

CONCERNING AN INVESTIGATION ON DICE (SOPRA LE SCOPERTE DEI DADI)

(GALILEO GALILEI, *Opere*, Firenze, Barbera, 8 (1898), pp. 591–594)

The fact that in a dice-game certain numbers are more advantageous than others has a very obvious reason, i.e. that some are more easily and more frequently made than others, which depends on their being able to be made up with more variety of numbers. Thus a 3 and an 18, which are throws which can only be made in one way with 3 numbers (that is, the latter with 6.6.6 and the former with 1.1.1, and in no other way), are more difficult to make than, e.g. 6 or 7, which can be made up in several ways, that is, a 6 with 1.2.3 and with 2.2.2 and with 1.1.4, and a 7 with 1.1.5, 1.2.4, 1.3.3, and 2.2.3. Nevertheless, although 9 and 12 can be made up in as many ways as 10 and 11, and therefore they should be considered as being of equal utility to these, yet it is known that long observation has made dice-players consider 10 and 11 to be more advantageous than 9 and 12. And it is clear that 9 and 10 can be made up by an equal diversity of numbers (and this is also true of 12 and 11): since 9 is made up of 1.2.6, 1.3.5, 1.4.4, 2.2.5, 2.3.4, 3.3.3, which are six triple numbers, and 10 of 1.3.6, 1.4.5, 2.2.6, 2.3.5, 2.4.4, 3.3.4, and in no other ways, and these also are six combinations. Now I, to oblige him who has ordered me to produce whatever occurs to me about such a problem, will expound my ideas, in the hope not only of solving this problem but of opening the way to a precise understanding of the reasons for which all the details of the game have been with great care and judgment arranged and adjusted.

And to achieve my end with the greatest clarity of which I am capable, I will begin by considering how, since a die has six faces, and when thrown it can equally well fall on any one of these, only 6 throws can be made with it, each different from all the others. But if together with the first die we throw a second, which also has six faces, we can make 36 throws each different from all the others, since each face of the first die can be combined with each of the second, and in consequence can make 6 different throws, whence it is clear that such combinations are 6 times 6, i.e. 36. And if we add a third die, since each one of its six faces can be combined with each one of the 36 combinations of the other two dice, we shall find that the combinations of three dice are 6 times 36, i.e. 216, each different from the others. But because the numbers in the combinations in three-dice throws are only 18, that is, 3.4.5, etc. up to 18, among which one must divide the said 216 throws, it is necessary that to some of these numbers many throws must belong; and if we can find how many belong to each, we shall have prepared the way to find out what we want to know, and it will be enough to make such an investigation from 3 to 18, because what pertains to one of these numbers, will also pertain to that which is the one immediately greater.

Three special points must be noted for a clear understanding of what follows. The first is that that sum of the points of 3 dice, which is composed of 3 equal numbers, can only be produced by one single throw of the dice: and thus a 3 can only be produced by the three ace-faces, and a 6, if it is to be made up of 3 twos, can only be made by a single throw. Secondly: the sum which is made up of 3 numbers, of which two are the same and the third different, can be produced by three throws: as e.g., a 4 which is made up of a 2 and of two aces, can be produced by three different throws; that is, when the first die shows 2 and the second and third show the ace, or the second die a 2

and the first and third the ace; or the third a 2 and the first and second the ace. And so e.g., an 8, when it is made up of 3.3.2, can be produced also in three ways: i.e. when the first die shows 2 and the others 3 each, or when the second die shows 2 and the first and third 3, or finally when the third shows 2 and the first and second 3. Thirdly the sum of points which is made up of three different numbers. can be produced in six ways. As for example, an 8 which is made up of 1.3.4. can be made with six different throws: first, when the first die shows 1, the second 3 and the third 4; second, when the first die still shows 1, but the second 4 and the third 3; third, when the second die shows 1, and the first 3 and the third 4; fourth, when the second still shows 1, and the first 4 and the third 3; fifth, when the third die shows 1, the first 3, and the second 4; sixth, when the third shows 1, the first 4 and the second 3. Therefore, we have so far declared these three fundamental points; first, that the triples, that is the sum of three-dice throws, which are made up of three equal numbers, can only be produced in one way; second, that the triples which are made up of two equal numbers and the third different, are produced in three ways; third, that those triples which are made up of three different numbers are produced in six ways. From these fundamental points we can easily deduce in how many ways, or rather in how many different throws, all the numbers of the three dice may be formed, which will easily be understood from the following table

	10	9	8	7	6	5	4	3
108	433	3	333	1				
108		27		25		21		15
216							10	
							6	
								3
								1

on top of which are noted the points of the throws from 10 down to 3, and beneath these the different triples from which each of these can result; next to which are placed the number of ways in which each triple can be produced, and under these is finally shown the sum of all the possible ways of producing these throws. So, e.g., in the first column we have the sum of points 10, and beneath it 6 triples of numbers with which it can be made up, which are 6.3.1, 6.2.2, 5.4.1, 5.3.2, 4.4.2, 4.3.3. And since the first triple 6.3.1 is made up of three different numbers, it can (as is declared above) be made by six different dice-throws, therefore next to this triple 6.3.1 a 6 is noted: and since the second triple 6.2.2 is made up of two equal numbers and a third which is different, it can only be produced by 3 different throws, and therefore a 3 is noted next to it: the third triple 5.4.1, being made up of three different numbers, can be produced by 6 throws, therefore a 6 is noted next to it: and so on with all the other triples. And finally at the bottom of the little column of numbers of throws these are all added up: there one can see that the sum of points 10 can be made up by 27 different dice-throws but the sum of points 9 by 25 only, the 8 by 21, the 7 by 15, the 6 by 10, the 5 by 6, the 4 by 3, and finally the 3 by 1: which all added together amount to 108. And there being

a similar number of throws for the higher sums of points, that is, for the points 11, 12, 13, 14, 15, 16, 17, 18, one arrives at the sum of all the possible throws which can be made with the faces of the three dice, which is 216. And from this table anyone who understands the game can very accurately measure all the advantages, however small they may be, of the *zare*,¹ the *incontri*,¹ and of any other special rule and term observed in this game.

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¹*Zara* was a game played with 3 dice, the rules of which were possibly those of the game known to us as Hazard. *Incontri* is used here probably as a technical term in *Zara*.