number of results are obtained from the analysis. Firstly, consumers pay a higher price in adverse economic conditions because they prefer a lower

[2]For example, Ofwat (1991) and WSA/WCA (1991) contain a particularly extensive exchange of views on these issues that took place between the regulator and the water industry shortly after its privatization.

expected price but, in contrast to De Fraja and Stones (2004), this result holds regardless of the firm's capital structure and not just once leverage reaches a certain level.

Secondly, a key finding from the model presented in this paper is that that the risks and returns for shareholders do not necessarily have to be tied to the firm's underlying business risk. In other words, even though the price is higher in adverse conditions, the returns to shareholders need not be lower. Consequently, the interesting possibility emerges that it might be optimal for the regulator to set prices so that profits are higher in adverse economic conditions. While this may appear to be counter-intuitive, it has a quite natural explanation viz. when consumers have a relatively low aversion to risk they may be willing to be paid (i.e. charged a lower expected price) in order to provide shareholders with insurance against market risk.

Thirdly, even in the absence of corporate taxation, the regulated firm has a socially optimal capital structure that depends both on the consumers' aversion to the risk of price variations and the shareholders' trade-off between risk and returns. This is important as the connection between consumer and shareholder risk aversion has not been explored in the regulatory literature.

Finally, there is only one very special set of conditions where it is socially optimal for the regulated firm to be wholly reliant on debt finance and operate on a 'not-for-profit' basis. In all other cases there should either be a combination of equity and debt or no debt at all. This result is of more than just theoretical interest. In the UK, for example, 'not-for-profit' companies have been established to take over the assets and operations of privatized utilities that have encountered financial difficulties. However, these developments remain controversial and not all proposals to introduce such structures have been successful.[3]

The format of this paper is as follows. Section 2 sets out the model. Section 3 considers the regulator's pricing decision when the firm's capital structure is given exogenously and leads to a number of Lemmas and Propositions about the conditions for achieving an optimum. The detailed analysis is given in appendix A while the proofs of the Lemmas and Propositions are provided in appendices B and C respectively. In section 4 the regulator can vary the firm's capital structure and the conditions under which a social optimum is achieved

[3]For example, Stones (2001) provides a commentary on proposals that have been put forward in the water industry in England and Wales.

are also set out in a series of Propositions. Detailed analysis of the social optimum is contained in appendix D and the proofs of the associated Propositions are provided in appendix E. Finally some conclusions are presented in section 5.

2 The model

2.1 Demand and variable costs

As in De Fraja and Stones (2004), it is assumed that a monopoly firm supplies a product that is fixed in quality. Demand for the product is price inelastic and normalized to 1. The regulator's objective is to maximize the representative consumer's expected utility by choosing the price p that the firm can charge. Assuming the consumer has a standard von Neumann-Morgenstern utility function in income, the consumer's indirect utility function is $U(p)$ with $U'(p)$, $U''(p) < 0$ to reflect risk aversion.

The variable cost of production is made up of two components:[4]

- an exogenously given component $\theta > 0$; and
- a random cost reducing component, which is either $-c$ or 0, with $c > 0$. The probability of this component being $-c$ is $x \in [0, 1]$.

Consequently, there are only two states of the world in cost terms. The variable cost is either high (i.e. the random component is 0) or low (i.e. the random component is $-c$). Subscripts are used to indicate the values of variables in each state of the world (e.g. p_H is the price when cost is high and p_L is the price when cost is low.) This is a simple but standard way of capturing exogenous market uncertainty and the analysis in the paper extends to more complex scenarios.

2.2 Investment and financing

Production also requires a capital investment $M > 0$, which is exogenously given. Debt and equity are the only sources of finance. The extent of debt financing is denoted by $D \in [0, M]$.

[4] De Fraja and Stones (2004) also allow for the firm to be run by a management which has its own utility function and can reduce costs by exerting effort. However, in this model management play no specific role and may be assumed to act merely as the agent of the shareholders. This simplifies the analysis and has no material effect on its conclusions.

The cost of debt is exogenously given at the market interest rate $r_D > 0$. Moreover, lenders are guaranteed that the debt and the interest will be paid under all circumstances and so the firm cannot go bankrupt. Consequently, r_D is also the risk free rate of return.

The firm's shareholders are risk averse and have limited liability in the sense that they cannot be obliged to finance a shortfall in revenue.

2.3 The cost of equity

De Fraja and Stones (2004) make the simplifying assumption that the cost of equity is a function of the level of debt as a proxy for the relationship between the firm's cost of equity and investment risk. This paper, however, considers a more general case. In asset pricing models such as the Capital Asset Pricing Model (CAPM) developed by Sharpe (1964) and Lintner (1965a) and (1965b), the cost of equity r_E depends on the covariance of the shareholders' rate of return R_E with the return on the market portfolio R_m. Accordingly, it is assumed that:

$$r_E = r_E\left(cov\left(R_E, R_m\right)\right), \tag{1}$$

and, using the notation $r_E\left(cov\left(R_E, R_m\right)\right) = r_E\left(.\right)$, that the cost of equity varies directly with the covariance:

$$r_E'\left(.\right) > 0. \tag{2}$$

It is also assumed that the rate of return on the market portfolio when the firm's costs are low, R_{mL} is higher than R_{mH}, the return when its costs are high, so that:[5]

$$\Delta R_m = \left(R_{mL} - R_{mH}\right) > 0. \tag{3}$$

The above assumptions lead to the following Lemma.

Lemma 1 *The covariance of the shareholders' rate of return R_E with the return on the market portfolio R_m is:*

$$cov\left(R_E, R_m\right) = \frac{x\left(1-x\right)}{M-D}\left(c - \left(p_H - p_L\right)\right)\Delta R_m. \tag{4}$$

[5] It should be noted that this does not imply there are two states of the world for the return on the market portfolio. It is only necessary to assume that each outcome for the firm's costs always coincides with a particular level of R_m.

The cost of equity in this model is, therefore, not only a function of the level of debt but, crucially, it is also a function of the regulator's decision on prices. This allows both effects to be taken into account independently.

It can also be seen from Lemma 1 that, under a price cap system of regulation where the upper limit on prices is invariant to cost levels and the firm charges at the limit (i.e. $p_H = p_L$), the cost of equity does not depend on the level of the price cap set by the regulator. At first sight this might seem counterintuitive as it suggests that a tightening of the regulatory contract and the consequent fall in revenue would not result in higher rates of return being required by shareholders. The point to note here is that cost of equity is determined by the covariance of the shareholders' returns with the market and this would be unaffected by a 'one-off' reduction in revenue.[6] Of course, the decrease in revenue would lead to a reduction in the market value of the equity but the cost of equity would not rise.[7]

3 Prices with a given capital structure

3.1 The regulator's problem

It is assumed that the regulator has complete information and there is no information asymmetry between the regulator and the firm. Although this is a strong assumption, it provides a useful starting point from which to investigate the issues raised in this paper. The regulator's problem when the capital structure is given exogenously is to choose p_L and p_H in order to maximize the representative consumer's expected utility subject to two break even constraints on the firm's profits, Π_H and Π_L, and the shareholders' participation constraint.[8] The former preserve the limited liability of shareholders and ensure that the firm meets its obligations to lenders while the latter ensures that the firm's expected profits, $E(\Pi)$, are high enough to cover the cost of equity

[6] This is because the size of the covariance between two variables is not affected if one of the variables is increased or decreased by a constant amount.

[7] Grout (1995) points out that the rate of return required by shareholders in normal times will exceed the cost of equity if the regulator's decision leads shareholders to expect that there will be negative shocks to the firm's revenues at future price cap reviews as a result of further tightening of the regulatory contract.

[8] Although the regulator must set the price before the random component of costs is realised it can be made conditional on the outcome.

and the repayment of the initial equity investment, that is:

$$\Pi_H = (1 + R_{EH})(M - D) \geqslant 0, \tag{5}$$
$$\Pi_L = (1 + R_{EL})(M - D) \geqslant 0, \tag{6}$$
$$E(\Pi) \geqslant (1 + r_E(.))(M - D), \tag{7}$$

where:

$$\begin{aligned} \Pi_H &= p_H - \theta - (1 + r_D) D, & (8) \\ \Pi_L &= p_L - (\theta - c) - (1 + r_D) D, & (9) \\ E(\Pi) &= x(p_L + c) + (1 - x) p_H + \theta - (1 + r_D) D. & (10) \end{aligned}$$

The regulator's problem is, therefore:

$$\max_{p_L, p_H} xU(p_L) + (1 - x) U(p_H) \text{ subject to (5) - (7)} \tag{11}$$

In the following p_L^* and p_H^* are used to represent the solution to the problem (11) while Π_L^* and Π_H^* are the firm's profits, R_{EL}^* and R_{EH}^* are the shareholders' rates of return and $r_E^*(.)$ is the cost of equity produced by that solution. The resulting covariance of shareholder returns with the market return is denoted $cov^*(.)$. The main results are presented in three Propositions.

3.2 Consumer preferences and risk

The first proposition concerns the regulator's decision on prices.

Proposition 1 *The prices chosen by the regulator are such that consumers pay a higher price when the firm's costs are high (i.e. $p_H^* > p_L^*$) and shareholders receive an expected return equal to the cost of equity (i.e. $E(R_E^*) = r_E^*(.)$).*

Proposition 1 states that consumers prefer some variation in prices, with prices being higher when the firm's costs are high. There are no feasible solutions to (11) in which $p_H^* \leq p_L^*$.

This conclusion applies irrespective of the firm's capital structure whereas in De Fraja and Stones (2004) it is only optimal for prices to vary once leverage reaches a certain level. Above that level the cost of equity is fixed in their model and so further increases in leverage reduce the equity capital available to absorb downside risks. This creates a trade-off for consumers in which a reduction in the expected price can only be achieved at the expense of greater price variability. In this model, however, price variations and the firm's capital structure have separate and independent effects on the cost of equity and this allows an optimal variation in consumer prices to be determined at any level of leverage.

As the regulator also wishes to minimize the expected price, the expected rate of return for shareholders at the optimum equals the cost of equity and so their participation constraint (7) is always binding. The firm, therefore, has no incentive to charge prices below those set by the regulator. The slope of this constraint describes the relationship between changes in p_L^* and p_H^* and the cost of equity, as shown by the following Lemma.

Lemma 2 *At the optimum the shareholders' participation constraint satisfies:*

$$\frac{dp_H^*}{dp_L^*} = -\frac{x\,(1 - r_E^{*\prime}\,(.)\,(1-x)\,\Delta R_m)}{(1-x)\left(1 + r_E^{*\prime}\,(.)\,x\Delta R_m\right)}. \tag{12}$$

3.3 Consumer risk and the cost of equity

Proposition 2 considers interior solutions to (11) while Proposition 3 relates to the 'corner' solutions.

Proposition 2 *At the optimum, if the consumer's marginal rate of substitution between a change in p_H^* and p_L^* equals the marginal rate of change in p_H^* and p_L^* required to maintain satisfactory returns for shareholders (i.e.$-\frac{xU'\left(p_L^*\right)}{(1-x)U'\left(p_H^*\right)} = \frac{dp_H^*}{dp_L^*}$), then the cost of equity is not necessarily higher than the cost of debt (i.e. $r_E^*\,(.) \gtreqless r_D$).*

At an internal optimum, the consumer's marginal rate of substitution between a change in p_L^* and p_H^* equals the slope of the shareholders' participation constraint given by (12). In other words, at the optimum the consumer's aversion to risk is such that the loss of utility from a marginal increase in the price variation is just matched by the benefit of the associated reduction in the cost of equity. Proposition 2 states that, at such an optimum, it is possible for the cost of equity to be higher than, lower than or equal to the cost of debt depending on the different risk profiles of consumers and shareholders. This arises because a key feature of this model is that it gives the regulator the option of setting prices which allow shareholders to receive higher profits when the firm's variable costs are high; an option that is unavailable to the regulator in De Fraja and Stones (2004). In other words, by manipulating consumer prices, the regulator can change the risks carried by shareholders and arrange for their returns to be positively correlated, uncorrelated or even negatively correlated with the market return (since the firm's costs are assumed in (3) to vary inversely with the market return). The last possibility would provide shareholders with a form of insurance against market risk and so the cost of equity would be lower than the cost of debt.

Proposition 3 *At the optimum, if the consumer's marginal rate of substitution between a change in p_H^* and p_L^* is greater (less) than the marginal rate of change in p_H^* and p_L^* required to maintain satisfactory returns for shareholders (i.e.$-\frac{xU'(p_L^*)}{(1-x)U'(p_H^*)} \gtrless \frac{dp_H^*}{dp_L^*}$), then the cost of equity is higher (lower) than the cost of debt (i.e. $r_E^*(.) \gtrless r_D$).*

If the consumer's marginal rate of substitution between a change in p_L^* and p_H^* is greater than the slope of the shareholders' participation constraint at the optimum, then consumers have a relatively high aversion to risk compared with shareholders. In these circumstances it is optimal for consumers to carry relatively less risk and there is a corner solution where profits are higher when the firm's costs are low and zero when its costs are high. The cost of equity is, therefore, higher than the cost of debt.

Conversely, if the consumer's marginal rate of substitution between a change in p_L^* and p_H^* is less than the slope of the shareholders' participation constraint, then consumers have a relatively low aversion to risk compared with shareholders. Consumers should, therefore, carry relatively more risk and the optimum is where profits are higher when costs are high and zero when costs are low. Since shareholder returns vary inversely with the market return, the cost of equity is lower than the cost of debt.

4 The socially optimal capital structure

4.1 The regulator's problem

In the previous section the level of debt was taken as given. However, since prices vary with the level of debt, so does welfare which is obtained by substituting the values of p_H^* and p_L^* into the regulator's payoff function in (11).

Consequently, the regulator's problem when the capital structure can be varied is to choose a level of debt that maximizes consumer welfare subject to the constraints that the capital investment is financed wholly by equity or debt or a mixture of both:

$$D \geqslant 0, \tag{13}$$

$$M - D \geqslant 0. \tag{14}$$

The problem can, therefore, be stated as:

$$\max_{D} \ xU\left(p_L^*\left(D\right)\right) + \left(1-x\right)U\left(p_H^*\left(D\right)\right) \text{ subject to (13) and (14).} \tag{15}$$

The solution to (15) is denoted by D^*, the socially optimal level of debt. By comparing the possible solutions to problem (15) with those relating to (11) the conditions under which an optimum is not only feasible but also represents a social optimum can be determined. The main results are set out in four propositions.

4.2 Zero leverage

Proposition 4 *At a social optimum, if the consumer's marginal rate of substitution between a change in p_H^* and p_L^* does not equal the marginal rate of change in p_H^* and p_L^* required to maintain satisfactory returns for shareholders (i.e. $-\frac{xU'\left(p_L^*\right)}{(1-x)U'\left(p_H^*\right)} \neq \frac{dp_H^*}{dp_L^*}$), then leverage is zero (i.e. $D^* = 0$).*

Proposition 4 states that the socially optimal level of leverage is zero when Proposition 3 applies, i.e. the consumer's marginal rate of substitution between a change in p_L^* and p_H^* does not equal the slope of the shareholders' participation constraint at the optimum. There are two possibilities.

Firstly, if the consumer's risk aversion is relatively high, the optimum is the lowest possible variation in prices. This is where the firm only makes profits when its costs are low, in which case the cost of equity exceeds the cost of debt. Consumers also benefit from lower levels of leverage. Although this increases the expected price, high risk aversion means the loss in the consumer's utility is more than offset by the benefit obtained from a smaller variation in prices.[9]

[9] When $\frac{dp_H^*}{dp_L^*} \lesseqgtr -\frac{xU'\left(p_L^*\right)}{(1-x)U'\left(p_H^*\right)}$ it can be shown that $\frac{d\left(E\left(p^*\right)\right)}{dD} \lesseqgtr 0$ and $p_H^{*\prime}\left(D\right) - p_L^{*\prime}\left(D\right) \gtreqless 0$.

Consequently, the socially optimal capital structure is a corner solution to (15) where the firm is wholly financed from equity.

Secondly, if the converse applies and consumers have a relatively low aversion to risk, then the optimum is the lowest possible expected price. This is where the firm only makes profits when costs are high and so the cost of equity is lower than the cost of debt. Again consumers gain from a reduction in leverage; the loss in utility being relatively small compared to the benefit of a lower expected price. The socially optimal capital structure is, therefore, achieved where the firm is, again, wholly financed from equity. The regulator's ability to manipulate prices so that the cost of equity can, if necessary, be lower than the cost of debt explains why there is no corner solution to (15) in which the social optimum is 100% debt finance.

4.3 A combination of debt and equity

Proposition 5 *At a social optimum, if the consumer's marginal rate of substitution between a change in p_H^* and p_L^* equals the marginal rate of change in p_H^* and p_L^* required to maintain satisfactory returns for shareholders (i.e. $-\frac{xU'(p_L^*)}{(1-x)U'(p_H^*)} = \frac{dp_H^*}{dp_L^*}$), then leverage is positive but below 100% when the cost of equity is not equal to the cost of debt (i.e. $0 < D^* < M$ when $r_E^*(.) \neq r_D$); otherwise the optimal level of leverage is 100% (i.e. $D^* = M$ when $r_E^*(.) = r_D$).*

Proposition 5 states that the socially optimal capital structure is a combination of debt and equity finance when the consumer's marginal rate of substitution between a change in p_L^* and p_H^* is equal to the slope of the shareholders' participation constraint at the optimum. In other words, the solution to (15) can only be an internal optimum when Proposition 2 also applies. The cost of equity can, therefore, be higher than, lower than or equal to the cost of debt. If the cost of equity at the optimum does not equal the cost of debt then the socially optimal level of leverage is less than 100%. Clearly, if the social optimum coincides with the position where the cost of equity equals the cost of debt, this is equivalent to the firm being wholly financed by debt with consumers carrying all the business risk.[10]

[10]When $cov^*(.) = 0$ it can be seen from Lemma 1 that $p_H^* - p_L^* = c$.

4.4 100% debt finance

The conditions under which 100% debt finance is the social optimum can be examined further by considering the relationship between the cost of equity and the cost of debt at a social optimum. This is the subject of Proposition 6.

Proposition 6 *If the relationship between the cost of equity and the covariance between the shareholders' returns and the market return is linear, then the risk premium received by shareholders at the social optimum is directly proportional to the covariance (i.e.* $r_E^*(.) - r_D = kcov^*(.)$ *where* $k = r_E^{*\prime}(.)$ *is a constant).*

Proposition 6 follows from Proposition 1 in that the shareholders' participation constraint (7) must be binding at all levels of debt and so, as the proof of this Proposition shows, the following relationship must hold at a social optimum:

$$r_E^*(.) = r_D + r_E^{*\prime}(.)\, cov^*(.) \tag{16}$$

This relationship demonstrates the linkage between the general form of the function (1) used in this model and asset pricing models such as the CAPM. Clearly, if (1) is linear (i.e. $r_E^{*\prime\prime}(.) = 0$) then, at a social optimum, shareholders receive a risk premium which is directly proportional to the covariance between the shareholders' returns and the market return. This is also the case in the CAPM.[11]

Proposition 6 leads to the following result which is the main finding of this section.

Proposition 7 *If the relationship between the cost of equity and the covariance between the shareholders' returns and the market return is linear, then the social optimum satisfies a leverage level of 100% (i.e.* $D^* = M$*) for a set of parameter values that has measure zero in the parameter space.*

Proposition 7 specifies the conditions under which it is socially optimal for the regulated firm to be wholly reliant on debt for its external finance. The Proposition can be explained by noting that the slope of (16) satisfies:

[11] If the CAPM applies in this model then $r_E^*(.) = r_D + \frac{cov^*(.)}{\sigma_m^2}(E(R_m) - r_D)$ and $r_E^*(.)$ is linear since $r_E^{*\prime}(.) = \frac{(E(R_m) - r_D)}{\sigma_m^2}$ is a constant.

$$r_E^{*\prime\prime}(.)\, cov^*(.) = 0. \tag{17}$$

Clearly, if (1) is non-linear (i.e. $r_E^{*\prime\prime}(.) \neq 0$), there can only be a social optimum where the cost of equity is equal to the cost of debt and this is equivalent to the firm being wholly financed by debt. Although the model makes no assumptions about the shape of (1), much of finance theory assumes that the shareholders' utility is determined by the mean and standard deviation of portfolio income. Under these assumptions the CAPM would apply and the socially optimal capital structure when (1) is non-linear would be of no significance.

If (1) is linear then (17) is satisfied by any value of the covariance. However, Propositions 4 and 5 show that if the cost of equity does not equal the cost of debt at the social optimum, then optimal level of leverage is less than 100%. They also show that 100% debt finance can only be a feasible solution at an internal optimum. Since the slope of the shareholders' participation constraint is constant when (1) is linear and the consumer's utility function is concave, it follows that there is only one very special case in which the consumer's marginal rate of substitution between a change in p_L^* and p_H^* is equal to the slope of the constraint at a point where the cost of equity is equal to the cost of debt. The set of parameter values that would produce such a solution, therefore, has measure zero in the parameter space. In other words, any change in the value of any of the parameters, however small, would move the social optimum away from the position where all investors are satisfied with a rate of return equal to the cost of debt.

5 Conclusions

Although the model discussed in this paper is highly stylized and assumes the regulator has complete information, it contains features that are of interest from a regulatory policy perspective.

Firstly, it is shown that consumers are willing to accept some variation in prices in exchange for a lower expected price. A price cap system in which prices are fixed or vary only in exceptional circumstances is, therefore, almost certainly sub-optimal. In practice, however, even under price cap systems, prices do change in response to changes in the firm's costs. For example, there are often 'cost pass through' arrangements which allow price adjustments in specified circumstances and there is generally a complete reassessment of costs at price cap reviews. While such arrangements are generally intended to ensure that efficient firms remain viable, this paper suggests that regulators should also consider the extent to which prices should be allowed to vary in response to systematic or undiversifiable risks.

Secondly, although prices should be higher in adverse economic conditions, a key finding from the analysis is that the returns to shareholders need not be lower. This novel result, which at first sight seems counter-intuitive, arises because the regulator has the option of setting prices so that the risks carried by shareholders and, therefore, the cost of equity do not have to be tied to the firm's business risks. For example, consumers might have such a low aversion to risk that prices should be set so that shareholders receive higher returns when economic conditions are unfavourable; the effect being to provide shareholders with insurance against market risk. In return, consumers would benefit from a lower expected price as the cost of equity would then be lower than the cost of debt.

An important implication of this finding is that regulators should only consider the cost of equity as being exogenously determined if the methodology for setting prices is itself fixed and not subject to alteration. Since regulators generally do have discretion about they way in which they reach decisions on prices, it follows that they should adjust the allowed rate of return to reflect any changes in methodology which affect the allocation of risk between consumers and investors. Such considerations should, however, be distinguished from the term 'regulatory risk'. This is normally used to describe the asymmetric downside risks of arbitrary regulatory or political interventions that tighten the regulatory contract. The model assumes that the allocation of systematic risks between shareholders and consumers is fully reflected in the allowed rate of return. A regulatory risk is that, in practice, this might not occur.

Thirdly, it is concluded that capital structure does matter for the regulated firm even in the absence of corporate taxation. This is because the amount of equity finance, in conjunction with the regulator's decision on prices, determines how risks are distributed between consumers and shareholders. There is, therefore, a socially optimal capital structure that depends on their respective attitudes to risk.

Finally, it is shown to be highly unlikely that a social optimum would be achieved at 100% leverage. When consumers have a relatively high marginal aversion to risk compared to shareholders they prefer the lowest possible price variation, even though this involves a higher expected price. Conversely, when the risk aversion of consumers is relatively low they prefer the lowest possible expected price and it is optimal to set prices so that the cost of equity is lower than the cost of debt. In either case, debt finance produces no benefit for consumers and the optimal capital structure is zero leverage. However, if consumers and shareholders both have the same marginal aversion to risk, there is only one very special set of conditions where it is socially optimal for the regulated firm to operate on a 'not-for-profit' basis and rely wholly on debt for its external finance. Apart from this unique case, the social optimum is the combination of debt and equity finance that balances the distribution of risks.

In the UK there have been two relatively recent cases where 'not-for-profit' companies have been established to replace privatized utilities. In 2001 Glas Cymru acquired Welsh Water and in the following year Network Rail was created to take over the operations of Railtrack. In both instances the replacement was a 'company limited by guarantee' and, because such companies have no equity interest, they are wholly reliant on the debt markets for external finance. Further, both of the former privatized utilities were in financial difficulties and a primary aim of the new arrangements was to secure the companies' long term finances. Consequently, when the replacement companies were being established, the attitude of lenders towards risk and, therefore, the terms on which the new debt finance could be raised, was a key consideration. While the outcome might have been satisfactory for the lenders, this paper shows it is very unlikely that the resulting risk profile will be optimal for consumers.

Appendix A Solutions for a given capital structure

The Lagrangian function for the problem (11) is:

$$\begin{aligned}\mathcal{L} &= xU(p_L) + (1-x)U(p_H) \\ &+ \pi_H(p_H - \theta - (1+r_D)D) + \pi_L(p_L - \theta + c - (1+r_D)D) \\ &+ \pi(x(p_L + c) + (1-x)(p_H) - \theta - (1+r_D)D - (1+r_E(.))(M-D)).\end{aligned} \tag{18}$$

where π_H, π_L and π are the multipliers associated with constraints (5) to (7).

Using (4) from Lemma 1 the first order conditions for an optimum are:

$$\frac{\partial \mathcal{L}}{\partial p_L} = xU'(p_L) + \pi_L + x\pi - \pi r'_E(.)\,x(1-x)\,\Delta R_m = 0, \tag{19}$$

$$\frac{\partial \mathcal{L}}{\partial p_H} = (1-x)U'(p_H) + \pi_H + (1-x)\pi + \pi r'_E(.)\,x(1-x)\,\Delta R_m = 0. \tag{20}$$

Rearranging these gives:

$$\pi_L = -xU'(p_L) - x\pi\left(1 - r'_E(.)(1-x)\Delta R_m\right), \tag{21}$$

$$\pi_H = -(1-x)U'(p_H) - (1-x)\pi\left(1 + r'_E(.)\,x\Delta R_m\right). \tag{22}$$

The possible solutions are determined by the values of π, π_L, and π_H . There are four Cases.

Case 1 ($\pi_H, \pi_L > 0$): Both (5) and (6) are binding. However, this is not feasible as it would violate the shareholders' participation constraint (7). Consequently, in all feasible solutions to the problem:

$$1 + r^*_E(.) > 0. \tag{23}$$

The conditions $\pi_H > 0$ and $\pi_L > 0$ in effect restrict the problem to finding a solution when there are no shareholders and, therefore, $D = M$. However, this restricted form of the problem can be considered as a special case of the more general problem in which shareholders do participate and the solution is such that $cov^*(.) = 0$ i.e. $r^*_E(.) = r_D$.

Case 2 ($\pi_L = \pi_H = 0$): Neither (5) nor (6) are binding and so $\Pi_L^*, \Pi_H^* > 0$. From (21) and (22) the optimum satisfies:

$$\pi = \frac{-U'(p_L^*)}{1 - r_E^{*\prime}(.)(1-x)\Delta R_m} = \frac{-U'(p_H^*)}{1 + r_E^{*\prime}(.)\,x\Delta R_m} = -\left(xU'(p_L^*) + (1-x)U'(p_H^*)\right). \tag{24}$$

Since $U'(p) < 0$ it follows that $\pi > 0$, i.e. (7) is binding.

Using (12) from Lemma 2 the optimum must, therefore, satisfy the conditions:

$$-\frac{xU'(p_L^*)}{(1-x)U'\left(p_H^*\right)} = \frac{dp_H^*}{dp_L^*}, \tag{25}$$

and:

$$1 > r_E^{*\prime}(.)(1-x)\Delta R_m. \tag{26}$$

From (2), (3) and (25) it also follows that $p_H^* > p_L^*$. The two prices and the expected price are:

$$\begin{aligned} p_H^* &= \theta + (1 + r_D)D + (1 + R_{EH}^*)(M - D), & (27)\\ p_L^* &= \theta - c + (1 + r_D)D + (1 + R_{EL}^*)(M - D), & (28)\\ E(p^*) &= \theta - xc + (1 + r_D)D + (1 + r_E^*(.))(M - D). & (29) \end{aligned}$$

As the price difference is:

$$p_H^* - p_L^* = c - (R_{EL}^* - R_{EH}^*)(M - D). \tag{30}$$

it follows that:

$$c > (R_{EL}^* - R_{EH}^*)(M - D). \tag{31}$$

The price difference, therefore, determines whether profits are higher or lower when the firm's costs are low (and vice versa).

Case 3 ($\pi_H > 0$,$\pi_L = 0$): The only binding break-even constraint is (5) and so $\Pi_L^* > 0$ and $\Pi_H^* = 0$. From (21) the optimum satisfies:

$$-U'(p_L^*) = \pi\left(1 - r_E^{*\prime}(.)(1-x)\Delta R_m\right). \tag{32}$$

Since $U'(p) < 0$ and $\pi < 0$ is not feasible, it follows that $\pi > 0$, i.e. (7) is binding. Further, (26) must also hold.

From (22), (32) and using (12) from Lemma 2 the optimum must, therefore, satisfy:

$$-\frac{xU'(p_L^*)}{(1-x)\,U'\left(p_H^*\right)} > \frac{dp_H^*}{dp_L^*}. \tag{33}$$

Consequently, Case 3 cannot be a feasible solution at the same time as Case 2.

From (2), (3) and (33) it follows that $p_H^* > p_L^*$. The prices are:

$$\begin{aligned} p_H^* &= \theta + (1 + r_D)\,D. && (34) \\ p_L^* &= \theta - c + (1 + r_D)\,D + \frac{1 + r_E^*(.)}{x}(M - D)\,, && (35) \end{aligned}$$

and the expected price is again given by (29).

As the price difference is:

$$p_H^* - p_L^* = c - \frac{1}{x}\left(1 + r_E^*(.)\right)(M - D)\,, \tag{36}$$

it follows that:

$$xc > \left(1 + r_E^*(.)\right)(M - D)\,. \tag{37}$$

In other words, when consumers carry some risk the expected variation in costs exceeds the expected return to shareholders.

Case 4 ($\pi_H = 0, \pi_L > 0$): The only binding break even constraint is (6) and so $\Pi_H^* > 0$ and $\Pi_L^* = 0$. From (22) the optimum satisfies:

$$-U'(p_H^*) = \pi\left(1 + r_E^{*\prime}(.)\,x\Delta R_m\right). \tag{38}$$

Since $U'(p) < 0$ it follows from (2), and (3) that $\pi > 0$, i.e. (7) is binding.

From (21), (38) and using (12) from Lemma 2 the optimum must, therefore, satisfy:

$$-\frac{xU'(p_L^*)}{(1-x)\,U'\left(p_H^*\right)} < \frac{dp_H^*}{dp_L^*}. \tag{39}$$

Consequently, Case 4 cannot be a feasible solution at the same time as Case 2 or Case 3.

Since (38) and (39) do not require (26) to apply in this Case then $\frac{dp_H^*}{dp_L^*} \lesseqqgtr 0$ according to whether $1 \gtreqqless r_E^{*\prime}(.)\,(1 - x)\,\Delta R_m$.

The prices are:

$$
\begin{aligned}
p_H^* &= \theta + (1 + r_D)\, D + \frac{1 + r_E^*(.)}{1 - x}\,(M - D)\,, && (40)\\
p_L^* &= \theta - c + (1 + r_D)\, D, && (41)
\end{aligned}
$$

and the expected price is again given by (29).

The price difference is:

$$
p_H^* - p_L^* = c + \frac{1 + r_E^*(.)}{1 - x}\,(M - D)\,, \tag{42}
$$

and it follows from (23) that $p_H^* > p_L^*$.

The risk carried by consumers is, therefore, greater than the variation in costs so that shareholders can receive a return when costs are high.

Appendix B Lemmas

Proof of Lemma 1: Using the notation $cov\,(.) = cov\,(R_E, R_m)$, by definition in this model:

$$
cov\,(.) = E\left((R_E - E\,(R_E))\,(R_m - E\,(R_m))\right).
$$

From (5) and (6):

$$
\begin{aligned}
cov\,(.) &= x\left(\left(\frac{\Pi_L - E\,(\Pi)}{M - D}\right)(R_{mL} - E\,(R_m))\right) + \\
&\quad (1 - x)\left(\left(\frac{\Pi_H - E\,(\Pi)}{M - D}\right)(R_{mH} - E\,(R_m))\right),
\end{aligned}
$$

and, from (8), (9) and (10):

$$
\begin{aligned}
\Pi_L - E\,(\Pi) &= (1 - x)\,(p_L + c - p_H)\,,\\
\Pi_H - E\,(\Pi) &= -x\,(p_L + c - p_H)\,.
\end{aligned}
$$

Consequently, from (3):

$$
cov\,(.) = \frac{x\,(1 - x)}{M - D}\,(c - (p_H - p_L))\,\Delta R_m.
$$

This ends the proof.■

Proof of Lemma 2: In Cases 2, 3 and 4 $\pi > 0$ at the optimum. The shareholders' participation constraint (7) then becomes:

$$x\left(p_L^* + c\right) + (1-x)\, p_H^* - \theta - (1+r_D)\, D = \left(1 + r_E^*(.)\right)(M-D)\,.$$

Differentiating with respect to p_L^* and p_H^* gives:

$$x dp_L^* + (1-x)\, dp_H^* = r_E^{*\prime}(.)\, \frac{\partial cov}{\partial p_L^*}(M-D)\, dp_L^* + r_E^{*\prime}(.)\, \frac{\partial cov}{\partial p_H^*}(M-D)\, dp_H^*.$$

Substituting $\frac{\partial cov}{\partial p_L^*}$ and $\frac{\partial cov}{\partial p_H^*}$ derived from (4) in Lemma 1 and rearranging gives:

$$\frac{dp_H^*}{dp_L^*} = -\frac{x\left(1 - r_E^{*\prime}(.)\,(1-x)\,\Delta R_m\right)}{(1-x)\left(1 + r_E^{*\prime}(.)\, x\Delta R_m\right)}.$$

This ends the proof.■

Appendix C Propositions for a given capital structure

Proof of Proposition 1: The only feasible solutions are Cases 2, 3 and 4. In all these solutions $p_H^* > p_L^*$ and $\pi > 0$, i.e. the shareholders' participation constraint (7) is binding.

This ends the proof.■

Proof of Proposition 2: Case 2 is the only optimum where (25) holds. Since (31) also applies in Case 2 then $R_{EL}^* \gtreqless R_{EH}^*$. From Lemma 1 the covariance can also be written as $cov\,(.) = x\,(1-x)\,(R_{EL} - R_{EH})\,\Delta R_m$. Consequently, from (3), $cov^*(.) \gtreqless 0$ when $R_{EL}^* \gtreqless R_{EH}^*$ and so $r_E^*(.) \gtreqless r_D$.

This ends the proof.■

Proof of Proposition 3: Case 3 is the only optimum where (33) holds. Since $\Pi_H^* = 0$, $\Pi_L^* > 0$ in Case 3 then $R_{EL}^* > R_{EH}^*$. Consequently, $cov^*(.) > 0$ and so $r_E^*(.) > r_D$. Case 4 is the only optimum where (39) holds. Since $\Pi_H^* > 0$, $\Pi_L^* = 0$ in Case 4 then $R_{EL}^* < R_{EH}^*$. Consequently, $cov^*(.) < 0$ and so $r_E^*(.) < r_D$.

This ends the proof.■

Appendix D Solutions for a socially optimal capital structure

The Lagrangian function for the problem (15) is:

$$\mathcal{L} = xU\left(p_L^*\left(D\right)\right) + \left(1-x\right)U\left(p_H^*\left(D\right)\right) + \lambda D + \gamma\left(M - D\right), \tag{43}$$

where λ and γ are the multipliers associated with constraints (13) to (14).

The first order condition for a social optimum is:

$$\frac{\partial \mathcal{L}}{\partial D} = xU'\left(p_L^*\right)p_L^{*\prime}\left(D\right) + \left(1-x\right)U'\left(p_H^*\right)p_H^{*\prime}\left(D\right) + \lambda - \gamma = 0. \tag{44}$$

The possible solutions are determined by the values of λ and γ. There are four Cases.

Case A (λ, $\gamma > 0$): Both (13) and (14) are binding and so $D^* = 0$ and $M - D^* = 0$. This is not feasible as $M > 0$.

Case B ($\lambda = \gamma = 0$): Neither (13) nor (14) are binding. $M > D^* > 0$ and so the solution is an internal optimum where $W'\left(D\right) = 0$. From (44) this is where:

$$\frac{p_H^{*\prime}\left(D\right)}{p_L^{*\prime}\left(D\right)} = -\frac{xU'\left(p_L^*\right)}{\left(1-x\right)U'\left(p_H^*\right)}. \tag{45}$$

Since $U'\left(p\right) < 0$ it follows that if $p_L^{*\prime}\left(D\right) > 0$ then $p_H^{*\prime}\left(D\right) < 0$ and vice versa. This is a global optimum if $W'\left(D\right) < 0$ when $D < D^*$ and $W'\left(D\right) > 0$ when $D > D^*$.

Case C ($\lambda > 0, \gamma = 0$): Only (13) is binding and so $D^* = 0$, i.e the optimum is 100% equity finance. This is a corner solution where $W'\left(D\right) < 0$. Since $U'\left(p\right) < 0$ then from (44) this is where:

$$\frac{p_H^{*\prime}\left(D\right)}{p_L^{*\prime}\left(D\right)} > -\frac{xU'\left(p_L^*\right)}{\left(1-x\right)U'\left(p_H^*\right)} \quad \text{if } p_L^{*\prime}\left(D\right) > 0 \quad \text{and vice versa.} \tag{46}$$

This is a global optimum if $W'\left(D\right) < 0$ when $D > 0$.

Case D ($\lambda = 0, \gamma > 0$): Only (14) is binding and so $D^* = M$, the optimum is 100% debt finance. This is a corner solution where $W'\left(D\right) > 0$.

Since $U'(p) < 0$ then from (44) this is where:

$$\frac{p_H^{*\prime}(D)}{p_L^{*\prime}(D)} < -\frac{xU'(p_L^*)}{(1-x)U'\left(p_H^*\right)} \quad \text{if } p_L^{*\prime}(D) > 0 \ \text{ and vice versa.} \tag{47}$$

This is a global optimum if $W'(D) > 0$ when $D > 0$.

Appendix E Propositions for a socially optimal capital structure

Proof of Proposition 4: From (25), (33) and (39) it can be seen that the solutions for Cases 2, 3 and 4 are mutually exclusive. Cases 3 and 4 are, therefore, the only feasible solutions in which (25) does not apply. In additon, from (12) in Lemma 2, if $\frac{dp_H^*}{dp_L^*}$ does not vary with D, then the solution for a social optimum satisfies:

$$\frac{p_H^{*\prime}(D)}{p_L^{*\prime}(D)} = \frac{dp_H^*}{dp_L^*} = -\frac{x\left(1 - r_E^{*\prime}(.)(1-x)\Delta R_m\right)}{(1-x)\left(1 + r_E^{*\prime}(.)\, x\Delta R_m\right)},$$

and this holds if:
$$\frac{d\left(r_E^{*\prime}(.)\right)}{dD} = r_E^{*\prime\prime}(.)\frac{d\left(cov^*(.)\right)}{dD} = 0$$

Firstly, consider Case 3. From (4) in Lemma 1 and (36):

$$cov^*(.) = (1-x)\left(1 + r_E^*(.)\right)\Delta R_m.$$

Differentiating with respect to D gives:

$$\frac{d\left(cov^*(.)\right)}{dD}\left(1 - r_E^{*\prime}(.)(1-x)\Delta R_m\right) = 0,$$

and, from (26), this only holds if $\frac{d(cov^*(.))}{dD} = 0$. Consequently, $\frac{dp_H^*}{dp_L^*}$ does not vary with D and so, from (33), Case 3 can only be a social optimum if:

$$\frac{p_H^{*\prime}(D)}{p_L^{*\prime}(D)} < -\frac{xU'(p_L^*)}{(1-x)U'\left(p_H^*\right)} < 0.$$

However, from (34), $p_H^{*\prime}(D) > 0$ and so Case 3 is a social optimum if $p_L^{*\prime}(D) < 0$ and, from (46) in Case C, this only holds where the social optimum is $D^* = 0$.

Secondly, consider Case 4. From (4) in Lemma 1 and (42):

$$cov^*(.) = -x(1 + r_E^*(.))\Delta R_m.$$

Differentiating with respect to D gives:

$$\frac{d(cov^*(.))}{dD}\left(1 + r_E^{*\prime}(.)\, x\Delta R_m\right) = 0,$$

which only holds if$\frac{d(cov^*(.))}{dD} = 0$. Consequently, $\frac{dp_H^*}{dp_L^*}$ does not vary with D and so, from (39) Case 4 can only be a social optimum if:

$$\frac{p_H^{*\prime}(D)}{p_L^{*\prime}(D)} > -\frac{xU'(p_L^*)}{(1-x)U'\left(p_H^*\right)}.$$

However, from (41), $p_L^{*\prime}(D) > 0$ and, from (46) in Case C, this only holds where the social optimum is $D^* = 0$.

This ends the proof.■

Proof of Proposition 5: From (25), (33) and (39) it can be seen that the solutions for Cases 2, 3 and 4 are mutually exclusive. Case 2 is, therefore, the only feasible solution in which (25) holds while Case B is the only feasible solution for a social optimum where $0 < D^* < M$. Proposition 4 shows that Cases 3 and 4 can only be a social optimum when Case C applies. Consequently, the solution for a social optimum in Case B can only be feasible if Case 2 also applies, that is, both (45) and (25) are satisfied.

In Case 2 the price difference determines whether profits are higher or lower when the firm's costs are low (and vice versa) and so whether $r_E^*(.) \gtreqless r_D$. It is, therefore, possible for Case B to apply when $cov^*(.) = 0$ and $r_E^*(.) = r_D$. In these circumstances the optimum for consumer prices is such that the returns received by shareholders in both states of the world are the same as for lenders and so the overall cost of finance is the same as when $D^* = M$. Consequently, a social optimum where $0 < D^* < M$ is only possible when $r_E^*(.) \neq r_D$.

This ends the proof.■

Proof of Proposition 6: From Proposition 1 $\pi > 0$ at the optimum and so differentiating the shareholders' participation constraint (7) with respect to D gives:

$$xp_L^{*\prime}(D) + (1-x)\, p_H^{*\prime}(D) = r_D - r_E^*(.) + r_E^{*\prime}(.)\,\frac{d(cov^*(.))}{dD}(M - D).$$

Further, differentiating (4) in Lemma 1 with respect to D at the optimum gives:

$$\frac{d\left(cov^*\left(.\right)\right)}{dD}=\frac{x\left(1-x\right)}{M-D}\left(p_L^{*\prime}\left(D\right)-p_H^{*\prime}\left(D\right)\right)\Delta R_m+\frac{cov^*\left(.\right)}{M-D}.$$

It follows that:

$$\begin{aligned}&xp_L^{*\prime}\left(D\right)\left(1-r_E^{*\prime}\left(.\right)\left(1-x\right)\Delta R_m\right)+\left(1-x\right)p_H^{*\prime}\left(D\right)\left(1+r_E^{*\prime}\left(.\right)x\Delta R_m\right)\\ =\ &r_D-r_E^*\left(.\right)+r_E^{*\prime}\left(.\right)cov^*\left(.\right).\end{aligned}$$

However, from Propositions 4 and 5 when $D=D^*$:

$$\frac{dp_H^*}{dp_L^*}=\frac{p_H^{*\prime}\left(D\right)}{p_L^{*\prime}\left(D\right)}=-\frac{x\left(1-r_E^{*\prime}\left(.\right)\left(1-x\right)\Delta R_m\right)}{\left(1-x\right)\left(1+r_E^{*\prime}\left(.\right)x\Delta R_m\right)}.$$

Consequently:

$$r_E^*\left(.\right)-r_D=r_E^{*\prime}\left(.\right)cov^*\left(.\right).$$

and so if $r_E^{*\prime\prime}\left(.\right)=0$ then $r_E^{*\prime}\left(.\right)=k$ is a constant and:

$$r_E^*\left(.\right)-r_D=kcov^*\left(.\right).$$

This ends the proof.■

Proof of Proposition 7: From Proposition 6:

$$r_E^*\left(.\right)=r_D+r_E^{*\prime}\left(.\right)cov^*\left(.\right).$$

Differentiating $r_E^*\left(.\right)$ with respect to $cov^*\left(.\right)$ gives:

$$r_E^{*\prime\prime}\left(.\right)cov^*\left(.\right)=0.$$

It follows that, if $r_E^{*\prime\prime}\left(.\right)\neq 0$, then $cov^*\left(.\right)=0$ and so $r_E^*\left(.\right)=r_D$. The overall cost of finance is then the same as when $D^*=M$. Conversely, if $r_E^*\left(.\right)\neq r_D$ then $cov^*\left(.\right)\neq 0$ and so $r_E^{*\prime\prime}\left(.\right)=0$, and $0<D^*<M$ from Proposition 5.

However, if $r_E^{*\prime\prime}\left(.\right)=0$ it is also possible that $cov^*\left(.\right)=0$ and $r_E^*\left(.\right)=r_D$ but, from Proposition 5, this can only be where Case 2 applies, that is, both

(45) and (25) are satisfied:

$$\frac{p_H^{*\prime}(D)}{p_L^{*\prime}(D)} = -\frac{xU'(p_L^*)}{(1-x)U'\left(p_H^*\right)} = -\frac{x\left(1-r_E^{*\prime}(.)(1-x)\Delta R_m\right)}{(1-x)\left(1+r_E^{*\prime}(.)x\Delta R_m\right)}.$$

Consequently, if $r_E^{*\prime\prime}(.) = 0$ then $r_E^{*\prime}(.)$is a constant and so is $\frac{p_H^{*\prime}(D)}{p_L^{*\prime}(D)}$. Since it is assumed that the consumer's utility function satisfies $U'(p), U''(p) < 0$, there can be only one combination of consumer prices p_H^* and p_L^* at which there is a social optimum when $r_E^{*\prime\prime}(.) = 0$ and $cov^*(.) = 0$.

This ends the proof.■

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